



Is Identity Really so Fundamental?

Décio Krause¹ · Jonas R. Becker Arenhart¹

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Abstract

We critically examine the claim that identity is a fundamental concept. According to those putting forward this thesis, there are four related reasons that can be called upon to ground the fundamental character of identity: (1) identity is presupposed in every conceptual system; (2) identity is required to characterize individuality; (3) identity cannot be defined; (4) the intelligibility of quantification requires identity. We address each of these points and argue that none of them advances compelling reasons to hold that identity is fundamental; in fact, most of the tasks that seem to require identity may be performed without identity. So, in the end, identity may not be a fundamental concept after all.

Keywords Identity · Indiscernibility · Fundamentality · Quantum mechanics

1 Introduction

In a recent paper, Bueno (2014) has raised several arguments to the effect that the concept of identity should be taken as fundamental in a specific sense of the word (to be made more precise soon). Closely associated with his claims about the fundamentality of identity, in particular, Bueno challenges the so-called Received View on quantum particles' non-individuality (see below), i.e. the interpretation of non-relativistic quantum mechanics according to which the theory deals with non-individual entities; that is, entities not satisfying a principle of individuality, in one of its many formulations (in fact, there are plenty of formulations and disputes on how precisely to formulate a reasonable principle of individuation; non-individuals are particular items that do not satisfy the conditions for being individuals in one of those characterizations; for details, see French and Krause 2006, Chap. 4; Arenhart 2017).¹ In this paper we shall examine Bueno's claims to the effect that identity

¹ For instance, individuality could be characterized by requiring that an individual must exemplify an intrinsic property that discerns it from every other item (following Caulton and Butterfield 2012). In this case, the claim that quantum entities are non-individuals (that is, that they fail this specific condition) could be assumed also in quantum field theories, at least if we agree with Wolfgang Ketterle (Ketterle 2007), for instance when he says that "Electrons everywhere in the world are excitations of the same field and therefore they are absolutely identical." Notice that here Ketterle uses "identical" in the physicists jargon, meaning "indiscernible". In this case, of course, what Ketterle means is that no such intrinsic property to grant individuation can be found.

✉ Jonas R. Becker Arenhart
jonas.becker2@gmail.com

¹ Department of Philosophy, Federal University of Santa Catarina (UFSC), Florianópolis, Brazil

is fundamental and argue that they are not enough to establish the general thesis intended by him. In particular, his claim that identity is fundamental involves the idea that identity should hold universally; to this we shall also oppose ourselves. Since Bueno's ideas seem to be representative of common beliefs held by most philosophers, we think that our arguments may be useful for questioning some of the most widely held conceptions about the role of identity in metaphysics.

There are four related claims that are put forward in order to establish the fundamentality of identity. Bueno (2014) claims that: (1) identity is presupposed in every conceptual system; (2) identity is required to characterize individuals; (3) identity cannot be defined; (4) the intelligibility of quantification requires identity. In the end, as we have already remarked, Bueno (2014) discusses the plausibility of an interpretation of non-relativistic quantum mechanics according to which it makes no sense to attribute identity to its basic entities. His claim, in a nutshell, is that this interpretation is implausible, given the fundamentality of identity he sought to establish with theses (1)–(4). So, as a by-product of the fundamentality of identity, the Received View, according to Bueno, fails in advancing an appropriate metaphysics for quantum mechanics.

In the next section we shall go through each of the four reasons advanced by Bueno in his attempt to grant the fundamental character of identity and we try to bring to light what we believe to be their weaknesses (henceforward, whenever we mention Bueno with no further specification, it is to the (2014) paper that we are referring). The very idea of what is meant by the “concept of identity” is not completely clear in Bueno's paper, and the precise notion of what does it mean for a concept to be “fundamental” is not presented either. The underlying idea seems to be that identity is fundamental because it has the features presented in the four mentioned cases, so that it cannot be eliminated and no metaphysical system—and as a result, no interpretation of quantum mechanics either—can be formulated without the use of a universally applicable relation of identity as a basic concept that applies to every entity concerned. In this sense, we shall consider that Bueno defends that identity is fundamental because of the four reasons he advances, and we shall not engage in the debate on fundamentality that relates to the more specific notion of grounding, although much of what we discuss here bears close relation to that debate (see Shumener 2017 for discussion of identity focused on fundamentality and grounding.)

2 Identity may not be so Fundamental

In this section, we present and discuss the merits of each of the arguments advanced by Bueno to establish the fundamentality of identity. Recall once again that according to Bueno, the fundamentality of identity consists precisely in the fact that these arguments hold.

2.1 Identity and Conceptual Systems

Bueno begins his paper by considering the role of identity in conceptual systems. The application of concepts, Bueno says, requires identity. He begins by describing the role of concepts: “[t]he most basic feature of concepts is to demarcate certain things from others, to draw a line between those things that fall under that concept and those that don't” (2014, p. 325). In order to achieve this, identity is thought to play an essential role: “[c]oncepts are used to classify objects, to make distinctions among them, and to group them together [...] [and this] demand[s] identity” (*idem*). There is also a second claim concerning objects

falling under the same concept, which requires identity because to “lump certain things together requires that they fall under the *same* concept” (*idem*).

To deal more precisely with those issues, from now on we would like to distinguish the two claims more sharply. The first claim concerns the *identity of objects* falling under a concept; the second claim concerns the *identity of the concepts* themselves. We shall treat the two cases separately.

