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Foundation of credibilistic logic

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Abstract In this paper, credibilistic logic is introduced as a new branch of uncertain logic system by explaining the truth value of fuzzy formula as credibility value. First, credibilistic truth value is introduced on the basis of fuzzy proposition and fuzzy formula, and the consistency between credibilistic logic and classical logic is proved on the basis of some important properties about truth values. Furthermore, a credibilistic modus ponens and a credibilistic modus tollens are presented. Finally, a comparison between credibilistic logic and possibilistic logic is given.

Keywords Fuzzy logic · Modus ponens · Fuzzy proposition · Fuzzy formula · Truth value

1 Introduction

Many artificial intelligence applications require to reason with uncertain information. For example, in expert systems, most of the rules obtained from the experts as well as the data provided by the users are not certain. In this case, a reasonable set of truth values of these rules and data should be [0, 1] instead of {0, 1}. However, classical logic system only makes sense when the truth value set is {0, 1}. Therefore, researchers began to extend classical logic to multi-valued logic which has truth value set [0, 1].

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In multi-valued logic, the connectives and the rules for constructing formula are those used in classical logic (Chang and Lee 1973), and the disjunction, conjunction and negation of formulas are defined by max, min operations together with the complementation to 1, respectively (Goguen 1969). Lee (1972) defined the concept of satisfiability and studied the resolution principle. A definition of logical inference of one formula based upon the assertion of some premise formulas was introduced by Yager (1985). Prade (1985) investigated the extension of modus ponens by using a continuous triangular norm to model the conjunction operation. In addition, Buckley et al. (1986) used the min operation to model the conjunction operation in a fuzzy expert system, and Bonissone (1987) suggested the use of other triangular conorms. For more information concerning the theory of multi-valued logic, the interested readers may consult the surveys by Skala (1978) and Bolc and Borowik (1992).

Although multi-valued logic is well developed, a practical interpretation of truth values of formulas is controversial. Nilsson (1986) explained it as the probability value, and proved that when the probability values of all propositions are either 0 or 1, the probabilistic truth value reduces to the classical truth value. Dubois and Prade (1987, 1991) considered it as possibility value and necessity value, and introduced a possibilistic logic as a new branch of uncertain logic system. A detailed survey about the meaning of truth value in multi-valued logic was given by Dubois et al. (1991).

In this paper, the truth value is considered as credibility value. Section 2 introduces some basic properties about credibility measures. In Sect. 3, a concept of truth value is defined, and the consistency between credibilistic logic and classical logic is proved. In Sect. 4, a concept of independence of fuzzy formulas is introduced and some properties about independent fuzzy formulas are proved. Section 5 introduces a credibilistic modus ponens and a credibilistic modus tollens as the counterparts of classical modus ponens and modus tollens in credibilistic logic. In Sect. 6, a comparison between credibilistic logic and possibilistic logic is given. At the end of this paper, a brief summary is given.

2 Preliminary

The concept of fuzzy set was introduced by Zadeh (1965) via membership function. In order to define a self-dual measure for fuzzy event, the concept of credibility measure was proposed by Liu and Liu (2002). In addition, a sufficient and necessary condition for credibility measure was given by Li and Liu (2006). Credibility theory was founded by Liu (2004) and refined by Liu (2007a), is a branch of mathematics for studying the behavior of fuzzy phenomena.

Let ξ be a fuzzy variable with membership function μ. Then for any set B of real numbers, we have

$$\mathrm{Cr}\{\xi \in B\} = \frac{1}{2}\left(\sup_{x\in B} \mu(x) + 1 - \sup_{x\in B^c} \mu(x)\right). \tag{1}$$

