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Variable fuzzy sets and its application in comprehensive risk evaluation for flood-control engineering system

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Abstract Due to significance of the engineering system for flood control and the multi-dimensional synthesis of the risk evaluation issue that the paper selects typical risk indexes to classify risk degree of flood-control engineering. Under global view of system that the variable fuzzy sets method is presented to set up comprehensive evaluation model for flood-control engineering system (FCES). The method can scientifically and reasonably determine membership degrees and relative membership functions of disquisitive indexes at level interval that relating to engineering, also it can fully use one's experience and knowledge, qualitative and quantitative information of index system to obtain weights of indexes for operating comprehensive risk evaluation for FCES. The numerical example shows that the proposed method is feasible and effective, and the evaluation results are reasonable.

Keywords Variable fuzzy sets · Flood-control engineering system · Integrated risk · Risk evaluation model

1. Introduction

It is well known that constructing flood control engineering system (FCES) can effectively alleviate flood calamity, yet we must admit that any FCES exist potential risk, collapse. Recently, due to frequent occurrence of flood that many hydraulic engineering are wrecked and bring serious threaten and great loss to people life and national economy, though people had been paid much attention to risk evaluation of FCES and obtained compelling fruits, owing to complexity of flood research and engineering system structure that the traditional theories and methods have obvious limitation on solving these problems, and the study on risk evaluation of FCES still had not made great advanced. Many researchers had operated much study on dam safety and levee overflow risk (Feng & Li, 1994; Wang, Yang, & Zhang

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1998; Zhang, Wang & yang, 1999; Li, 2001a; Mei & Tan, 2002) whereas the risk evaluation on FCES have just commenced (Li & Zhang, 2001b).

Generally, FCES is a synthesis of multi-dimensional factors, so its risk evaluation shall be operated from single factor to multi factors, which means that routine single factor evaluation often omit important information and can not obtain integrated risk evaluation for engineering system of global drainage area. Accordingly, under global view of system and founded upon characteristics of risk evaluation and risk management that the variable fuzzy sets (VFS) method (Chen, 2005a) is presented to evaluate the synthetic risk of FCES. The method can scientifically and reasonably determine membership degrees and relative membership functions of disquisitive objectives (or indexes) at level interval that relating to engineering system, also it can fully use one's experience and knowledge, qualitative and quantitative information of index system to obtain weights of objectives (or indexes) for operating comprehensive risk evaluation of FCES.

2. Principle of VFS

2.1. Definition of VFS

In defining the concept, let us suppose that U is a fuzzy concept (alternative or phenomenon) $\underset{\sim}{A}$, and to any elements $u(u \in U)$, $\mu_{\underset{\sim}{A}}(u)$ and $\mu_{\underset{\sim}{A}^c}(u)$ are relative membership degree (RMD) function that express degrees of attractability and repellency, respectively (Chen, 2002). Let

$$D_{\underset{\sim}{A}}(u) = \mu_{\underset{\sim}{A}}(u) - \mu_{\underset{\sim}{A}^c}(u) \tag{1}$$

Where $D_{\underset{\sim}{A}}(u)$ is defined as relative difference degree of u to $\underset{\sim}{A}$. Mapping

$$\begin{aligned} &D_{\underset{\sim}{A}} : D \to [-1, 1] \\ &u| \to D_{\underset{\sim}{A}}(u) \in [-1, 1] \end{aligned} \tag{2}$$

is defined as relative difference function of u to $\underset{\sim}{A}$. And we have

$$\mu_{\underset{\sim}{A}}(u) + \mu_{\underset{\sim}{A}^c}(u) = 1 \tag{3}$$

Then

$$D_{\underset{\sim}{A}}(u) = 2\mu_{\underset{\sim}{A}}(u) - 1 \tag{4}$$

or

$$\mu_{\underset{\sim}{A}}(u) = (1 + D_{\underset{\sim}{A}}(u))/2 \tag{5}$$

Where $0 \le \mu_{\underset{\sim}{A}}(u) \le 1, 0 \le \mu_{\underset{\sim}{A}^c}(u) \le 1$. Let

