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The Principle of Inertia in the History of Classical Mechanics

Danilo Capecchi[1]

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Abstract
Making a history of the principle of inertia, as of any other principle or concept, is a complex but still possible operation. In this work it has been chosen to make a back story which seemed the most natural way for a reconstruction. On the way back, it has been decided to stop at the 6th century CE with the contribution of Ioannes Philoponus. The principle he stated, although very different from the modern one, is certainly associated with it. Going back in time it is still possible and of course one could proceed to the origins of the homo sapiens who perhaps posed the problem of why a club thrown with his hand could go so far from him. But the similarities that one can find with the modern principle are very vague, too perhaps. Without going so far one could see an embryonic idea of the principle of inertia in the atomistic theory of Democritus in the 5th century BCE. The motion of atoms whirling with no apparent reason can suggest the idea that a body can also move with no reason. But the transition from the atom, a metaphysical being, to the body, an empirical being, is far from immediate.

Keywords Principle of inertia · Impetus · Absolute space · Dynamics

1 Introduction

The *principle of inertia* (often referred to as the law of inertia) is a fundamental truth of classical mechanics. Countless papers have been written on its history, and it seems inevitable that many more will be.[1] The following paper is one more work in this stream; it does not (strictly speaking) follow a purely historical approach, in the sense that there is no attempt to investigate or discover new documents. Rather, it is an attempt to report on the most significant among the already published original sources in some orderly fashion, with the hope that this ordering and discussion may present some new considerations. It is, after all, always possible to stand a little taller on the shoulders of giants.

[1] The argument has been considered both from a historical and epistemological points of view. No comprehensive study exists, for what I know, on the history of the principle of inertia, but very interesting suggestions can be found in Koyré (1978), Maier (1982) and Westfall (1971). Interesting epistemological considerations can be found in Nagel (1961) and Mach (1883).

Danilo Capecchi
danilo.capecchi@uniroma1.it

[1] Dipartimento di Ingegneria Strutturale e Geotecnica, Sapienza, University of Rome, Rome, Italy

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Making the history of a principle is a very complex affair. A primary necessity is elucidating the current shared formulation (if it exists) of the principle, a process that forces choice and therefore a margin of arbitrariness on the part of the researcher. It is then necessary to trace the various formulations of the principle over the years, based on some criteria of similarities that are also chosen with a certain margin of arbitrariness; in the following, these criteria are defined by the aspects listed below:

1. Wording.
2. Given explanation, or contextualization.
3. Usage in a theory.

It should be clear that if all of the above aspects remain consistently identical in time, there would be no history. Rather, history begins when some difference emerges in at least one of the criteria.

Some special attention must be paid to the criterion of similarity in wording, as the satisfaction of this criterion may not be sufficient if read on a superficial level, as two texts expressed with similar words may mean completely different concepts in different eras, places and situations, and it is not easy to disentangle the difference.

Once the formulations of the principle of inertia that satisfy the proposed criteria of similarity have been identified and listed, it remains to be established whether and in what ways the older formulations have influenced the more modern. This, as is well known, is a very difficult problem to solve for several reasons. Firstly, before the diffusion of the printing press and/or the publication of the first scientific journals in the 17th century, there was no general assurance that the scholarly contributions of one generation would survive or be passed on fully to the next. Moreover, in the treatises of the time, it was not common practice to cite the sources used in authoring the treatise; these sources often being kept hidden deliberately so as to present the newer treatise as being the premier work on a topic. However in this present work, as already mentioned, a purely historical approach has not been followed, and in the choice of the various formulations of the principle of inertia, only those scholars who are widely considered to have been the main pioneers and developers by the current historiography have been taken into consideration. There is also a level of unanimity among modern scholars in considering it likely that these listed older sources were transmitted in some form to the authors of newer sources, even those that are more remote, though there is sometimes no agreement on how these transmissions took place and of what nature they may have been.

The approach used in this paper is that of a backward-looking history, as justified by a common view of history as being an account of a current situation. In the present case, where the subject of research is limited in range, this inverse process will help achieve a better understanding, for the following reasons:

1. It is in this way (from present to past) that historical research proceeds, and it is therefore the natural way to proceed.
2. By approaching it in this way, it is simpler to avoid the misleading idea that the history of a concept is a process of continuous improvement, and if due care is taken then there is no danger of ascribing concepts to more ancient scholars which were invented only by later scholars, or to value science in the light of successive science.

In this backward history, we must begin with a properly chosen period, and will then trace back to the first discontinuity on the principle formulation (without posing too many epistemological questions).

In proceeding with this backward historical analysis, it is found that any attempted formulation of the principle of inertia that is a candidate for representation in this class of propositions is consistently characterized by a description of the following phenomenon: the persistence of motion (either rectilinear or circular) of a body without an evident cause. This phenomenon is classified as being *regular motion*,[2] and the history of the principle of inertia is also the history of the various formulations of regular motion. This approach may also be justified by the fact that the term *principle of inertia* dates back to the 18th century, after the introduction and specification of the concept of inertia by Isaac Newton. The term *regular motion* seems to be adequate, though even then a margin of ambiguity is maintained. By allowing this assumption, there is then greater freedom in the historical analysis and less danger of introducing anachronisms into that analysis. Indeed, in constructing the history of regular motion, it seems inevitable that the history of the entire system of dynamics will thus also be constructed.

As to the period that has been chosen to start from with a shared formulation of the principle of inertia, this paper has elected for the beginning of the 20th century, when relativistic mechanics started to replace classical mechanics as a subject of research by physicists. From this point, though some further researches have been carried out to found the principle in a rigorous way and to solve the many critical issues still present, no substantial change has occurred in the statement of the principle of inertia.[3]

Accepting a naive realism our principle can be stated as follows:

1. Wording.
   There is a class of space-time reference frames in which an isolated mass point moves with uniform velocity, forever.
2. Given explanation, or contextualization.
   If a reference frame, of a given ambient space, belongs to the class, any reference frame translating uniformly with respect to it, while time is invariant, belongs to the class. The representatives of this class are commonly called *inertial systems of reference*. Note that the velocity which the statement refers to is a vector, its constancy implies motion in a straight line with constant speed; rest is when speed is zero.
3. Use made in a theory.
   In modern treatises of mechanics the ‘principle’ of inertia is considered either a principle as in Truesdell (1971),[4] or a theorem as in Landau and Lifshitz (1976)[5] and in Arnold (1989).[6] Here, without giving too much weight to the matter, I will consider the principle of inertia as if it were actually a principle, because (following Newton lead) this was the prevailing point of view before the 20th century.

[2] I adopt the same term used by Maier (1982, p. 78).

[3] For an in depth discussion on the logical status of classical mechanics see the still topical text by Nagel (1961).

[4] p. 65.

[5] p. 3.

[6] p. 10.

There is no need for any particular insight to come to the realization that the principle must have an essentially abstract nature and that it would be verifiable only within the bounds of imperfect experiments.

It is not possible to find the class of reference frames postulated in which the principle holds exactly; however, there are some in which the principle holds in an approximate way, such as (in increasing order of approximation) the earth, the solar system, the fixed stars. But even if this ideal frame of reference were fully described, it would not be possible to check whether it holds true forever, since any empirical observation has a limited duration; moreover it cannot be ascertained whether the body is fully isolated because it is not possible to check an infinite space. However, it can be empirically verified that by gradually eliminating the interaction of other bodies with the body mentioned in the principle, the duration of the motion can be extended as desired. Moreover, the concept of a mass point is highly idealized, more a mathematical entity than a physical one; however a sufficiently small body should satisfy the requirements asked of a mass point well enough.

The purpose of this paper is:

1. To present the main instances of the principle of inertia, from ancient Greece up to the early 20th century.
2. To verify if the formulation of the principle of inertia has followed a continuous route since ancient Greece, or if there have been discontinuities along that route. If the latter is the case, then this paper will also show what these discontinuities are and when they occurred.
3. To understand how the evolution of the concept of space has influenced the formulation of the principle of inertia.

2 Instances of Regular Motion

Before expounding the various formulations of the laws of regular motion, it is worthwhile to begin with a brief overview concerning the history of the concept of space, as the concepts of space and motion are closely associated and the formulation of the principle of inertia cannot always be separated from a sufficiently precise notion of space, a term that even today has many meanings (Lucas, 1969)[7] and can be seen from different perspectives (Schemmel, 2016). In this paper, the point of view is assumed of natural philosophy and mathematics, disciplines which have generally seen space as the receptacle of motion.

In traditional natural philosophy as well as in modern physics a realistic point of view is generally assumed, that is, that our concept of space is a faithful mental reproduction of things that really exist outside of us. In such a case, there are two possible ways of looking at space that go beyond immediate perception. On the one hand there is (a) what is now called *relationalism*, which denies the existence of space as an object in its own right; seeing it rather as relations between material objects. On the other hand, there is (b) what is now called *substantivalism*, which sees space as an existing being (a substance), over and above the other material objects and processes of the world. By cancelling a body, the space it occupies becomes void, just as how the void is space where there are no bodies. A particular view of substantivalism has given rise to the concept of *absolute space*, a term initially popularized by Newton to indicate an external space, a space which is infinite,

[7] pp. 1–3.

isotropic, uniform, perfectly penetrable and immovable where he located motion and studied its variations.

The following section is but a short overview of the concept of absolute space and its evolution, particularly focusing on the Renaissance. For a more in-depth analysis the reader has a vast resource of pre-existing literature at their disposal; see for instance the following monographs of general characters (Jammer, 1993; Grant, 1981; Sklar, 1974; Cornford, 1976; Friedman, 1983; Duhem, 1913; Koyré, 1957; Sambursky, 1962; Bakker et al., 2018) and my recent paper (Capecchi, 2022).

The reflections of the Greek philosophers and mathematicians on space were fundamental for Western civilization later understandings. In these works, one finds the positions of the Atomists who conceived of infinite space as being a void populated by full, very small, rigid atoms, as well as the concepts of Plato and Aristotle who considered the universe to be a plenum of finite dimension, and finally the Stoics, who postulated a finite cosmos that was immersed in an infinite void space.

Both the Atomists and Stoics conceived of space as being a receiver. However, only the Atomists paid any great attention to motion, being concerned also with considering aspects of kinematics. Space, beyond accommodating atoms (matter) was also the place where rectilinear and uniform motion occurred. Between Aristotle and Plato, only the former had a relationalist view of space, which reflected his conception of geometrical objects as being simple abstractions. Plato, on the other hand, gave geometry preeminence over matter. His was a geometrical space; though also a space centred locally; which is to say that in having little interest in conceiving space as containing the whole world, Plato's concept does not differ much from Aristotle's. Both philosophers gave a fundamental role to local (spatial) motion to explain changes in the world, but when motion was described kinematically they viewed space intuitively and in the manner of their day.

In the Middle Ages, the dominant view on space was relationalism, based on Aristotelian philosophy. During the Renaissance, though, many philosophers considered substantivalism further, among them Fancesco Patrizi (1529–1597) and Giordano Bruno (1548–1600). Patrizi spoke in depth about his ideas on space in the first three books of the *Pancosmia—De spatio physico, De spatio mathematico, De physici ac mathematici spacii, affection nibus*—a section of his opus magnus *Nova de universis philosophia*, which was published twice in 1591 and in 1593 (Patrizi, 1591). The first book of the *Pancosmia* introduces space as having existed since the beginning. It is what God created first, before all the other things: "Now what did the Supreme Maker create before all other things apart from Himself? [...] But this is Space itself. For all things, whether corporeal or incorporeal, if they are not somewhere, are nowhere; and if they are nowhere they do not even exist" (Patrizi, 1591).[8]

Having ascertained the existence and priority of space, Patrizi goes on to wonder about its ontological nature:

> First space is a three dimensional being. It is something incorporeal having all the three dimensions of a body, though it is not a body. [...] What then is it, a body or an incorporeal substance? Neither, but a mean between the two [...]. Therefore it is an incorporeal body and a corporeal non-body (Patrizi, 1591)[9]

[8] f. 61r, translation into English in Patrizi and Brickman (1943).

[9] f. 65r-v. English translation in Patrizi and Brickman (1943).

A further characterization of space is obtained by considering the presence of bodies within it; of them Patrizi claimed that no one can doubt they are in place (locus) since they are surrounded by space on all sides (Patrizi, 1591).[10] He used the term *locus*, but criticized Aristotle own pre-existing definition of the term, arguing that Aristotle usage of the word was internally contradictory, because after asserting its three-dimensional character he goes on to reduce it to bi-dimensionality (Patrizi, 1591).[11] "Hence locus, not being a body, will by necessity be a space possessed of three dimensions, of length, breadth, and depth—with which it then receives into itself and holds the length, breadth, and depth of the enclosed body" (Patrizi, 1591).[12] In completing his characterization of space, Patrizi also examined its relationship with vacuum. It is important to note that Patrizi's vacuum is not an absolute emptiness, but simply the absence of corporeal substances. This vacuum was in fact a plenum, filled with incorporeal light (then considered to be heat and fluid).

In conclusion, space is entirely unmoved and unmovable, infinite and uniform in Patrizi's view. However, this ontological uniformity is broken by his stoic cosmology. For Patrizi space has a centre that coincides with the centre of the cosmos: "the centre of the space therefore is at the midpoint of the infinite universum space" (Patrizi, 1591).[13] The uniformity of space is also undone by Patrizi's attempt to reconcile his cosmology with that of Aristotle, in particular with the existence of natural place, that is privileged portions of space.

