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# A displayed inventory model with L–R fuzzy number

**Nirmal Kumar Mandal · Tapan Kumar Roy**



**Abstract** In this study, we formulate a multi-item displayed inventory model under shelf-space constraint in fuzzy environment. Here demand rate of an item is considered as a function of the displayed inventory level. The problem is formulated to maximize average profit. In real life situation, the goals and inventory parameters are may not precise. Such type of uncertainty may be characterized by fuzzy numbers. Here, the constraint goal and the inventory cost parameters are assumed to be triangular shaped fuzzy numbers with different types of left and right membership functions. The fuzzy numbers are then approximated to a nearest interval number. Using arithmetic of interval numbers, the problem is described as a multi-objective inventory problem. The problem is then solved by fuzzy geometric programming approach. Finally a numerical example is given to illustrate the problem.

**Keywords** Inventory · Interval number · Membership function · Geometric programming

## 1. Introduction

Multi-item classical inventory models under various types of constraints such as capital investment, available storage area, number of orders and available set-up time are presented in well-known books (Churchman, Ackoff, & Arnoff 1957; Hadley & Whitin 1958; Lewis 1970; Silver & Peterson 1985; Naddor 1996, etc).

While modelling an inventory problem, generally three types of demands are considered. These are (1) Constant demand, (2) time-dependent demand and

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(3) stock-dependent demand. Among these stock dependent, specially displayed inventory level demand has an effect on sales for many retail products.

According to Whitin (1957), "For retail stores the inventory control problem for style goods is further complicated with the fact that inventory and sales are not independent of one another. An increase in inventory may bring about increased sales of some items".

In 1968, Wolfe noted that the sales of style merchandize, such as women's dresses or sports' clothes are proportional to the amount of inventory displayed. Levin, McLaughln, Lamone, and Kottas (1972) stated that the presence of inventory has a motivational effect on the customer. The most of the retailers displayed some products on shelf following the product variety, customer's choice of brand quality, and physical size of the product to influence the customer's attention. According to Silver and Peterson (1985) the sale at the retail level is proportional to the amount of displayed inventory. Larson and De Marais (1990) named this phenomenon as "psychic stock" and quoted that "psychic stock is retail display inventory carried to stimulate demand". In formulating the inventory model, the demand rate is considered as a function of the shelf-space allocated to the product. Urban (1969) developed a model to identify those products, which should be included in a firm's product line; the model formulated the demand rate as a polynomial function of price, advertising and distribution (represented by the number of shelf facing in the empirical application) considering both main and cross-elasticity of the marketing variables. Corstjens and Doyle (1981) developed a shelf-space allocation model in which demand rate is a function of shelf-space allocated to the product.

But all these inventory problems are solved with the assumptions that the coefficients or cost parameters are specified in a precise way, i.e. in crisp environment. In real life, there are many diverse situations due to uncertainty in judgments, lack of evidence, etc. Sometimes it is not possible to get relevant precise data. This type of imprecise data is not always well represented by random variables selected from a probability distribution. So decision-making methods under uncertainty are needed. In fuzzy programming problem (1965, 1970, 1976, 1978) the constraints and the goals are taken as fuzzy sets. It is also assumed that their membership functions are known. But it is not always easy for the decision maker to specify them. The fuzzy numbers describe the imprecise coefficients. These imprecise coefficients are then approximated to crisp set of interval numbers. Grzegorzewski (2002) suggests a method to substitute a fuzzy set by a crisp one. Chanas and Kutchta (1996) defined a transportation problem with fuzzy cost coefficients and developed an algorithm to solve the problem. Ishibuchi and Tanaka (1990) developed a concept for optimization of multi-objective programming problem with interval objective function.

The GP method is an effective method to solve a non-linear programming problem. It has certain advantages over the other optimization methods. Here the advantage is usually much simpler to work with the dual than the primal. To solve a non-linear programming problem by GP method, degree of difficulty (DD) plays a significant role (it is defined as DD = total number of terms in objective function and constraints − total number of decision variables −1). It will be difficult to solve the problem for higher values of DD. If DD = 0, the dual variables can be uniquely determined from the normality and orthogonality conditions. If DD > 0, there are infinite number of solutions of the system of constraint equations in the dual problem. This method is now widely used to solve the optimization problem in inventories. After the first introduction by Zener (1961), Duffin, Peterson, and Zener (1967) developed the GP method. Worral and Hall (1982) analysed the inventory models with some constraints and solved by GP technique. Abou-el-Ata, Fergany, and El-Wakeel (2003) and Chen (2000) developed some inventory problems and solved these by GP method. In 1989, Cheng solved an EOQ model for single product with demand dependent unit price. Following the same idea Roy and Maiti (1997) developed a fuzzy EOQ model. Recently, Jung and Klein (2005), Mandal, Roy, and Maiti (2005, 2006) formulated the inventory models and solved by GP technique.

In this paper, we consider a multi-item inventory model with full-shelf merchandising policy. In this policy, there are backroom storage and one showroom with shelves for display. Shelves are kept always fully stocked, i.e. demands are met from the stock of backroom warehouse and items are replenished as soon as the backroom inventory reaches zero. Demand rate is a function of the displayed inventory level and there is a limitation on total display space. Here inventory cost parameters and available display area are imprecise, i.e. fuzzy in nature. The said parameters are expressed in fuzzy numbers. The fuzzy numbers are approximated to interval numbers with the help of linear and non-linear membership functions. The problem is then reduced to multi-objective decision-making problem. Fuzzy geometric programming method is applied to solve the problem. Finally a numerical example is given to illustrate the problem.

## 2. Mathematical model

A multi-item displayed inventory model is formulated under the following notations and assumptions

**Notations**

| | | |
|---|---|---|
| $W$ | = | Total display-shelf space, |
| $n$ | = | number of items, |
| | | Parameters for $i(=1,2,\ldots,n)$th item are, |
| $S_i$ | = | number of display quantity (decision variable), $(S \equiv (S_1, S_2, ..., S_n)^T)$, |
| $Q_i$ | = | number of order quantity (decision variable), $(Q \equiv (Q_1, Q_2, ..., Q_n)^T)$, |
| $\theta_i$ | = | instantaneous inventory level of the entire system including both the backroom storage and the displayed inventory, |
| $P_i$ | = | selling price of each product, |
| $c_i$ | = | purchasing price of each product, |
| $c_{1i}$ | = | holding cost per unit product, |
| $c_{2i}$ | = | display shelf-space cost per unit product, |
| $c_{3i}$ | = | set-up cost, |
| $D_i$ | = | demand rate, |
| $P_{ri}$ | = | production rate. |

**Assumptions**

1. Lead time is zero,
2. shortages are not permitted,
3. demand rate $D_i$ is display inventory level dependent of $i$th item

$$D_i = d_i S_i^{d_i'} \quad (d_i > 0,\ 0 < d_i' < 1).$$

Here, $d_i$ and $d_i^{'}$ $(i = 1, 2, \ldots, n)$ are the scale and shape parameters of the demand function.

The inventory model is formulated to maximize the average net profit, which includes the gross revenues, unit purchasing cost, set-up cost, holding cost and the display cost under the limited display-space constraint.

Average net profit = Gross revenues per unit time – purchasing price per unit time – set-up cost per unit time – holding cost per unit time – display shelf-space cost per unit time.