$$\underset{\sim}{V} = \left\{ (u, D)|\, u \in U, D_{\underset{\sim}{A}}(u) = \mu_{\underset{\sim}{A}}(u) - \mu_{\underset{\sim}{A}^c}(u), D \in [-1, 1] \right\} \tag{6}$$

$$A_+ = \{u|\, u \in U,\ \mu_{\underset{\sim}{A}}(u) > \mu_{\underset{\sim}{A}^c}(u)\ \} \tag{7}$$

$$A_- = \{u|\, u \in U,\ \mu_{\underset{\sim}{A}}(u) < \mu_{\underset{\sim}{A}^c}(u)\ \} \tag{8}$$

$$A_0 = \{u|\, u \in U,\ \mu_{\underset{\sim}{A}}(u) = \mu_{\underset{\sim}{A}^c}(u)\ \} \tag{9}$$

Here $\underset{\sim}{V}$ is just defined as VFS of U; A_+, A_- and A_0 are defined as attracting (as priority) sets, repelling (as priority) sets and balance boundary or qualitative change boundary of VFS $\underset{\sim}{V}$, respectively. Assume that C is variable factors sets of $\underset{\sim}{V}$

$$C = \{C_A, C_B, C_C\} \tag{10}$$

Here C_A are variable model sets, C_B are variable model parameters sets and C_C are variable other factors sets except model and its parameters. Let

$$A^+ = C(A_-) = \{u | u \in U,\ \mu_{\underset{\sim}{A}}(u) < \mu_{\underset{\sim}{A}^c}(u), \quad \mu_{\underset{\sim}{A}}(C(u)) > \mu_{\underset{\sim}{A}^c}(C(u))\} \tag{11}$$

$$A^- = C(A_+) = \{u | u \in U,\ \mu_{\underset{\sim}{A}}(u) > \mu_{\underset{\sim}{A}^c}(u), \quad \mu_{\underset{\sim}{A}}(C(u)) < \mu_{\underset{\sim}{A}^c}(C(u))\} \tag{12}$$

We generally define these two subsets as qualitative change sets of VFS $\underset{\sim}{V}$ to variable elements sets C. Let

$$A^{(+)} = C(A_{(+)}) = \{u | u \in U,\ \mu_{\underset{\sim}{A}}(u) > \mu_{\underset{\sim}{A}^c}(u), \quad \mu_{\underset{\sim}{A}}(C(u)) > \mu_{\underset{\sim}{A}^c}(C(u))\} \tag{13}$$

$$A^{(-)} = C(A_{(-)}) = \{u | u \in U,\ \mu_{\underset{\sim}{A}}(u) < \mu_{\underset{\sim}{A}^c}(u), \quad \mu_{\underset{\sim}{A}}(C(u)) < \mu_{\underset{\sim}{A}^c}(C(u))\} \tag{14}$$

We generally define these two subsets as quantitative change sets of VFS $\underset{\sim}{V}$ to variableelements sets C.

VFS models include fuzzy optimization model, fuzzy pattern recognition model and fuzzy clustering iteration model, etc. (Chen, 2002). Variable parameters sets of model include indexes weights, standard indexes values and other important parameters. We will illustrate changeability of model and parameters in application of risk evaluation.

2.2. Methods of relative difference function

We suppose that $X_0 = [a, b]$ are attracting (as priority) sets of VFS $\underset{\sim}{V}$ on real axis, i.e.interval of $\mu_{\underset{\sim}{A}}(u) > \mu_{\underset{\sim}{A}^c}(u)$, $X = [c, d]$ is a certain interval containingX_0, i.e.$X_0 \subset X$(see Fig. 1).