Bruno, a younger contemporary of Patrizi, was a member of the naturalist Italian school alongside Bernardino Telesio (1509–1588) and Tommaso Campanella (1568–1639). His philosophizing was concerned with the infinite, infinite worlds in infinite space, and his conception of space and the cosmos greatly influenced his contemporaries; not the least due to his eventual and dramatic condemnation to be burnt at the stake. He surpassed both Cusanus and Copernicus in his achieving a vision of innumerable worlds moving within an infinite space, a space that manifests as the sole container of bodies and that unites them all. He removed the stoic elements present in Patrizi, namely, the existence of a centre wherein the cosmos was located, as well as any kind of non-uniformity. According to Bruno, space was essentially an infinite and uniform three-dimensional continuum, that ontologically precedes, contains, and receives all things indifferently. His main writings concerning space are the *Cena delle ceneri* (Bruno, 1830a), *De l'infinito universo e mondi* (Bruno, 1830b), and *De immenso et innumerabilibus* (Bruno, 1884).

Bruno's view is that of the Atomists; indeed it is well documented that Bruno was acquainted with Lucretius' *De rerum natura*, though this was likely not the main source of his atomism (Rossini, 2019).[14] Notably, and in a striking difference from the Atomists, Bruno's conception of space was as a plenum, filled with an aether, sometimes named air (but not common air) or chaos (Grant, 1981). Moreover, Bruno held that atoms had only one shape, that of a sphere, instead of existing in a great variety of forms, as argued by the Atomists.

Bruno often described space as being a vacuum or void; either as nothing or as a very tiny substance that can be penetrated without opposing any resistance, such as the aether

[10] f. 61r.

[11] f. 62r.

[12] f. 62v.

[13] f. 64v.

[14] p. 29.

(for the various meanings of vacuum in Bruno see Amato (1997).[15]). Indeed, it sometimes seems as if Bruno identified space and aether as being synonymous. Below is a quotation which synthesizes Bruno's ideas of space:

> It is therefore not necessary to seek whether in the external sky there are place, vacuum or time; because the general place is one, the immense space that we can freely call void; in which there are innumerable and infinite globes, such as this one in which we live and vegetate. We say such space infinite, because there is no reason, convenience, possibility, sense or nature that must make it finite: in it there are infinite worlds similar to this one, and not different in general from this; because there is neither reason nor defect of natural faculties, I say, as much passive power as active, for which, as there are in this space around us, likewise there are in all the other spaces whose by nature are not different from this one (Bruno, 1584).[16]

2.1 Isaac Newton First Law of Motion

Going back into the past one finds different formulations and interpretations of the law of regular motion. However, it seems natural to stop at Newton's first law of motion as reported in the *Principia*—first edition 1687, last 1726—also commonly known as the principle of inertia, which is substantially similar to the modern statement. Going further back in time one would find formulations very distant from this; Newton's law therefore represents a discontinuity in the historical process backward, the first which is found.

> Law 1. Every body perseveres in its state of being at rest or of moving uniformly straight forward, except insofar as it is compelled to change its state by forces impressed (Newton, 1726).[17]

Newton's statement and the modern one are quite similar. In the first, however, there are more implicit or explicit references to philosophy of nature. This becomes clear by observing that when instead of dealing with an isolated body Newton considered one on which a null (resultant) force acted, what requires the use of a concept, that of force, that creates epistemological and ontological problems (Jammer, 1957); there is however an advantage, the principle is more general and valid for not isolated bodies also. Notice that the straightness of motion and the constancy of speed were put on an equal foot for the first time in the history of mechanics. Furthermore, instead of a material point, Newton spook about a body; a modern reader would be tempted to identify the two concepts; but this is not correct. Indeed referring to a body instead than to a mass point creates ambiguities, as for an isolated body it is only the center of mass that necessarily moves in a straight line, all the other points may partecipate of a rotatory motion around the center of mass; this was however perfectly clear to Newton (see below) who chose to simplify the statement of his principle for the sake of brevity.

Notice that, even if in the statement there is no reference to a space-time reference system, this was specified by Newton before the enunciation of his law: the reference is that of absolute space and time, concepts with a very impegnative ontology but, once accepted, they render the interpretation of Law 1 not problematic.

[15] p. 187.

[16] Dialogue 5, p. 93. My translation.

[17] p. 13. Translation into English in Newton (1999).

Many doubts have been raised about the epistemological status should be attributed to Newton Law 1; that is, whether it has an empirical nature, it is a definition, a theorem or a tautology; they are clearly exposed in Nagel (1961). Newton considered it as an empirical law; or better using his language a law deduced from phenomena (Newton, 1726).[18]

In commenting on his law Newton considered as examples of inertial motion also the rotation of a body around its axis and of the planets around the sun in which there is certainly, within certain limits, the conservation of speed but not of direction. It is not clear why Newton introduced the reference to rotatory motion. He gave some hints in the corollary 3, lemma 3, section 1 of book 1, concerning the conservation of the quantity of motion: "The circular motions of bodies about their own centers also generally arise from reflections of this sort. But I do not consider such cases in what follows, and it would be too tedious to demonstrate everything relating to this subject (Newton, 1726).[19]

Newton expounded his ideas of space and motion in the *De gravitatione* (Newton, 1962; Biener, 2017) a text of controversial dating, with a metaphysical view and, with a more physical mathematical approach, in the Scholium on space and time of the *Principia* (1687); and lastly on the General scholium to the second edition of the *Principia* (1713) (Capecchi, 2021).[20] Recently a new short writing has been discovered, known as *Tempus and locus*, dated in the 1680s, which presents most succinct statement of how place and time relate to existing things occurs (McGuire, 1978a, b; Sadaillan, 2022).

Below a quotation from the Scholium of definitions found in the *Principia*:

> Absolute space, of its own nature without reference to anything external, always remains homogeneous and immovable. Relative space is any movable measure or dimension of this absolute space; such a measure or dimension is determined by our senses from the situation of the space with respect to bodies and is popularly used for immovable space (Newton, 1726).[21]

It is not clear what was the contribution Newton gave to the conception of space; he was certainly strongly influenced by Gassendi, although other influences such as More's were someway important.[22] In any case in the background there was Euclid; Newton was a mathematician. Certainly he devoted much room to the idea of space, he discussed it in depth and justified it on mechanical basis. The idea of absolute space was not dictated only by theological or logical reasons but was the only way to justify an absolute motion. After conceiving space and absolute motion Newton could think of Law 1 in some way as an empirical principle: in the absolute space bodies not subject to forces remain in motion with constant velocity and force clearly becomes the cause of the change in motion, not of its maintenance.

The weak ontological commitment on the conception of space of the *Principia* and the fact it was introduced in a scholium of the section devoted to definitions, leave a modern reader the impression Newton did not try to answer the metaphysical question if space is actually absolute or not; on the contrary, he did not even take for granted that such a question was well-posed. His primary aim, much probably, was instead to define *absolute*

18 p. 530.

19 p. 18.

20 pp. 30–35.

21 Scholium to definitions; p. 6.

22 Notice that both and Gassendi were influenced by Patrizi, thus one can say Newton derived his ideas on space from Patrizi. The role of Henry More for Newton's conception of space is discussed in depth in Hall (1990).

space, *absolute time* and *absolute motion* for applying these concepts and to reveal the roles that they play in solving the problems of mechanics; this point of view was for instance sustained in Stein (1967); Biener (2017); Disalle (2016).

Granted that at the beginning, in the first draft of the *Principia*, the introduction of absolute space and time had a predominantly instrumental character, subsequently Newton intended to give a sense of reality to his absolute space of the *Principia*, also influenced by the accusations of atheism of his conception of the universe. He thus made God intervene who, with his omnipresence, guaranteed the existence and the infinity of space. This relationship between God and space is reported in a sufficiently clear way in the *Scolium generale* added to the second edition of the *Principia* of 1713: "He endures always and is present everywhere, and by existing always and everywhere he constitutes duration and space" (Newton, 1726).[23] But already in the Latin edition of the *Optics* of 1706 he had expressed clear concepts on the role of God, introducing the analogy of the the sensorium of God (Henry, 2020).

If it is simple to accept that the first law of motion enunciated by Newton has an unambiguous meaning within Newton's natural philosophy, it is more complicated to understand on what basis it was formulated. Newton claimed to have deduced his laws of motion (and therefore also the first) from phenomena. But this is hardly credible; it is true that once enunciated, it is empirically verifiable, but it is also true that it seems contrary to any common experience, and so scarcely intuitive. Newton provided a historical justification: the law was first derived by Galileo. But this justification is not completely convincing for the reasons that will appear clear in the following. So many historians, especially the French ones, find this attribution too generous toward Galileo—when for them it would have been more natural for Newton to refer to Descartes or Gassendi—and explain Newton's claim as dictated by nationalistic reasons or to detach himself from a cumbersome author as Descartes. Personally I do not trust these charges and think Newton had his right to assume Galileo and not Descartes or Gassendi the discoverer of the law of inertia; for an interesting paper on the authorship of Galileo refer to Nicotra (2022).

Some of Newton's conceptions of inertial motion is found in Definition III of the *Principia*.

> Definition III. Inherent force (vis insita) of matter is the power of resisting by which every body, so as far as it is able, perseveres in its state either of resting or of moving uniformly straight forward (Newton, 1726).[24]

And in his commentary, in which Newton tried without too much success to treat the *vis insita* as a nominal definition.

> This exercise of force is, depending on the viewpoint, both resistance and impetus: resistance insofar as the body, in order to maintain its state, strives against the impressed force, and impetus insofar as the same body, yielding only with difficulty to the force of a resisting obstacle, endeavors to change the state of that obstacle. Resistance is commonly attributed to resting bodies and impetus to moving bodies; but motion and rest, in the popular sense of the terms, are distinguished from each other only by point of view (Newton, 1726).[25]

[23] p. 528.

[24] p. 2. Translation into English in Newton (1999).

[25] p. 2. Translation into English in Newton (1999).

Richard Westfall in one of his still actual book (Westfall, 1971), reconstructs the evolution of the concept of *vis insita* from the earliest times. According to him, Newton gradually changed from a conception of vis insita as internal force to a concept of *vis insita* as inertia. At the same time gradually he introduced the concept of force as an external cause of change of motion. The two concepts of force—internal and external—are incompatible, the latter rejects inevitably the former, since one conceives the force as the cause of motion the other as the cause of its variation; Newton will never be able to fully carry out the separation of the two concepts, and here and there also in the *Principia* tracks of the *vis insita* conceived as internal force remained.

Newton's fluctuations are clearly evidenced by the latest reworking of one of his manuscripts, commonly known as *De motu* (Westfall, 1971).[26]

> Law 1. By innate force [vi insita] every body perseveres in its state of resting or of moving uniformly in a right line unless it is compelled to change that state by impressed force. Moreover this uniform motion is of two sorts, progressive motion in a right line which the body describes with its center which is borne uniformly, and circular motion about any one of its axes which either rests or remains ever parallel to its prior position as it is carried with a uniform motion (Newton, 1684).[27]

The above quotation specifies that the center of gravity of a body moves with uniform rectilinear motion, while the whole body can also rotate around one of its axes (Hecht, 2015)[28] But what is more interesting, it specifies that the body moves as a result of an innate force, making the sentence much less 'modern' than that of the *Principia*.

2.2 Descartes and the Laws of Nature

The second discontinuity in the statements on regular motion one encounters in the backward path of the history is noticed in two laws of regular motion due to Descartes, fully made explicit in the 1644 edition of the *Principia philosophiae*.

> (A) [First law]. The first of these is that each thing, *so far as it is simple and undivided* [emphasis added], insofar as it is in itself, always remains in the same state, and never it is ever changed except by external causes.
> (A*) [First law]. First law of nature. Each thing, for what is in itself, perseveres in the same state; and thus what is moved once continues to move for ever (Descartes, 1644).[29]

> (B) [Second law]. The other law of nature is that each part of matter, considered separately, does not tend at any time to continue to move along any curved lines, but only along straight lines (Descartes, 1644).[30]

(A) and (B) are referred to as the first and second laws of nature and are presented in two different wordings, as titles of the sections that introduce them and in the body of the text. The wordings (A) and (B) are those of the body of the text. The wording (A*) is that of the

[26] Footnote 10, pp. 514–515.

[27] Translation into English in Westfall (1971).

[28] p. 82.

[29] p. 54.

[30] p. 55.

title of the section; a synthesis of the whole section; in particular the expression "simple and undivided" is avoided, and explicit reference to motion is made. Most probably Descartes would grasp the main meaning of the law. To note that in the French edition of the *Principia philosophiae* it is specified what the external causes are: "unless through collisions with other" (Descartes, 1668).[31] There is a third law also which regulates the collision among bodies, not reported here for the sake of simplicity.