Hence the profit function is

$$\mathrm{PF}(S, Q) = \sum_{i=1}^{n} \left[ D_i \left\{ (P_i - c_i) - \frac{c_{3i}}{Q_i} \right\} - \left(1 - \frac{D_i}{P_{ri}}\right) \frac{c_{1i}\theta_i}{2} - c_{2i} S_i \right]. \tag{1}$$

Now consider the situation in which the retailer follows a "full-shelf merchandising" policy, i.e. the display area is always kept fully stocked, so the inventory is replenished as soon as the backroom inventory reaches zero. The displayed inventory will always be at its maximum value. The inventory level decreases at a constant rate.

The average inventory is $\theta_i = S_i + \frac{Q_i}{2}$ and the cycle time is $T_i = \frac{Q_i}{d_i S_i^{d_i^{'}}}$. Therefore, the average net profit function is reduced to

$$\begin{aligned} \mathrm{PF}(S, Q) = \sum_{i=1}^{n} \Bigg[ & d_i S_i^{d_i^{'}} \left\{ (P_i - c_i) - \frac{c_{3i}}{Q_i} \right\} - \left(1 - \frac{d_i S_i^{d_i^{'}}}{P_{ri}}\right) \frac{c_{1i} Q_i}{2} \\ & - \left( \left(1 - \frac{d_i S_i^{d_i^{'}}}{P_{ri}}\right) c_{1i} + c_{2i} \right) S_i \Bigg]. \end{aligned} \tag{2}$$

The problem is then stated as

$$\begin{aligned} &\text{Max} && \mathrm{PF}(S, Q) \\ &\text{subject to} && \sum_{i=1}^{n} W_i S_i \le W, \\ & && S,\ Q > 0. \end{aligned} \tag{3}$$

## 3. Fuzzy model

When the cost parameters and total display shelf-space parameters are fuzzy numbers then the problem (3) is transformed to

$$\begin{aligned} \widetilde{\text{Max}}\ \mathrm{PF}(S, Q) = \sum_{i=1}^{n} \Bigg[ & d_i S_i^{d_i^{'}} \left\{ (\widetilde{P}_i - \widetilde{c}_i) - \frac{\widetilde{c}_{3i}}{Q_i} \right\} \\ & - \left(1 - \frac{d_i S_i^{d_i^{'}}}{\widetilde{P}_{ri}}\right) \frac{\widetilde{c}_{1i} Q_i}{2} - \left( \left(1 - \frac{d_i S_i^{d_i^{'}}}{\widetilde{P}_{ri}}\right) \widetilde{c}_{1i} + \widetilde{c}_{2i} \right) S_i \Bigg] \end{aligned} \tag{4}$$

subject to

$$\sum_{i=1}^{n} W_i S_i \leq \widetilde{W},$$
$$S,\ Q > 0.$$

where $\sim$ represents the fuzzification of the parameters.

## 4. Fuzzy number and its nearest interval approximation

*Fuzzy number:* A real fuzzy number $\widetilde{A}$ described as a fuzzy subset on the real line $\Re$ whose membership function $\mu_{\widetilde{A}}(x)$ has the following characteristics with $-\infty < a_1 \leq a_2 \leq a_3 < \infty$

$$\mu_{\widetilde{A}}(x) = \begin{cases} \mu^L_{\widetilde{A}}(x), & \text{if } a_1 \leq x \leq a_2, \\ \mu^R_{\widetilde{A}}(x), & \text{if } a_2 \leq x \leq a_3, \\ 0, & \text{otherwise,} \end{cases}$$

where $\mu^L_{\widetilde{A}}(x)\colon\ [a_1, a_2] \to [0,1]$ is continuous and strictly increasing; $\mu^R_{\widetilde{A}}(x)\colon\ [a_2, a_3] \to [0,1]$ is continuous and strictly decreasing.
*$\alpha$-level set:* The $\alpha$-level of a fuzzy number $\widetilde{A}$ is defined as a crisp set $A(\alpha) = [x : \mu_{\widetilde{A}}(x) \geq \alpha,\ x \in X]$ where $\alpha \in [0,1]$. $A(\alpha)$ is a non-empty bounded closed interval contained in $X$ and it can be denoted by $A_\alpha = [A_{\mathrm{L}}(\alpha),\ A_{\mathrm{R}}(\alpha)]$. $A_{\mathrm{L}}(\alpha)$ and $A_{\mathrm{R}}(\alpha)$ are the lower and upper bounds of the closed interval, respectively.
*Note:* $\alpha_1$ level set of $\widetilde{A}$ is $A(\alpha_1) = [A_{\mathrm{L}}(\alpha_1),\ A_{\mathrm{R}}(\alpha_1)]$ and that of $\alpha_2$ level set is $A(\alpha_2) = [A_{\mathrm{L}}(\alpha_2),\ A_{\mathrm{R}}(\alpha_2)]$. If $\alpha_2 \geq \alpha_1$ then $A_{\mathrm{L}}(\alpha_2) \geq A_{\mathrm{R}}(\alpha_1)$ and $A_{\mathrm{R}}(\alpha_1) \geq A_{\mathrm{R}}(\alpha_2)$.
*Interval number:* An interval number $A$ is defined by an ordered pair of real numbers as follows $A = [a_{\mathrm{L}}, a_{\mathrm{R}}] = \{x : a_{\mathrm{L}} \leq x \leq a_{\mathrm{R}},\ x \in \Re\}$ where $a_{\mathrm{L}}$ and $a_{\mathrm{R}}$ are the left and right bounds of interval $A$, respectively. The interval $A$, is also defined by centre $(a_c)$ and half-width $(a_w)$ as follows

$A = \langle a_c, a_w\rangle = \{x :\ a_c - a_w \leq x \leq a_c + a_w,\ x \in \Re\}$ where $a_c = \frac{a_{\mathrm{R}}+a_{\mathrm{L}}}{2}$ is the centre and $a_w = \frac{a_{\mathrm{R}}-a_{\mathrm{L}}}{2}$ is the half-width of $A$.
*Nearest interval approximation:* Here we want to approximate a fuzzy number by a crisp model. Suppose $\widetilde{A}$ and $\widetilde{B}$ are two fuzzy numbers with $\alpha$-cuts are $[A_{\mathrm{L}}(\alpha), A_{\mathrm{R}}(\alpha)]$ and $[B_{\mathrm{L}}(\alpha), B_{\mathrm{R}}(\alpha)]$, respectively. Then the distance between $\widetilde{A}$ and $\widetilde{B}$ is

$$d(\widetilde{A},\ \widetilde{B}) = \sqrt{\int_0^1 \Big(A_{\mathrm{L}}(\alpha) - B_{\mathrm{L}}(\alpha)\Big)^2 d\alpha + \int_0^1 \Big(A_{\mathrm{R}}(\alpha) - B_{\mathrm{R}}(\alpha)\Big)^2 \mathrm{d}\alpha}.$$

Given $\widetilde{A}$ is a fuzzy number. We have to find a closed interval $C_d(\widetilde{A})$, which is the nearest to $\widetilde{A}$ with respect to metric $d$. We can do it since each interval is also a fuzzy number with constant $\alpha$-cut for all $\alpha \in [0,1]$. Hence $(C_{\mathrm{d}}(\widetilde{A}))\alpha = [C_{\mathrm{L}}, C_{\mathrm{,R}}]$. Now we have to minimize

$$d(\widetilde{A},\ C_d(\widetilde{A})) = \sqrt{\int_0^1 \Big(A_{\mathrm{L}}(\alpha) - C_{\mathrm{L}}\Big)^2 d\alpha + \int_0^1 \Big(A_{\mathrm{R}}(\alpha) - C_{\mathrm{R}}\Big)^2 d\alpha}$$

with respect to $C_{\mathrm{L}}$ and $C_{\mathrm{R}}$.