The first argument looks like a transcendental argument, attempting to establish that without a notion of identity for objects there would be no possibility of applying concepts. We may formulate it as follows: in order to determine the extension of a concept, we must determine also its complement. Things that fall under a concept cannot be also in the complement of the concept: those are *distinct* things. So, identity seems to be required in order to *distinguish* the items in the extension of a concept from the items belonging to its complement. To illustrate this point, let us assume that a concept C is given together with objects o_1 and o_2 , so that o_1 falls under C and o_2 does not fall under C. In this case, o_1 is distinct from o_2 . Identity is required in order to make sense of that, given that it is identity that enables meaningful application of concepts (by allowing such a distinction between the extension and its complement).

So, if this represents correctly what Bueno had in mind with his first claim, our first complaint against this line of reasoning is that it begs the question against those that do not recognize the fundamental character of identity in the sense under scrutiny now. In fact, for those not wanting a commitment with identity, the situation may be analyzed in an alternative way that dispenses with identity. If that is correct, then identity is not required in order to apply concepts meaningfully, and the first part of the first argument fails. Let us see. Our suggestion is that everything Bueno claims identity should do may be done by employing a weaker notion of discernibility: given that C distinguishes between o_1 and o_2 , they are discernible. Does discernibility imply distinctness, so that the use of identity is really unavoidable? Well, it depends on our understanding of the relation between identity and (in)discernibility (more on the relation between identity and indiscernibility also in the next topic).

On a first attempt to spell the relation between identity and indiscernibility, one could be a reductionist on identity and analyze identity in terms of indiscernibility, as Quine did (and more recently Saunders 2003; see also Ketland 2006; Caulton and Butterfield 2012; and the discussion in Shumener 2017). In this case, we have identity, but it is no longer fundamental, it is rather a derived concept. Against this move Bueno would complain that identity is undefinable, and we shall deal with this claim later (see Sect. 2.3).

There is another option, however, more radical than defining identity through indiscernibility: assume indiscernibility as a primitive term and recognize that it does not collapse with identity. We believe that the fact that indiscernibility can be analyzed without necessarily implying identity in some systems of logic shows that there is not a necessary equivalence between these notions: the fact that two items are indiscernible does not imply that they are identical. At the very least, it is logically possible that the relations of discernibility and difference are not the same, with discernibility being a weaker notion. In this case, there is an alternative way to understand the situation envisaged by Bueno without necessarily using identity. If this is correct, then identity is not really fundamental in the sense of being required for the meaningful application of concepts. The transcendental appeal of the argument goes by the board.

Even though we do not enter into the details of the difference between identity and indiscernibility in this moment, there are several systems of logic which we could call upon here to substantiate our claim, and which keep discernibility and numerical difference

apart: Schrödinger logics and quasi-set theory (see French and Krause 2006, Chaps. 7 and 8; Arenhart and Krause 2017), and also Wittgenstein logics, in which there is no identity (see a discussion in Wehmeier 2012). In quasi-set theory, for instance, indiscernibility is a primitive binary relation. It does not collapse on identity, because it is not compatible with set membership. In other words: two items may be indiscernible, but that does not mean that they necessarily belong to the same collections. So, one item may be indiscernible from items inside a collection, without itself being there. Recall that identity demands compatibility with every predicate and relation, in particular with set membership. Given that in quasi-set theory this demand is not met by indiscernibility, it is indeed a relation that is weaker than identity.

But one needs not even change logic in order to dissociate identity from indiscernibility. Even first-order classical logic with identity using so-called *non-normal models* sometimes interprets the symbol of identity by a relation that is mere indiscernibility (see Mendelson 2010, p. 93; and again, see Sect. 2.3). That means, roughly, that in such models the symbol of identity is interpreted in a relation R that is “coarser” than the diagonal of the domain; entities a and b in the domain of interpretation may be related by the relation R (which plays the role of identity in the model, recall), *without* it being the case that a is the same as b . In this sense, a and b are indiscernible in the model in question.

Now, that last point may be strengthened to an even more interesting underdetermination argument. Suppose that one could really translate our natural language (or fragments of science and metaphysics that one would happen to be interested in) to a first-order language *with identity*, so that we could “talk” in such language just as a natural language (call this idiom *logiquese* or *formalese*, if you will). Does that automatically entitle us to say that identity would hold overall? Not really: by simply judging from the sentences employed there is no way one could discern a normal model from a non-normal model (again, see Mendelson *loc. cit.*); in short, there is no set of sentences capable of discerning a model in which the identity symbol refers to identity and a model in which it refers to a weaker notion of indiscernibility. No evidence can be adduced in favor of one interpretation against the other. So, when using identity in order to apply concepts, if Bueno is correct, there would be no evidence that it is really identity that is being meant: one cannot grant in this scenario that one is not part of the underdetermined situation we have just described and that is allowed for first-order logic with identity sign. To settle that issue, what is required is that further metaphysical issues concerning reference be settled, because this point is not settled by appealing to our linguistic practices.

So, summing up, our first point against the first argument is that it fails to establish its point. Everything that identity is supposed to do for the application of concepts may be done also by a relation of indiscernibility, which, as we have argued, needs not collapse with identity.

As our second complaint about the first argument, we point to one undesirable consequence of that view: it is too strictly tied to a conception of negation as complement. This view is obviously the one held by the majority of philosophers in the actual philosophical scenario, but it would render an intuitive interpretation of paraconsistency—and along with it, certain versions of dialetheism—untenable. In fact, consider a “contradictory object”, like Russell’s set R , one of the favorite examples of paraconsistentists (one can take as example any of the so-called “contradictory objects” available in the literature; Russell’s set is a good one because even those paraconsistentists not accepting that a contradiction may obtain in concrete reality would accept that Russell’s set is a contradictory object). Now, R both satisfies the concept defining R and does not satisfy it (the concept defining R is “does not belong to itself”, obviously). So, if paraconsistent set theories allowing

sets like R are supposed to make sense, then they cannot accept Bueno's account of how concepts are applied. Otherwise, they won't be able to make sense of their contradictory objects, because by definition the set R must belong to its extension and also not belong to its extension, a situation ruled out by the view on concepts advanced by Bueno. Alternatively, if one does not want to be committed with extensions in this pathological case, sticking only with concepts, one may keep with R falling under the concept defining R and also not falling under this concept. On Bueno's account of the use of concepts, it seems, this would imply either that this application of concepts is meaningless or else that R is distinct from itself. None of the options is good enough.