Even though the world of Descartes was corpuscular the possibility of vacuum was not accepted; the matter that fills the world has geometric characteristic only; it is completely defined by the extension (res-extensa). In this world completely filled with matter, motion involves necessarily the formation of vortices of corpuscles or particles. Motion—which in this world completely filled with matter involves necessarily the formation of vortices that concern any corpuscle or particle— according to Descartes is one of the modes of the extended substance, like shape and size: "motion and rest are just different modes of the motion of bodies". That is motion is a matter of fact whose causes are not of interest. Considered independently of causes, motion is simply the transference of a part of matter from the neighborhood of those bodies that immediately touch it and are regarded as being at rest, into the neighborhood of others (Descartes, 1644).[32] When investigating the causes of motion Descartes involved God: the persistence of motion like that of any mode directly stems from God's persistence, if he abandoned the mode it would no longer exist. Garber (1992, 2001) it is discussed the possibility that Good continuously recreates the world and thus motion consists simply in a sequence of creations of equal bodies in different places.

At the time of creation God gave to the material substance a certain amount of motion that is preserved for ever by God himself:

> Now as far as the general cause is concerned, *it seems clear to me that this is no other than God himself* [emphasis added]. In the beginning, in his omnipotence, he created matter, along with its motion and rest; and now, merely by his regular concurrence, he preserves the same amount of motion and rest in the material universe as he put there in the beginning. Admittedly motion is simply a mode of the matter which is moved. But nevertheless it has a certain determinate quantity; and this, we easily understand, may be constant in the universe as a whole while varying in any given part (Descartes, 1644).[33]

Descartes' laws (A) and (B) for some 20th century historians would represent the first explicit formulation of the principle of inertia in the history of mechanics (Pav, 1966).[34] A position strongly supported by Alexandre Koyré in his *Études galiléennes*. It must be conceded that Koyré judgment is not only dictated by nationalistic reasons, but also by his particular rationalistic conception of science, in which the experiment is somehow placed in the background to metaphysics.

Although a superficial reading of the text, perhaps influenced by secondary literature, seems to support this position, a more careful reading shows how Descartes's laws had a very different meaning for him than that Newton could attribute to his principle. The following questions arise:

[31] p. 85.

[32] pp. 46–47.

[33] p. 53. My translation.

[34] p. 24.

1. Why are the two laws credited as laws of nature if in the first of them motion is not even explicitly mentioned?
2. What do the restrictions on simple, “undivided things” mean, and then what does it mean by “in itself”?
3. Why are there two laws? What is the role of the second law of nature, this time regarding motion alone, which seems to be contained in the first?
4. What is the space-time reference system in which the two laws should apply?
5. Is there any justification for these laws?
6. What is their epistemological status?

Let’s now try to answer the above questions:

1. Descartes considered motion as the fundamental object of his philosophy of nature, and therefore the laws of nature and laws of motion for him were substantially synonymous. The first law, even if it appears more general, mainly refers to motion; the purpose of the generality serves to underline that motion is a mode like any other of extended matter, as the shape of a body, its size and even its temperature. In this way Descartes tried to eliminate the difficulties raised by the natural philosophy of Aristotelian mold on the possibility of the continuation of motion without an engine. He declared that explaining motion is no more difficult than explaining rest. It is a fact, a way of existence, a characteristic of the body like many others.
2. The restriction to simple and undivided things and in itself serves to limit the validity of the law to elementary corpuscles, eliminating the assembly of bodies. “In itself” then refers to a trend that would be realized if there were no interaction with other bodies which, instead, always exists.
3. The first law of nature in Descartes head refers to the constance of speed and thus the second law, concerning the constancy of direction, is independent of it. Both of them are however justified in the same way: the immutability of God for which everything tends to keep the same. According to Garber (1992), the difference of the two laws is due more to the evolution over time of Descartes’ ideas, than to his complete philosophy of nature. Since his early studies, Descartes had focused his attention on speed rather than on direction. Only starting from the mid 1620s in his studies on vortices did he raise the problem of rectilinear motion and mainly of the resistances opposed to its change (Garber, 1992).[35] In any case, in some passages Descartes seems to believe that the conservation of the speed of motion is more basic than that of its direction. For example souls can interact with the direction of bodies but not with their speed, at least according to Leibniz’s reading of Descartes (McLaughlin, 1993).[36]
4. The question of the nature of motion, and of the space where to place it, is one of the most complex themes in Descartes’ philosophy of nature. According to him, motion is always local even when it comes to cosmological issues. With regard to the motion of a body Descartes writes: “the transference [motion] takes place from the neighborhood of those bodies that immediately touch it into the neighborhood of others” (Descartes, 1644).[37] Descartes’s statement is of problematic interpretation and much has been written about this, with some authors believing that his theory is inconsistent on the

35 p. 225.

36 p. 155.

37 p. 42; II 25.

whole; while others argue that one must strive to read Descartes with the ideas of his time and that basically his writings are coherent (Garber, 1992, 2001). Recent studies support the thesis that the space of Descartes should be classified as relationalist (in the sense that his space does not have a structure that supports absolute motion), but not in the strict sense; in some way some preferred reference system can be chosen (Slowik, 1999; Garber, 1992). As motion is relational and 'forces' in this space depends on its geometry, Einstein saw to Descartes space time to justify his theory of general relativity (Slowik, 2005).

5. Descartes, as already mentioned, justified his laws with the invariance of God who in every instant reproduces conditions of motion equal to the pre-existing ones. As a further reflection, it should be pointed out that Descartes' corpuscular conception of matter had close analogies with the atomistic theories which in his time were receiving a strong attention by philosophers of nature (the discovery of Lucretius's *De rerum natura* dates back to the 15th century). In this theory, the motion of atoms is considered as natural.

   In a letter to Henry More of August 1649 Descartes said: "I consider matter left to itself and receiving no impulse from anything else as plainly at rest. But it is impelled by God, conserving the same amount of motion or translation in it as he put there from the first" (Descartes, 1964).[38] In substance Descartes like the schoolmen thought that motion requires a cause. But instead of air or impetus he individuated it in God (Garber, 1992).[39]

6. It is clear that having recourse to God to explain the laws of motion makes them indubitable. It is Descartes himself who said that his laws have an ideal nature and cannot be verified in his plenum, where the interaction between the corpuscles is always present. Even though Descartes' statements do not differ much from those of Galileo and Cavalieri (apart from its much more explicit form, see below), they have a different status. For their validity there is no need to refer to everyday experience, as Galileo did, but only to the persistence of God, always engaged with the same activity; Descartes' are thus metaphysical theorems. Descartes tried to justify his metaphysical principle with reference to a daily experience, that of projectiles thrown through a medium "the fact that they keep their motion a few moments after they have been launched, is a proof that once they are moved they continue to move" (Descartes, 1644).[40] That they actually stop at a certain point, as a result of the resistance opposed by the medium, and law (A) of motion is falsified, is irrelevant because this law is necessarily true. What a difference with respect to Galileo!

Descartes started to deal with regular motion at least from 1618 during his apprenticeship with Beeckman who had embraced the new mechanistic philosophy and as a supporter of the atomistic theses accepted the existence of a vacuum. Beeckman expounded his ideas on regular motion in several places in his diaries, some of the most significant ones are given in the following quotations:

> That which is once moved, is always moved in a vacuum, either along a straight line or circular one, as much over its center as the diurnal motion of the Earth (Beeckman, 1939–1953).[41]

[38] vol. 5, p. 440.

[39] p. 227.

[40] pp. 55.

[41] p. 254.

> The following theorem holds: what is once moved is always moved in the same way as long as it is hindered from. In a vacuum, however, no such consideration should be taken: for a large body, small heavy, light, with large or small surface, with this or that configuration, etc, it continues to move always in the way in which it is when it has been moved, it continues to move, if nothing will get close to it bearing an impediment (Beeckman, 1939–1953).[42]

As it can be seen, the first statement is not as precise as the Cartesian one because it does not specify that the motion develops with constant speed. Furthermore, it refers to a circular motion also, which makes problematic the interpretation of Beeckman's thought. The second statement is more explicit, once a motion has been impressed it is kept straight in the same way at constant speed or uniformly circular.

Descartes resumed his studies on the regular motion many years later. They reached a mature stage in the treatise *Le monde ou le traité de la lumiére* of 1632–1633, substantially contemporary to Galileo's *Dialogo sopra i due massimi sistemi*, published only in 1664 after his death. Below the two laws corresponding to the above (A) and (B),

> (A′).The first [law of nature] is that each individual part of matter always continues to remain in the same state unless collision with others constrains it to change that state. That is to say, if the part has some size, it will never become smaller unless others divide it; if it is round or square, it will never change that shape without others forcing it to do so; if it is stopped in some place, it will never depart from that place unless others chase it away; and if it has once begun to move, it will always continue with an equal force until others stop or retard it (Descartes, 1664).[43]

> (B′). I will add as a third rule that, when a body is moving, even if its motion most often takes place along a curved line and (as has been said above) can never take place along any line that is not in some way circular, nevertheless each of its individual parts tends always to continue its motion along a straight line. And thus their action, i.e. the inclination they have to move, is different from their motion (Descartes, 1664).[44]

What a modern reader notices first is that the laws are presented in a different order. In the *Principia philosophiae* the law concerning the impulsion is the third one, in *Le monde* it is the second (not reported here). In any case the first law is similar to that of the *Principia philosophiae*, the main differences being: the lack of specification that a body should be simple and undivided, the specification of the cause of change—that is impact—and the use of the expression of "equal force" instead of equal motion, to qualify what is persisting.

No wonder for this change; the concept of force in Descartes was indeed as complex as that of space. Generally it is conceded that his was not only kinematics but also dynamics, thus the concept of force, as cause of motion, should have some role; possibly force could be identified with God's action. The alternation of force and speed in describing motion indicates a struggle that can also be found in Newton.

[42] p. 256.

[43] pp. 81–82.

[44] p. 94.

2.3 Gassendi's Infinite Space and Motion

Going further back in time, by a few years, the third discontinuity is found in the writings of Pierre Gassendi (1592–1655). Below you find two of the most interesting formulations of the law of regular motion:

> (A) Let us suppose, that the space, through which a stone should be Projected, were absolute Inane, or such as the Imaginary spaces; and then we must acknowledge, that it would be carried in a direct and invariate line, through the same space, and with an Uniforme and Perpetual motion, until it should meet with some other space, full of magnetique rayes, Aer, or some other resisting substance (Gassendi, 1658).[45]

> (B) The wise Creator of the World, would have any motion Perpetual; He ordained it to be Circular; as that, which being equally distant from the Centre in all parts, and wanting both beginning and end, might be continued with one constant tenour, and also uncessantly (Gassendi, 1658).[46]

Before discussing on these two laws, it is worth commenting on the ideas of Gassendi on natural philosophy; that of space in particular. His spatial conception was a synthesis of the two major traditions of the 17th century, the Greek atomists (but also Plato and the Stoics), Francesco Patrizi[47]—who added to the ancient heritage—and the scholastic. From the Greek he derived the basic properties of space, namely infinity, three-dimensionality, incorporeality, wholly passive nature including an incapacity to resist bodies and therefore an ability to coexist with material dimensions, and finally, from Patrizi, the exclusion of space from the traditional categories of substance and accident. But he also derived much from the scholastic tradition: the doctrine of divine annihilation and the concept of God's omnipresence, by which the deity was said to be present in every part of space which is therefore indivisible. Using the latter device, Gassendi could follow scholastic tradition and proclaim God as omnipresent in infinite void space (Grant, 1981).[48]

The influence of Patrizi's view of space is testified in the *Sintagma philosophicum* where Gassendi gave a brief account of earlier ideas about space; when he got to Patrizi, he recognized his debit with him: "about this space, or locus, to which three dimensions length, breadth, and depth belongs, he [Patrizi] propounds nothing other than that what we ourselves have argued about it above" (Gassendi, 1658).[49]

Gassendi accepted vacuum and denied the existence of a centre. Thus, in this respect, his ideas differ from Patrizi's. The main difference, however, is in the motivations that pushed the two scholars to take care of the space. For Patrizi, religious and metaphysical motivations were dominant, while for Gassendi the stimulus was offered by the study of motion. Both to support his version of Epicurean atomism and to insert himself in the wake of the new Galilean science. Apart from the purely geometrical characteristics of space such as infinity, isotropy and uniformity, there are two characteristics with a mechanical

[45] Vol. 1, *Syntagma philosophicum*, Physica, sec. 1, book 3, p. 355a. Translation into English in Charleton (1654).

[46] Vol. 1, *Syntagma philosophicum*, Physica, sec. 1, book 3, p. 345a. Translation into English in Charleton (1654).

[47] To signal that Gassendi quoted Patrizi and also named Campanella—who was one of his correspondent—and Telesio (Gassendi, 1658), vol. 1, *Syntagma philosophicum*, Physica, sec. 1, book 3, p. 246a,b.

[48] p. 213.

[49] Vol. 1, *Syntagma philosophicum*, Physica, sec. 1, book 3, pp. 246v.

nature that have fundamental importance for the study of motion: immobility and complete penetrability. With these two characteristics, derived from Patrizi, Gassendi could formulate a more general law of inertia than that of Galileo and substantially coincident with that of Newton (Koyré, 1978).[50]

Gassendi reported his conceptions of space and motion in various writings; in the following reference is made to the *Syntagma philosophicum*, published posthumously in 1658 in the first two volumes of his *Opera omnia* (Gassendi, 1658), and to *De motu impressed with a translated motor* [hereinafter more simply *De motu*] of 1642 (Gassendi, 1642). A synthesis of Gassendi's thought is found in the *Abrégé de la philosophie de Gassendi* of 1684 (Bernier, 1684) and especially in the *Physiologia Epicuro-Gassendo-Charltoniana* of 1654, edited by Walter Charleton (Charleton, 1654), the text which mostly contributed to the spreading of the ideas of Gassendi in Europe.