According to definition of VFS we know that interval $[c, a]$ and $[b, d]$ all are repelling (as priority) sets of VFS, i.e. interval of $\mu_{\underset{\sim}{A}}(u) < \mu_{\underset{\sim}{A}^c}(u)$. Suppose that M is point value of $\mu_{\underset{\sim}{A}}(u) = 1$ in attracting (as priority) sets $[a, b]$, and M can be determined by actual problem or selected as midpoint value of interval $[a, b]$. x is value of random point in interval$X$, then if x locates at left side of M, its difference function is

$$\begin{aligned} D_{\underset{\sim}{A}}(u) &= \left(\tfrac{x-a}{M-a}\right)^{\beta} \quad x \in [a, M] \\ D_{\underset{\sim}{A}}(u) &= -\left(\tfrac{x-a}{c-a}\right)^{\beta} \quad x \in [c, a] \end{aligned} \tag{15}$$

And if x locates at right side of M, its difference function is

$$\begin{aligned} D_{\underset{\sim}{A}}(u) &= \left(\tfrac{x-b}{M-b}\right)^{\beta} \quad x \in [M, b] \\ D_{\underset{\sim}{A}}(u) &= -\left(\tfrac{x-b}{d-b}\right)^{\beta} \quad x \in [b, d] \end{aligned} \tag{16}$$

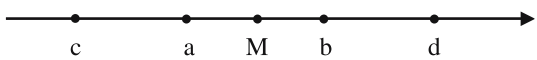

Fig. 1 Relationship between points x,M and internals [a,b], [c,d]

Where β is index that bigger that 0, usually we take it as $\beta = 1$, viz. Eqs. (15) and (16) become linear functions. Equations (15) and (16) satisfy: (i) $x = a, x = b, \mu_{\underset{\sim}{A}}(u) = \mu_{\underset{\sim}{A}^c}(u) = 0.5$; (ii) $x = M, \mu_{\underset{\sim}{A}}(u) = 1$; (iii) $x = c, x = d, \mu_{\underset{\sim}{A}}(u) = 0$. Then according to Eqs. (15) (or (16)) and (5) we can obtain values of relative membership function $\mu_{\underset{\sim}{A}}(u)$of disquisitive indexes.

3. Selection and classification of FCES risk indexes

The FCES is mainly comprised of reservoir, levee, lake, detention area and river regulation engineering, though the variety of engineering is not much, its actual circs are quite complex. For example, as levee, due to its different constructed time, rank and type thus the risk is diverse; and as lake, it has functions of detention and diversion of flood, but it also has levee and exists flood risk. Therefore, it's difficult and complicated to determine risk indexes of FCES; here we simply introduce selecting process of risk indexes for above five flood control engineering (Ji, Li & Zhang, 2005).

3.1. Reservoir risk indexes

When we build reservoir at upstream of flood control area where have developed economy and dense population, it can regulate and store flood and enhance standard of flood control, yet it also brings potential danger, and once reservoir collapsed it will take huge disaster. Flood control faculty of reservoir not only has relation with natural inflow, also it has relation with some uncertain factors, such as construction layout, construction quality control and hydraulic parameters selection; and that routing mode, priming level and flood forecasting all can take risk to reservoir flood control. Consequently, evaluating indexes selected for reservoir flood control shall reflect above hydrologic risk, hydraulic risk and structure risk, etc. its integrated risk ratio can be expressed as (Xie, Yuan & Guo, 1997)

$$R_0 = P\{Q_{\mathrm{A}} < Q_{\mathrm{D}}\} \tag{17}$$

Where R_0 is integrated risk ratio, Q_A represents actual discharge faculty, Q_{D} represents design discharge faculty.