One way to get out of this situation would be to bite the bullet and rule out paraconsistent logics along with such an intuitive semantics. That would mean that one renders analytic the fact that paraconsistent logics are meaningless. Now, although some philosophers would find this the correct conclusion, this is not a palatable move, we think, in the age of logical pluralism, although we shall not pursue it here. An alternative option consists in changing the interpretation of how we apply concepts, so that paraconsistent objects can make sense after all. But this would rule out Bueno's account of how concepts work (along with its allegedly fundamental use of identity). In fact, one would have to adopt an approach like Priest's (2006, p. 75ff), in which it is suggested that the overlap of a concept and its negation can sometimes mean something, but not everything and not nothing. That, of course, is not how Bueno describes concepts. A third way would be to keep the conclusion that R is really distinct from itself, but it is very difficult to make sense of this claim from an intuitive point of view, independently of whether one thinks that identity is fundamental or not. In any case, either one gives up the idea that identity is fundamental by changing the approach to the use of concepts, or else one accepts the unpalatable consequences.

Now, going directly to the case that concerns us most, that is, the case of quantum mechanics, we shall point out that the use of concepts in this domain is illustrative of how identity is not fundamentally involved or required, contrarily to what Bueno defends. In fact, it is usually held that the properties of quantum objects are not discovered by inspection; for instance, Dalla Chiara and Toraldo di Francia have suggested that quantum objects are *nomological*, given by physical law, and that all objects of the same kind obey exactly the same laws that characterize them, so they could be discerned by none of such qualities (this notion, and its difficulties, is further discussed in French and Krause 2006, p. 221ff). In other words, we have clear classifications of these entities inside a conceptual system, even facing the fact that they are indiscernible from each other (without being identical). Clusters of properties (denoted by concepts) are formed and constitute the kind of a particle. Everything instantiating those properties is an example of such a kind. No two particles of the same kind are identical, although there is nothing to discern them. Identity, in the described sense, is not required in the quantum domain, for what we need is a criterion for something to be, for instance, a positron, or a Z particle, and we do not require the identity (in the above sense) of each of these particles themselves. But then identity is surely required in order to distinguish positrons from Z particles, right? Not really; this can be taken as being merely a way of speech. As suggested above, all we need is that positrons and Z particles be *discernible*, a weaker notion, and this is granted by their nomological characterization. In regimenting natural language for metaphysical purposes, it suffices to use discernibility, and not necessarily identity (again, see Krause and Arenhart 2015; also French and Krause 2006, Chaps. 7 and 8). That is, one could make sense of the claim that concepts are applied and that some entities have discerning kinds without the need of identity.

This claim may be generalized when we follow one of the most typical approaches to quantum properties. According to this approach, in quantum mechanics we do not attribute properties to particular entities, but to *species* of entities. The preparation of a state does not put a particular entity in a particular state. The measurement process must be repeated many times, and all we can do is to ascribe a property to a class of entities of the same species (see the discussion in French and Krause 2006, pp. 106–107). In this case, again, all that is needed is discernibility, and we don't have to (and cannot, in fact) select a unique particular to be the bearer of a property.

Before we proceed to the second claim concerning the identity of concepts, notice that there is something very strange in the approach to the understanding of concepts developed by Bueno. In fact, let us concede that the identity of the objects in the extension of a concept is required in order for us to make sense of that concept. But now, what can we say about the concept of identity itself? It applies to everything, if Bueno's claims are correct. However, in order to make sense of this concept we need... the concept of identity! Then, the options are clear: either we don't understand the concept of identity (because it presupposes itself and we are trapped in a vicious circle), or else Bueno's account of the application of concepts does not require identity in some cases (like the case of identity itself), so that identity is not really fundamental as claimed. You choose.

Now, let us go to the *second* of Bueno's claims, the claim concerning the identity of concepts. According to this claim, recall, when objects o_1 and o_2 are similar on one specific aspect (described by a concept), this would only happen because they fall under the *same* concept. So *identity of concepts* would be required for the very application of concepts. For instance, when we say that Plato and Aristotle are philosophers, they must fall under the same concept "being a philosopher". In this sense, there must be identity for concepts too.

Our view on this issue is, once again, that identity is not really fundamental. First of all, if concepts are understood extensionally, then their identity will depend on the identity of the objects that fall under them (on an intuitive understanding of extensionality). This won't fit very well with the claim that individual objects must be discerned by properties, because then the identity of objects depends on their properties, and the identity of properties depends on objects, giving rise to a circularity (see our discussion on individuality in the next section). Now, supposing that the identity of objects is taken as primitive, this suggestion also does not fit very well with Bueno's claim that identity is fundamental for concepts, because in this case the identity of concepts would be *definable* in terms of the identity of the objects falling under them, something that cannot be done in the case of fundamental concepts (see further ahead). On an intensional understanding of concepts, on the other hand, it is notoriously difficult to account for the identity of concepts (recall the discussions by Quine, for instance), and given that no such account was advanced by Bueno, we believe that he unintentionally was thinking in extensional terms.

Consider now more carefully the extensional understanding of concepts. In this case, the claim that concepts require the identity of objects as a fundamental feature (or that identity of concepts is fundamental for the application of concepts in some way) has some interesting consequences that are well-known in the literature. First of all, what we would intuitively classify as distinct concepts are in fact the same. Concepts that apply to nothing, like being "different from itself" or "being green and not green" are just the same concept (from an extensional point of view). The same holds for concepts that apply to everything, like "being green or not green" and "being green, if green". That seems to be bad news for someone claiming that concepts have some kind of identity. There is nothing that distinguishes those concepts in this extensional approach, as required by the view.