This formula is also called the *credibility inversion theorem*. Conversely, the membership function may be calculated by credibility measure via

$$\mu(x) = (2\text{Cr}\{\xi = x\}) \wedge 1, \quad \forall x \in \Re.$$

Different from possibility measure and necessity measure, credibility measure has self-duality property. That is, for any fuzzy event A, we have

$$\text{Cr}\{A\} + \text{Cr}\{A^c\} = 1. \tag{2}$$

Credibility measure is increasing and sub-additive. That is, for any fuzzy events A and B, we have

$$\text{Cr}\{A\} \le \text{Cr}\{B\}, \quad \text{if } A \subset B, \tag{3}$$

$$\text{Cr}\{A \cup B\} \le \text{Cr}\{A\} + \text{Cr}\{B\}. \tag{4}$$

In addition, if $\text{Cr}\{A\} + \text{Cr}\{B\} > 1$, then we have

$$\text{Cr}\{A \cap B\} = \text{Cr}\{A\} \wedge \text{Cr}\{B\}. \tag{5}$$

Recently, credibility measure is applied successfully to defining fuzzy entropy (Li and Liu 2007, 2008; Liu 2007b). For more information concerning credibility theory, the interested readers may consult the survey by Liu (2006) and the book by Liu (2008).

3 Truth value

A *fuzzy proposition* is a statement with credibility value belonging to $[0, 1]$. For example,

"Tom lives in Beijing with credibility 0.8"

is a fuzzy proposition, where "Tom lives in Beijing" is a statement, and its truth value is 0.8 in credibility. In addition,

"Ellis is a man with credibility 0.1"

is a fuzzy proposition, where "Ellis is a man" is a statement, and its truth value is 0.1 in credibility.

Generally speaking, we use p to express the fuzzy proposition and use c to express its credibility value. In fact, the fuzzy proposition p is essentially a fuzzy variable

$$p = \begin{cases} 1, & \text{with credibility } c \\ 0, & \text{with credibility } 1 - c \end{cases}$$

where $p = 1$ means p is true and $p = 0$ means p is false.

In addition to proposition symbols p and q, negation symbol $\neg$ and disjunction symbol $\vee$ are also the primitive symbols in credibilistic logic. If p and q are fuzzy propositions, then $\neg p$ means "the negation of p" and $p \vee q$ means "p or q". Based on these symbols, *fuzzy formula* is defined as a member of the minimal set $\mathcal{S}$ of finite sequence of primitive symbols satisfying:

(a) $p \in \mathcal{S}$ for each fuzzy proposition p;
(b) if $X \in \mathcal{S}$, then $\neg X \in \mathcal{S}$;
(c) if $X_1 \in \mathcal{S}$ and $X_2 \in \mathcal{S}$, then $X_1 \vee X_2 \in \mathcal{S}$.

In fact, a fuzzy formula X is essentially a fuzzy variable taking values 0 or 1. First, it is clear that each fuzzy proposition is a fuzzy variable. Furthermore, if a fuzzy formula X is a fuzzy variable, then $\neg X$ is also a fuzzy variable defined as

$$\neg X = 1 - X.$$

Finally, since fuzzy formulas X_1 and X_2 are fuzzy variables, $X_1 \vee X_2$ is a fuzzy variable defined as

$$X_1 \vee X_2 = \max\{X_1, X_2\}$$

where $X = 1$ means X is true and $X = 0$ means X is false.

For any fuzzy formulas X_1 and X_2, we define conjunction symbol $\wedge$ and implication symbol $\rightarrow$ as

$$X_1 \wedge X_2 = \neg(\neg X_1 \vee \neg X_2), \quad X_1 \rightarrow X_2 = \neg X_1 \vee X_2.$$

It is clear that $X_1 \wedge X_2$ and $X_1 \rightarrow X_2$ are fuzzy formulas.