In order to minimize $d(\widetilde{A}, C_d(\widetilde{A}))$ it is sufficient to minimize the function $D(C_{\mathrm{L}}, C_{\mathrm{R}})$ $(= d^2(\widetilde{A}, C_d(\widetilde{A})))$. The first partial derivatives are

$$\frac{\partial D(C_{\mathrm{L}}, C_{\mathrm{R}})}{\partial C_{\mathrm{L}}} = -2\int_0^1 A_{\mathrm{L}}(\alpha)\,\mathrm{d}\alpha + 2C_{\mathrm{L}}$$
$$\text{and } \frac{\partial D(C_{\mathrm{L}}, C_{\mathrm{R}})}{\partial C_{\mathrm{R}}} = -2\int_0^1 A_{\mathrm{R}}(\alpha)\,\mathrm{d}\alpha + 2C_{\mathrm{R}}.$$

Solving $\frac{\partial D(C_{\mathrm{L}}, C_{\mathrm{R}})}{\partial C_{\mathrm{L}}} = 0$ and $\frac{\partial D(C_{\mathrm{L}}, C_{\mathrm{R}})}{\partial C_{\mathrm{R}}} = 0$ we get

$C_{\mathrm{L}}^* = \int_0^1 A_{\mathrm{L}}(\alpha)\,\mathrm{d}\alpha$ and $C_{\mathrm{R}}^* = \int_0^1 A_{\mathrm{R}}(\alpha)\,\mathrm{d}\alpha$.

Again since $\frac{\partial D^2(C_{\mathrm{L}}^*, C_{\mathrm{R}}^*)}{\partial C_{\mathrm{L}}^2} = 2 > 0$, $\frac{\partial D^2(C_{\mathrm{L}}^*, C_{\mathrm{R}}^*)}{\partial C_{\mathrm{R}}^2} = 2 > 0$ and

$$H(C_{\mathrm{L}}^*, C_{\mathrm{R}}^*) = \frac{\partial D^2(C_{\mathrm{L}}^*, C_{\mathrm{R}}^*)}{\partial C_{\mathrm{L}}^2}.\frac{\partial D^2(C_{\mathrm{L}}^*, C_{\mathrm{R}}^*)}{\partial C_{\mathrm{R}}^2} - \left(\frac{\partial D^2(C_{\mathrm{L}}^*, C_{\mathrm{R}}^*)}{\partial C_{\mathrm{L}}, \partial C_{\mathrm{R}}}\right)^2 = 4 > 0.$$

So $D(C_{\mathrm{L}}, C_{\mathrm{R}})$, i.e. $d(\widetilde{A}, C_d(\widetilde{A}))$ is global minimum. Therefore, the interval $C_d(\widetilde{A}) = \left[\int_0^1 A_{\mathrm{L}}(\alpha)\,\mathrm{d}\alpha, \int_0^1 A_{\mathrm{R}}(\alpha)\,\mathrm{d}\alpha\right]$ is the nearest interval approximation of fuzzy number $\widetilde{A}$ with respect to the metric $d$.

Let $\widetilde{A} = (a_1, a_2, a_3)$ be a fuzzy number. The $\alpha$-level interval of $\widetilde{A}$ is defined as $A_\alpha = [A_{\mathrm{L}}(\alpha), A_{\mathrm{R}}(\alpha)]$.

When $\widetilde{A}$ is a linear fuzzy number (LFN) then $A_{\mathrm{L}}(\alpha) = a_1 + \alpha(a_2 - a_1)$ and $A_{\mathrm{R}}(\alpha) = a_3 - \alpha(a_3 - a_2)$. By the nearest interval approximation method the lower limit of the interval is

$$\begin{aligned} C_{\mathrm{L}} &= \int_0^1 A_{\mathrm{L}}(\alpha)\,\mathrm{d}\alpha \\ &= \int_0^1 [a_1 + \alpha(a_2 - a_1)]\,\mathrm{d}\alpha = \frac{1}{2}(a_1 + a_2) \end{aligned}$$

and the upper limit of the interval is

$$\begin{aligned} C_{\mathrm{R}} &= \int_0^1 A_{\mathrm{R}}(\alpha)\,\mathrm{d}\alpha \\ &= \int_0^1 [a_3 - \alpha(a_3 - a_2)]\,\mathrm{d}\alpha = \frac{1}{2}(a_2 + a_3). \end{aligned}$$

Therefore, the interval number considering $\widetilde{A}$ as a LFN, is $\left[\frac{1}{2}(a_1 + a_2), \frac{1}{2}(a_2 + a_3)\right]$. In the centre and half-width form the interval number of $\widetilde{A}$ is defined as

$$\left\langle \frac{1}{4}(a_1 + 2a_2 + a_3),\ \frac{1}{4}(a_1 - a_3) \right\rangle.$$

Similarly, when $\widetilde{A}$ is a parabolic fuzzy number (PFN) then

$$A_{\mathrm{L}}(\alpha) = a_2 - (a_2 - a_1)\sqrt{(1-\alpha)} \quad \text{and} \quad A_{\mathrm{R}}(\alpha) = a_2 + (a_3 - a_2)\sqrt{(1-\alpha)}.$$

Following the same way stated above the interval number is $\left[\frac{1}{3}(a_2 + 2a_1),\ \frac{1}{3}(a_2 + 2a_3)\right]$. In the centre and half-width form the interval number of $\widetilde{A}$ is defined as

$$\left\langle \tfrac{1}{3}(a_1 + a_2 + a_3),\ \tfrac{1}{3}(a_1 - a_3) \right\rangle.$$

When $\widetilde{A}$ is a exponential fuzzy number (EFN) then

$$A_{\mathrm{L}}(\alpha) = a_1 - \frac{(a_2 - a_1)}{\beta_1} \log\left(1 - \frac{\alpha}{\nu_1}\right) \quad \text{and} \quad A_{\mathrm{R}}(\alpha) = a_3 + \frac{(a_3 - a_2)}{\beta_2} \log\left(1 - \frac{\alpha}{\nu_2}\right).$$

Following the same way stated above the interval number is

$$\left[ a_1 + \frac{a_2 - a_1}{\beta_1} \frac{1}{\nu_1(\nu_1 - 1)},\ a_3 - \frac{a_3 - a_2}{\beta_2} \frac{1}{\nu_2(\nu_2 - 1)} \right].$$

In the centre and half-width form the interval number of $\widetilde{A}$ is defined as

$$\left\langle \frac{1}{2}\left( a_1 + a_3 + \frac{a_2 - a_1}{\nu_1\beta_1(\nu_1 - 1)} - \frac{a_3 - a_2}{\nu_2\beta_2(\nu_2 - 1)} \right), \frac{1}{2}\left( a_1 - a_3 + \frac{a_2 - a_1}{\nu_1\beta_1(\nu_1 - 1)} + \frac{a_3 - a_2}{\nu_2\beta_2(\nu_2 - 1)} \right) \right\rangle.$$

$\widetilde{A}$ are triangular shaped fuzzy numbers (TiFNs) with different types of left and right branch of the membership functions. They may be of linear, parabolic, exponential, etc., type membership functions (Table 1).

