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Sensitivity and stability analysis in fuzzy data envelopment analysis

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Abstract As a useful management and decision tool, data envelopment analysis (DEA) has become a pop area of research. One of the topics of interests in DEA is sensitivity and stability analysis of decision making units (DMUs). Due to the uncertainty of the data in real life, this paper will give some DEA models in fuzzy environment. It is followed by a series analysis of sensitivity and stability for all DMUs. Finally a numerical example will be presented to give an illustration of the sensitivity and stability analysis.

Keywords Data envelopment analysis · Credibility measure · Efficiency · Sensitivity · Stability

1 Introduction

Data envelopment analysis (DEA), introduced by Charnes et al. (1978) and extended by Banker et al. (1984), Charnes et al. (1985), Petersen (1990), Tone (2001) and Cooper (1999a), is a useful method to evaluate the relative efficiency of decision making units (DMUs). Its goal is to classify the DMUs into two classes: efficient or inefficient ones. However, uncertainty such as a measurement error should be

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incorporated in observed data. This indicates the necessity to assess the sensitivity of classifications in DEA.

During the recent years, the issue of sensitivity and stability of DEA has been extensively studied. The first DEA sensitivity analysis paper by Charnes et al. (1985) examined change in a single output. This is followed by a series of sensitivity analysis articles. Charnes and Neralic (1990) gave some sufficient conditions in preserving efficiency. Charnes et al. (1992, 1996) developed a sensitivity analysis technique on super-efficiency DEA model for the situation where simultaneous proportional change is assumed in all inputs and outputs for a specific DMU under consideration. This data variation condition is relaxed in Zhu (1996) and Seiford and Zhu (1998) to a situation, where inputs or outputs can be changed individually and the largest stability region is obtained. The DEA sensitivity analysis methods we have just reviewed are all developed for the situation, where data variations are only applied to the efficient DMU under evaluation and the data for the remaining DMUs are assumed to be fixed. Obviously, this assumption may not be realistic, since possible data errors may occur in each DMU. Seiford and Zhu (1999) generalized the technique in Zhu (1996) and Seiford and Zhu (1998) to the case, where the efficiency of the under evaluation efficient DMU is deteriorating while the efficiencies of the other DMUs are improving.

The original DEA models assume that inputs and outputs are measured by exact values. Recently, many researchers addressed the problem with fuzzy data (Cooper et al. 1999b, 2001a,b; Kao and Liu 2000; Entani et al. 2002; Guo and Tanaka 2001; Lertworasirikul et al. 2003). In this paper, we discuss a technique for assessing the sensitivity of efficiency and inefficiency classifications in DEA with fuzzy data. Having identified the efficient and inefficient DMUs in fuzzy DEA analysis, we will introduce how sensitive these identifications are to possible variations in data.

This paper is organized as follows: In Sect. 2, some basic concept and results on credibility measure will be introduced. Some introduction to one of the DEA models are given in Sect 3. Section 4 will give some fuzzy models for determine the radius of the stability for all the DMUs. Finally, the analysis of the sensitivity and stability is introduced through a numerical example in Sect. 5.

2 Credibility measure

Credibility theory, which is founded by Liu (2004) in 2004 and refined by Liu (2007a) in 2007, is a branch of mathematics for studying the behavior of fuzzy phenomena. This section will state some basic concepts and results on credibility measure, fuzzy variable and membership function. For the up-to-date credibility theory, the interested reader can consult (Liu 2002a,b, 2004, 2007a,b).

Let Θ be a nonempty set, and $\mathcal{P}$ the power set of Θ. The triplet $(\Theta, \mathcal{P}(\Theta), \text{Cr})$ is called a credibility space. For any $A \in \mathcal{P}$, Liu and Liu (2002) presented the credibility measure $\text{Cr}\{A\}$ to express the chance that fuzzy event A occurs. Li and Liu (2006) gave a sufficient and necessary condition for credibility measure: $(i)\,\text{Cr}\{\Theta\} = 1$; $(ii)\,\text{Cr}\{A\} \le \text{Cr}\{B\}$ whenever $A \subset B$; $(iii)\,\text{Cr}\{A\} + \text{Cr}\{A^c\} = 1$ for any event A; $(iv)\,\text{Cr}\{\cup_i A_i\} = \sup_i \text{Cr}\{A_i\}$ for any events $\{A_i\}$ with $\sup_i \text{Cr}\{A_i\} < 0.5$.