Below I refer to Gassendi's conception of space as expressed in the *Syntagma philosophicum*:

> The first is that there were immense spaces before God created the World, that these would continue to exist were He perchance to destroy the world; and that of these God has chosen for his own god pleasure the specific region in which to create the World [...].
> Secondly, that these spaces are entirely immobile. For it is not the case that If God were to move the World from its present location, that space would follow accordingly and move along with it [...].
> Thirdly, that spatial dimensions, without which these spaces would be endlessly open in length, width and depth, as they are immobile, are thus incorporeal, and so have no resistance, or can be penetrated by bodies, or, as it is even commonly said, can coexist with them (Gassendi, 1658).[51]

It is clear from that quotation the similarity between Gassendi and Patrizi.

Let us now pass to comment on the laws of motion, starting with (A). This law is strikingly similar to Newton's and its meaning, given Gassendi's notion of space, looks the same. This formulation is considered by some historians, for example by Koyré (1978),[52] as the first expression of the law of regular motion ever published, even before Descartes' most controversial one reported in the *Principia philosophiae* only in 1644 (see Sect. 2.2). This claim does not seem very credible, both because it minimizes Galileo's formulation and his school, and because Gassendi's law (A) is presented together with the law of circular inertial motion (B) which suggests a certain inconsistency on Gassendi's part; what is the true law of regular motion?

How was Gassendi able to enunciate law (A)? Certainly his atomistic conception must have helped him a lot. In his physics the weight of the body was not a substantial attribute, as it could be explained mechanically with the interaction between atoms. It is therefore possible to think of a weightless body; one for example that if launched into the air should not fall toward the center of the world. Moreover Gassendi was a profound connoisseur and admirer of Galileo and was aware of his thought experiments to justify the perpetual motion of a body moving on a horizontal plane, once the impediments are eliminated. The reference to Galileo is clear from Gassendi's *De motu* of 1642:

[50] pp. 244–245.

[51] Vol. 1, *Syntagma philosophicum*, Physica, sec. 1, book 3, pp. 183a,b. Translation into English in Capek (1976).

[52] pp. 244–245.

> [Y]ou ask what would happen to that stone which I said could be conceived in empty space if, having been disturbed from rest, it were impelled by some force. I reply that it is likely that it would move equably and without ceasing, and either slowly or quickly, depending on whether a small or great impetus had been impressed. *I derive support for this from the equability of the horizontal motion that has already been explained*. For it seems that it would not stop except due to the addition of perpendicular motion. Therefore, because there would be no addition of perpendicular motion in that space, in whatever direction the motion started, it would be like horizontal motion, and would not be accelerated or retarded, and so would never cease (Gassendi, 1642, 1658).[53]

When Gassendi said "I derive support for this from the equability of the horizontal motion that has already been explained" he though to a section of his *De motu* where the arguments of Galileo are summarized (Gassendi, 1642).[54]

Note that Gassendi did not resort to the idea of an internal force as was the case in medieval explanations of regular motion (see Sect. 2.5). He did still use the word *impetus* but without any ontological implication.

> We, with the Vulgar, say, that there is an Imprest Force remaining, for some time, in the thing moved, or projected; we could thereby understand no other than the Impetus, or motion it self (Gassendi, 1658).[55]

At a first sight law (A) seems of cosmological kind, that is suitable for an infinite void space. There are however hints suggesting that Gassendi thought the law valid in an ambient circumscribed to the earth neighbors. Just after having introduced law (A), Gassendi added that as the atmospheric sphere is filled by the air and swarms of "magnetic rays" [terrenis radiis], no body can be projected in an absolutely perfec straight line. For, if the projection be made either obliquely, or parallel to the horizon the projected body suddenly begins to be deflected from the mark at which it was aimed, and so describes not a straight, but crooked line (Gassendi, 1658).[56] A reflection that Newton did not make, to underline his principle had a universal value.

The presence of the law of free circular motion (B), puts the modern reader into crisis and probably would also have put into crisis a contemporary. Gassendi seemed to consider the constancy of the speed more important and fundamental than that of its direction for the continuation of regular motion. This idea has ancient roots. And all considered it is quite reasonable because actually on an intuitive level it seems more difficult to explain the continuation of motion than the maintenance of its direction, especially for philosophers who were not mathematicians and had no idea, of what a vector quantity was.

The motion of celestial bodies Gassendi imagined in law (B) may take place on a circular orbit with constant speed expresses in itself a matter of fact, but Newton's explanation of this puts into play the force of attraction of a planet toward its satellite or of the sun toward a planet. Gassendi, instead, did not believe that there was a force of attraction that extended over great distances as cosmic space is a vacuum and no mechanical actions can exist.

[53] Epistola 1, p. 62; vol. 3, p. 495b.

[54] Epistola 1, pp. 39–40.

[55] Vol. 1, *Syntagma philosophicum*, Physica, sec. 1, book 3, p. 354a. Translation into English in Charleton (1654).

[56] Vol. 1, *Syntagma philosophicum*, Physica, sec. 1, book 3, p. 355a.

Law (B) is stated in a cosmological context, but it is also valid in the terrestrial environment even though in this case the circular motion results from the action of a 'force' (gravity) which acts on bodies preventing them from following a straight trajectory. Below I refer to an example of free circular motion on a terrestrial environment, that take much from Galileo *Dialogo sopra i due massimi sistemi del mondo*:

> Why may you not lawfully conjecture that if the Terrestrial Globe were of a superfice exquisitely polite, or smooth as the finest Venice Glass; and another small Globe as polite were placed in any part of its superfice, and but gently impelled any way, it would be moved with constant Uniformity quite round the Earth, according to the line of its first direction; and having rowled once round the Earth, it would, without intermission again begin, or rather continue another Circuit, and so maintain a perpetual Circulation upon the surface of the Earth? (Gassendi, 1658).[57]

The justification given by Gassendi for the regular motion of the small globe is not very coherent; on the one hand he noticed that many parts of the small globe, during the motion thereof, tend toward the centre of the earth, just so many are, at the same time elevated from it: so that a full compensation being made in all points of the motion, the same cannot but perpetually continue (and this is a interesting but inconsistent explanation for the regular motion on a horizontal plane); on the other hand he noticed that there is no declivity, whereby it should be accelerated, no acclivity, whereby it should be retarded, which is the Galilean explanation for a perpetual regular motion.

The double way of conceiving inertial motion, laws (A) and (B), is internally contradictory because it cannot be said that in a space devoid of matter the motion of a body remains rectilinear while instead in the presence of matter (the sun for example) the motion is circular even though there is no action exerted on the body. A most striking thing is that Gassendi in the same text, the *Syntagma philosophicum*, first formulated law (B) (p. 345a), then (A) (p. 355a), then again (B) (p. 355a) (Gassendi, 1658).[58] It must be said however that this kind of internal contradiction can be found in Galileo too, and in all his predecessors as well. Only with Newton there would be a unique law for regular motion, the one with constant speed and along a straight line, valid for the whole cosmos.

Another reason for Gassendi's inconsistencies is internal to his atomistic philosophy. He imagined macroscopic bodies made up of atoms. To decipher exactly what are the properties of the atomic motion according to Gassendi is unfortunately quite hard. For him atoms have a constant degree of mobility, or force, or weight, imparted by God at the creation. This however does not imply that atoms maintains a constant speed, between a collision and another, and of course the direction change after a collision. The constancy of speed is admitted for instance in Lolordo (2007),[59] but is contrasted in Fisher (2005),[60] where it is assumed as possible that atoms could have a rest, still maintaining the same degree of mobility by spinning or vibrating. Moreover atoms move in any direction. How can then be explained the regular motion of macroscopic bodies along a straight line with constant speed? A possibility is that the free motions result from a polarization of the motion of the atoms that, as an aggregate, made a

[57] Vol. 1, *Syntagma philosophicum*, Physica, sec. 1, book 3, p. 355a. Translation into English in Charleton (1654).

[58] Vol. 1.

[59] pp. 149–152.

[60] p. 264.

macroscopic body and admitting that even though there were rest the mean speed of each atom is constant. This could be someway inferred from the following quotation:

> There seems to exist in an innate motion on account of the atoms. And either these atoms are those from which the mobile body is composed, as when it is moved by itself or per se; or they are those that make up the moving body, as when something is moved by something else, which thing, while it does the moving, would in some measure be moved by itself. For, since the atoms in some body are variously agitated, if some that are more mobile and quicker conspire together to press toward some place, then the whole body is itself moved in that direction (Gassendi, 1658).[61]

The mechanism of polarization could be explained by reflecting on the fact that before a body, a stone, is hurled into the air, it is meantime joined to the hand for a certain time and may be considered with it as a single moving object because one and the same motion applies jointly to them both (Gassendi, 1642).[62] More precisely: "when a thing projected is impelled, it is first touched by the proficient only in those parts, which are in its surface or outside and that those outward parts, being pressed by the impulse, do drive inward or press upon the parts next to them; and those again impel the parts next to them, and those again the next to them, till the impulse be by succession propagated quite through the body of the thing projected, to the superficial parts in the opposite side [and then begins the motion of the whole]" (Gassendi, 1658).[63]

Notice that the idea of polarization is the only convincing mechanicistic causal explanation one knows to justify regular motion; it only needs to assume as not problematic the motion of atoms. However, the regular motion seen in this perspective is no longer only inert(ial) as due to the intrinsic mobility, or force, of atoms, and differently from what Gassendi has declared impetus is no longer motion in itself. Gassendi was not fully explicit about the relation between motions at a microscopic level and that at a macroscopic level. In the end he followed two distinct paths in his natural philosophy; on the one hand he moved in the footprint of mathematical physics in the attempt to justify the experience at a phenomenological level, on the other hand he managed to develop a coherent atomistic view. It is not known if he had actually the purpose to join the two approaches; if it was the case he did not reach completely his scope.

Before leaving two comments are needed. First; the motion of celestial bodies, planets, exhibits a great difference at a phenomenological level with respect to that of a stone launched by a thrower. In this latter case experience shows that motion comes at rest after a while. Assuming it will last for ever is an idealization, a principle of natural philosophy. Planets instead show 'experimentally' a permanent motion. In both cases the causes are hidden; Gassendi assumed for them an imparted impulse, for the stone by man, for the planets by God. Second; Gassendi knew Kepler's studies and accepted that planets had an elliptic orbit; he expressly cited Kepler in the second epistle of his *De motu* (Gassendi, 1642, 1658),[64] so declaring the orbits were circular seems quite at odds. Most probably Gassendi glossed over the fact because only with a circular orbits his idea of an impressed motion could work.

[61] Vol. 1, *Syntagma philosophicum*, Physica, sec. 1, book 3, p. 338a. Translation into English in Lolordo (2007).

[62] p. 75.

[63] Vol. 1, *Syntagma philosophicum*, Physica, sec. 1, book 3, p. 354a, b. Translation into English in Charleton (1654).

[64] Epistola 1, pp. 141–142; vol. 3, p. 515b.

2.4 Galileo and Motion on a Horizontal Plane

Going further back in time it is found the formulations of the laws of regular motion by Galileo Galilei and his pupils. Below some formulations due to Galileo.

> (A) *Salviati*. Now tell me, what would befall the same moveable upon a superficies that had neither acclivity nor declivity? [...] *Simplicius*. I cannot tell how to discover any cause of acceleration, or retardation, there being no declivity or acclivity. *Salviati*. Therefore if such a space were interminate, the motion upon the same would likewise have no termination, that is, *would be perpetual* [emphasis added] (Galilei, 1632).[65]

> (B) *Scholium*. It may also be noted that whatever degree of speed is found in the moveable, this is by its nature [suapte natura] indelibly impressed on it when external causes of acceleration or retardation are removed, which occurs only on the horizontal plane; for on declining planes there is cause of more [maioris] acceleration, and on rising planes, of retardation. From this it likewise follows that motion in the horizontal is also eternal since if it is indeed equable it is not [even] weakened or remitted: much less removed (Galilei, 1638).[66]

> (C) *De motu proiectorum*. I mentally conceive of some moveable projected on a horizontal plane, all impediments being put aside. Now it is evident from what has been said elsewhere at greater length that equable motion on this plane would be perpetual if the plane were of infinite extent; but if we assume it to be ended, and [situated] on high, the moveable (which I conceive of as being endowed with heaviness), driven to the end of this plane and going on further, *adds on to its previous equable and indelible motion* [emphasis added] that downward tendency which it has from its own heaviness (Galilei, 1638).[67]

> (D) *Sagredo*. There is no doubt that to maintain the optimum placement and perfect order of the parts of the universe as to local situation, nothing will do but circular motion or rest. As to motion by a straight line, I do not see how it can be of use for anything except to restore to their natural location such integral bodies as have been accidentally removed and separated from their whole, as we have just said (Galilei, 1632).[68]

The first thing a modern reader notice is that the above statements, unlike those presented previously, had no cosmological implications. Actually, at least law (A) was used to justify a cosmological stance, the Copernican view of solar system, by discussing the dropping of a stone from the mast of a ship in motion (Galilei, 1632).[69] Even though Galileo had probably a substantialist view of space—being acquainted with atomists, Giordano Bruno's and Copernicus' cosmologies—he did not need such an ontological commitment about space and did not need to go beyond the intuitive idea of it. It was enough for him the possession of a metric and a reference frame; in his case the ground/earth; though he knew it moved in space.