That also spells trouble for the claim that concepts apply by distinguishing between their extension and its complement. There is simply nothing to be distinguished in those cases: everything is in the complement of those concepts that apply to nothing; everything is in the extension of those concepts that apply to everything. However, it seems that we understand the use of such concepts without requiring identity.

To advance even further our claim that identity is not fundamental in this case, we hold that the described situation could be analyzed in an alternative way. To say that a concept like “being a philosopher” applies to Aristotle and Plato does not require anything like the identity of the concept “being a philosopher”. For, changing now to the material mode, going from concepts to their metaphysical representatives, one could for instance be a trope theorist, and deny that it is the same trope that applies to both individuals, as trope theorists in fact do. What is needed in this case is a relation of similarity between tropes that accounts for their “looking alike”, a relation which, however, does not collapse on identity. We are aware that trope theories are not without their own problems; anyway, they serve to emphasize that one cannot use the fact that the same linguistic entity (a concept) is being applied to distinct names to ground the claim that such linguistic fact has an ontological counterpart (a universal?) that is the same in both cases, thus requiring identity for concepts.

For those committed with universals, there is also an alternative to the claim that a linguistic concept requires a unique metaphysical counterpart: one could accept that there are indiscernible universals, so that items completely alike in one respect do not necessarily share one universal, but rather exemplify indiscernible universals (see Rodriguez-Pereyra 2015). In that sense, a concept may well refer to two or more universals. In any case, whether one takes concepts to refer to tropes or universals, Bueno’s conclusion does not follow so straightforwardly. In fact, if the possibilities we have just raised are workable (and we think they are), it would be plausible to say that Socrates and Plato are philosophers, without requiring that there is identity between that entity that is the denotation of “is a philosopher” in both cases.

Of course, one could complain of our change to the material mode. Perhaps in the formal mode, linguistically, there must be a single concept applying to each particular object that falls under it, so that at least this linguistic entity must have an identity. That is, even if a concept refers to various tropes or indiscernible universals when it is applied, as a concept (i.e. a linguistic entity) it is just one and the same. But notice that this only shows one thing: that some abstract linguistic entities, types of concepts, have multiple instantiations on various tokens. To recognize that a token is an instance of a type, no use of identity is required.

2.2 Identity and Individuality

As a second point concerning the fundamentality of identity, Bueno argues for the claim that identity is required to define individuals (Bueno 2014, pp. 326–328). With the search for a principle of individuality as a central issue in metaphysics, this is certainly an important aspect to be considered (see Lowe 2003 for a general discussion on individuality). According to Bueno, individuality is defined as comprising two minimal conditions: (i) an individual is discernible from every other individual (*discernibility condition*) and (ii) individuals must be re-identifiable through time. Now, both conditions are said to involve identity: discernibility requires *difference*, while re-identification through time requires that an item must be such that one can identify it as *the same item* at two distinct instants of time.

Our first point is that this line of argument is off the mark: one could accept that individuals are characterized by *at least* these two conditions and still hold that identity is not fundamental. In fact, for the conclusion that identity is fundamental to go through, one would still have to add the premise that every object is an individual, or some other claim to that effect. However, if there are objects that are not individuals (according to the above definition), then, *they* do not obey the conditions for individuality (by definition of a non-individual), so that they may be characterized according to conditions that do not require identity (and then identity is not fundamental if that is really the case). In fact, assuming that there are non-individuals amounts to such an option: some objects “have identity” (in some sense to be specified), while others do not.

Perhaps one could claim that the burden of proof is on us to show that some items are not individuals according to the definition above. However, even if that were the case, we believe we could meet the challenge: quantum entities are notoriously known for not obeying the condition of re-identification over time. Furthermore, discernibility is a very contentious matter for them too (again, see French and Krause 2006, Chap. 4). So, assuming Bueno’s characterization of individuals, in the face of the failure of identity over time for quantum particles and the controversy over quantum (in)discernibility, we think it is much more plausible to consider quantum entities as non-individuals than as individuals. So, the burden of proof is on Bueno to show that quantum particles are re-identifiable over time, for instance, and that the controversy over discernibility can be solved by establishing above all doubts quantum discernibility. Unless that is done, we can consider quantum entities as non-individuals, and the claim that identity is fundamental fails for them, precisely because the conditions requiring identity do fail in their case.

So, to establish his conclusion on the fundamentality of identity, Bueno has to produce reasons for us to believe in the following two claims: (a) that the requirements of identity over time and discernibility are in fact minimal for individuality and (b) that there are no items that could be objects without being individuals. The second point seems crucial for us if the thesis that identity is fundamental is to be established. This would be the implicit premise granting universal applicability of identity. However, Bueno does not present any argument to that effect, so that it is difficult to see why identity should be fundamental just because it is used to define individuality. Of course, one could still hold that a concept can be of restricted application and still be fundamental; however, if this is the case, once again we don’t have to worry, because we can simply leave individuals as being those things having identity and non-individuals as those things that do not have identity. If that is the case, identity is still not fundamental in the sense envisaged by Bueno, and the claim that identity is fundamental in this restricted sense does not have the consequences he advanced; for instance, this restricted thesis does not undermine the interpretation of quantum mechanics according to which some of its entities are non-individuals (the Received View). In this sense, perhaps this restricted fundamentality is not what Bueno has in mind, and we shall leave this option aside for now.