Assume X is a fuzzy formula containing fuzzy propositions $p_1, \ldots, p_n$. It is well-known that there is a function $f\colon \{0,1\}^n \rightarrow \{0,1\}$ such that $X = 1$ if and only if $f(p_1, \ldots, p_n) = 1$. In credibilistic logic, we will call f the *truth function* of X. For example, the truth function of fuzzy formula $(p \vee q) \rightarrow \neg r$ is

$$f(1,1,1) = 0, \quad f(1,0,1) = 0, \quad f(0,1,1) = 0, \quad f(0,0,1) = 1,$$

$$f(1,1,0) = 1, \quad f(1,0,0) = 1, \quad f(0,1,0) = 1, \quad f(0,0,0) = 1.$$

Definition 3.1 Let X be a fuzzy formula. Then its truth value is defined as

$$T(X) = \mathrm{Cr}\{X = 1\}. \tag{6}$$

For any fuzzy proposition p, it is easy to prove that $\mathrm{Cr}\{p = 1\} = c$. That is, the truth value of fuzzy proposition is just its credibility value. If $T(X) > 0.5$, then fuzzy formula X is somewhat true, and $T(X) = 1$ means X is certainly true. If $T(X) < 0.5$, then X is somewhat false, and $T(X) = 0$ means X is certainly false. Especially, $T(X) = 0.5$ means X is totally uncertain. The higher the truth value is, the more true the fuzzy formula is. For any fuzzy formula X containing fuzzy propositions

$p_1, \ldots, p_n$ with truth function f, we have

$$T(X) = \mathrm{Cr}\{f(p_1, \ldots, p_n) = 1\}.$$

Theorem 3.1 *Let X be a fuzzy formula containing fuzzy propositions $p_1, \ldots, p_n$ whose truth function is $f(x_1, \ldots, x_n)$. Then*

$$T(X) = \frac{1}{2}\left(\max_{f(x_1,\ldots,x_n)=1} \mu(x_1, \ldots, x_n) + 1 - \max_{f(x_1,\ldots,x_n)=0} \mu(x_1, \ldots, x_n)\right)$$

where μ is the joint membership function of fuzzy variables $p_1, \ldots, p_n$.

Proof The theorem follows immediately from the credibility inversion theorem (1).

Theorem 3.2 (Self-duality) *For any fuzzy formula X, we have*

$$T(\neg X) = 1 - T(X).$$

Proof It follows from the self-duality of credibility measure (2) that

$$\begin{aligned} T(\neg X) &= \mathrm{Cr}\{\neg X = 1\} = \mathrm{Cr}\{X = 0\} \\ &= 1 - \mathrm{Cr}\{X = 1\} \\ &= 1 - T(X). \end{aligned}$$

The proof is complete.

If p is the fuzzy proposition "Tom lives in Beijing with credibility 0.8", then $\neg p$ means "Tom does not live in Beijing". It follows from the self-duality theorem that $T(\neg p) = 1 - T(p) = 1 - 0.8 = 0.2$.

Theorem 3.3 (Monotonicity and subadditivity) *For any fuzzy formulas X_1 and X_2, we have*

$$T(X_1) \vee T(X_2) \leq T(X_1 \vee X_2) \leq T(X_1) + T(X_2)$$

Proof It follows from the monotonicity of credibility measure (3) that

$$\begin{aligned} T(X_1 \vee X_2) &= \mathrm{Cr}\{X_1 \vee X_2 = 1\} = \mathrm{Cr}\{\{X_1 = 1\} \cup \{X_2 = 1\}\} \\ &\geq \mathrm{Cr}\{X_1 = 1\} \vee \mathrm{Cr}\{X_2 = 1\} \\ &= T(X_1) \vee T(X_2). \end{aligned}$$

Furthermore, it follows from the subadditivity of credibility measure (4) that

$$\begin{aligned} T(X_1 \vee X_2) &= \mathrm{Cr}\{\{X_1 = 1\} \cup \{X_2 = 1\}\} \\ &\leq \mathrm{Cr}\{X_1 = 1\} + \mathrm{Cr}\{X_2 = 1\} \\ &= T(X_1) + T(X_2). \end{aligned}$$

The proof is complete.