65 p. 173. Translation into English in Galilei (1967).

66 p. 207. Translation into English in Galilei (1974).

67 p. 236. Translation into English in Galilei (1974).

68 p. 56. Translation into English in Galilei (1967).

69 Second day.

The writings of Galileo on regular motion are now well known, but it is worth summarizing them. Before 1600 he had set out his ideas in *De motu antiquiora* (Galilei, 1590) and in *Le mecaniche* (Galilei, 1634, 2002). Here he reported the *indubitable axiom*: "we can assume as an indubitable axiom this conclusion: that heavy bodies, all the external impediments being removed, can be moved in a horizontal plane by any minimal force" (Galilei, 1634).[70]

A result that had been obtained previously by other scholars, Hero of Alexandria in the antiquity, but also Girolamo Cardano, Michel Varron, Giovanni Battista Benedetti had already postulated it with more or less rigorous reasons in the 16th century, a result that in some literature is known as the principle of minimum force PFM (Festa & Roux, 2013). Galileo however was more precise and convincing. He obtained his result both with a thought experiment, imagining inclined planes of decreasing slope, up to a null value, and with a rigorous argument, finding the equilibrium law of the inclined plane that indicates zero displacement force for a horizontal plane. Neither Galileo nor the scholars that preceded him were however able at the time to pass from conceiving displacement with minimal force to motion with null force, an only apparently small step.

In the following the attention is limited to Galileo's mature treatises, *Discorsi e dimostrazioni matematiche sopra due nuove scienze* (1638, hereinafter *Discorsi*) and *Dialogo sopra i due massimi sistemi del mondo* (1632, hereinafter *Dialogo*). However, it is worth noting that Galileo already in 1607 was credited with the thesis that "to start motion is very necessary a mover, but to continue it is enough not to have contrast" (Galilei, 1890–1909).[71] And in 1612 writing to Mark Welser (1558–1614) about to some sunspots declared that: "All external impediments being removed, a heavy body on the spherical surface concentric with the earth will be indifferent to rest or to motion toward any part of the horizon, and in this state will remain" (Galilei, 1890–1909).[72]

Formulation (A) is the best known and also the most interesting, in my opinion. In addition to the statement of the law, it also contains the proof, suggested with a maieutic technique by Salviati (Galileo) to Simplicius. Galileo was a great writer, but he was also a mathematician who tended to avoid traditional philosophical (metaphysical) digressions, and his prose was simple, fascinating, convincing. He presented the law of the horizontal regular motion with a thought experiment. A modern reader finds this experiment absolutely convincing, perhaps thinking to people skating on ice or curling players.

A contemporary of Galileo would have had more difficulty in accepting the thought experiment; an engineer would possibly have but a philosopher or a mathematician maybe would not. Roberval for instance did not validate Galileo's experimental thought. He assumed that the experimenter is not authorized to assert that a motion is lasting for ever, but only for a long time (for a more in depth discussion about Roberval's position see Roux (2006)).[73] Roberval's is the same position of the supporters of the impetus theory who admitted it will eventually exhaust also in the void (see for instance Sect. 2.5).

Galileo could propose this thought experiment only because he was strengthened by a result he had already obtained in his youth according to which a minimum force, in the limit of zero value, was sufficient to move a body on a horizontal plane. Furthermore, he most likely had an a priori idea about the outcome of the experiment; it is in fact most

[70] p. 180.

[71] Vol. 10, p. 170. Letter of Benedetto Castelli to Galileo, 1st April 1607.

[72] Vol. 5, p. 134. Letter of Galileo to Mark Welser, 14th August 1612.

[73] p. 495.

probable that he was aware of medieval theories on regular motion, in particular that of impetus, which postulated the persistence of motion. One more stimulus came probably from the need to justify the Copernican cosmology, what needed a law similar to (A). However, there are conflicting opinions on Galileo's cultural background on motion; for a comparison see Weisheipl (1985).

To understand exactly what Galileo meant by formulation (A), one needs to continue reading the *Dialogo*. Here it is suggested that a surface that is devoid of declivity or acclivity must be equidistant from the center of the earth, that is, it must be a sphere. As an example of an ideal surface, Salviati suggested the surface of the sea (Galilei, 1632).[74] To note that an observer positioned on the earth would see it happen on a horizontal plane given the greatness of the earth's radius.

Given the above 'matter of fact', Alexandre Koyré considers himself authorized to speak of a proto principle of inertia or an incomplete principle of inertia because it postulates only continuity but not also straightness of motion (Koyré, 1957). And, according to Koyré, Galileo's conclusion could not be avoided by him since he could not conceive of bodies without weight.

It should be added that Galileo in the *Dialogo* clearly stated that the only perennial motions possible in nature were the circular ones (see above, law (D) of regular motion). Moreover in his myth on the origin of solar system Salviati/Galileo imagined that all the planets have been created by God in the same place and assigned tendencies of motion, descended toward the sun until they had acquired those degrees of velocity which originally seemed good to the Divine mind. The planets were set in rotation, each retaining in its circolar orbit its predetermined speed; the closer to sun, being fallen from a higher height have a greater speed (Galilei, 1632).[75]

Concluding in the *Dialogo*:

1. Galileo presented a law of regular motion as purely experimental without any reference to metaphysical principles of natural philosophy, differently from Descartes, and to some extent also from Gassendi.
2. He postulated only the perseverance of motion and not its straightness.
3. He used his law to 'justify' his principle of relativity, that allowed him to validate the Copernican cosmology. This is an important move that possibly only a mathematician could pursue, because it needed a comprehension of the composition of motions.

In the *Discorsi* the cosmological stance is no longer relevant and a terrestrial mechanics is carried out. Formulation (B) proposes again (A) in a more concise and precise way. It appears on the third day of the *Discorsi* where the accelerated motion of falling bodies is discussed. The same goes for the first part of formulation (C) found at the beginning of the fourth day where the parabolic motion of projectiles is studied. In the second part Galileo stated that if the plane had an end, the horizontal motion would continue with the same speed. Of course this last statement has no root in formulation (A), because a though experiment without a material plane has no meaning; it is so quite arbitrary and one can think it was justified by Galileo's understanding of the theory of impetus. However, Galileo offered an experimental proof in some of his manuscripts that described his experiment to

[74] p. 174.

[75] p. 53.

prove that the trajectory of a projectile is a parabola (Capecchi, 2014).[76] Indeed if the experience shows that the motion of a projectile is a parabola and the downward natural motion follows the law of the square of times, then it is necessary that the horizontal motion has a constant speed.

In the fourth day Galileo imagined the parabolic motion of a projectile as the superposition of two motions, one horizontal and one vertical uniformly accelerated. The horizontal motion is rectilinear with constant speed; its possibility is justified by formulation (C). The parabolic motion Galileo was considering is not that of projectiles fired from a cannon, but the less interesting motion of a horizontally launched object. Galileo was aware of his shortcoming and also addressed the motion of a projectile launched obliquely. In the *Discorsi* this topic is left to a cryptic corollary to proposition VII (Galilei, 1638).[77] Galileo was more explicit in a manuscript in which the composition of an upward decelerated motion and a horizontal motion is implied. Galileo/Salviati introduced the horizontal component of the velocity along the inclined as follows: "which inclination is sufficient to make the projectile in equal times approach horizontally for spaces equal to the axis of the parabola" (Galilei, 1890–1909).[78] For the upward motion it can be said that Galileo could hardly have considered it as a superposition of a upward uniform motion and an accelerated downward motion. It is more likely that he was referring to the ideas he had expressed in *De motu antiquiora*, in which the decelerated upward motion was thought to be due to the gradual loss of the lightness imparted to the body at the moment of launch (according to Hipparcus' theory of impetus) (Capecchi, 2014).[79] It is appropriate to signal another manuscript, actually only a sheet with a drawing (folio 175v), which in Damerow et als (Damerow et al., 1992) is interpreted as the construction of the trajectory of a projectile launched with oblique motion (Damerow et al., 1992).[80] According to Damerow, Galileo would be composing a decelerated motion along the direction of launch, considered as an inclined plane, and a downward accelerated motion, leading to a non-symmetrical and therefore not a parabolic trajectory (the same approach followed by Thomas Harriot, see Sect. 2.5). It is difficult to pronounce in front of only hypothetical reconstructions, but if Damerow is even only partially correct one would be faced with the following two facts: (1) Galileo did not have a clear idea on how to compose the motions; (2) the motion in the inclined direction is not treated as inertial.

The superposition of motions became the subject of discussion between Salviati and Simplicius. The latter raised interesting objections to the possibility of horizontal uniform motion. Basically Simplicius suggested that it is assumed a horizontal and a straight line as if every part of such a line could be at the same distance from the center, which is not true. Indeed as moving away from its midpoint toward its extremities, this line departs from the center of the earth, and hence it is always rising. Consequently it is impossible that the motion perseveres; rather, it would be always weakening (Galilei, 1638).[81] Moreover the vertical motion is not vertical (that is perpendicular to the horizontal plane) but it is radial, directed toward the center of the earth. In substance the two motions to compose would be not horizontal and vertical, but rather circular and radial respectively.

[76] pp. 154–155

[77] p. 289.

[78] p. 447.

[79] p. 169.

[80] pp. 206–209.

[81] pp. 243–244.

However Salviati objected to Simplicius and to Sagredo that he was following the Archimedean method according to which one can accept idealizations if they do not alter the substance of the analysis of the phenomenon, and to consider the earth's surface perfectly flat is legitimate, for practical purposes, such as the launch of a projectile, given the large radius of the terrestrial sphere. The same holds true for the motion that can be assumed orthogonal to the plane and not converging toward the center of the earth: "I admit that the conclusions demonstrated in the abstract are altered in the concrete [...]. But on the other hand, I ask you not to reject in our Author what other very great men have assumed, despite its falsity. The authority of Archimedes alone should satisfy everyone" (Galilei, 1638).[82]

The Archimedean method to which Galileo is referring to is the mathematical physics approach. Here some propositions derived from experiments are assumed as principles of a deductive theory; the principles when given by a mathematical statement are often charged with imprecision and sometimes approximations; however from them important results can be derived. One can imagine Galileo writing a treatise on motion on a terrestrial environment in an axiomatic way; he most probably would have assumed a principle like the following one:

> The horizontal component of the motion of an insulated (heavy or not heavy) body occurring on the earth would maintain constant its value, unless impediments such as friction from the air makes speed to decrease.

Concluding what lacks in Galileo to propose a law of regular motion close to Newton's principle of inertia is not so much that the motion is circular instead of straight—indeed in the mathematical theory it will be assumed as straight—but rather its being limited to the horizontal motion and to a terrestrial physics.

Before leaving Galileo and his laws of regular motion, it is a must to report some considerations from the sixth day of the *Discorsi* (published posthumously), where he considered the motion of two heavy bodies suspended with two ropes wrapped around a pulley; essentially a system that today goes by the name of Atwood's machine. Galileo assumed the particular case of two bodies of equal weight. In this case, if an impetus is imparted to one of the two bodies, both bodies will begin to move—one upward the other downward—with uniform speed which will be maintained if there are no impediments (in particular the friction of the rope on the pulley and the air resistance). Galileo assimilated this situation of inertial motion to that which occurs when an impetus is imparted to a body located on a horizontal plane.

> If the plane were not inclined, but horizontal, then this round solid placed on it would do whatever we wish; that is, if we place it at rest, it will remain at rest, and given an impetus in any direction, it will move in that direction, maintaining always the same speed that it shall have received from our hand and having no action [by which] to increase or diminish this, there being neither rise nor drop in that plane. And in this same way the two equal weights, hanging from the ends of the rope, will be at rest when placed in balance, and if impetus downward shall be given to one, it will always conserve this equably (Galilei, 1890–1909).[83]

Bonaventura Cavalieri and Evangelista Torricelli resumed Galileo's studies on the motion of projectiles, considering explicitly the possibility of a regular motion in any direction

[82] p. 244. Translation into English in Galilei (1974).

[83] Vol. 8, pp. 336–337. Translation into English in Galilei (1974)

in the case no weight would act on bodies. Notice that they did not claim that bodies can really exist deprived of weight but simply that one could imagine them as they did not have. Another Galileo's pupil, Giovanni Battista Baliani declared his adherence to the theory of impetus before in a letter to Galileo of 1639 then in the second edition of his *De motu naturali gravium* of 1646: "The impetus is the force for which a mobile is able to move without being acted upon by gravity or any other thing" (Baliani, 1646).[84]

Cavalieri investigated the trajectory of projectiles in *Lo specchio ustorio, overo trattato delle settioni coniche* (Cavalieri, 1632) of 1632 before Galileo published his own results in the *Discorsi*. In this text, in Chapter XXXIX devoted to the conic sections, Cavalieri wrote without giving it much emphasis that the regular motion of a projectile can be in any direction and not only horizontally.