Let us consider now the two main claims Bueno has to ground, the claims we labelled (a) and (b) in the previous paragraph. The first point, the one concerning the minimal conditions for individuality, also involves great controversies. Both the requirement of discernibility and the re-identification requirement seem to be just too strict to demand on something to be an individual. We begin by discussing the demand of discernibility.

Traditionally, discernibility is treated as a distinct notion from numerical difference. Discernibility concerns epistemology, dealing only with what we are able to discern, while difference concerns metaphysics, having to do with the numerical distinctness of items, even if never discovered or known by us. To make the difference clearer,

philosophers tend to present a thought experiment according to which we are asked to imagine a possible world in which there is only one object. This object is identical to itself, but not discernible from anything else, so that the concepts of difference and discernibility do not coincide (in the same way, identity and indiscernibility do not coincide).

Bueno claims that the example is controversial, and that there are already lots of arguments in the literature against possible worlds with only one object. Anyway, he claims, even if the thought-experiment holds, it is not so easy to separate numerical difference from discernibility: the single object o could have modal properties, like “being discernible from every other object that could have existed”, or, if indiscernible objects could have existed, the class of the indiscernibles from object o would have to be distinct from the class of the indiscernibles from other objects discernible from o . So, identity would be required anyway.

We should point out that this is already going too far, amounting perhaps to a change of subject: the fact that an object could have such and such modal properties does not help us in characterizing its individuality. For instance, to say that Socrates could have been a truck driver or a soccer player does not help us in characterizing his ‘actual’ individuality. Recall that the individuality of an individual, intuitively, is precisely that which makes a thing being what it is, not what it could be. So, modal properties seem to be of little help to the original problem.

Also, the claim that modal properties should be included in the discussion is obviously troublesome. When we consider that the modal property “being discernible from every other object that could have existed” holds of a lone object o existing at a world w , we are actually begging the question against the defender of the example of the world with a single object. In fact, it is assumed to begin with that the object does have that property allowing for its “modal” discernibility. So, discernibility is assumed to hold overall. However, as far as we know, there could exist an object o' in w that is *indiscernible* from o . Furthermore, why can't o have the modal property “*being indiscernible from at least some objects that could have existed*”? Well, that is precisely what is in question. To assume that this property doesn't hold is to beg the question, or so it seems to us, against those who defend that identity and indiscernibility are distinct concepts.

Furthermore, Bueno has nothing to say about symmetrical universes comprising indiscernible objects. Those universes, like Max Black's universe comprising only two indiscernible spheres (Black 1952), seem to require that individuality must be characterized by something not involving qualities and discernibility, but rather in terms of other features which could grant individuality without appealing to discernibility. The case of quantum particles is also a useful example. As Bueno himself acknowledges, sometimes quantum particles are interpreted as being individuals. However, their individuality in these interpretations is not understood as grounded on discernibility, but rather is conferred through some other individuation principle that allows for indiscernible individuals, like a primitive thisness, a haecceity, or a substratum (see French and Krause 2006, Chap. 4).

To make this point even clearer, we could distinguish between two senses of identity: (i) identity as a relation, which says that there is one thing (and whose negation says that there are two things), and (ii) identity as a metaphysical notion, in the sense of a thisness or a haecceity which every individual is supposed to have and which characterizes each thing as the very thing it is. In the first sense, identity is not required to characterize an individual, it merely expresses the fact that there is just one or that there are more things. In the second sense, identity is required to characterize individuality *according to some accounts of individuality* (again, see Lowe 2003). However, some philosophers prefer to attribute some

form of primitive identity to things and not commit themselves with concepts such as haecceity and thisnesses (see Dorato and Morganti 2013).

Now, given that distinction, perhaps here Bueno could refine his discernibility requirement and adopt some of the recent *weak discernibility* strategies to ground his claim that individuality involves identity. Recall that according to some philosophers (beginning with Saunders 2003) some entities—and in particular some quantum particles (fermions)—that were usually presented as paradigm cases of indiscernibility are not really indiscernible; perhaps philosophers have confused themselves by conflating diverse kinds of discernibility. In fact, quantum particles, for instance, may share every *property*, but at least for some kinds of particles (fermions), there are *irreflexive and symmetric relations* that may be employed to discern such entities. The most famous example concerns the two fermions in a singlet state; the relation “have opposite spin to” discerns the two fermions in such a state, because none has spin opposed to itself, and when the relation obtains we must have in fact two fermions. Black’s spheres, mentioned above, are in a similar situation, because the relation “being one mile apart” is irreflexive and symmetric, and it holds between the two spheres. So, when discernibility by properties fails and when no asymmetric relation obtains to discern things, we can still look for an irreflexive and symmetric relation that is able to discern those things; entities that are discerned by such relations are said to be *weakly discernible*. Fermions then are indiscernible by their properties and by asymmetric relations, but are nonetheless weakly discernible, so they could still satisfy one of Bueno’s requirement for individuality. With that in hand, can’t we grant that identity is fundamental?

Not really. To begin with, as we have already mentioned, Saunders (2003) employs the weak discernibility strategy in the context of a reductive account of identity, so that the two theses (fundamentality and weak discernibility) may be kept apart. Furthermore, as later discussions have shown (see in particular Ladyman 2007; Shapiro 2008; Caulton and Butterfield 2012), *weak discernibility does not imply individuality and weak discernibility is not even always available*. Concerning the first point, one may accept that weak discernibility is available for some entities and still not claim that it grants those entities any kind of individuality. As we have claimed earlier, most people think that individuality is concerned with attributing an intrinsic feature (qualitative or not) that should grant the identity and individuality of particulars. So, as Caulton and Butterfield argue (2012), if individuality is defined as involving this attribution of intrinsic features, entities that are merely weakly discernible—like fermions—are not individuals (and entities that are not even weakly discernible are not individuals either). So, in the end, weak discernibility does not necessarily save individuality unless it is already established that it is mere discernibility that is concerned with attributing individuality. The majority view is that weak discernibility is not enough for individuality, and if Bueno uses weak discernibility as a last resource, as we have seen, he cannot grant that there are no non-individuals. But let us concede, contrary to the tradition, that weak discernibility is enough to grant individuality. Is that enough to grant that there are not non-individuals? We shall argue now that even if that is conceded, the issue is not over.