## 5. Interval approximation of the inventory model with fuzzy number

In our multi-item displayed inventory model, we have considered that the cost parameters $P_i$, $c_i$, $c_{1i}$, $c_{2i}$, $c_{3i}$, $P_{ri}$ and total display-shelf space ($W$) as a fuzzy number. The fuzzy numbers are

$$\begin{aligned}
&\widetilde{P}_i = (P_{1i}, P_{2i}, P_{3i}), && \widetilde{c}_i = (c_{1i}, c_{2i}, c_{3i}),\\
&\widetilde{c}_{1i} = (c_{11i}, c_{12i}, c_{13i}), && \widetilde{c}_{2i} = (c_{21i}, c_{22i}, c_{23i}),\\
&\widetilde{c}_{3i} = (c_{31i}, c_{32i}, c_{33i}), && \widetilde{P}_{ri} = (P_{r1i}, P_{r2i}, P_{r3i}),\\
&\widetilde{W} = (W_1, W_2, W_3).
\end{aligned}$$

We now form interval numbers for each fuzzy parameters with the help of the procedure of the nearest interval approximation of a fuzzy number stated in Section 4.

**Table 1** Nearest interval approximations of coefficient parameters

| Br | $\mu^{\mathrm{L}}_{\tilde{A}}(x)$ | Br | $\mu^{\mathrm{R}}_{\tilde{A}}(x)$ | $A_\alpha = [C_{\mathrm{L}},\, C_{\mathrm{R}}]$ |
|---|---|---|---|---|
| L: | $1-\frac{a_2-x}{a_2-a_1}$ | L: | $1-\frac{x-a_2}{a_3-a_2}$ | $\left[\frac{1}{2}(a_1+a_2),\ \frac{1}{2}(a_2+a_3)\right]$ |
| L: | $1-\frac{a_2-x}{a_2-a_1}$ | P: | $1-\left(\frac{x-a_2}{a_3-a_2}\right)^2$ | $\left[\frac{1}{2}(a_1+a_2),\ \frac{1}{3}(a_2+2a_3)\right]$ |
| L: | $1-\frac{a_2-x}{a_2-a_1}$ | E: | $\nu_2\left(1-\mathrm{e}^{-\beta_2\left(\frac{a_3-x}{a_3-a_2}\right)}\right)$ | $\left[\frac{1}{2}(a_1+a_2),\ a_3-\frac{a_3-a_2}{\nu_2\beta_2(\nu_2-1)}\right]$ |
| P: | $1-\left(\frac{a_2-x}{a_2-a_1}\right)^2$ | L: | $1-\frac{x-a_2}{a_3-a_2}$ | $\left[\frac{1}{3}(a_2+2a_1),\ \frac{1}{2}(a_2+a_3)\right]$ |
| P: | $1-\left(\frac{a_2-x}{a_2-a_1}\right)^2$ | P: | $1-\left(\frac{x-a_2}{a_3-a_2}\right)^2$ | $\left[\frac{1}{3}(a_2+2a_1),\ \frac{1}{3}(a_2+2a_3)\right]$ |
| P: | $1-\left(\frac{a_2-x}{a_2-a_1}\right)^2$ | E: | $\nu_2\left(1-\mathrm{e}^{-\beta_2\left(\frac{a_3-x}{a_3-a_2}\right)}\right)$ | $\left[\frac{1}{3}(a_2+2a_1),\ a_3-\frac{a_3-a_2}{\nu_2\beta_2(\nu_2-1)}\right]$ |
| E: | $\nu_1\left(1-\mathrm{e}^{-\beta_1\left(\frac{x-a_1}{a_2-a_1}\right)}\right)$ | L: | $1-\frac{x-a_2}{a_3-a_2}$ | $\left[a_1-\frac{a_2-a_1}{\nu_1\beta_1(\nu_1-1)},\ \frac{1}{2}(a_2+a_3)\right]$ |
| E: | $\nu_1\left(1-\mathrm{e}^{-\beta_1\left(\frac{x-a_1}{a_2-a_1}\right)}\right)$ | P: | $1-\left(\frac{x-a_2}{a_3-a_2}\right)^2$ | $\left[a_1-\frac{a_2-a_1}{\nu_1\beta_1(\nu_1-1)},\ \frac{1}{3}(a_2+2a_3)\right]$ |
| E: | $\nu_1\left(1-\mathrm{e}^{-\beta_1\left(\frac{x-a_1}{a_2-a_1}\right)}\right)$ | E: | $\nu_2\left(1-\mathrm{e}^{-\beta_2\left(\frac{a_3-x}{a_3-a_2}\right)}\right)$ | $\left[a_1-\frac{a_2-a_1}{\nu_1\beta_1(\nu_1-1)},\ a_3-\frac{a_3-a_2}{\nu_2\beta_2(\nu_2-1)}\right]$ |

where L, P and E stand for linear, parabolic and exponential membership functions, respectively, $\nu_1, \nu_2 > 1$; $\beta_1, \beta_2 > 0$.

$$
\begin{aligned}
\text{Max PF}(S, Q) = \sum_{i=1}^{n} \Bigg[ & d_i S_i^{d_i'}([P_{i\mathrm{L}},\, P_{i\mathrm{R}}]) - [c_{1i\mathrm{L}},\, c_{1i\mathrm{R}}] - \frac{1}{Q_i}[c_{3i\mathrm{L}},\, c_{3i\mathrm{R}}] \\
& - \left(1 - \frac{d_i S_i^{d_i'}}{[P_{ri\mathrm{L}}, P_{ri\mathrm{R}}]}\right)\frac{Q_i}{2}[c_{1i\mathrm{L}},\, c_{1i\mathrm{R}}] \\
& - \left(\left(1 - \frac{d_i S_i^{d_i'}}{[P_{ri\mathrm{L}}, P_{ri\mathrm{R}}]}\right)[c_{1i\mathrm{L}},\, c_{1i\mathrm{R}}] + [c_{2i\mathrm{L}},\, c_{2i\mathrm{R}}]\right) S_i \Bigg] \qquad (5) \\
= \Big[ & \mathrm{PF_L}(S, Q),\ \mathrm{PF_R}(S, Q) \Big]
\end{aligned}
$$

$$
\begin{aligned}
\text{subject to} \quad & \sum_{i=1}^{n} W_i S_i \le [W_{\mathrm{L}},\, W_{\mathrm{R}}] \\
& S,\ Q > 0,
\end{aligned}
$$

$$
\begin{aligned}
\text{where } \mathrm{PF_L}(S,\, Q) = \sum_{i=1}^{n} \Bigg[ & d_i S_i^{d_i'} \left\{ \left(P_{i\mathrm{L}} - c_{i\mathrm{R}}\right) - \frac{c_{3i\mathrm{R}}}{Q_i} \right\} - \frac{c_{1i\mathrm{R}}}{2} Q_i + \frac{d_i c_{1i\mathrm{L}}}{2P_{ri\mathrm{R}}} S_i^{d_i'} Q_i \\
& - \left(c_{1iR} + c_{2iR}\right) S_i + \frac{d_i c_{1i\mathrm{L}}}{P_{ri\mathrm{R}}} S_i^{d_i'+1} \Bigg]
\end{aligned}
$$

$$\text{and } \mathrm{PF_R}(S,\,Q)=\sum_{i=1}^{n}\left[d_iS_i^{d_i'}\left\{\left(P_{i\mathrm{R}}-c_{i\mathrm{L}}\right)-\frac{c_{3i\mathrm{L}}}{Q_i}\right\}-\frac{c_{1i\mathrm{L}}}{2}Q_i+\frac{d_ic_{1i\mathrm{R}}}{2P_{ri\mathrm{L}}}S_i^{d_i'}Q_i\right.$$
$$\left.-\left(c_{1i\mathrm{L}}+c_{2i\mathrm{L}}\right)S_i+\frac{d_ic_{1i\mathrm{R}}}{P_{ri\mathrm{L}}}S_i^{d_i'+1}\right].$$

## 6. Cases of proposed inventory problem with fuzzy number

Three major cases that may arise in proposed inventory model.