Remark 3.1 The credibilistic logic and classical logic are consistent. (a) For any fuzzy proposition p with credibility value c. If p is true, we have $T(p) = c = 1$, and if p is false, we have $T(p) = c = 0$. (b) For any fuzzy formula X, if $T(X) = 1$, it follows from Theorem 3.2 that

$$T(\neg X) = 1 - T(X) = 0,$$

and if $T(X) = 0$, we have $T(\neg X) = 1$. (c) For any fuzzy formulas X_1 and X_2, if $T(X_1) = 1$ or $T(X_2) = 1$, it follows from Theorem 3.3 that

$$T(X_1 \vee X_2) \geq T(X_1) \vee T(X_2) = 1,$$

which implies that $T(X_1 \vee X_2) = 1$. If $T(X_1) = 0$ and $T(X_2) = 0$, it follows from Theorem 3.3 that

$$T(X_1 \vee X_2) \leq T(X_1) + T(X_2) = 0.$$

That is, $T(X_1 \vee X_2) = 0$.

Theorem 3.4 (Normality) *For any fuzzy formula X, we have*

$$T(X \vee \neg X) = 1, \quad T(X \vee X) = T(X).$$

Proof It follows from the normality of credibility measure that

$$T(X \vee \neg X) = \text{Cr}\{X \vee \neg X = 1\} = \text{Cr}\{X = 1 \text{ or } 0\} = 1.$$

Furthermore, it follows from Definition 3.1 that

$$T(X \vee X) = \text{Cr}\{X \vee X = 1\} = \text{Cr}\{X = 1\} = T(X).$$

The proof is complete.

Theorem 3.5 *For any fuzzy formulas X_1 and X_2, we have*

$$T(X_1) + T(X_2) - 1 \leq T(X_1 \wedge X_2) \leq T(X_1) \wedge T(X_2).$$

Proof It follows from Theorem 3.3 that

$$\begin{aligned} T(X_1 \wedge X_2) &= 1 - T(\neg X_1 \vee \neg X_2) \leq 1 - T(\neg X_1) \vee T(\neg X_2) \\ &= T(X_1) \wedge T(X_2), \\ T(X_1 \wedge X_2) &= 1 - T(\neg X_1 \vee \neg X_2) \geq 1 - (T(\neg X_1) + T(\neg X_2)) \\ &= T(X_1) + T(X_2) - 1. \end{aligned}$$

The proof is complete.

Theorem 3.6 *For any fuzzy formula X, we have*

$$T(X \wedge \neg X) = 0, \quad T(X \wedge X) = T(X).$$

Proof It follows from Theorem 3.4 that

$$T(X \wedge \neg X) = 1 - T(\neg X \vee \neg\neg X) = 1 - 1 = 0,$$
$$T(X \wedge X) = 1 - T(\neg X \vee \neg X) = 1 - T(\neg X) = T(X).$$

The proof is complete.

Theorem 3.7 *For any fuzzy formulas X_1 and X_2, we have*

$$(1 - T(X_1)) \vee T(X_2) \leq T(X_1 \to X_2) \leq 1 - T(X_1) + T(X_2).$$

Proof It follows immediately from Theorem 3.3 and the fact $X_1 \to X_2 = \neg X_1 \vee X_2$.

Theorem 3.8 *For any fuzzy formula X, we have*

$$T(X \to X) = 1, \quad T(X \to \neg X) = 1 - T(X).$$

Proof It follows from Theorem 3.4 that

$$T(X \to X) = T(\neg X \vee X) = 1,$$
$$T(X \to \neg X) = T(\neg X \vee \neg X) = T(\neg X) = 1 - T(X).$$

The proof is complete.

Theorem 3.9 *For any fuzzy formulas X_1 and X_2, we have*

$$T(\neg X_2 \to \neg X_1) = T(X_1 \to X_2).$$

Proof It follows from definition of implication symbol that

$$T(\neg X_2 \to \neg X_1) = T(\neg\neg X_2 \vee \neg X_1) = T(\neg X_1 \vee X_2) = T(X_1 \to X_2).$$

The proof is complete.