The reason for that statement concerns our second claim made before, viz. that weak discernibility is not always available; it is not clear that it is available to bosons (most people, Saunders included, think it is not) and it is certainly not available for certain mathematical structures, such as Ladyman’s (2007, p. 34) example of unlabeled edgeless graphs or Shapiro’s (2008, p. 287) “finite cardinality structures” (see also Ketland 2006). In those cases, when no discernibility relation is available, then the entities involved are not individuals in Bueno’s sense. So, if we stick to Bueno’s definition, we must accept that there

are non-individuals even if we adopt weak discernibility as a condition for individuality. But, leaving discernibility aside for a moment, can we really jump to the conclusion that those entities that are not even weakly discernible are non-individuals and that identity is not fundamental? There is one further issue to be discussed before we can grant that conclusion.

Let us consider Shapiro's example of a structure comprised only by a set of four objects in the domain. Nothing in the structure allows us to discern such objects (there are no properties or relations to do so), but they are four. What are we supposed to do? How do we ground this cardinality statement? In the face of structures in which some entities are indiscernible (indiscernible by properties *and* by relations), such as this finite cardinality structure, philosophers like Shapiro (2008) and Ladyman (2007) have not declared those entities as non-individuals, but rather they are said to have what Ladyman calls "primitive contextual individuality". Their identity is primitive, and is *given* by the structure in the sense that there are four objects in the domain and no intrinsic feature of those objects granting their cardinality. Or, alternatively, as Shapiro (2008) points, mathematical practice just points to the fact that we deal with structures like the one mentioned above, and the identity of the elements of the set must be primitive. Mathematical practice has precedence over metaphysical principles. Shapiro and Ladyman conflate the identity/individuality attribution, so that a collection with a cardinal greater than zero is a collection of individuals, entities having identity. So, in this case, identity is primitive and fundamental, as Bueno claims, isn't it? It grants individuality through bare numerical difference, so individuality requires identity, even though through a distinct route than the one envisaged by Bueno.

We shall discuss in the next section some claims about the definability of identity, but for now let us consider the cardinality statement, the association of having a cardinal and being an individual/having identity. As it is clear from the structuralist perspectives of Ladyman and Shapiro, the primitive identity attributed by them to the objects in a structure is not an intrinsic identity, or identity in the metaphysical sense as we discussed above. It is just identity in the numerical difference sense. Recall that identity in the numerical difference sense needs not be grounded in a substantive metaphysical account of identity, according to which what makes two items distinct is that they have some intrinsic thisness or haecceity making the difference. In fact, for the structuralist it is precisely this metaphysical sense of identity which must be left out of the account (again, see the discussion in Ladyman 2007; also Caulton and Butterfield 2012). So, we are left with the thin notion of identity as a relation granting bare numerical difference (and contextual individuality). According to Ladyman and Shapiro, it is this notion that must be primitive in most cases where any kind of discernibility is absent. But that claim does not vindicate the fundamentality of identity, as we shall argue now.

We ground our claim that identity in that sense is not fundamental by advancing the claim that cardinality statements may be alternatively analyzed without involving identity; that is, one needs not to conflate the claim that there is a certain number of entities with the claim that identity is primitive. Of course, that move is really important when it comes to quantum entities, the paradigm of non-individuals. To establish the reasonableness of that claim dissociating identity from cardinality for quantum objects (at least), we would like to make clearer the broader thesis according to which numerical identity, understood in the sense that any collection having a cardinal greater than one necessarily entails that the elements of that collection are different (so that identity in the thin sense is required). Our point is that one can in fact dissociate those notions, so that claims to the fact that identity (in the sense of cardinality) is primitive may be understood as also not involving the relation of identity, but merely a cardinal attribution. Granted that, we need not join Ladyman

and Shapiro in calling every collection endowed with a cardinal a collection of individuals (even if they are only “contextual individuals”).

To begin with, we must make sure that it is the notion of cardinal that we are discussing now. Both Shapiro and Ladyman are worried with the fact that a plurality of items is available in the domain of quantification but there is no metaphysical feature available to ground this numerical difference. So, it is a notion of cardinal that they are dealing with. The concept of a cardinal, as we shall claim now, is neutral as to whether it applies to individuals or to non-individuals, and it can be understood in the absence of identity (we return to this issue with more details in Sect. 3). We may say that the very notion of cardinality is common to collections both of individuals and of non-individuals. Individuals may have identity given by some form of primitive identity, some kind of haecceity, and may even be discernible from every *other* individual (it all depends on which definition is adopted), while non-individuals do not have identity, which does not imply that they cannot be aggregated in collections with *many* of them. Both individuals and non-individuals can be aggregated in collections with a cardinal, but only individuals may be in principle discerned from other individuals of the same kind. In fact, let us recall the origins of modern chemistry. John Dalton explicitly claimed, long time ago, that “[t]herefore we may conclude that *the ultimate particles of all homogeneous bodies are perfectly alike in weight, figure, &c.* In other words, every particle of water is like every other particle of water, every particle of hydrogen is like every other particle of hydrogen, &c.” (Dalton 1808, p. 143). From this time on, it was noticed the importance of the notion of *number*. With Dalton, we started writing (in present day notation) things like H_2O , C_2H_4 , etc., emphasizing that it is not the individuality of the components that matters, but their species and number! Indeed, in a typical chemical reaction, such as in the combustion of methane, we have $CH_4 + 2 O_2 \rightarrow CO_2 + 2 H_2O$, plus energy. In the reaction, four Oxygen atoms move to form a molecule of carbon dioxide and two water molecules. It doesn’t matter *which* of the four atoms move to the carbon dioxide molecule; the result is the same independently of which particular atom is involved. Entities of this kind should not be treated as having identity in the standard sense.