3.2. Levee risk indexes

River levee is one of important measure of FCES, it can prevent flood overflow, enhance river flood discharge capacity and improve flood control standard for big protect region. Risk of levee flood-control capacity can be represented by risk degree, which reflects relationship between maximal levee flood control level and inflow flood level, so we can adopt analysis methods of chance variable (Liang & Li, 2001) (such as checking point method of linear second moment) to calculate:

$$P_f = \lambda \frac{\mu}{\alpha} \tag{18}$$

Where μ is first moment of difference series between flood-control level and flood level (viz. mathematic expective value), α is second moment, i.e., mean quadratic error of difference series between flood-control level and flood level, λ is risk coefficient.

3.3. Risk indexes of detention area

Programming and management of detention area are influenced by uncertain factors of engineering technology, natural environment, economy and society. Accordingly, if we evaluate its risk, we shall select key influenced factors, set up corresponding indexes system and take representative indexes as risk indexes of detention area. Concretely, fatalness indexes of detention area have (Ye, Yang & Liu, 2001): d_{11} is service frequency, d_{12} is diversion floodwater amount, d_{13} is average submerged depth, d_{14} is average submerged time; flood losing indexes have: d_{21} is population density of flooding area, d_{22} is output value density of industry and agriculture (output value of industry and agriculture of unit area of flooding area), d_{23} is traffic main stem density (length of traffic main stem of unit area); indexes that reflected bearing flood: d_{31} is sheltered area from flood (safety platform area), d_{32} is possessed boats of per 100 people, d_{33} is retreat path area, d_{34} is ratio of preventive alarm system up to standard, d_{35} is popularization ratio of knowledge sheltered from flood. Synthetically analyzing above indexes we can get relative indexes for reflecting flood risk:

$$\begin{aligned} K &= \xi(d_1 + d_2 - d_1 \cdot d_2) - \eta \cdot d_3' \\ &= f(d_{13}, d_{14}, d_{21}, d_{22}, d_{23}) \left(d_{11} + d_{12} - \frac{d_{11} \cdot d_{12}}{D} \right) - g(d_{32}, d_{34}, d_{35}) \frac{dd_{31} + d_{33}}{S} \end{aligned} \tag{19}$$

Where d_1 is service frequency, d_2 is diversion ratio (ratio of diversion amount to usable quantity of diversion area), d_3' is safety sheltered ratio (percentage of sum of sheltered area and retreat area to submerged area), D is useable diversion capacity, S is submerged area of diversion, ξ is coefficient of flood fatalness and damage of diversion and reflects flood submerged depth, time and damage; η is coefficient of disaster control faculty and reflects safety precautions of diversion and flood control.

3.4. Lake risk indexes

When cataclysm happened, lake in drainage basin (or region) can also store some water quantity to alleviate pressure of river discharge, and if water quantity exceed useable capacity of lake, it exist flood risk too. The risk can be calculated as (Min, 1994):

$$R = 2\alpha \sqrt{\frac{H_m - H_0}{H_{\max} - H_{\min}}} + \beta \tag{20}$$

Where R is risk degree of lake flood, $R \in [1, 12]$, α is coefficient expressed feature difference, β is starting value, H_m is peak level, H_0 is alarm level, H_{max} is annual maximal peak level, $H_{\min}$ is annual minimal peak level.

3.5. Risk indexes of river regulation engineering

River regulation is to widen or increase discharge section for enhancing discharge capacity. Commonly, the effect of river regulation engineering can be expressed by degree of smooth discharge, and risk be expressed by suffocating degree. From Ben, Qian & Zhao (2003) we know that, if change of roughness coefficient is 1% discharge can alter 20–25m^3/s, therefore, its risk indexes can be expressed by change ratio K_h of riverbed roughness:

$$K_h = \Delta n = n' - n_0 \tag{21}$$

Table 1 Risk indexes and their classification of flood-control engineering system

	Reservoir Integrated risk R_0	Levee Risk degree P_f	Detention area Flood risk degree K	Lake Flood risk degree R	River regulation Change ration of roughness K_h
Tiny	0.00–0.25	0.00–0.25	0.00–0.25	1–5	−0.070–0.045
Light	0.25–0.50	0.25–0.50	0.25–0.50	5–7	−0.045–0.025
Intermediate	0.50–0.75	0.50–0.75	0.50–0.75	7–9	−0.025–0.010
Strong	0.75–0.90	0.75–0.90	0.75–0.90	9–10	0.010–0.045
Special	0.90–1.00	0.90–1.00	0.90–1.00	10–12	0.045–0.070

Where K_h is risk ratio of river regulation, $n'$is regulated riverbed roughness. The n_0 is river roughness before regulation.