> Moreover I say, that considering that the motion [of the body] pushed by a projector toward a given direction, if it had not other motive virtue that would push it toward another direction, [the body] should go in the place pointed by the projector for a straight line, because of the virtue impressed to it for a straight line. It is not reasonable that the mobile deviates from that direction, as long as there is no other motive virtue that deflects it, and if when between the two terminal points there is not impediment (Cavalieri, 1632).[85]

Cavalieri's law is closer to the Newtonian law, than the Galilean one. The main difference, I think, is its explicitness. Particularly when talking about the motion of a body on a plane Galileo never said explicitly that it moves/would move in a straight line. And in the fourth day he never spoke explicitly of motions launched in a different direction from the horizontal one. Even though Galileo could imagine a uniform rectilinear motion in any direction in a terrestrial if not cosmological context, "leaving the cannon its motion would follow a straight line continuing the alignment of the cannon" (Galilei, 1632).[86] if there was no weight.

Torricelli also studied the motion of projectiles in Book II of the *De motu gravium*.[87] In his hands the composition of motion became just a matter of geometry based on two motions which hardly challenge the adherence to reality: a uniformly accelerated motion directed downward and a uniform motion in any direction. His treatment of motion of projectiles is very similar to that found in modern treatises of mathematical physics. The proof of Proposition 2, concerning the motion of a bullet, is opened by the statement: "A mobile is thrown from A at any angle. It is clear that without the attraction of gravity it would proceed in a straight and equal motion along the direction AB" (Torricelli, 1644).[88]

Baliani studied the motion of a projectile even if limited to a horizontal throw, arriving at the conclusion that its motion is not described by a parabola but by a curve that lies below it. He divided the duration of the motion into small intervals of equal duration Δt; in each interval he assumed the superposition of two motions; an inclined motion with uniform velocity tangent to the described path and a vertical motion accelerated downward. The procedure is correct but Baliani was misled by a qualitative reasoning; if he

84 p. 57. The letter to Galileo is of 19 August 1639, see Galilei (1890–1909), vol. 18, p. 88.

85 pp. 154–157. My translation.

86 p. 201.

87 Published as a part of *Opera geometrica* in 1644 but almost certainly based on a manuscript dating at least to 1641.

88 *De motu proiectorum*, p. 156.

had written precise mathematical relations, as it was in his possibility even if he was a less good mathematician than Cavalieri and Torricelli, he would have also found a parabolic trajectory (Baliani, 1646).[89]

Cavalieri and Torricelli (and partially Baliani) followed Galileo in the route of mathematical physics. They could benefit from the results of the master and were able to get ahead of him because their mathematics had got ahead too. They were more mathematically oriented than Galileo and for them the mathematical physic theory (broad meaning) had it own reality. However, Cavalieri and Torricelli were more prudent than Newton; that is they made a limited use of the quantifiers *for all* and avoided the quantifier *forever* with regard to time used by Newton, and essentially their was a terrestrial mechanics without any commitment to cosmological considerations.

2.5 Jean Buridan and the Theory of Impetus

Going further back in time, the discontinuity it is decided to point out is found in the following propositions due to Jean Buridan in the 14th century:

> (A) A projectile, after leaving the thrower, is moved by impetus imparted by the thrower and moves as long as the impetus remains stronger than the resistance; and that the impetus would last forever if it were not diminished and destroyed by the opposing resistance or by the tendency to contrary motion (Buridan, 1518).[90]
>
> (B) In celestial motions there is no opposing resistance; therefore when God, at the Creation, moved each sphere of the heavens with just the velocity he wished, he [then] ceased to move them himself, and since then those motions have lasted forever due to the impetus impressed on those spheres (Buridan, 1518).[91]
>
> (C) And you have an experiment: If you cause a large and very heavy smith's mill to rotate and you then cease to move it, it will still move a while longer by this impetus it has acquired. Nay, you cannot immediately bring it to rest, but on account of resistance from the gravity of the mill, the impetus would be continually diminished until the mill would cease to move. *And if the mill would last forever without some diminution or alteration of it, and there were no resistance corrupting the impetus, perhaps the mill would be moved perpetually by that impetus* [emphasis added] (Buridan, 1942).[92]

Law (A) is similar, but only apparently, to the modern statement of the principle of inertia. Meanwhile it speaks of an impetus and not of a motion that lasts forever, which is at least an ambiguous statement. Indeed it is in fact probable that Buridan considered it less demanding the persistence of impetus than that of motion. Moreover in (A) it is not explicitly stated that the motion of the projectile is rectilinear (and with constant speed). While the statements (A) and (B) express metaphysical principles of natural philosophy, the statement (C) claims in the incipit to be of an empirical nature: "And you have an experiment". Actually the experiment in question is a thought experiment, similar to that of Galileo

[89] pp. 80–81.

[90] Book 12, question 9, f. 73r. Translation into English in Maier (1982).

[91] Book 12, question 9, f. 73r. Translation into English in Maier (1982).

[92] p. 180. Translation into English in Clagett (1959).

relating to the motion of a body on a horizontal plane. However, the reference to experience of statement (C) gives it a certain scientific character, at least according to modern standards.

The centuries preceding Galileo are complex enough to analyze; meanwhile, it must be said that the term regular motion is no longer suitable and can also be misleading, in the sense that in this period, in which Aristotle's ideas on the philosophy of nature prevailed, motion was classified only in two ways: violent and natural. The first was that of a bullet thrown by someone; it roughly corresponds to our regular motion, only it was not considered free but forced as will be clarified below. The second type of motion was the natural downward motion due to gravity.

The 16th century saw a flourishing of studies on ballistic due to the spreading of the modern artillery, that had some relevance for clarifying concepts about regular motions. They were carried out by engineers/mathematicians starting from Niccoló Tartaglia (1499–1557) (Capecchi, 2014),[93] until Thomas Harriot (1560–1621). The latter in one of his notes wrote this sentence: "Now I say because of the bullets gravity the crooked line is made. If the gravity be abstracted the motion would be only in the right line '*ad*': & if the resistence of the ayre [&] or medium be also abstracted his motion would be *infinitely onward* [emphasis added]" (Schemmel, 2008).[94]

What is of interest in the previous sentence is not so much the reference to a motion in straight line—this is a common stance, Tartaglia too made an assertion like this—but rather to "infinitely onward", that is the persistence of motion. Harriot's seems like a law of regular motion more advanced than Galileo's; but this is not the case as it appears clear by browsing Harriot's notes. Indeed, in the effort to get the trajectory of bullets, when composing the rectilinear inclined 'violent' motion and the vertical 'natural' motion due to gravity, the former was assumed as not uniform but retarded like the motion of an heavy body raising along an inclined plane (Schemmel, 2001).[95] This is a criticism that some historians raised against Galileo too (see previous section).

The various theories on violent motion, the only ones to be considered here, discussed from the 16th to the 14th century, differ both in how the phenomenon is described and in how it is explained. Regarding the first aspect, there were theories arguing that without impediments, for example in a vacuum, violent motion would have had no end, while others said that it would. The first position is the most interesting from the point of view of the present paper, because it is opposed to common sense and is the one that will most influence the scholars of the 16th and 17th centuries. Regarding the causes adduced to explain the violent motion, one can distinguish two categories; those that strictly followed the Aristotelian theses, for which it was the medium (the air for instance) to transfer the motion to the projectile and those that instead believed that it was the projectile that contained within itself the ability to move. In the following only the latter is considered which fall within the vast class of the theories of impetus.

About the history of the spreading of the theories of impetus there is a large bibliography; Clagett (1959)[96] reports a concise but well done summary, especially on the first ideas up to the 14th century. A more philosophically oriented analysis can be found in the writings of Annelise Maier, whose relevant passages are translated into English in Maier

[93] pp. 96–112.

[94] p. 35.

[95] pp. 235–236.

[96] pp. 505–515.

(1982). Though most studies on the history of impetus regard its relation of motion, also other situations have been explored, as signaled in Fritsche (2011).[97]

The concept of impetus is rooted in the 5-6th century CE with a Christian commentator of Aristotle, Ioannes Philoponus (c. 490-c. 570). Akin concepts are attributed to the pens of writers such as Hipparchus of Nicaea (2nd century BCE) and Synesius of Cyrene (4th century BCE), but Philoponus, who nowhere intimated he was influenced by any of them, was the most known. His writings, in particular his commentary on Aristotle *Physica* were translated into Latin only in the 16th century; the Greek text if ever known could have been read by only a few schoolmen; thus he was virtually unknown in the Middle Ages, even though his idea reached the western scholars through the commentaries to Aristotle's physics by Avicenna, Avempace and other Arabian commentators (Clagett, 1959).[98]

The theory of impetus found followers in the West with its most passionate supporters in the faculty of arts of the university of Paris in the 14th century. The protagonist was certainly Jean Buridan (deceased c. 1360), probably the most prolific of Aristotle's commentator of the entire 14th century. He made major contributions to logic, physics, metaphysics, and ethics (Thijssen, 2004); exposed his theory on impetus in the *Quaestiones super octo phisicorum libros Aristotelis* (Buridan, 1509) and in the *Quaestiones super libris quattuor de caelo et mundo* (Buridan, 1942). How much did he inherit from his predecessors, apart possibly from the reading of Tommaso d'Aquino's, Roger Bacon's and may be Franciscus de Marchia's (Francesco della Marchia c. 1290-a 1344) comments, is not known. To signal anyway two of Buridan predecessors, Joannes Canonicus and Nicholas Bonetus (c.a. 1280-1343) who were active in Paris in 1320 (Grant, 1981).[99] For a possible history of the theory of impetus in the West before Buridan see Schabel (2006).

Buridan was not alone to discuss on impetus; he confronted his ideas with other people who constituted, if not a school having him as a master, an intellectual community or network with him as a leader, at least for what the theory of impetus is concerned. The components of this community where Jean Buridan, Nicole Oresme (c. 1320–1382), Albertus de Saxonia (d. 1390), Themon Judes (fl. 1349–1360) and Marsilus von Inghen (c. 1330–1396).

Buridan shared an essentially Aristotelian cosmology. His world was finite, spherical and mainly it was a plenum. As Aristotle he thought unnecessary a container of matter (that is an external space), thus his space was internal; for an interesting discussion on some different conceptions of space and place of Buridan's time see Trifogli (2011). As many Aristotelians of his time he supposed the earth at rest, planets and stars to be moved by concentrical spheres.

Even though Buridan supposed that a vacuum was not allowed in the cosmos, he did not assume it as contradictory. According to Buridan there were two ways to conceive void; one as the container of matter, the other as actual void made possible by God, but that actually he did not make.

> There are two ways, then, that the void can exist by divine power. That is for me an item of faith and not a proof based on natural reason. I therefore do not intend to prove this, but merely to state how this seems possible to me. With respect to the first way of conceiving the void, I concede that God can make an accident without a subject, and that He can separate the accidents from the subjects that have them and

[97] pp. 22–23.

[98] p. 510. See also Franco (2003).

[99] p. 42.

> conserve them after having separated them; He can therefore create a simple volume (dimensio) without there being any substance coexisting with this volume [...] With respect to the second way of conceiving the void, I believe that God can annihilate the world below and conserve heaven as it is now with regard to its size and shape; hence the cavity of the lunar orb would be empty (Buridan, 1509).[100]

Buridan in his *Subtilissimae quaestiones super octo physicorum libros Aristotelis* of 1509 criticized the soundness of the various causes proposed by Aristotle and his commentators to explain the violent motion, arriving to propose his own explanation, by introducing what is now known as the *theory of impetus*:

> Thus we can and ought to say that in the stone or other projectile there is impressed something which is the motive force (*virtus motiva*) of that projectile. And this is evidently better than falling back on the statement that the air continues to move that projectile. For the air appears rather to resist. Therefore, it seems to me that it ought to be said that the motor in moving a moving body impresses in it a certain impetus (impetus)[101] or a certain motive force (*vis motiva*) of the moving body, in the direction toward which the mover was moving the moving body, either up or down, or laterally, *or circularly* [emphasis added] [...]. But that impetus is continually decreased by the resisting air and by the gravity of the stone, which inclines it in a direction contrary to that in which the impetus was naturally predisposed to move it. Thus the movement of the stone continually becomes slower, and finally that impetus is so diminished or corrupted that the gravity of the stone wins out over it and moves the stone down to its natural place (Buridan, 1509).[102]

The impetus is impressed in the direction toward which the motor moves, tends to decrease because of the resistance of air, gravity, frictions, obstacles and any other inclination to a contrary motion, and it is proportional to both the speed and the "quantity of matter".

> And by the amount the motor moves that moving body more swiftly, by *the same amount* [emphasis added], it will impress in it a stronger impetus. It is by that impetus that the stones is moved after the projector ceases to move.
> [...]
> Hence by the *amount* more there is of *matter* [emphasis added], *by that amount can the body receive more of that impetus and more intensely* [emphasis added] [...] And so also if light wood and heavy iron of the same volume and of the same shape are moved equally fast by a projector, the iron will be moved farther because there is impressed in it a more intense impetus, which is not so quickly corrupted as the lesser impetus would be corrupted (Buridan, 1509).[103]

The locution *amount of matter* in the previous passage deserves a comment. It resembles the concept of mass of a modern uneducated man; it is not the inertial mass of Newtonian mechanics and in addiction it is not a physical magnitude. If one wanted to assign it a measure he should make recourse to the property of matter of having weight. The more the

[100] Book 4, Question 7, f. 73v. Translation into English in Duhem (1985).

[101] Buridan introduced the term impetus only after 1352 in his final reading of the *Quaestiones super octo phisicorum libros Aristotelis*. This is most probably the reason for the absence of the term in (and Oresme)'s commentaries on Aristotle's *Physica* written before this date (Thijssen, 2004).