4 Independence

The independence of fuzzy variables were defined by Liu and Gao (2007). That is, the fuzzy variables $\xi_1, \xi_2, \ldots, \xi_n$ are independent if

$$\mathrm{Cr}\{\cup_i \{\xi_i \in B_i\}\} = \max_{1 \leq i \leq n} \mathrm{Cr}\{\xi_i \in B_i\}$$

for any sets $B_1, B_2, \ldots, B_n$ of $\Re$.

Definition 4.1 Fuzzy formulas X_1 and X_2 are called independent if they are independent fuzzy variables.

Let $X_1 = p \vee q$, $X_2 = \neg r$ and $X_3 = p \wedge r \to s$. Then X_1 and X_2 are independent. However, X_1 and X_3 are not independent because if $X_1 = 0$, it is clear that $X_3 = 1$. That is, they are dependent.

Theorem 4.1 *If fuzzy formulas X_1 and X_2 are independent, then*

$$T(X_1 \vee X_2) = T(X_1) \vee T(X_2). \tag{7}$$

Proof Since fuzzy variables X_1 and X_2 are independent, we have

$$\begin{aligned} T(X_1 \vee X_2) &= \mathrm{Cr}\{X_1 \vee X_2 = 1\} = \mathrm{Cr}\{\{X_1 = 1\} \cup \{X_2 = 1\}\} \\ &= \mathrm{Cr}\{X_1 = 1\} \vee \mathrm{Cr}\{X_2 = 1\} \\ &= T(X_1) \vee T(X_2). \end{aligned}$$

The proof is complete.

Example 4.1 If fuzzy formulas X_1 and X_2 are not independent, we may have

$$T(X_1 \vee X_2) > T(X_1) \vee T(X_2).$$

For example, let p be a fuzzy proposition with credibility value 0.5. It follows from Definition 3.1 that $T(p) = 0.5$, $T(\neg p) = 0.5$ and

$$T(p \vee \neg p) = \mathrm{Cr}\{p \vee \neg p = 1\} = \mathrm{Cr}\{p = 1 \text{ or } 0\} = 1.$$

Therefore, we get $T(p \vee \neg p) > T(p) \vee T(\neg p)$.

Theorem 4.2 *If fuzzy formulas X_1 and X_2 are independent, then*

$$T(X_1 \wedge X_2) = T(X_1) \wedge T(X_2).$$

Proof Since X_1 and X_2 are independent, we have $\neg X_1$ and $\neg X_2$ are also independent. It follows from Theorem 4.1 that

$$\begin{aligned} T(X_1 \wedge X_2) &= T(\neg(\neg X_1 \vee \neg X_2)) = 1 - T(\neg X_1 \vee \neg X_2) \\ &= 1 - T(\neg X_1) \vee T(\neg X_2) \\ &= T(X_1) \wedge T(X_2). \end{aligned}$$

The proof is complete.

Theorem 4.3 *If fuzzy formulas X_1 and X_2 are independent, then*

$$T(X_1 \to X_2) = (1 - T(X_1)) \vee T(X_2).$$

Proof Since X_1 and X_2 are independent, we have $\neg X_1$ and X_2 are also independent. It follows from Theorem 4.1 that

$$\begin{aligned} T(X_1 \to X_2) &= T(\neg X_1 \vee X_2) = T(\neg X_1) \vee T(X_2) \\ &= (1 - T(X_1)) \vee T(X_2). \end{aligned}$$

The proof is complete.

5 Credibilistic modus ponens

As a rule of inference, the modus ponens plays an important role in classical logic and allows us to infer Y from X and $X \to Y$. When X and $X \to Y$ are assumed fuzzy formulas, how do we infer the truth value $T(Y)$ from $T(X)$ and $T(X \to Y)$? We will answer this question by credibilistic modus ponens.