**Case 1** The cost parameters $(P_i,\,c_i,\,c_{1i},\,c_{2i},\,c_{3i},\,P_{ri})$ are in the form of fuzzy number whereas total displayed shelf-space parameter $(W)$ is deterministic.

**Case 2** The total displayed shelf-space parameter $(W)$ is a fuzzy number but the cost parameters are deterministic.

**Case 3** The cost parameters and total displayed shelf-space parameter are all fuzzy numbers.

**Case 1** When the cost parameters $P_i,\,c_i,\,c_{1i},\,c_{2i},\,c_{3i},\,P_{ri}$ are all fuzzy numbers then these can be transformed to the nearest approximation interval numbers. With the interval numbers the problem can be stated as

$$\begin{aligned}&\text{Max} && \mathrm{PF}(S,\,Q)=\left[\mathrm{PF_L}(S,Q),\,\mathrm{PF_R}(S,Q)\right]\\ &\text{subject to} && \sum_{i=1}^{n}W_iS_i\le W\\ &&& S,\,Q>0.\end{aligned}\tag{6}$$

The centre of the objective function $\mathrm{PF}(S,Q)$ is defined by $\mathrm{PF}_c(S,Q)=\frac{1}{2}\left(\mathrm{PF_L}(S,Q)+\mathrm{PF_R}(S,Q)\right)$. The problem is now reduced to a multi-objective non-linear programming problem

$$\begin{aligned}\text{Max } \mathrm{PF_L}(S,\,Q)&=\sum_{i=1}^{n}\left[d_iS_i^{d_i'}\left\{\left(P_{i\mathrm{L}}-c_{i\mathrm{R}}\right)-\frac{c_{3i\mathrm{R}}}{Q_i}\right\}-\frac{c_{1i\mathrm{R}}}{2}Q_i+\frac{d_ic_{1i\mathrm{L}}}{2P_{ri\mathrm{R}}}S_i^{d_i'}Q_i\right.\\&\qquad\left.-\left(c_{1i\mathrm{R}}+c_{2i\mathrm{R}}\right)S_i+\frac{d_ic_{1i\mathrm{L}}}{P_{ri\mathrm{R}}}S_i^{d_i'+1}\right],\\ \text{Max } \mathrm{PF}_c(S,\,Q)&=\sum_{i=1}^{n}\left[d_iS_i^{d_i'}\left\{\left(P_{ic}-c_{ic}\right)-\frac{c_{3ic}}{Q_i}\right\}-\frac{c_{1ic}}{2}Q_i+\frac{d_ic_{1ic}}{2P_{ric}}S_i^{d_i'}Q_i\right.\\&\qquad\left.-\left(c_{1ic}+c_{2ic}\right)S_i+\frac{d_ic_{1ic}}{P_{ric}}S_i^{d_i'+1}\right]\\ \text{subject to}\quad&\sum_{i=1}^{n}W_iS_i\le W\\&S,\,Q>0.\end{aligned}\tag{7}$$

We now solve the multi-objective inventory problem (7) by geometric programming technique and form a pay-off matrix of order $2\times 2$. To find the pay-off matrix, solve the multi-objective inventory problem as a single objective inventory problem (7) using each time only one objective function with the constraint and ignoring other objective function. From the optimal results, determine the corresponding value of other objective function at each solution derived.

To find the optimal value of $\mathrm{PF_L}(S, Q)$ subject to the given constraint by geometric programming technique the corresponding primal problem is

$$\begin{aligned}
\text{Min } \mathrm{PF}'_{\mathrm{L}}(S,\,Q) = \sum_{i=1}^{n}\Bigg[& d_i S_i^{d'_i}\left\{\frac{c_{3i\mathrm{R}}}{Q_i} - \Big(P_{i\mathrm{L}} - c_{i\mathrm{R}}\Big)\right\} + \frac{c_{1i\mathrm{R}}}{2}Q_i - \frac{d_i c_{1i\mathrm{L}}}{2P_{ri\mathrm{R}}}S_i^{d'_i}Q_i \\
&+\Big(c_{1i\mathrm{R}} + c_{2i\mathrm{R}}\Big)S_i - \frac{d_i c_{1i\mathrm{L}}}{P_{ri\mathrm{R}}}S_i^{d'_i+1}\Bigg] \qquad (8)\\
\text{subject to} \quad & \sum_{i=1}^{n} W_i S_i \le W \\
& S,\, Q > 0,
\end{aligned}$$

where $\mathrm{PF}'_{\mathrm{L}}(S,Q) = -\mathrm{PF_L}(S,Q)$.

The primal problem (8) is a constrained signomial problem with $3n-1$ degree of difficulty. The corresponding dual problem is

$$\begin{aligned}
\text{Max } d_{\mathrm{L}} = -\Bigg[\prod_{i=1}^{n}&\left(\frac{d_i c_{3i\mathrm{R}}}{w_{1i}}\right)^{w_{1i}}\left(\frac{d_i(P_{i\mathrm{L}} - c_{i\mathrm{R}})}{w_{2i}}\right)^{-w_{2i}}\left(\frac{c_{1i\mathrm{R}}}{2w_{3i}}\right)^{w_{3i}}\left(\frac{d_i c_{1i\mathrm{L}}}{2P_{ri\mathrm{R}}w_{4i}}\right)^{-w_{4i}} \\
&\times\left(\frac{c_{1i\mathrm{R}} + c_{2i\mathrm{R}}}{w_{5i}}\right)^{w_{5i}}\left(\frac{d_i c_{1i\mathrm{L}}}{P_{ri\mathrm{R}}w_{6i}}\right)^{w_{6i}}\left(\frac{w_i}{Ww_{7i}}\right)^{w_{7i}}\left(\sum_{i=1}^{n} w_{7i}\right)^{\sum_{i=1}^{n} w_{7i}}\Bigg]^{-1} \qquad (9)
\end{aligned}$$

subject to the normality and orthogonality conditions are

$$\begin{aligned}
& w_{1i} - w_{2i} + w_{3i} - w_{4i} + w_{5i} - w_{6i} = -1, \\
& d'_i w_{1i} - d'_i w_{2i} - d'_i w_{4i} + w_{5i} - (d'_i + 1)w_{6i} + w_{7i} = 0, \\
& -w_{1i} + w_{3i} - w_{4i} = 0, \\
\text{where} \quad & w_{1i},\, w_{2i},\, w_{3i},\, w_{4i},\, w_{5i},\, w_{6i} \text{ and } w_{7i} > 0, \quad i = 1,2,\ldots,n.
\end{aligned}$$

Solving the dual problem (9) we find the optimal values $S^*$, $Q^*$, $\mathrm{PF}^*_{\mathrm{L}}(S^*,\,Q^*)$ and hence $\mathrm{PF}^*_c(S^*,\,Q^*)$.