A fuzzy variable is defined as a function from a credibility space $(\Theta, \mathcal{P}, \mathrm{Cr})$ to the set of real numbers (Liu 2006). Let ξ be a fuzzy variable defined on the credibility space $(\Theta, \mathcal{P}, \mathrm{Cr})$. Then its membership function is derived from the credibility measure by

$$\mu(x) = (2\mathrm{Cr}\{\xi = x\}) \wedge 1, \ x \in \mathcal{R}. \tag{1}$$

Membership function represents the degree of possibility that the fuzzy variable ξ takes some prescribed value. Conversely, let ξ be a fuzzy variable with membership function μ. Then for any set B of real numbers, we have

$$\mathrm{Cr}\{\xi \in B\} = \frac{1}{2}\left(\sup_{x\in B} \mu(x) + 1 - \sup_{x\in B^c} \mu(x)\right). \tag{2}$$

3 DEA model

This section will introduce one of the DEA models, proposed by Charnes et al. (1985). The symbols and notations are given as follows: DMU_i is the ith DMU, $i = 1, 2, \ldots, n$. DMU_0 is the target DMU. The vectors $x_i = (x_{i1}, x_{i2}, \ldots, x_{ip})$ and $y_i = (y_{i1}, y_{i2}, \ldots, y_{iq})$ are the inputs and outputs of DMU_i, $i = 1, 2, \ldots, n$. The inputs and outputs vectors of the target DMU_0 are $x_0 = (x_{01}, x_{02}, \ldots, x_{0p})$ and $y_0 = (y_{01}, y_{02}, \ldots, y_{0q})$. The production possibility set (PPS), consisting of all convex combinations of $(x_k,\ y_k)$, $k = 1, 2, \ldots, n$, can be formulated as

$$PPS = \left\{(x, y) | x = \sum_{k=1}^{n} x_k\lambda_k,\ y = \sum_{k=1}^{n} y_k\lambda_k,\ \sum_{k=1}^{n}\lambda_k = 1,\ \lambda_k \geq 0,\ 1 \leq k \leq n\right\}.$$

The model Charnes et al. (1985) is given as:

$$\begin{aligned}
\max \quad & \sum_{i=1}^{p} s_i^- + \sum_{j=1}^{q} s_j^+ \\
\text{subject to :} \quad & \\
& \sum_{k=1}^{n} x_{ki}\lambda_k = x_{0i} - s_i^-, \quad i = 1, 2, \ldots, p \\
& \sum_{k=1}^{n} y_{kj}\lambda_k = y_{0j} + s_j^+, \quad j = 1, 2, \ldots, q \\
& \sum_{k=1}^{n} \lambda_k = 1 \\
& \lambda_k \geq 0, \quad k = 1, 2, \ldots, n \\
& s_i^- \geq 0, \quad i = 1, 2, \ldots, p \\
& s_j^+ \geq 0, \quad j = 1, 2, \ldots, q.
\end{aligned} \tag{3}$$

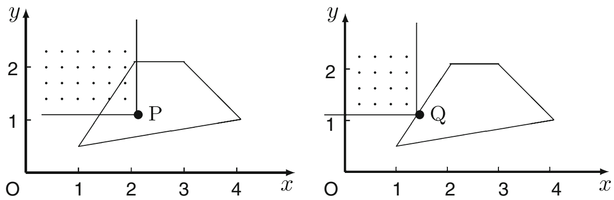

Fig. 1 This figure shows the DMUs with one input and one output. The quadrangle is the PPS, in which all points are feasible. Let us look at the point P. It can be easily seen that many points in PPS are better than the point P. Thus the point P is inefficient. However, no point in PPS is better than the point Q, so it is efficient

Definition 1 (*Efficiency*) DMU_0 is efficient if s_i^{-*} and s_j^{+*} are zero for $i = 1, 2, \ldots, p$ and $j = 1, 2, \ldots, q$, where s_i^{-*} and s_j^{+*} are optimal solutions of (3).