Suppose that $X_1, \ldots, X_n$ and Y are fuzzy formulas. An assignment $X_1 = x_1, \ldots, X_n = x_n$ may be illogical. Otherwise, there are three possible outcomes about Y. That is, $Y = 1$ (true), $Y = 0$ (false) and $Y = 1$ or 0 (undetermined). For example, $X = 1$ and $X \vee Y = 0$ is illogical because X is true implies that $X \vee Y$ is also true. Otherwise, if X is false and $X \vee Y$ is true, then Y is true; if X is true and $X \vee Y$ is true, then Y is undetermined; if X and $X \vee Y$ are all false, then Y is false.

The *inference function* $g\colon \{0, 1\}^n \to \{0, 0.5, 1\}$ of Y on $X_1, \ldots, X_n$ is defined as follows: (a) $g(x_1, \ldots, x_n) = 1$ if $Y = 1$ with $X_1 = x_1, \ldots, X_n = x_n$; (b) $g(x_1, \ldots, x_n) = 0.5$ if $Y = 1$ or 0 with $X_1 = x_1, \ldots, X_n = x_n$; (c) $g(x_1, \ldots, x_n) = 0$ if $X_1 = x_1, \ldots, X_n = x_n$ are illogical or $Y = 0$ with $X_1 = x_1, \ldots, X_n = x_n$. For example, the inference function of Y on X and $X \vee Y$ is $g(0, 1) = 1$, $g(1, 1) = 0.5$, $g(0, 0) = 0$ and $g(1, 0) = 0$.

Inference rule Suppose that $X_1, \ldots, X_n$ and Y are fuzzy formulas, and g is the inference function of Y on $X_1, \ldots, X_n$. Then we infer

$$\mathrm{Cr}\left\{g(X_1, \ldots, X_n) = 1\right\} \le T(Y) \le \mathrm{Cr}\left\{g(X_1, \ldots, X_n) > 0\right\}.$$

Remark 5.1 Let X be a fuzzy formula containing fuzzy propositions $\xi_1, \ldots, \xi_n$. It is clear that any assignment $\xi_1 = x_1, \ldots, \xi_n = x_n$ is logical. In addition, for each assignment, there are only two possible outcomes about X. That is, $X = 0$ and $X = 1$. Hence, the inference function g of X on $\xi_1, \ldots, \xi_n$ degenerates to its truth function f. Then we have $\mathrm{Cr}\left\{g(\xi_1, \ldots, \xi_n) > 0\right\} = \mathrm{Cr}\left\{f(\xi_1, \ldots, \xi_n) = 1\right\} = \mathrm{Cr}\left\{g(\xi_1, \ldots, \xi_n) = 1\right\}$. It follows from the inference rule that

$$T(X) = \mathrm{Cr}\left\{f(\xi_1, \ldots, \xi_n) = 1\right\} = \mathrm{Cr}\{X = 1\}.$$

Therefore, the inference rule and credibilistic truth value are consistent.

Theorem 5.1 (Modus ponens) *From* $T(X) + T(X \to Y) > 1$ *we infer*

$$T(X) \wedge T(X \to Y) \le T(Y) \le T(X \to Y). \tag{8}$$

Proof Since the inference function of Y on X and $X \to Y$ is $g(1,1) = 1$, $g(1,0) = 0$, $g(0,1) = 0.5$ and $g(0,0) = 0$, it follows from the inference rule that $T(Y) \leq \mathrm{Cr}\{X \to Y = 1\} = T(X \to Y)$. In addition, since $T(X) + T(X \to Y) > 1$, we have $\mathrm{Cr}\{X = 1\} + \mathrm{Cr}\{X \to Y = 1\} > 1$ and it follows from (5) that

$$\begin{aligned} T(Y) &\geq \mathrm{Cr}\{X = 1, X \to Y = 1\} = \mathrm{Cr}\{X = 1\} \wedge \mathrm{Cr}\{X \to Y = 1\} \\ &= T(X) \wedge T(X \to Y). \end{aligned}$$

The proof is complete.