Another typical example is the case of two electrons of an Helium atom in the fundamental state; according to quantum mechanics, as is well known, we can say that one of them has spin up in a given direction, while the other one has spin down (in the same direction), but *it makes no sense to ask for which is which*. Some philosophers claim that once a collection of objects has a cardinal, they necessarily are individuals, with primitive identity (see also Dorato and Morganti 2013). However, this move already presupposes those accounts of cardinality that are closely related to identity, something we are not required to do; in fact, quasi-set theory shows that there may exist collections (quasi-sets) of objects having a cardinal greater than one, but being so that the elements are non-individuals (see French and Krause 2006, Chap. 7 and, for a more developed argumentation against the view that cardinality and identity must be related, see Arenhart 2012; Arenhart and Krause 2014). What is really relevant for us is that individuality and non-individuality may live together, and that even if some things are individuals and do have identity, from this it does not follow that identity is fundamental (we discuss further the claim that cardinality is related to identity in Sect. 3; there we complement the argument of this section by arguing that cardinals may well be defined without the resources of identity).

Before we move ahead, perhaps we should address a further concern that may be raised by the friends of weak discernibility. It could be argued that even if weak discernibility is not enough to grant individuality in the sense required by Bueno, it grants that quantum particles are weakly discernible, and so, are non-identical. Taken together with the claims

by Muller and Saunders (2008) and Muller and Seevinck (2009) that weak discernibility holds for every kind of quantum particles, identity holds for them, after all. So, what about our claims that identity statements may be replaced by mere indiscernibility statements? Did we overlook such a *quantum mechanical fact*?

Not really. Weak discernibility is certainly important when studying identity and discernibility in quantum mechanics, but it is not without difficulties as its defenders seem to argue; it is not a straightforward kind of *metaphysical theorem* read off from quantum mechanics. It is not clear, for instance, that weak discernibility does ground discernibility in a reasonable sense and even that it grounds numerical diversity, as usually claimed. Here is how French and Krause (2006, pp. 170–171) put the issue (the classical objection may be found forcefully put in Hawley 2009; see also Ladyman and Bigaj 2010):

[...] the worry is that in order to appeal to such relations, one has already had to individuate the particles which are so related and the numerical diversity of the particles has been presupposed by the relation, which hence cannot account for it.

So, even without going into the details here, we need not take weak discernibility as imposing identity as a consequence in quantum mechanics. However, what is more relevant for us is that for those already convinced that weak discernibility forces identity on us in quantum mechanics, there is an identity-free option: to provide for an alternative formalism of quantum mechanics in which identity is not present and in which usual identity claims are framed in terms of cardinality. There are constructions of Fock spaces in quasi-set theory in which identity plays no role for the entities dealt with, so we may grant that in these cases, identity is dispensable (see Domenech et al. 2008). In the end, however one takes it, weak discernibility does not necessarily settle the matter in favor of identity.

Now, after that discussion on discernibility, let us discuss a little bit the requirement of re-identification. Recall that Bueno demands that individuals be not only discernible from each other, but also re-identifiable over time. What we can say is that it is very unusual to characterize individuality by appealing to such a feature. In fact, it seems strange to say that what makes Socrates exactly what he is and nothing else somehow depends on re-identification. Furthermore, notice that once again, by the way it is posed by Bueno, the condition has an epistemic connotation, conflating metaphysics and epistemology: individuals must be re-identifiable, at least in principle. So, that makes individuality (a metaphysical notion) dependent on our identification abilities (an epistemic notion).

Also, recall that the kind of identity required to make such identifications through distinct instants of time possible is just another kind of identity than the one we have already discussed: it is identity over time. The demands for identity over time are distinct from the demands on synchronic identity, which is the one required for individuality. As we mentioned, it is not clear which kind of identity is to be fundamental, nor whether all these distinct senses of identity are the same, or even equally fundamental.

2.3 The Indefinability of Identity

Bueno's next step is to consider the definition of identity or, as he says, its "indefinability" (Bueno 2014, pp. 328–329). According to Bueno, identity is not definable, not even in languages which are usually thought to have the resources to provide for such a definition (see Ketland 2006 for more discussions on such issues). He is right in saying that the notion of numerical identity cannot be defined within the contexts of standard logic for some structures. Classical logic (both of first and of higher-order) deals with domains of objects that

are usually thought to be sets (without loss of generality) in a standard set theory like ZFC. In standard semantics, the alleged relation of identity that one would like to define in the syntactical counterpart of our logic would be a relation symbol which should be interpreted in the *diagonal* of the domain D (let us deal with first-order logic only), namely, the set $\text{Diag}(D) = \{ \langle x, x \rangle \mid x \text{ is in } D \}$. As is well known, there is no way of defining or giving suitable first-order postulates for a binary predicate that has this set as its sole interpretation (the proof is reproduced in French and Krause 2006, pp. 253–3). Recall that in higher-order languages identity is usually introduced by Leibniz Law:

$x = y :=$ For every P , $(Px \text{ iff } Py)$, where P is a variable of suitable type and x and y are of the same type.

However, even in such higher-order languages, that definition truly does not define identity in the required sense, for we can easily present Henkin models comprising things that obey this definition but which are not *the very same object* (again, French and Krause, *op. cit.* present an example at p. 257).