In a similar way, we find optimal value of $\mathrm{PF}_c(S,\,Q)$ subject to the given constraint. The problem is then written to standard geometric programming problem as

$$\begin{aligned}
\text{Min } \mathrm{PF}_c(S,\,Q) = \sum_{i=1}^{n}\Bigg[& d_i S_i^{d'_i}\left\{\Big(P_{ic} - c_{ic}\Big) - \frac{c_{3ic}}{Q_i}\right\} - \frac{c_{1ic}}{2}Q_i + \frac{d_i c_{1ic}}{2P_{ric}}S_i^{d'_i}Q_i \\
&-\Big(c_{1ic} + c_{2ic}\Big)S_i + \frac{d_i c_{1ic}}{P_{ric}}S_i^{d'_i+1}\Bigg] \qquad (10)\\
\text{subject to} \quad & \sum_{i=1}^{n} W_i S_i \le W, \\
& S,\, Q > 0,
\end{aligned}$$

where $\mathrm{PF}'_c(S,Q) = -\mathrm{PF}_c(S,Q)$.

The primal problem (10) is also a constrained signomial problem with $3n-1$ degree of difficulty. The corresponding dual problem is

$$\text{Min } d_c = -\Bigg[\prod_{i=1}^{n}\left(\frac{d_i c_{3ic}}{w_{1i}}\right)^{w_{1i}}\left(\frac{d_i(P_{ic}-c_{ic})}{w_{2i}}\right)^{-w_{2i}}\left(\frac{c_{1ic}}{2w_{3i}}\right)^{w_{3i}}\left(\frac{d_i\,c_{1ic}}{2P_{ric}\,w_{4i}}\right)^{-w_{4i}}$$
$$\times\left(\frac{c_{1ic}+c_{2ic}}{w_{5i}}\right)^{w_{5i}}\left(\frac{d_i\,c_{1ic}}{P_{ric}\,w_{6i}}\right)^{w_{6i}}\left(\frac{w_i}{W\,w_{7i}}\right)^{w_{7i}}\left(\sum_{i=1}^{n}w_{7i}\right)^{\sum_{i=1}^{n}w_{7i}}\Bigg]^{-1} \tag{11}$$

subject to the normality and orthogonality conditions are

$$\begin{aligned}&w_{1i}-w_{2i}+w_{3i}-w_{4i}+w_{5i}-w_{6i}=-1,\\&d_i^{'}w_{1i}-d_i^{'}w_{2i}-d_i^{'}w_{4i}+w_{5i}-(d_i^{'}+1)w_{6i}+w_{7i}=0,\\&-w_{1i}+w_{3i}-w_{4i}=0,\end{aligned}$$

where $w_{1i}$, $w_{2i}$, $w_{3i}$, $w_{4i}$, $w_{5i}$, $w_{6i}$ and $w_{7i}>0$, $\quad i=1,2,\ldots,n.$

Solving the dual problem (11) we find the optimal values $S^*$, $Q^*$, $\mathrm{PF}_c^*(S^*,\,Q^*)$ and hence $\mathrm{PF}_\mathrm{L}^*(S^*,\,Q^*)$.

Using the optimal solutions construct a pay-off matrix of size $2\times 2$ as follows:

$$\begin{array}{c c} & \begin{array}{cc}\mathrm{PF}_\mathrm{L} & \mathrm{PF}_c\end{array}\\ \begin{array}{c}1\\2\end{array} & \begin{pmatrix}\mathrm{PF}_\mathrm{L}^{1*} & \mathrm{PF}_c^{1}\\ \mathrm{PF}_\mathrm{L}^{2} & \mathrm{PF}_c^{2*}\end{pmatrix}.\end{array}$$

From the pay-off matrix, lower bounds are

$$\begin{aligned}L_\mathrm{L}&=\mathrm{Min}(\mathrm{PF}_\mathrm{L}^{1*},\ \mathrm{PF}_\mathrm{L}^{2}),\\L_c&=\mathrm{Min}(\mathrm{PF}_c^{1},\ \mathrm{PF}_c^{2*})\end{aligned}$$

and upper bounds are

$$\begin{aligned}U_\mathrm{L}&=\mathrm{Max}(\mathrm{PF}_\mathrm{L}^{1*},\ \mathrm{PF}_\mathrm{L}^{2}),\\U_c&=\mathrm{Max}(\mathrm{PF}_c^{1},\ \mathrm{PF}_c^{2*}).\end{aligned}$$

Hence $L_\mathrm{L}<\mathrm{PF}_\mathrm{L}<U_\mathrm{L}$ and $L_c<\mathrm{PF}_c<U_c$.

Now solve the problem by fuzzy programming technique. According to Zimmermann (1976) the linear membership functions are taken as follows:

$$\mu_{\mathrm{PF}_\mathrm{L}}(S,Q)=\begin{cases}1, & \text{if } \mathrm{PF}_\mathrm{L}(S,Q)\ge U_\mathrm{L},\\ 1-\dfrac{U_\mathrm{L}-\mathrm{PF}_\mathrm{L}(S,Q)}{U_\mathrm{L}-L_\mathrm{L}}, & \text{if } L_\mathrm{L}\le \mathrm{PF}_\mathrm{L}(S,Q)\le U_\mathrm{L},\\ 0, & \text{otherwise,}\end{cases}$$

$$\mu_{\mathrm{PF}_c}(S,Q)=\begin{cases}1, & \text{if } \mathrm{PF}_c(S,Q)\ge U_c,\\ 1-\dfrac{U_c-\mathrm{PF}_c(S,Q)}{U_c-L_c}, & \text{if } L_c\le \mathrm{PF}_c(S,Q)\le U_c,\\ 0, & \text{otherwise.}\end{cases}$$

Following Bellman and Zadeh's (1970) max–min operator or convex combination operator the fuzzy goal programming problem may be reduced to a crisp primal geometric programming (PGP) problem. To reduce the DD, here convex combination operator is used. So, the problem can be formulated as

$$\begin{aligned} &\text{Max } V(S,\, Q) = \left[\mu_{\mathrm{PFL}}(S,Q) + \mu_{\mathrm{PF}_c}(S,Q)\right] \\ &\text{subject to} \quad \sum_{i=1}^{n} W_i S_i \le W, \\ &\qquad\qquad\quad S,\, Q \,>\, 0. \end{aligned} \tag{12}$$

Omitting the constant terms

$$\left[1 - \frac{U_{\mathrm{L}}}{U_{\mathrm{L}} - L_{\mathrm{L}}} + 1 - \frac{U_c}{U_c - L_c}\right]$$

from the objective function of the problem (12) for the time being may be restated as

$$\text{Max } V^{'}(S,Q) = \left[\frac{\mathrm{PF}_{\mathrm{L}}(S,Q)}{U_{\mathrm{L}} - L_{\mathrm{L}}} + \frac{\mathrm{PF}_c(S,Q)}{U_c - L_c}\right] \tag{13}$$

subject to the same constraint of the problem (12) where

$$V(S,Q) = V^{'}(S,Q) + \left[1 - \frac{U_{\mathrm{L}}}{U_{\mathrm{L}} - L_{\mathrm{L}}} + 1 - \frac{U_c}{U_c - L_c}\right].$$