For example, let $X \to Y$ be a fuzzy formula "If someone lives in Beijing, then he works in Beijing with credibility 0.9", and X a fuzzy formula "Tom lives in Beijing with credibility 0.8". Then it follows from (8) that "Tom works in Beijing with credibility belonging to [0.8, 0.9]".

Remark 5.2 Assume $T(X) = 1$ and $T(X \to Y) = 1$. It follows from the credibilistic modus ponens that $T(Y) = 1$. That is, if X and $X \to Y$ are all true, then Y is true. Hence, credibilistic modus ponens is consistent with the classical one.

Theorem 5.2 (Modus tollens) *From* $T(\neg Y) + T(X \to Y) > 1$ *we infer*

$$T(\neg Y) \wedge T(X \to Y) \leq T(\neg X) \leq T(X \to Y). \tag{9}$$

Proof It follows immediately from credibilistic modus ponens and the fact $T(X \to Y) = T(\neg Y \to \neg X)$.

For example, if we know "If someone lives in Beijing, then he works in Beijing with credibility 0.9", and "Tom does not work in Beijing with credibility 0.8". Then it follows from (9) that "Tom does not live in Beijing with credibility belonging to [0.8, 0.9]".

Remark 5.3 If $T(\neg Y) = 1$ and $T(X \to Y) = 1$, it follows from (9) that $T(\neg X) = 1$. In other words, if Y is false and $X \to Y$ is true, then X is false. Hence, the credibilistic modus tollens is consistent with the classical one.

6 Credibilistic logic versus possibilistic logic

In possibilistic logic (Dubois and Prade 1991), fuzzy proposition is defined as a pair (p, α), where p is a statement and α is its necessity value. Let X be a fuzzy formula containing fuzzy propositions $(p_1, \alpha_1), \ldots, (p_n, \alpha_n)$. If its truth function is f, then its possibility value is defined as

$$\mathrm{Pos}(X) = \max_{f(x_1,\ldots,x_n)=1} \min_{1\leq i\leq n} \mu_i(x_i) \tag{10}$$

where $\mu_i(x_i) = 1$ if $x_i = 1$ and $\mu_i(x_i) = 1 - \alpha_i$ if $x_i = 0$. Furthermore, its necessity value is defined as

$$\mathrm{Nec}(X) = 1 - \mathrm{Pos}(\neg X). \tag{11}$$

Let p be a fuzzy proposition "Ellis is a man with truth value 0.1". In this case, it is clear that "Ellis is a woman" is more likely than "Ellis is a man". It follows from (10) that

$$\mathrm{Pos}(p) = 1, \quad \mathrm{Pos}(\neg p) = 1 - 0.1 = 0.9,$$

i.e., $\mathrm{Pos}(p) > \mathrm{Pos}(\neg p)$, which implies that "Ellis is a man" is more likely than "Ellis is a woman". Furthermore, it follows from (11) that

$$\mathrm{Nec}(p) = 1 - 0.9 = 0.1, \quad \mathrm{Nec}(\neg p) = 1 - 1 = 0,$$

i.e., $\mathrm{Nec}(p) > \mathrm{Nec}(\neg p)$, which implies that "Ellis is a man" is more possible than "Ellis is a woman". Hence, it is not reasonable to explain truth value as necessity value.

However, if 0.1 is explained as credibility value, then it follows from Definition 3.1 that

$$T(p) = 0.1, \quad T(\neg p) = 0.9,$$

which implies that "Ellis is a woman" is more possible than "Ellis is a man". This is a reasonable result.

Finally, let p be a fuzzy proposition with necessity value 0. Then we get $\mathrm{Nec}(p) = 0$ and $\mathrm{Pos}(p) = 1$. In addition, it is easy to prove that $\mathrm{Nec}(\neg p) = 0$ and $\mathrm{Pos}(\neg p) = 1$, which is inconsistent with classical logic. Hence, compared with possibilistic logic, one advantage of credibilistic logic is the well consistency with classical logic.