4 Fuzzy DEA model

The model in Sect. 3 assumes the inputs and outputs are all exactly known. But in many cases, the inputs and outputs cannot be given exactly, which can be considered to be fuzzy. Wen and You (2007) have given a fuzzy DEA model, in which the vectors $\widetilde{x}_i = (\widetilde{x}_{i1}, \widetilde{x}_{i2}, \ldots, \widetilde{x}_{ip})$ and $\widetilde{y}_i = (\widetilde{y}_{i1}, \widetilde{y}_{i2}, \ldots, \widetilde{y}_{iq})$ represent the fuzzy inputs and outputs vectors of DMU_i, $i = 1, 2, \ldots, n$, respectively. The model Wen and You (2007) is given as follows:

$$
\begin{cases}
\max \quad \sum_{i=1}^{p} s_i^- + \sum_{j=1}^{q} s_j^+ \\
\text{subject to:} \\
\qquad \text{Cr}\left\{ \sum_{k=1}^{n} \widetilde{x}_{ki}\lambda_k \le \widetilde{x}_{0i} - s_i^- \right\} \ge \alpha, \quad i = 1, 2, \ldots, p \\
\qquad \text{Cr}\left\{ \sum_{k=1}^{n} \widetilde{y}_{kj}\lambda_k \ge \widetilde{y}_{0j} + s_j^+ \right\} \ge \alpha, \quad j = 1, 2, \ldots, q \\
\qquad \sum_{k=1}^{n} \lambda_k = 1 \\
\qquad \lambda_k \ge 0, \quad k = 1, 2, \ldots, n \\
\qquad s_i^- \ge 0, \quad i = 1, 2, \ldots, p \\
\qquad s_j^+ \ge 0, \quad j = 1, 2, \ldots, q.
\end{cases} \tag{4}
$$

Definition 2 (α-*efficiency*) DMU_0 is α-efficient if $s_i^{-*}(\alpha)$ and $s_j^{+*}(\alpha)$ are zero for $i = 1, 2, \ldots, p$ and $j = 1, 2, \ldots, q$, where $s_i^{-*}(\alpha)$ and $s_j^{+*}(\alpha)$ are optimal solutions of (4).

Theorem 1 *If DMU_0 is α-inefficient, then the optimal solution satisfying $\lambda_0^*(\alpha) = 0$.*

Proof For a fixed α, suppose the optimal solution is $(\lambda_j^*, \lambda_0^*, s_i^{-*}, s_j^{+*})$ and the optimal objective value is $\sum_{i=1}^p s_i^{-*} + \sum_{j=1}^q s_j^{+*}$. If $\lambda_0^* = 0$, then the Theorem has been proved. Otherwise let $\lambda_0 > 0$. Since DMU_0 is inefficient, there exists at least one $s_i^{-*} > 0$ or $s_j^{+*} > 0, i = 1, 2, \ldots, p, j = 1, 2, \ldots, q$. Without loss of generality, we assume $s_1^{-*} > 0$. If $\lambda_0^* = 1$, then $\text{Cr}\left\{\widetilde{x}_{01} \le \widetilde{x}_{01} - s_1^{-*}\right\} = 0$. The contradiction implies that $\lambda_0^* \ne 1$.

$$\begin{aligned}
&\text{Cr}\left\{\sum_{k=1}^{n} \widetilde{x}_{ki}\lambda_k^* \le \widetilde{x}_{0i} - s_i^{-*}\right\}\\
&\quad= \text{Cr}\left\{\sum_{k=1,k\ne 0}^{n} \widetilde{x}_{ki}\lambda_k^* + \widetilde{x}_{0i}\lambda_0^* \le \widetilde{x}_{0i} - s_i^{-*}\right\}\\
&\quad= \text{Cr}\left\{\frac{(1-\lambda_0^*)\left(\sum_{k=1,k\ne 0}^{n} \widetilde{x}_{ki}\lambda_k^*\right)}{(1-\lambda_0^*)} \le (1-\lambda_0^*)\widetilde{x}_{0i} - s_i^{-*}\right\}\\
&\quad= \text{Cr}\left\{\frac{\sum_{k=1,k\ne 0}^{n} \widetilde{x}_{ki}\lambda_k^*}{1-\lambda_0^*} \le \widetilde{x}_{0i} - \frac{s_i^{-*}}{(1-\lambda_0^*)}\right\}\\
&\quad\ge \alpha,\ \ i = 1, 2, \cdots, p.
\end{aligned}$$