In the standard signomial geometric programming form it can be stated as

$$\begin{aligned} &\text{Min } V^{''}(S,Q) = \sum_{i=1}^{n}\left[-k_{1i}S_i^{d_i^{'}} + k_{2i}\frac{S_i^{d_i^{'}}}{Q_i} + k_{3i}Q_i - k_{4i}S_i^{d_i^{'}}Q_i + k_{5i}S_i - k_{6i}S_i^{d_i^{'}+1}\right] \\ &\text{subject to} \quad \frac{1}{W}\sum_{i=1}^{n} W_i S_i \le 1, \\ &\qquad\qquad\quad S,\, Q \,>\, 0,\;\; i = 1,2,...,n, \end{aligned} \tag{14}$$

where $k_{1i} = d_i\left(\frac{P_{i\mathrm{L}} - c_{i\mathrm{R}}}{U_{\mathrm{L}} - L_{\mathrm{L}}} + \frac{P_{ic} - c_{ic}}{U_c - L_c}\right)$, $k_{2i} = d_i\left(\frac{c_{3i\mathrm{R}}}{U_{\mathrm{L}} - L_{\mathrm{L}}} + \frac{c_{3ic}}{U_c - L_c},\right)$,

$$k_{3i} = \frac{c_{1i\mathrm{R}}}{2(U_{\mathrm{L}} - L_{\mathrm{L}})} + \frac{c_{1ic}}{2(U_c - L_c)},\quad k_{4i} = \frac{d_i}{2}\left(\frac{c_{1i\mathrm{L}}}{P_{ri\mathrm{R}}(U_{\mathrm{L}} - L_{\mathrm{L}})} + \frac{c_{1ic}}{P_{ric}(U_c - L_c)}\right),$$

$$k_{5i} = \frac{c_{1i\mathrm{R}} + c_{2i\mathrm{R}}}{U_{\mathrm{L}} - L_{\mathrm{L}}} + \frac{c_{1ic} + c_{2ic}}{U_c - L_c},\quad K_{6i} = d_i\left(\frac{c_{1i\mathrm{L}}}{P_{ri\mathrm{R}}(U_{\mathrm{L}} - L_{\mathrm{L}})} + \frac{c_{1ic}}{P_{ric}(U_c - L_c)}\right)$$

and Min $V^{''}(S,Q) = -\mathrm{Max}V^{'}(S,Q)$.

The primal problem (14) is a constrained signomial problem with $5n - 1$ degree of difficulty and is solved by geometric programming method.

**Case 2** When the cost parameters of the objective function are deterministic but the total display-shelf space parameter $W$ is a fuzzy number. Then problem is

$$\begin{aligned} \mathrm{MaxPF}(S,\, Q) = \sum_{i=1}^{n}\Bigg[& d_i S_i^{d_i^{'}}\left\{\left(P_i - c_i\right) - \frac{c_{3i}}{Q_i}\right\} - \frac{c_{1i}}{2}Q_i + \frac{d_i c_{1i}}{2P_{ri}}S_i^{d_i^{'}}Q_i - \left(c_{1i} + c_{2i}\right)S_i \\ &+ \frac{d_i c_{1i}}{P_{ri}}S_i^{d_i^{'}+1}\Bigg] \end{aligned}$$

$$\text{subject to } \sum_{i=1}^{n} W_i S_i \leq W_{\mathrm{R}},$$
$$\sum_{i=1}^{n} W_i S_i \geq W_{\mathrm{L}},$$
$$S, Q > 0.$$

In standard geometric programming form

$$\text{Min PF}'(S, Q) = \sum_{i=1}^{n} \left[ d_i S_i^{d_i'} \left\{ \frac{c_{3i}}{Q_i} - \left(P_i - c_i\right) \right\} + \frac{c_{1i}}{2} Q_i - \frac{d_i c_{1i}}{2P_{ri}} S_i^{d_i'} Q_i \right.$$
$$\left. + \left(c_{1i} + c_{2i}\right) S_i - \frac{d_i c_{1i}}{P_{ri}} S_i^{d_i'+1} \right] \tag{15}$$

$$\text{subject to} \sum_{i=1}^{n} W_i S_i \leq W_{\mathrm{R}},$$
$$\frac{W\mathrm{L}}{W_1} S_1^{-1} - S_1^{-1} \textstyle\sum_{i=2}^{n} W_i S_i \geq W_{\mathrm{L}},$$
$$S, Q > 0.$$

The primal problem (15) is a constrained signomial problem with $4n-1$ degree of difficulty and is solved by geometric programming.

**Case 3** Here we assume that the cost parameters $P_i$, $c_i$, $c_{1i}$, $c_{2i}$, $c_{3i}$, $P_{ri}$ and total display-shelf space parameter $W$ are fuzzy numbers. First we derive the nearest approximation interval numbers from said fuzzy numbers. Then the multi-objective inventory problem is stated as

$$\text{Max PF}_{\mathrm{L}}(S, Q) = \sum_{i=1}^{n} \left[ d_i S_i^{d_i'} \left\{ \left(P_{i\mathrm{L}} - c_{i\mathrm{R}}\right) - \frac{c_{3i\mathrm{R}}}{Q_i} \right\} - \frac{c_{1i\mathrm{R}}}{2} Q_i + \frac{d_i c_{1i\mathrm{L}}}{2P_{ri\mathrm{R}}} S_i^{d_i'} Q_i \right.$$
$$\left. - \left(c_{1i\mathrm{R}} + c_{2i\mathrm{R}}\right) S_i + \frac{d_i c_{1i\mathrm{L}}}{P_{ri\mathrm{R}}} S_i^{d_i'+1} \right]$$
$$\text{Min PF}_c(S, Q) = \sum_{i=1}^{n} \left[ d_i S_i^{d_i'} \left\{ \left(P_{ic} - c_{ic}\right) - \frac{c_{3ic}}{Q_i} \right\} - \frac{c_{1ic}}{2} Q_i + \frac{d_i c_{1ic}}{2P_{ric}} S_i^{d_i'} Q_i \right.$$
$$\left. - \left(c_{1ic} + c_{2ic}\right) S_i + \frac{d_i c_{1ic}}{P_{ric}} S_i^{d_i'+1} \right]$$
$$\text{subject to } \sum_{i=1}^{n} W_i S_i \leq W_{\mathrm{R}},$$
$$\sum_{i=1}^{n} W_i S_i \geq W_{\mathrm{L}},$$
$$S, Q > 0.$$

Following cases 1 and 2, we solve multi-objective inventory problem and forming a pay-off matrix construct the membership function for the objective functions.

According to Zimmermann (1976), the inventory problem in fuzzy environment is stated as

$$\text{Min} \sum_{i=1}^{n}\Big[-k_{1i}S_i^{d_i'}+k_{2i}\frac{S_i^{d_i'}}{Q_i}+k_{3i}Q_i-k_{4i}S_i^{d_i'}Q_i+k_{5i}S_i-k_{6i}S_i^{d_i'+1}\Big] \tag{16}$$
$$\text{subject to} \sum_{i=1}^{n} W_iS_i \le W_{\mathrm{R}},$$
$$\sum_{i=1}^{n} W_iS_i \ge W_{\mathrm{L}},$$
$$S, Q > 0.$$

The problem (16) is a signomial primal geometric programming problem with DD $= 6n - 1$. After reducing it to the standard GP form we can solve and find the optimal solutions.

## 7. Numerical example

A company produces and sells two types of items. The necessary informations for the concerned items are given as follows

**For item** $A$
from the past records it is seen that demand $(D_1) = 20\,S_1^{0.6}$ per unit of item,
production cost $(c_1) = \$\,8$ per unit of item,
holding cost $(c_{11}) = \$\,1$ per unit of item,
shortage cost $(c_{21}) = \$\,2.5$ per unit of item,
set up cost $(c_{31}) = \$\,63$ per batch,
selling price $(P_1) = \$\,11$ per unit of item,
production rate $(P_{r1}) = 4{,}010$ units per unit time,
storage space $(w_1) = 5\,\mathrm{m}^2$.