7 Conclusions

In this paper, credibilistic logic was designed as a generalization of classical logic for dealing with fuzzy knowledge. The concept of credibilistic truth value was introduced in the framework of credibility theory, and credibilistic modus ponens and credibilistic modus tollens were also presented. The consistency between credibilistic logic and classical logic was proved on the basis of some important properties about truth values.

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References

Bolc, L., & Borowik, P. (1992). *Many-valued logics*. Berlin: Springer.

Bonissone, P. (1987). Summarizing and propagating uncertain information with triangular norms. *International Journal of Approximate Reasoning, 1*, 71–101.

Buckley, J., Siler, W., & Tucker, D. (1986). A fuzzy expert system. *Fuzzy Sets and Systems, 20*, 1–16.

Chang, C., & Lee, R. (1973). *Symbolic logic and mechanical theorem proving*. New York: Academic Press.

Dubois, D., & Prade, H. (1987). Necessity measure and resolution principle. *IEEE Transactions on Man Cybenet, 17*, 474–478.

Dubois, D., & Prade, H. (1991). Possibilistic logic, preference models, non-monotonicity and related issues. *Proceedings of 12th International Joint Conference on Artificial Intelligence* (pp. 419–424), Sydney, Australia, August 24–30.

Dubois, D., Lang, J., & Prade, H. (1991). Fuzzy sets in approximate reasoning, Part 2: Logical approaches. *Fuzzy Sets and Systems, 40*, 203–244.

Goguen, J. (1969). The logic of inexact concepts. *Synthese, 19*, 325–373.

Lee, R. (1972). Fuzzy logic and the resolution principle. *Journal of the Association for Computing Machinery, 19*(1), 109–119.

Li, P., & Liu, B. (2008). Entropy of credibility distributions for fuzzy variables. *IEEE Transactions on Fuzzy Systems, 16*(1), 123–129.

Li, X., & Liu, B. (2006). A sufficient and necessary condition for credibility measures. *International Journal of Uncertainty, Fuzziness & Knowledge-Based Systems, 14*(5), 527–535.

Li, X., & Liu, B. (2007). Maximum entropy principle for fuzzy variables. *International Journal of Uncertainty, Fuzziness & Knowledge-Based Systems, 15*(Suppl 2), 40–48.

Liu, B. (2004). *Uncertainty Theory*. Berlin: Springer.

Liu, B. (2006). A survey of credibility theory. *Fuzzy Optimization and Decision Making, 5*(4), 387–408.

Liu, B. (2007a). *Uncertainty Theory*, 2nd ed., Berlin: Springer.

Liu, B. (2007b). A survey of entropy of fuzzy variables. *Journal of Uncertain Systems, 1*(1), 4–13.

Liu, B. (2008). *Uncertainty Theory*, 3rd ed. http://orsc.edu.cn/liu/ut.pdf.

Liu, B., & Liu, Y. (2002). Expected value of fuzzy variable and fuzzy expected value models. *IEEE Transactions on Fuzzy Systems, 10*(4), 445–450.

Liu, Y., & Gao, J. (2007). The independence of fuzzy variables with applications to fuzzy random optimization. *International Journal of Uncertainty, Fuzziness & Knowledge-Based Systems, 15*(Suppl 2), 1–20.

Nilsson, N. (1986). Probabilistic logic. *Artificial Intelligence, 28*, 71–87.

Prade, H. (1985). A computational approach to approximate and plausible reasoning with applications to expert systems. *IEEE Transactions on Pattern Anal. Machine Intelligence, 7*(3), 260–283.

Skala, H. (1978). On multi-valued logics, fuzzy sets, fuzzy logics and their applications. *Fuzzy Sets and Systems, 1*, 129–149.

Yager, R. (1985). Inference in a multivalued logic system. *International Journal of Man-Machine Studies, 23*, 27–44.

Zadeh, L. (1965). Fuzzy sets. *Information and Control, 8*, 338–353.