Similarly we can get

$$\begin{aligned}
&\text{Cr}\left\{\sum_{k=1}^{n} \widetilde{y}_{kj}\lambda_k^* \ge \widetilde{y}_{0j} + s_j^{+*}\right\}\\
&\quad= \text{Cr}\left\{\frac{\sum_{k=1,k\ne 0}^{n} \widetilde{y}_{kj}\lambda_k^*}{1-\lambda_0^*} \ge \widetilde{y}_{0j} + \frac{s_j^{+*}}{(1-\lambda_0^*)}\right\}\\
&\quad\ge \alpha,\ \ j = 1, 2, \cdots, q.
\end{aligned}$$

It is easy to prove that $\dfrac{\sum_{k=1,k\ne 0}^{n} \lambda_k^*}{1-\lambda_0^*} = 1$. Thus $\left(\dfrac{\lambda_1^*}{\sum_{k=1,k\ne 0}^{n} \lambda_k^*}, \cdots, \dfrac{\lambda_{0-1}^*}{\sum_{k=1,k\ne 0}^{n} \lambda_k^*}, 0, \dfrac{\lambda_{0+1}^*}{\sum_{k=1,k\ne 0}^{n} \lambda_k^*}, \cdots, \dfrac{\lambda_n^*}{\sum_{k=1,k\ne 0}^{n} \lambda_k^*}\right)$ is a feasible solution and the objective value is $\frac{1}{1-\lambda_0^*}\left(\sum_{i=1}^p s_i^{-*} + \sum_{j=1}^q s_j^{+*}\right) > \sum_{i=1}^p s_i^{-*} + \sum_{j=1}^q s_j^{+*}$, since $0 < \lambda_0^* < 1$, which leads to a contradiction with the assumption. Thus $\lambda_0^* = 0$.

Theorem 2 (Wen and You 2007) *An α-inefficient DMU_0 becomes α-efficient if $(\widehat{x}_0, \widehat{y}_0) = (\widetilde{x}_0 - s^{-*}(\alpha), \widetilde{y}_0 + s^{+*}(\alpha))$, in which $s^{-*}(\alpha)$ and $s^{+*}(\alpha)$ are optimal solution of* (4).

Theorem 2 has given the stable region when the DMU is inefficient. But we also want to know the efficient radius of efficient DMUs. For this purpose, the following model is proposed:

$$\begin{aligned}
&\min \quad \sum_{i=1}^{p} t_i^+ + \sum_{j=1}^{q} t_j^- \\
&\text{subject to:} \\
&\qquad \mathrm{Cr}\left\{ \sum_{k=1,k\neq 0}^{n} \widetilde{x}_{ki}\lambda_k \le \widetilde{x}_{0i} + t_i^+ \right\} \ge \alpha, \quad i = 1, 2, \ldots, p \\
&\qquad \mathrm{Cr}\left\{ \sum_{k=1,k\neq 0}^{n} \widetilde{y}_{kj}\lambda_k \ge \widetilde{y}_{0j} - t_j^- \right\} \ge \alpha, \quad j = 1, 2, \ldots, q \\
&\qquad \sum_{k=1}^{n} \lambda_k = 1 \\
&\qquad \lambda_k \ge 0, \quad k = 1, 2, \ldots, n \\
&\qquad t_i^+ \ge 0, \quad i = 1, 2, \ldots, p \\
&\qquad t_j^- \ge 0, \quad j = 1, 2, \ldots, q.
\end{aligned} \tag{5}$$

Theorem 3 *The α-efficient DMU_0 stays α-efficient if $(\widehat{x}_0, \widehat{y}_0) = (\widetilde{x}_0 + t^{+*}(\alpha), \widetilde{y}_0 - t^{-*}(\alpha))$, where $t^{+*}(\alpha)$ and $t^{-*}(\alpha)$ are optimal solution of (5).*