[102] Book 8, Question 12, ff. 120v–121r. Translation in Clagett (1959).

[103] Book 8, Question 12, f. 120v. Translation into English in Clagett (1959).

weight the more the quantity of matter. To note that in the science of weight, a science subalternate to mathematics, the locution quantity of matter is always substituted with weight, a better defined term.

Buridan also made comments on the ontology of impetus. Firstly impetus is not motion; otherwise the same thing could produce itself. Secondly impetus is a permanent thing, apart from that destroyed by resistance. Impetus is a (transient) form generated in the body which transfers to it the property of motion. It has an analogy with the form impressed on a heavy body while it is generated, but it may be destroyed by resistance (Weisheipl, 1965).[104] Buridan also made a comparison between the impetus impressed on a projectile and the quality impressed in iron by a magnet that moves it.

> The first [conclusion] is that that impetus is not the very local motion in which the projectile is moved, because that impetus moves the projectile and the mover produces motion.
> [...] The third conclusion is that that impetus is a thing of permanent nature (*res naturae permanentis*), distinct from the local motion in which the projectile is moved. And it is probable (*verisimile*) that that impetus is a quality naturally present and predisposed for moving a body in which it is impressed, just as it is said that a quality impressed in iron by a magnet moves the iron to the magnet. And it also is probable that just as that quality (the impetus) is impressed in the moving body along with the motion by the motor; so with the motion it is remitted, corrupted, or impeded by resistance or a contrary inclination (Buridan, 1509).[105]

Nowhere did Buridan say that the impetus was an internal engine, an internal efficient cause, or a soul, as many subsequent philosophers and modern historians attributed to him (Weisheipl, 1965).

As light, impetus can be reflected, so if a body encounters an obstacle with a given speed, it is reflected with the same speed; references to elasticity for the reflection are of interest. For instance a ball thrown to the hard ground is compressed by the impetus of its motion; and immediately after striking, it returns swiftly to its sphericity by elevating itself upward. From this elevation it acquires an impetus which moves it upward a long distance. The same holds with a cither cord which, put under strong tension and percussion, remains a long time in vibration.

Buridan suggested, but not said explicitly, that in the void where there are no impediments to motion, a projectile once it has left the projector, would last for ever because “impetus is a thing of permanent nature”. Thus a modern reader is led to see in his writings a formulation extraordinarily close to Newton’s principle of inertia. One would indeed be tempted to attribute to Buridan the following law of regular motion:

> A projectile once launched in a void space will continue its motion uniformly along a straight line for ever.

A closer examination, and also a warning on the fact that ideas from the past should be correctly framed in the historical period under examination, resizes the judgment. First of all Buridan denied the existence of a vacuum, and so in any real situation a projectile eventually will come at rest. Furthermore, there is not only the resistance of the medium that Buridan can imagine, but also that offered by gravity that opposes the motion. Furthermore,

[104] p. 44.

[105] Book 8, Question 12, f. 121r. Translation into English in Clagett (1959).

even if Buridan does not mention it there is an internal resistance due to an 'inertia' of the body, a tendency toward rest. At least this is the opinion of Annelise Maier who considers the tendency toward rest a widespread idea in the Middle Ages (Maier, 1982).[106] Basically, speaking of a motion of infinite duration expresses an extreme idealization that has nothing to do with reality and therefore is not very demanding ontologically. Second, in the imaginary situation of a cosmos reduced to a vacuum by God, an infinite rectilinear trajectory is at odds with a finite spherical cosmos. Could the trajectory eventually drill the first mobile? As a matter of fact, as in Gassendi and Galileo, in Buridan the theory of violent motion is restricted to a terrestrial environment; its laws are different from those of the heavens.

Furthermore one must consider that Buridan is not a 18th century mathematician, his world is not framed in Euclidean geometry and to interpret impetus, being proportional to both 'weight' and 'speed', as the product mv mass (m) times speed (v), that it with the modern momentum means to assume that he possessed a way to measure speed and mass; which probably was not the case.

In any case Buridan's message was very strong: no external force is needed to keep a body in motion. External forces, for example resistant ones, only serve to change the state of motion of the body. This message will be transmitted more or less directly to the following centuries. This even though the causal explanation of Aristotelian mould, in-between the 16th and 17th centuries, when the mechanicism was spreading, appeared at least questionable and for many a no sense.

Other laws hold for the regular motion of the heavens (see law B above). For them there is no cause which could diminish the impetus imparted by God at the creation, neither the gravity nor the resistance of the air; so Buridan could postulate for the possibility of a permanent motion. Though he considered it only a possibility, leaving the theological masters to state if this is true or not.

> Also, since the Bible does not state that appropriate intelligences move the celestial bodies, it could be said that it does not appear necessary to posit intelligences of this kind [...] But I do not say assertively, but [rather tentatively] so that I might seek from the theological masters what they teach me in these matters so to how these things take place (Buridan, 1509).[107]

The circular permanent motion could also be allowed in a terrestrial environment, without coming into contradiction with Buridan's cosmology, in a particular circumstance. This is the case for motion of bodies around a fixed axis (as the concentric sphere of the heaven), a smith's mill for instance free of friction (Law C). Here an infinite motion could not disturb Buridan, as there is no infinite trajectories. Notice that Buridan's inertial rotating motion could be framed into modern categories. Here a body rotating about a fixed axis is considered to behave exactly as a mass point free in the space. In the first case motion is described by means of torque and angle of rotation, in the second case by force and displacement. And as no force acts on a mass point it moves uniformly along a straight line so a body constrained to an axis with no torque rotates uniformly.

[106] p. 92, footnote 15.

[107] Book. 8, Question 12, f. 120v–121r. Translation into English in Clagett (1959).

2.6 Ioannes Philoponus: The Beginning

Going further back in time, the most evident discontinuity that is encountered is represented by the following proposition due to the Greek Neoplatonic commentator of Aristotle, Ioannes Philoponus of the 6th century CE:

> If things thrown by force are moved [...] it is surely clear that even if someone were to throw an arrow or a stone by force and unnaturally in a void [...] it will not need something pushing from outside (Philoponus, 2012).[108]

Ioannes Philoponus (ca. 490-575) was the first and probably the only universal mind of the culture of Christian Egypt, as evident from recent studies. For a quite exhaustive bibliography on him see Sorabji (1990, 2010).[109] This notwithstanding his writings were substantially unknown in the Latin Middle Ages, even though some of his ideas were known thanks to the Arab commentators of Aristotle, Avempace, Avicenna, Averroé, as already noted. The writings were translated into Latin and read by the scholars of the time together with those of Simplicius only from the 16th century on and they were appreciated much more than the Arabic excerpts due to the authority of a Greek tradition.

In this section I report some of Philoponus' ideas on space and motion that represent a break with the traditional philosophy not only of Aristotle. There were and there are scholars, a minority indeed, that do not agree on this view. In the Arab and also in the medieval Latin worlds there was a tendency to insert the theory of impetus in the Aristotelian tradition. After all it was argued that Aristotle had conceived a transmission of some energy into the medium while Philoponus into the projectile; thus there is only a different subject to which transfer the energy/impetus.[110]

According to a modern reader this reasoning is substantially correct, in both cases it is dealt with a transmission of kinetic energy but for a medieval and for a scholar of the 17th century, apart from the Aristotelian philosophers, the transmission to the air was much less mysterious than that to the projectile; it had a mechanistic explanation: the transmission of motion was due to successive impacts of the particles of air. In the scholastic language of causality it was distinguished between *causality by contact* and *causality by transference*, Aristotle's theory of motion would require the former kind of causality, while the impetus theory the latter. If one proved that in Aristotle there are examples of causality by transference, he could say that he had inspired the theory of impetus. This is what is suggested in Fritsche (2011), where the Aristotelian causality of transference is located in the biological writings regarding the sexual reproduction. Personally I am not very convinced by this kind of argumentation as the very root of an idea can be difficulty located and symptoms more or less evident of it can be found everywhere.

It must be said that the cosmological ideas of Philoponus were quite traditional and not so different from those of Aristotle; there were four elements (again earth, water, air and fire) even though their role was very different, and the most recent astronomical theories were taken into account, including those of Ptolemy. Philoponus' cosmos was a finite spherical plenum, with the earth spherical and at rest in the center; the heavens, mainly made of fire, rotate around the earth; outside the universe there was nothing. Moreover

[108] 642-1.

[109] pp. 497–499; 271–293.

[110] See for instance the argumentations of Franciscus de Marchia quoted in Clagett (1959, p. 529).

according the Christian theology, differently from the Greek tradition, Philoponus' cosmos was not eternal.

Fundamental to the formation of Philoponus' ideas on space and motion was the discord on Aristotle's criticisms of the existence of the void, referred to in that part of his commentary on physics known as *Corollaries on place and void* (Philoponus, 2010). However, despite the criticisms about Aristotle's thesis against void, Philoponus agreed with him that its existence was actually impossible. The difference between Aristotle and Philoponus lies in the fact that the former thought that the idea of a vacuum was inconceivable in itself, the latter instead thought it conceivable but did not believe that in reality it could exist.

Philoponus used the term void (*χενόν*) in two acceptances. On the one hand, void meant vacuum, on the other it simply meant space, conceived as the container of all bodies; he had thus an external conception of space. That was probably an idea not completely original with Philoponus, see for instance (Sedley, 2010),[111] but he was very clear on the matter. According to Philoponus the concept of space, although ontologically preceding that of void as the container of material bodies, could only be intuitively grasped by referring to vacuum; for this reason Philoponus could use void indifferently to indicate vacuum and space, as space in itself is a potential void.

With his idea of void-space Philoponus could affirm that the motion of a body was possible only if the existence of void is admitted as explained in the following passage in which the movement of a body through the air is imagined.

> An argument to show that motion is impossible without void. When a body moves through air, it displaces a volume of air equal to its own volume. But it would not need to displace anything, if it did not need a space equal to itself in volume to occupy, and the air would not need to be displaced from this extension if it did not need another space to move into. Hence the locomotion of a body requires a succession of void spaces. These extensions are not, of course, ever actually void: the volatile air fills them too quickly for a void to be left (Philoponus, 2010).[112]

Empty space is necessary for the displacement of the body, but this displacement does not determine a vacuum due to the presence of the air that takes the place of the body.

Closely related to Philoponus' notions of space are those of motion. These derived from the criticism toward the explanation of Aristotle on violent motion in which the medium (air) played a fundamental role. One of the possible explanations suggested by Aristotle for the continuation of motion was the *antiperistasis*, an explanation also accepted by Plato, based on the horror vacui: when a bullet moves, it tends to leave an empty space behind it, as void is not possible (for Aristotle) this space is immediately filled with air in a violent way which causes the bullet to be pushed forward. It was not difficult for Philoponus to find inconsistencies with this perpetual motion mechanism.

But if the presence of the medium does not serve to explain the violent motion, asked Philoponus, what is the cause of the continuation of motion? Would the motion also occur in a vacuum? In this case it would find itself in an ideal situation because there would be no resistance opposed by the medium. These considerations were the occasion for Philoponus to give his own explanation of motion, introducing what after Pierre Duhem has been called the *theory of impetus*, even though Philoponus did not use the term impetus.

[111] p. 181.

[112] 693,28, p. 180.

> From these and many other arguments it is possible to understand that it is impossible for things moved by force to move in this way, but it is necessary for some incorporeal power to be given by the thrower to the thrown thing, and that either the air pushed does not contribute at all to the movement or very little. If things thrown by force are moved in this way, it is surely clear that even if someone were to throw an arrow or a stone by force and unnaturally in a void, the same thing will happen much more, and it will not need something pushing from outside. Surely this argument will not be more difficult, being supported by what is obvious (I mean that some incorporeal kinetic force is given by the thrower to what is thrown, from which the thrower ought to touch the object being thrown) (Philoponus, 2012).[113]

In essence, according to Philoponus, the thrower endows the thrown thing with some energy, or force or impetus, that makes it possible to continue the motion. To justify this explanation Philoponus gave the example of the energies arriving on the eyes from the things seen, attributing this idea to Aristotle. For one sees from colors some energies arriving incorporeally and coloring the solid bodies existing in front of them. Next, added Philoponus, "if what is thrown is the stone or the arrow what prevents our throwing something with what is between being void" (Philoponus, 2012)?[114] To conclude: "It is clear therefore that some *energies* [emphasis added] come bodilessly from some things to others" (Philoponus, 2012).[115]

A modern reader finds Philoponus explanation quite naive, but fascinating. Much more interesting than the explanation given by Buridan with his complex convolution using the Aristotelian category of form to introduce impetus. Energy in Philoponus has not a technical meaning, but calls forth something similar and refers to a known causal mechanism. As in Buridan there is no explicit reference to a trajectory along a straight line, differently from him there is no quantification of energy with respect to mass and speed. As Buridan, even though in a less explicit way, the energy would last for ever without impediments. A *medieval reader*, educated in the philosophy of Aristotle, found the explanation of Philoponus quite a bizarre one, a play of words, and many criticisms were raised to it. Tommaso d'Aquino in particular rejected the existence of a force impressed into the projectile for the fact that this theory would place the origin of motion within the projectile, with the result that in this case a violent motion would have an internal source, which appears contrary to the nature of the violent motion (Clagett, 1959).[116]

As thrown bodies continue their motion, so do the heavens. Indeed Philoponus in his early writings supposed that heavens were moved by a soul, but in his mature writings he assumed that God at the creation imparted the heaven an energy which lasted for ever. On the purpose he said that there were nothing more ridiculous than to think of angels having the patience to carry along so many and so heavy bodies for such a long time and with the use of a great force (Sarnowsky, 2008).[117]

In the *de Opificio mundi*—a mature text, largely devoted to the refutation of the presence of souls in heavenly bodies—Philoponus completely gives up the doctrine of celestial soul. According to him it is impossible to say that the celestial bodies are moved by angels. For how could bodies of that size continue to be moved over such distances by an unnatural

[113] 642-1, 642-10.