After determining above risk indexes, we can calculate risk indexes values by using risk analysis and classify risk degree of FCES, usually we classify it into five level: tiny, light, intermediate, strong and special (see Table 1).

When applying above risk evaluation of FCES for a certain drainage basin (or region), based on constitutes of engineering system, we firstly calculate values of engineering risk indexes and setup parameters of VFS risk evaluating system; then, according to risk values of evaluating engineering, we get classification of the engineering system risk and that offer reference for correction or decision making of FCES.

4. Application

In this paper, the author takes FCES of a certain drainage basin as example, adopts data of Ji et al. (2005) to show application of VFS method in risk evaluation of FCES.

At present, in this certain drainage basin, the FCES of mid- and downstream mainly comprised of reservoir, detention area, levee and natural lake, then according to characteristics and calculating risk indexes values of FCES, we have Table 2.

According to Table 1 and Chen (2002), we set up values matrix of parameters (a–d, M) for calculating difference function of VFS:

$$I_{[a,b]}=\begin{bmatrix} [0.00, 0.25] & [0.25, 0.50] & [0.50, 0.75] & [0.75, 0.90] & [0.90, 1.00] \\ [0.00, 0.25] & [0.25, 0.50] & [0.50, 0.75] & [0.75, 0.90] & [0.90, 1.00] \\ [0.00, 0.25] & [0.25, 0.50] & [0.50, 0.75] & [0.75, 0.90] & [0.90, 1.00] \\ [1, 5] & [5, 7] & [7, 9] & [9, 10] & [10, 12] \\ [-0.007, -0.045] & [-0.045, -0.025] & [-0.025, 0.01] & [0.01, 0.045] & [0.045, 0.07] \end{bmatrix}$$

$$I_{[c,d]}=\begin{bmatrix} [0.00, 0.50] & [0.00, 0.75] & [0.25, 0.90] & [0.50, 1.00] & [0.75, 1.00] \\ [0.00, 0.50] & [0.00, 0.75] & [0.25, 0.90] & [0.50, 1.00] & [0.75, 1.00] \\ [0.00, 0.50] & [0.00, 0.75] & [0.25, 0.90] & [0.50, 1.00] & [0.75, 1.00] \\ [1, 7] & [1, 9] & [5, 10] & [7, 12] & [9, 12] \\ [-0.007, -0.025] & [-0.007, 0.01] & [-0.045, 0.045] & [-0.025, 0.07] & [0.01, 0.07] \end{bmatrix}$$

Table 2 Actual statues of evaluation index for flood-control system

R_0	P_f	K	R	K_h
0.189	0.268	0.313	6.2	0.019

$$\vec{M} = \begin{bmatrix} 0.00 & 0.25 & 0.625 & 0.90 & 1.00 \\ 0.00 & 0.25 & 0.625 & 0.90 & 1.00 \\ 0.00 & 0.25 & 0.625 & 0.90 & 1.00 \\ 1 & 5 & 8 & 10 & 12 \\ -0.07 & -0.045 & -0.0075 & 0.045 & 0.07 \end{bmatrix}$$

Based on matrixes $I_{[a,b]}$,$I_{[c.d]}$and $\overrightarrow{M}$, we judge that evaluating index x locates at left side or right side of point M, and according these to select equation (15) or (16) for calculating difference function$\mu_h(u_{ij})$of indexes to standards. Here h is grade number and $h = 1, 2, 3, 4, 5$; i is indexes number and $i = 1, 2, 3, 4, 5$.