Proof Consider the following DEA model for evaluating the relative efficiency of the adjusted DMU_0:

$$\begin{aligned}
&\max \quad \sum_{i=1}^{p} t_i^+ + \sum_{j=1}^{q} t_j^- \\
&\text{subject to :} \\
&\mathrm{Cr}\left\{ \sum_{k=1,k\neq 0}^{n} \widetilde{x}_{ki}\lambda_k + (\widetilde{x}_{0i} + t_i^{+*})\lambda_0 \le (\widetilde{x}_{0i} + t_i^{+*}) - s_i^- \right\} \ge \alpha, \quad i = 1, 2, \ldots, p \\
&\mathrm{Cr}\left\{ \sum_{k=1,k\neq 0}^{n} \widetilde{yj}_{kj}\lambda_k + (\widetilde{y}_0 - t_j^{-*})\lambda_0 \ge (\widetilde{yj}_0 - t_j^{-*}) + s_j^+ \right\} \ge \alpha, j = 1, 2, \ldots, q \\
&\sum_{k=1}^{n} \lambda_k = 1 \\
&\quad \lambda_k \ge 0, \quad k = 1, 2, \ldots, n \\
&s_i^- \ge 0, \quad i = 1, 2, \ldots, p \\
&s_j^+ \ge 0, \quad j = 1, 2, \ldots, q.
\end{aligned} \tag{6}$$

For a fixed α, let the optimal solution to be $(\lambda_j^*, \lambda_0^*, s^{-*}, s^{+*})$ and assume that the DMU_0 is inefficient. From Theorem 1, we get $\lambda_0^* = 0$. Thus this optimal solution is a feasible solution for (5). Hence $t_i^{+*} - s_i^{-*} \geq t_i^{+*}$ and $t_j^{-*} - s_j^{+*} \geq t_j^{-*}$, which means $s_i^{-*} = 0$ and s_j^{+*}, $i = 1, 2, \ldots, p,\ j = 1, 2, \ldots, q$. This leads to a contradiction with the assumption.

From above analysis, the ranges of inputs and outputs and radius of stability of DMU_0 are identified as follows:

1. If DMU_0 is α-inefficient by solving model (4), then DMU_0 stays α-inefficient if $(\widehat{x}_0, \widehat{y}_0) = (\widetilde{x}_0 - s^-, \widetilde{y}_0 + s^+)$, in which $s^- = \{(s_1^-, \ldots, s_p^-) | 0 \leq s_i^- < s_i^{-*}(\alpha),\ i = 1, 2, \ldots, p\}, s^+ = \{(s_1^+, \ldots, s_q^+) | 0 \leq s_j^+ < s_j^{+*}(\alpha),\ j = 1, 2, \ldots, q\}$, where $s_i^{-*}(\alpha)$ and $s_j^{+*}(\alpha)$ are optimal solutions of (4).
2. If DMU_0 is α-efficient by solving model (4), then we use model (5) to account the efficient radius. DMU_0 stays α-efficient if $(\widehat{x}_0, \widehat{y}_0) = (\widetilde{x}_0 + t^+, \widetilde{y}_0 - t^-)$, in which $t^+ = \{(t_1, \ldots, t_p) | 0 \leq t_i \leq t_i^{+*}(\alpha),\ i = 1, 2, \ldots, p\}$ and $t^- = \{(t_1, \ldots, t_q) | 0 \leq t_j \leq t_j^{-*}(\alpha),\ j = 1, 2, \ldots, q\}$, where $t_i^{+*}(\alpha)$ and $t_j^{-*}(\alpha)$ are optimal solutions of (5).