[114] 642-10.

[115] 642-20.

[116] p. 517.

[117] p. 125.

moving force? Why would it be impossible for God, who created heavenly bodies, at the same time to put in them a natural force to make them move in a circle, a force similar to the heaviness and lightness in bodies that move in a straight line? (Verrychen, 1990).[118]

The energy acquired by the moving body is exhausted by the resistance opposed by the medium, for example air. But would it be exhausted even if there were no such resistance, for example in a vacuum? On this point Philoponus is not explicit. In the following quotation, where one of Aristotle's arguments against the existence of the vacuum is commented, according to which if there was a vacuum the motion of a moving body could never be stopped and therefore would last forever (Aristotle, 2018a),[119] Philoponus declares that even in a vacuum the energy impressed to a projectile could decrease, thus opening without being very explicit, to the hypothesis of an impetus that is exhausting.

> I say that just as you who think that the cause of unnatural movement is the thrust of the air, say that it moves so far until the kinetic power given to the air from what originally pushed it expires, in this way clearly even if something were to move unnaturally, in the void it would move so far until the kinetic power given to it by the original thruster was exhausted (Philoponus, 2012).[120]

Historians have long wondered how Philoponus came to his theory of motion. One of the most popular ideas is that he took his ideas from Hipparcus of Nicaea, an astronomer who lived in 2nd century BCE, seven centuries before him. This hypothesis is supported by some historians, but is contrasted by many others. It, although suggestive, does not seem well founded. The main point is that the ideas of Hipparcus are only known through the commentaries of Simplicius, and refers to accelerated natural motion. According to Hipparcus, as reported by Simplicius, any acceleration in the fall of bodies presupposes a cause which acts contrary to downward motion. When dropping a heavy bodies the 'force' which held them back remains with them up to a certain point, and this accounts for a slower motion at the start of the fall. A modern reader can see in the above as if some energy were transmitted to the bodies which vanishes gradually generating an accelerated downward motion. And this energy has some analogy with the energies of Philoponus. A careful reading suggests however that this interpretation, the same of that held by Capecchi (2014),[121] hardly can support Philoponus theory (Wolff, 2010).[122]

Another explanation, with enforced the assumption of Philoponus' originality, is equally suggestive, but perhaps even more difficult to sustain, uses sociological motivations. According to Wildberg (1999), a German scholar whose pioneering work on Philoponus have been very influential, the impetus theory is not at all the result of exclusively philosophical and cosmological speculations, the origin rather is to be found in a change of the social and economic status of slaves which took place in late antiquity. The impetus theory is based on two very different ideas which together constitute it. The first is that whenever an agent generates a motion, a force is being parted with; this idea springs from the common experience of physical labour being exhausting. The other component of the theory is the idea that a force is transmitted onto the moved object and subsequently resides in it for the rest of the motion. This last assumption is, according to Wolff, quite fantastic and

[118] p. 269.

[119] 215a.

[120] 644-17, 644-20.

[121] p. 169.

[122] pp. 140–145.

cannot at all be derived from the context of common experience. According to him, it is exclusively conditioned by its economical counterpart, that is the notion that the value and price of a product is dependent upon the amount of work needed to produce it (Wildberg, 1999).[123]

It must be said that if it is a dogma of the history that there is always an antecedent to something, in the case of Philoponus it is difficult to find a plausible predecessor and from what it is known today his is the most ancient statement with some similitude with the modern principle of inertia. At least for macroscopic bodies.

2.7 The Metaphysical View of the Atomists

Going further back in time the following propositions due to the atomist Democritus 5th century BCE is encountered, meaning by "Democritus" not so much a historical character but rather a labels of ancient atomism.

> (A) These atoms [...] travelled about in the void overtaking one another and colliding, and some rebounded at random (Taylor, 1999).[124]

> (B) That motion of atoms must be understood as having had no beginning, but as having gone on from eternity (Taylor, 1999).[125]

It could be contested that they represent laws valid for a macroscopic body and that they hold for invisible atoms only and therefore they are propositions of a metaphysical nature. Therefore the oldest documented formulation on the regular motion of a macroscopic body would remain that of Philoponus. This is true, but it is also true that here for the first time a motion in the void is postulated that perseveres without external intervention, which is the essence of the principle of inertia.

To try to understand what Democritus actually meant by the above proposition one has only a limited number of fragments to be considered reliable. Moreover, these fragments are not autographs, but reported, no one knows how accurately, by authors who lived in a period ranging from the 4th century BCE to the 6th century CE, such as Aristotle, Sextus Empiricus, Cicero, Simplicius and so on. Even in this situation very general statements sufficiently reliable are possible. After all it is not important what Democritus meant exactly, but how he was read by the scholars who followed him. And certainly the message that speaks of bodies moving forever without any cause was received by the scholars of the 17th century as a strong statement to oppose the dominant point of view according to which a body could move only if pushed by a force. And this message is still received today in the same way; for instance Federigo Enriques (1871–1946) collects in a section entitled "principle of inertia" the most significant fragments concerning the motion of atoms (Enriques & Mazziotti, 2016).[126]

[123] pp. 112–113.

[124] p. 87. Simplicius, commentary on *De caelo*.

[125] p. 89. Cicero, *De finibus*.

[126] pp. 93–97.

Democritus considered space as external,[127] unlimited, empty, populated by invisible hard atoms,[128] that are infinite both in number and in shape.

> Democritus thought the nature of the eternal things consisted of small substances infinite in number; these he placed in space, *separate from them and infinite in extent* [emphasis added]. He called space by the following names: *the void*, *nothing*, and *the infinite*, and each of the substances he calls *thing*, *the solid*, and *what is*. He thought that the substances were so small as to escape our senses. They have all kinds of forms and all kinds of shapes and differences of size. Now from these as elements the visible and perceptible bodies are generated. He says that they conflict with one another and travel about in the void because of their unlikeness (Taylor, 1999).[129]

In the above quotation Simplicius assumed that space, void and nothing synonyms. Actually things are much more complex, or if you like not very clear. Void (or nothing) had some degree of reality for Democritus; 'nothing' is not really nothing but something of real, something that can possibly occupy space, and the debate on this subtle concept—and consequently on that of space—has been in progress since the time the atomist assumed the existence of void. In any case, even if there was a difference between space and void the two concepts inevitably flow together, and Simplicius's flaw is not simply a misunderstanding.

Atoms move in every direction, courting each other until they collide. There is no explicit reflection on the nature of space in the above quotation; the fundamental concept is in fact that of motion, considered real (absolute) without posing any problem about its origin. But, even if not specified, it seems reasonable that Democritus thought of an infinite space—because it has to contain infinite atoms –, isotropic, uniform like Newton's absolute space. But even it were not his thought, the reading of his writings by early modern philosophers and mathematicians, gave the impression of an absolute motion in an absolute space.

Many historians believe that Democritus did not pose the problem of the cause of motion and considered it as natural. This is for instance the opinion of Aristotle accusing Democritus not to have faced the theme of the cause of motion (Aristotle, 2018b);[130] others identify the cause of motion in the weight following the testimony of Simplicius for instance: "The followers of Democritus and later Epicurus say that all atoms are of the same nature and have weight, but because some are heavier they sink down and in so doing push the lighter ones up, thus, they say, making it appear that some are light and some heavy" (Taylor, 1999).[131]

The above term weight must be interpreted; it may be a macroscopic effect as referred to in the mechanicistic explanation of gravity in the 17–18th centuries. But it may be an "occult faculty"; according to Vittorio Enzo Alfieri it would be associated with the

[127] It should be noticed that according to some interpretations Democritus probably could not conceive atoms immersed in an empty space as a container but rather he thought about an alternation of void and atoms; where void had some form of reality for Democritus, and both atoms and void were in a place (Jammer, 1993; Sedley, 1982), p. 11; pp. 179-182.

[128] Johannes Stobaeus (after 5th century CE) stated however: "And he [Democritus] said that it was possible for an atom to be as big as a world" (Taylor, 1999, p. 88). Note that Democritus did not use the very word atoms, but rather solids, beings, substances; see Pagano and Pagano (2022, p. 7).

[129] pp. 70–71. Simplicius, commentary on *De caelo*.

[130] 1071b.

[131] p. 89. Simplicius, commentary on *De caelo*.

tendency for atoms to approach each other which holds only for atoms of similar shape—given that the shape together with the size is the only characterization of atoms—according to the criterion also accepted by Plato for which the like calls for the like (the so called like to like principle). This mechanism to work, would require that the atoms are not too distant from each other (Alfieri, 1953).[132]

However, there are fragments that allow us to state that both motion without a cause and motion with a cause exist. The former situation would occur in the pre-cosmic situation, when all atoms are far from each other (a situation of isolation as required by the modern statement of the principle of inertia); the latter situation is valid in the newborn worlds where atoms are close to each other and weight can act. More precisely:

1. There is pre-cosmic motion, which is the original condition of isolated atoms which move in all directions: in this sort of motion the atoms can collide, bounce from the impact, just like the dust one sees dancing in a ray of sun.
2. There is the cosmogonic motion, which is precisely shaped in vortex and is produced when, due to the presence of a vacuum of considerable size there is an influx and a concurrence of atoms of various shapes and sizes in that free space: this concurrence of material elements of different mass produces a whirlwind movement, in which, by operating the law of aggregation, the vortex works as a sieve, so that the heavier elements are arranged in the center of the vortex while the more minute ones are dispersed toward the external void. Thus the earth is formed, which Democritus conceived of a non-spherical drum-shaped one as, indeed, Anaximander.
3. There is the motion of atoms in the cosmos, as far as they can move as individuals, detaching themselves from the aggregation of any compound body of which they are part: such are the scents, for example, of perfumes (Alfieri, 1953).[133]

3 Conclusions

In this work examples of statements that can be associated with the principle of inertia for a period of time of over 1000 years from Philoponus to Newton are considered. The scholars of reference are all well studied and known for their important contributions to the principle of inertia: Newton, Descartes, Gassendi, Galileo, Buridan, Philoponus; for the sake of space, many other philosophers and mathematicians have been neglected, some of whom, like Huygens for example, might have had the same title of merit as those considered.

Countless works by historians of science have been written on the principle of inertia as stated by the considered scholars; the paper, however, has not summarized their conclusions but rather has critically reviewed the various statements as reported in the original works, arriving to some new achievement.

From the examination of the various formulations associated to the principle of inertia it can be concluded:

[132] p. 91. According to Alfieri there are no reason to believe in atoms embedded in vast empty spaces; atoms could be quite close to each other. That is Democritus' universe would be not so empty of matter as one is led to think.

[133] pp. 84–85.

1. The modern version of the principle of inertia is due to Newton, the only one who gave the same value to the persistence of motion and to the maintenance of direction.
2. Since Philoponus there has been a constant reference to the persistence of both rectilinear and circular motion, the persistence of direction being considered less interesting.
3. Many of the statements have a metaphysical character; that is they are justified without reference to observable situations. One of the first statements that refers to a thought experiment is due to Buridan, who gave the examples of the persistence of the motion for a body rotating around its axis. Galileo, however, was the first to introduce the principle of inertia on an experiential basis for a linear motion. He did it with a very convincing thought experiment, taking a simple situation: a sphere that rolls on the horizontal plane and therefore in a terrestrial context. Galileo's law of inertia was not an end in itself but rather a principle of a mathematical physic for a primordial dynamics.
4. Newton and Gassendi made quite demanding assumptions about the ontology of space, taking a clear substantialist position: space is empty and immobile. Philoponus also took a position not too far from that. Descartes and Buridan had instead an relationalist view of space. For the latter, there were no major problems in defining the motion of a body since it has the fixed earth as its reference; the situation of Descartes who did not have this support was more complicated. Galileo is the one who posed less difficulties and in fact ignored the problem of the ontology of space, considering it from an intuitive point of view.

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Danilo Capecchi Born 1948, Italy. Degree on engineering, mathematics and philosophy. He has been full professor of solid mechanics of the University of Rome La Sapienza until 2018. He his author of many papers on structural mechanics, mathematical physics, history of mechanics and history of physics with particular focus on foundation aspects. He wrote books about structural mechanics and history of mechanics and physics. Among them: D. Capecchi (2012) History of virtual work laws. Birckhäuser, Milan D. Capecchi (2104) The problem of motion of bodies, Springer. Doerdrecht D. Capecchi, G. Ruta (2014) Strength of materials and theory of elasticity in 19th century Italy. Springer, Dordrecht D. Capecchi (2018) The path to post-Galilean epistemology. Reinterpreting the birth of modern science. Springer, Dordrecht D. Capecchi (2022) Epistemology and .natural philosophy in the 18th century. The roots of modern physics. Springer, Dordrech.