From Table 2, for reservoir integrated risk R_0, when $h = 1$, its attracting (as priority) matrix$[a, b]$, interval matrix$[c, d]$ and point values matrix $\overrightarrow{M}$,respectively, are

$$\begin{aligned} [a, b] &= ([0.0, 0.25]\ [0.25, 0.5]\ [0.5, 0.75]\ [0.75, 0.9]\ [0.9, 1]) \\ [c, d] &= \big([0.00, 0.50]\ [0.00, 0.75]\ [0.25, 0.90]\ [0.50, 1.00]\ [0.75, 1.00]\big) \\ \vec{M} &= (0.0\ 0.25\ 0.625\ 0.90\ 1) \end{aligned}$$

When $i = 1$, the value of reservoir integrated risk is $\mathrm{R}_0 = 0.189$, and that $c_{11} = 0.0$, $a_{11} = 0.0$, $b_{11} = 0.25$, d_{11}=0.50, $M_{11} = 0.0$, then we can see that index value (0.189) locates at right side of point M_{11} and belongs to interval$[M_{11}, b_{11}]$, so we select equation $D_{\underset{\sim}{A}}(u_{11}) = (x_{11} - b_{11})^{\beta} \big/ (M_{11} - b_{11})^{\beta}$ in Eq. (16). Substituting β=1 and other relevant parameters into this equation then we obtain $D_{\underset{\sim}{A}}(u_{11}) = 0.244$; according to Eq. (5) we obtain $\mu_{\underset{\sim}{A}}(u_{11}) = 0.622$. Analogously, we get relative membership function $\mu_{\underset{\sim}{A}}(u_{ih})$ of each single index under $i = 1, 2, 3, 4, 5$ to degrees $h = 1, 2, 3, 4, 5$ as:

$$\mu_{\underset{\sim}{A}}(u_{ih})_{5\times 5} = \begin{bmatrix} 0.622 & 0.378 & 0 & 0 & 0 \\ 0.464 & 0.964 & 0.036 & 0 & 0 \\ 0.374 & 0.874 & 0.126 & 0 & 0 \\ 0.2 & 0.7 & 0.3 & 0 & 0 \\ 0 & 0 & 0.372 & 0.871 & 0.129 \end{bmatrix}$$

To get synthetic RMD of each index, we use variable fuzzy recognition model presented by Chen (1998)

$$u_{ih} = \frac{1}{1 + \left\{ \frac{\sum_{i=1}^{m} [w_i(1-\mu_{\underset{\sim}{A}}(u_{ih}))]^p}{\sum_{i=1}^{m} (w_i \mu_{\underset{\sim}{A}}(u_{ih}))^p} \right\}^{\alpha/p}} \tag{22}$$

Through it we obtain synthetic RMD of each index for FCES by using Eq. (22), after normalizing them that we get normalized synthetic RMD of each index. Here w_i is index weight; $m$is number of recognition indexes; $\alpha$is rule parameter of model optimization, $\alpha = 1$ is least single method and $\alpha = 2$ is least square method; p is distance parameter, $p = 1$ is hamming distance and $p = 2$ is Euclidean distance.

To determine weights of five indexes to five standards, we use consistency theorem of taxis on importance of determining indexes weights (Chen 1998) and get qualitative scribe of four indexes by their influence to comparison between elements:

Table 3 Relationships between mood operator relative membership degrees of quantitative scale