**For item** $B$
from the past records it is seen that demand $(D_2) = 19\,S_2^{0.5}$ per unit of item,
production cost $(c_2) = \$\,7$ per unit of item,
holding cost $(c_{12}) = \$\,1.3$ per unit of item,
shortage cost $(c_{22}) = \$\,2.2$ per unit of item,
set up cost $(c_{32}) = \$\,72$ per batch,
selling price $(P_2) = \$\,12$ per unit of item,
production rate $(P_{r2}) = 3{,}233$ units per unit time,
storage space $(w_2) = 4\,\mathrm{m}^2$
and
total available storage space $(W) = 2{,}500\,\mathrm{m}^2$.
But practically, the cost parameters and total available storage space are imprecise in nature. The production cost of item $A$ is nearly $\$\,7$ but never less than $\$\,5$ and above $\$\,10$ (i.e. $\widetilde{c}_1 = \$\,(5, 7, 10)$). Similarly, production cost of item $B$ is $\widetilde{c}_2 = \$\,(4, 6, 8)$. The holding, shortage, set up, selling costs of items $A$ and $B$ are $\widetilde{c}_{11} = \$\,(1, 1.2, 1.6)$, $\widetilde{c}_{12} = \$\,(1.1, 1.4, 1.6)$, $\widetilde{c}_{21} = \$\,(1.5, 2.1, 2.7)$, $\widetilde{c}_{22} = \$\,(1.4, 1.9, 2.5)$, $\widetilde{c}_{31} = \$\,(45, 55, 70)$,

$\widetilde{c}_{32} = \$\,(60, 65, 80)$, $\widetilde{P}_1 = \$\,(10, 13, 15)$ and $\widetilde{P}_2 = \$\,(12, 14, 17)$. The production rate of concerned items are $(3,500, 3,800, 4,200)$ and $(2,600, 2,900, 3,400)$ units per unit time. The total available storage space is $\widetilde{W} = [2,000, 2,500, 3,000]\,\mathrm{m}^2$ (Tables 2, 3,4, 5).

**Table 2** Left and right branches of fuzzy parameters

| Br | $\widetilde{c}_1$ | $\widetilde{c}_2$ | $\widetilde{c}_{11}$ | $\widetilde{c}_{12}$ | $\widetilde{c}_{21}$ | $\widetilde{c}_{22}$ | $\widetilde{c}_{31}$ | $\widetilde{c}_{32}$ | $\widetilde{P}_1$ | $\widetilde{P}_2$ | $\widetilde{P}_{r1}$ | $\widetilde{P}_{r2}$ | $\widetilde{W}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Left | L | E | P | E | P | P | L | L | L | P | P | E | L |
| Right | E | P | L | P | P | L | E | L | E | P | E | P | P |

**Table 3** Values of $(\nu_1, \beta_1)$ and $(\nu_2, \beta_2)$ for the membership functions of $\widetilde{c}_1, \widetilde{c}_2, \widetilde{c}_{12}, \widetilde{c}_{31}, \widetilde{P}_1, \widetilde{P}_{r1}, \widetilde{P}_{r2}$

| Br | $c_1$ | $c_2$ | $c_{12}$ | $c_{31}$ | $P_1$ | $P_{r1}$ | $P_{r2}$ |
|---|---|---|---|---|---|---|---|
| $(\nu_1, \beta_1)$ | – | (1.6,1.8) | (1.7,1.5) | – | – | – | (1.8,1.9) |
| $(\nu_2, \beta_2)$ | (1.5,2.1) | – | – | (1.8,1.6) | (1.4,2.3) | (1.6,2.2) | – |

**Table 4** Nearest interval approximation to fuzzy numbers

| | $\widetilde{c}_1$ | $\widetilde{c}_2$ | $\widetilde{c}_{11}$ | $\widetilde{c}_{12}$ |
|---|---|---|---|---|
| $[c_{\mathrm{L}}, c_{\mathrm{R}}]$ | [6,8.1] | [5.2,7.3] | [1.1,1.4] | [1.3,1.5] |
| $\langle a_c, a_w \rangle$ | $\langle 7, 1 \rangle$ | $\langle 6.2, 1.1 \rangle$ | $\langle 1.2, .2 \rangle$ | $\langle 1.4, .13 \rangle$ |
| | $\widetilde{c}_{21}$ | $\widetilde{c}_{22}$ | $\widetilde{c}_{31}$ | $\widetilde{c}_{32}$ |
| $[c_{\mathrm{L}}, c_{\mathrm{R}}]$ | [2.4,2.5] | [1.6,2.2] | [50,63.5] | [62.5,72.5] |
| $\langle a_c, a_w \rangle$ | $\langle 2.45, .05 \rangle$ | $\langle 1.88, .32 \rangle$ | $\langle 56.75, 6.75 \rangle$ | $\langle 67.5, 5 \rangle$ |
| | $\widetilde{P}_1$ | $\widetilde{P}_2$ | $\widetilde{P}_{r1}$ | $\widetilde{P}_{r2}$ |
| $[c_{\mathrm{L}}, c_{\mathrm{R}}]$ | [11.5,13.5] | [12.7,16] | [3,600,4,010.7] | [2,709.6,3,233.3] |
| $\langle a_c, a_w \rangle$ | $\langle 12.47, .98 \rangle$ | $\langle 14.33, 1.67 \rangle$ | $\langle 3,805.35, 205.35 \rangle$ | $\langle 2,971.45, 261.85 \rangle$ |
| | | | $\widetilde{W}$ | |
| $[W_{\mathrm{L}}, W_{\mathrm{R}}]$ | | | [2250,2833.33] | |
| $\langle a_c, a_w \rangle$ | | | $\langle 2541.7, 291.7 \rangle$ | |

**Table 5** Optimal solution

| Cases | $i$ | $S_i^*$ | $Q_i^*$ | $D_i^*$ | PF*($S^*$, $Q^*$) ($) |
|---|---|---|---|---|---|
| Case 1 | 1 | 361.4280 | 269.2826 | 685.2406 | [1,142.380, 5,759.586] |
| | 2 | 173.2150 | 159.5484 | 250.0612 | |
| Case 2 | 1 | 364.8681 | 323.8085 | 689.1465 | 1,038.606 |
| | 2 | 171.5156 | 172.8038 | 248.8315 | |
| Case 3 | 1 | 408.6430 | 283.9342 | 737.6464 | [1,137.579, 8,390.173] |
| | 2 | 197.5288 | 166.4451 | 267.0354 | |

The optimal results of the inventory problem (5) is

## 8. Conclusion

In this paper, we have designed and solved the multi-item displayed inventory model. The cost parameters and the total display area are taken as fuzzy numbers. The fuzzy numbers are described by linear/non-linear type membership functions. Fuzzy numbers are then approximated to an interval number. Hence the problem has been converted into multi-objective inventory problem where the objective functions are represented by left and right interval functions which are maximized. These objective functions can be considered as the maximization of the worst case and the average case. The problem is solved by fuzzy geometric programming technique. We have discussed here three different situations that arise in our problem. Each case is illustrated by numerical examples. The displayed inventory model can be extended to a shortage level inventory model. One may use the presented method to a multi-product multi-constraint inventory systems. The nearest interval approximation method may widely be used in case of transportation problem.

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