For any $\alpha \in (0, 1]$, the α-optimistic and α-pessimistic values of a fuzzy variable ξ are defined as

$$\xi_{\sup}^{\alpha} = \sup \{\text{Cr}\{\xi \geq r\} \geq \alpha\},\ \ \xi_{\inf}^{\alpha} = \inf \{\text{Cr}\{\xi \leq r\} \geq \alpha\}.$$

They play an important role in fuzzy programming. It has been proved that (a) $\xi_{\inf}^{\alpha}$ is an increasing and left-continuous function of α; (b) $\xi_{\sup}^{\alpha}$ is a decreasing and left-continuous function of α; (c) if $c \geq 0$, then $c\xi_{\sup}^{\alpha} = c\xi_{\sup}^{\alpha}$ and $c\xi_{\inf}^{\alpha} = c\xi_{\inf}^{\alpha}$; (d) if $c < 0$, then $c\xi_{\sup}^{\alpha} = c\xi_{\inf}^{\alpha}$ and $c\xi_{\inf}^{\alpha} = c\xi_{\sup}^{\alpha}$; (e) if ξ and η are independent, $(\xi + \eta)_{\sup}^{\alpha} = \xi_{\sup}^{\alpha} + \eta_{\sup}^{\alpha}$. The more details can refer to Liu (2007a).

From the properties of the α-optimistic and α-pessimistic values, we can rewrite the fuzzy DEA models (4) and (5) as follows:

$$\begin{aligned}
&\max \quad \sum_{i=1}^{p} s_i^- + \sum_{j=1}^{q} s_j^+ \\
&\text{subject to :} \\
&\quad \sum_{k=1}^{n} \lambda_k (\widetilde{x}_{ki})_{\inf}^{\alpha} + \lambda_0 \left[(\widetilde{x}_{0i})_{\sup}^{\alpha} - (\widetilde{x}_{0i})_{\inf}^{\alpha} \right] \leq (\widetilde{x}_{0i})_{\sup}^{\alpha} - s_i^-, \quad i = 1, 2, \ldots, p \\
&\quad \sum_{k=1}^{n} \lambda_k (\widetilde{y}_{kj})_{\sup}^{\alpha} + \lambda_0 \left[(\widetilde{y}_{0j})_{\inf}^{\alpha} - (\widetilde{y}_{0j})_{\sup}^{\alpha} \right] \geq (\widetilde{y}_{0j})_{\inf}^{\alpha} + s_j^+, \quad j = 1, 2, \ldots, q \\
&\quad \sum_{k=1}^{n} \lambda_k = 1 \\
&\quad \lambda_k \geq 0, \quad k = 1, 2, \ldots, n \\
&\quad s_i^- \geq 0, \quad i = 1, 2, \ldots, p \\
&\quad s_j^+ \geq 0, \quad j = 1, 2, \ldots, q
\end{aligned} \tag{7}$$

Table 1 DMUs with two fuzzy inputs and two fuzzy outputs

DMU_i	1	2	3	4	5
Input 1	(3.5,4.0,4.5)	(2.9,2.9,2.9)	(4.4,4.9,5.4)	(3.4,4.1,4.8)	(5.9,6.5,7.1)
Input 2	(1.9,2.1,2.3)	(1.4,1.5,1.6)	(2.2,2.6,3.0)	(2.1,2.3,2.5)	(3.6,4.1,4.6)
Output 1	(2.4,2.6,2.8)	(2.2,2.2,2.2)	(2.7,3.2,3.7)	(2.5,2.9,3.3)	(4.4,5.1,5.8)
Output 2	(3.8,4.1,4.4)	(3.3,3.5,3.7)	(4.3,5.1,5.9)	(5.5,5.7,5.9)	(6.5,7.4,8.3)

and

$$\begin{cases} \min \quad \sum_{i=1}^{p} t_i^{+} + \sum_{j=1}^{q} t_j^{-} \\ \text{subject to:} \\ \qquad \sum_{k=1,k\neq 0}^{n} \lambda_k (\widetilde{x}_{ki})_{\inf}^{\alpha} \le (\widetilde{x}_{0i})_{\sup}^{\alpha} + t_i^{+}, \quad i = 1, 2, \ldots, p \\ \qquad \sum_{k=1,k\neq 0}^{n} \lambda_k (\widetilde{y}_{kj})_{\sup}^{\alpha} \ge (\widetilde{y}_{0j})_{\inf}^{\alpha} - t_j^{-}, \quad j = 1, 2, \ldots, q \\ \qquad \sum_{k=1}^{n} \lambda_k = 1 \\ \qquad \lambda_k \ge 0, \quad k = 1, 2, \ldots, n \\ \qquad t_i^{+} \ge 0, \quad i = 1, 2, \ldots, p \\ \qquad t_j^{-} \ge 0, \quad j = 1, 2, \ldots, q \end{cases} \tag{8}$$

which are all linear programming. Thus they can be easily solved by many traditional methods.