Mood operator	Equal		Slight		Some-what		Rather	Obvious			
Quantitative scale	0.50	0.525	0.55	0.575	0.60	0.625	0.65	0.675	0.70		0.725
RMD	1.0	0.905	0.818	0.729	0.667	0.60	0.538	0.481	0.429		0.379
Mood operator	Remark-able		Very		Extra		Exceeding		Extreme		Incomp-arable
Quantitative scale	0.75	0.775	0.80	0.825	0.85	0.875	0.90	0.925	0.95	0.975	1
RMD	0.333	0.290	0.250	0.212	0.176	0.143	0.111	0.081	0.053	0.026	0

$$F = \begin{array}{c} \\ \begin{bmatrix} 0.5 & 0 & 0 & 0 & 1 \\ 1 & 0.5 & 1 & 0 & 1 \\ 1 & 0 & 0.5 & 0 & 1 \\ 1 & 1 & 1 & 0.5 & 1 \\ 0 & 0 & 0 & 0 & 0.5 \end{bmatrix} \end{array} \begin{array}{c} \text{Taxis} \\ (4) \\ (2) \\ (3) \\ (1) \\ (5) \end{array}$$

We take lake flood risk degree, whose ranking is first, as comparison standard and get under consideration: lake flood risk degree is on the way from “slight” to “somewhat” important than levee risk degree; lake flood risk degree is on the way from “rather” to “obvious” important than detention area risk degree; lake flood risk degree is on the way from “obvious” to “remarkable” important than reservoir integrated risk; lake flood risk degree is on the way from “exceeding” to “extreme” important than river regulation engineering (see Table 3).

And according to Table 3 (Chen, 1998) we obtain weights of five evaluating indexes as:

$$w' = (\,0.379\ 0.739\ 0.481\ \ 1\ 0.081\,) = (w'_i)$$

Then normalized weights vector of indexes is :

$$w = (\,0.141\ 0.276\ 0.181\ 0.373\ 0.030\,) = (w_i)$$

Therefore, we may use variable fuzzy recognition model (22) to calculate synthetic RMD of each index of FCES. When taking rule parameter of model optimization $\alpha = 1$, distance parameter $p = 1$ and substituting relative data into model (22) we get synthetic RMD as

$$u' = (\,0.358\ 0.738\ 0.156\ 0.026\ 0.004\,)$$

After normalized it is: $u = (0.279, 0.576, 0.122, 0.020, 0.003)$

Using rank feature values (RFV) equation (Chen, 1998) and we get RFV of FCES as

$$H = (1, 2, 3, 4, 5) \cdot (0.279, 0.576, 0.122, 0.020, 0.003)^T = 1.892 \approx 1.9$$

For FCES(as disquisitive objective), due to its standard is five grades, so we have

a. If $1.0 \le H \le 1.5$, then risk degree belongs to tiny (□grade),
b. If $1.5 < H \le 2.5$, then it belongs to slight (□grade),
c. If $2.5 < H \le 3.5$, then it belongs to Intermediate (□ grade),
d. If $3.5 < H \le 4.5$, then it belongs to strong (□grade),
e. If $4.5 < H \le 5.0$, it belongs to special (□grade).

Hence, we judge that comprehensive risk evaluation of FCES of this certain drainage basin belongs to IIgrade (slight). Comparing results with Ji et al. (2005), we find that the conclusions of two methods are basically coincident, yet VFS use RFV to operate evaluation, so it can intuitively reflect risk degree of FCES partial to another rank, and the conclusion of VFS are more reasonable and appropriately. Ji et al. (2005) put forward that the matter elements model can be used for comprehensive evaluation of flood control engineering system, yet the dependence function (matter elements model and matter elements analysis) (Cai, 1999)has mathematical mistake, and the judgment rule, $K(x) > 0, K(x) = 0, K(x) < 0$, has logistic mistake, all these make Ji et al. (2005) lose scientific foundation (Chen, 2005b; Chen and Guo,. 2005c).

5. Conclusion

(a) The paper introduces elementary application of VFS in risk evaluation and presents application, calculating results show that VFS can be applied in risk evaluation of FCES. The VFS provides an abundant and meaningful improvement or extension of conventional logic, the mathematics generated by this theory is consistent, and it may be generalization of classic fuzzy sets. The VFS not only can be used in FCES, but also be applied in other engineering field, and that are our next work.

(b) We can see that calculation of difference function is just arithmetic, so the method is simpler and practical. Values scope of corresponding RMD function can be adjusted neatly based on need of actual cases, and the method has no limit on specimen modeling, neither its precision influence by specimen number modeling.

(c) The information on FCES includes so much data and that the author advances VFS to evaluate the risk degree. The method can scientifically and reasonably determine membership degrees and relative membership functions of disquisitive objectives at level interval that relating to FCES, also it can fully use one's experience and knowledge, qualitative and quantitative information of index system to obtain weights of objectives (or indexes) for operating comprehensive risk evaluation. The numerical example has shown that the proposed method is feasible and effective.

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References

Ben, L., Qian, X., & Zhao, Z. (2003). Determination on the roughness coefficient of Wutong River natural river course (in Chinese). *Journal of Heilongjiang Hydraulic Engineering College*, *30*(1), 49–51.

Cai, W. (1999). Extension theory and its application. *Chinese Science Bulletion*, *44*(17), 1538–1548.

Chen, S. (1998). *Theory and application engineering fuzzy sets*. Beijing (in Chinese): National Defense Industry Press.

Chen, S. (2002). *Fuzzy Recognition Theory and Application for Complex Water Resources System Optimization*. Changchun, China (in Chinese): Jilin University Press.

Chen, S. (2005a). Theory and model of engineering variable fuzzy sets - mathematical basis for fuzzy hydrology and water resources (in Chinese). *Journal of Dalian University of Technology*, *45*(2), 308–312.

Chen, S. (2005b). Variable fuzzy methods and mistake demonstration in extension dependence function (in Chinese). *Water Resources and Power 23*(5), 1–4.

Chen, S., & Guo, Y. (2005c). Application of variable fuzzy sets method in power load prediction (in Chinese). *Water Resources and Power 23*(5), 29–31.

Feng, P., & Li, Y. (1994). Hydrological risk assessment for flood control levee in reservoir protect area (in Chinese). *Journal of Hohai University*, *22*(6), 98–100.

Ji, C., Li, J., & Zhang, Y. (2003). Matter-element model of integrated risk evaluation for flood–control engineering system. (in Chinese). *Journal of North China Electric Power Universtiy*, *32*(1), 86–90.

Li, Q. (2001a). Evaluation methods for reservoir collapsed-dam risk (in Chinese). *Design of Hydroelectric Power Station*, 17(4), 43–44.

Li, Q., & Zhang, J. (2001b). On the risk analysis and safety assessment research of the Yangtze river main levee (in Chinese). *China Soft Science*, 10(11), 112–115.

Liang, Z., & Li, T. (2001). Risk analysis and computation of flood control capacity for river levee (in Chinese). *Journal of Yangtze River Scientific Research Institute*. *18*(1), 7–10.

Mei, Y., & Tan, G. (2002). Risk criteria for assessment of dam hydrological safety (in Chinese). *International Journal of Hydroelectric Energy*, 20(4), 8–10.

Min, Q. (1994). Classification of flood level and its meaning in disaster reduction (in Chinese). *Journal of Natural Disasters*, *3*(1), 34–39.

Wang, Z., Yang, G., & Zhang, Z. (1998). Risk Calculation model for flood levee structure. (in Chinese). *Shuili Xuebao* 29(7),64–67.

Xie, C., Yuan, H., & Guo, Y. (1997). Overall risk model of reservoir flood control (in Chinese). *Journal of Wuhan University of Hydr. and Electrical Engineering 30*(2), 71–74.

Ye, M., Yang, Z., & Liu, C. (2001). Study on parameter evaluation for identifying the situation of sustainable development in flood storage and detention areas (in Chinese). *China Water Resources*, 38(3), 29–30.

Zhang, Z., Wang, Z., & Yang, G. (1999). Flood levee risk analysis for Yangtze river in Nanjing (in Chinese). *Journal of Hohai University*, *27*(2), 17–21.