5 A numerical example

This numerical example is presented to give an illustration of the sensitivity and stability analysis. The example is taken from Guo and Tanaka (2001). Table 1 provides the information of the DMUs. There are two fuzzy inputs and two fuzzy outputs which are all triangular fuzzy variables.

Table 2 shows the results of evaluating DMUs using model (4) with confidence level $\alpha = 0.6$. The results can be interpreted in the following way: DMU_1 and DMU_3 are inefficient, whereas DMU_2, DMU_4 and DMU_5 are efficient.

Tables 3 reports the sensitivity analysis results for inefficient DMUs by model (4). In Table 3, the columns 2 and 3 report lower bounds of variation ranges of inputs and the columns 4 and 5 are upper bounds of variation ranges of outputs. For instance consider DMU_1 in Table 3. It stays inefficient when $(\widehat{x}_{11}, \widehat{x}_{12}, \widehat{y}_{11}, \widehat{y}_{12}) = (\widetilde{x}_{11} - r_{x1}, \widetilde{x}_{12}, \widetilde{y}_{11}, \widetilde{y}_{12} + r_{y2})$, in which $0 \le r_{x1} < 0.15$ and $0 \le r_{y2} < 0.60$.

Table 4 reports the sensitivity analysis results for efficient DMUs by model (5). In Table 4, the columns 2 and 3 report upper bounds of variation ranges of inputs and the columns 4 and 5 are lower bounds of variation ranges of outputs. For instance

Table 2 Results of evaluating the DMUs

DMUs	$(\lambda_1^*, \lambda_2^*, \lambda_3^*, \lambda_4^*, \lambda_5^*)$	$\sum_{i=1}^{p} s_i^{-*} + \sum_{j=1}^{q} s_j^{+*}$	The result of evaluating
DMU_1	(0,0.74,0,0.09,0.17)	0.75	Inefficiency
DMU_2	(0,1,0,0,0)	0	Efficiency
DMU_3	(0,0,1,0,0)	0.19	Inefficiency
DMU_4	(0,0,0,1,0)	0	Efficiency
DMU_5	(0,0,0,0,1)	0	Efficiency

Table 3 Sensitivity analysis results of inefficient DMUs by model (4)

DMUs	s_1^{-*}	s_2^{-*}	s_1^{+*}	s_2^{+*}
DMU_1	0.15	0.00	0.00	0.60
DMU_3	0.47	0.05	0.00	0.83

Table 4 Sensitivity analysis results of efficient DMUs by model (5)

DMUs	s_1^{-*}	s_2^{-*}	s_1^{+*}	s_2^{+*}
DMU_2	1.20	0.66	0.00	0.00
DMU_4	0.00	0.17	0.00	1.01
DMU_5	0.00	0.00	2.42	1.92

consider DMU_4 in Table 4. It stays efficient when $(\widehat{x}_{41}, \widehat{x}_{42}, \widehat{y}_{41}, \widehat{y}_{42}) = (\widetilde{x}_{41}, \widetilde{x}_{42} + r_{x2}, \widetilde{y}_{41}, \widetilde{y}_{42} - r_{y2})$, in which $0 \leq r_{x2} \leq 0.17$ and $0 \leq r_{y2} \leq 1.01$.

The similar interpretation can be stated for other rows in Table 3 and Table 4.

6 Conclusion

Due to its widely practical used background, data envelopment analysis (DEA) has become a pop area of research. Since the data cannot be precisely measured in some practical cases, many paper have been published when the inputs and outputs are uncertain. Fuzzy methods are one of them. This paper has given some sensitivity and stability analysis in fuzzy DEA. Based on credibility measure, we found radius of stability for all DMUs when the inputs and outputs were fuzzy variables. Finally a numerical example was presented to give an illustration of sensitivity and the stability analysis.

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