However, Bueno is not considering such well-known results when he says that identity cannot be defined. In fact, in higher-order languages restricted to so-called standard models, identity can be defined following the Whitehead-Russell definition above, and for some first-order structures identity can be defined (again, see Ketland 2006). So, how can Bueno claim that identity is not definable in such languages? The idea is not that these definitions violate some condition on definability or that they do not have the correct models. Bueno makes his point with a remark concerning Leibniz Law (the formula used in the Whitehead-Russell definition) in saying that in formulating the definition, identity is already *presupposed* “given that the variables occurring on the left-hand side of the biconditional [our ‘:=’ above] need to be the same as those occurring in the right-hand side”. In this criticism, Bueno follows McGinn (2000): we must use identity in order to state the definition of identity and in order to understand the definition. So, identity is not definable, and being not definable, it is fundamental, in the sense of fundamentality under consideration by Bueno.²

However, this argument is not really a problem for those questioning the fundamental character of identity. The two x ’s in Leibniz Law are instances of the same abstract object (a variable). Of course, in describing our conceptual schemes through the use of some language or other we need to discern things such as the letters a and b . We should mention that we do it in an almost constructive way. We start with objects we are used to, and step by step we move to more and more sophisticated conceptual schemes until we arrive, say, at a strong theory such as the ZFC system. Then, as suggested by Kunen (2009), we employ the resources of this system in order to repeat our steps, perhaps understanding and rigorously grounding what we have done before. As an example, in order to elaborate Peano first-order arithmetics, we need to have the notion of ‘two’ (in order to be aware that we have, say, two different symbols in our language), but only after having developed arithmetics itself we can (supposedly) get an understanding about what ‘two’ is intended to

² F. P. Ramsey has questioned Whitehead and Russell’s definition of identity in terms of indistinguishability in their *Principia Mathematica*, claiming that “the definition makes self-contradictory for two [different] things to have all their elementary properties in common. Yet this is really perfectly possible, even if, in fact, it never happens. Take two things a and b . Then there is nothing self-contradictory in a having any self-consistent set of elementary properties, nor in b having this set, nor therefore, obviously, in both a and b having all their elementary properties in common. Hence, since this is logically possible, it is essential to have a symbolism which allows us to consider this possibility and does not exclude it by definition.” (Ramsey 1950, p. 31).

mean. As Kunen says, “formal logic must be developed *twice*” (Kunen 2009, p.191). This is so also with other systems of logic and mathematics. For instance, paraconsistent logics and paraconsistent set theories make use of the basic idea that a proposition and its negation can both be true (Béziau 2003; Arenhart and Krause 2017). But in formulating such a logic, we assume that nothing is an axiom and not an axiom at once. That is, in the meta-level, we assume something that resembles classical logic (or at least a kind of constructive logic). But even assuming the validity of things like the Principle of Contradiction, we arrive at systems that supposedly violate it. Furthermore, the definition of identity given by Leibniz Law can be said to be formulated in a part of our framework where identity makes sense, although it does not hold for some objects of our intended domain. This happens in particular in quasi-set theory, where in its ‘classical’ part, the objects obey classical logic (see French and Krause 2006, p. 285; see also Arenhart 2014).

Kunen’s claim holds also in the case of identity. We may start by using a very rough intuitive notion of identity and difference, of course, or use a notion of discernibility, and by using them we may arrive at strong logical systems in which these very notions can be questioned and even eliminated for some of the objects the theories are supposed to apply. The fact that we use identity in elaborating our conceptual scheme does not force upon us the identity of the objects we are dealing with, and this is the point to be emphasized. This, we think, answers Bueno’s related claims concerning propositional logic. In fact, in the language of classical propositional logic, the occurrences of A in a tautology like $(A \text{ or not } A)$ are occurrences of the same variable, but we could simply say that they are two occurrences of the variable A without mentioning identity at all, just by emphasizing the number (as we made before, by distinguishing the various tokens of a type). Anyway, this use of identity is in another level than the one which questions its applicability to a certain realm. Indeed, this notion does not matter for the possible consideration of a metaphysics involving objects like quantum non-individuals. As a further remark, let us mention that there is a theory of *multisets* (Bliziau 1988); roughly speaking, a multiset is a collection of objects where a certain element may appear more than once, and the number of occurrences of the elements are relevant for the cardinal of the collection. For instance, while $\{1, 1, 2, 3, 3, 3\}$ has cardinal 3 in a standard set theory like ZFC, in multiset theory it has cardinal 6. A quasi-set is not a multiset. In a multiset, it is *the same* element that is counted more than once, while in a quasi-set, due to the fact that some of the elements may lack identity, we cannot say that, but only that a certain *kind* of entity may appear more than once. Anyway, the cardinal number of the collection makes sense, even without identity conditions, as the theory makes clear.

Just to stress the same point a little more, we may even use identity to build a conceptual system, as we are mentioning, and by studying the resulting system we may discover that some of the assumptions we began with do not hold. That would happen, for instance, with identity. As Feynman (1985, p. 127) claims:

In the beginning of the history of experimental observation, or any kind of observation on scientific things, it is intuition, which is really based on simple experience with everyday objects, that suggests reasonable explanations for things. But as we try to widen and make more consistent our description of what we see, as it gets wider and wider and we see a greater range of phenomena, the explanations become what we call laws instead of simple explanations. One odd characteristic is that they always seem to become more and more unreasonable and more and more intuitively far from obvious.

[...]

There is no reason why we should expect things to be otherwise, because the things of everyday experience involve large numbers of particles, or involve things moving very slowly, or involve other conditions that are special and represent in fact a limited experience with nature.

So, this quote from Feynman indicates the same suggestion as ours (although he does not mention identity, of course). Even though we begin by judging only from our everyday experience, as theories evolve we may be required to revise some of the concepts we began with. Identity may be just such a concept. So, we may begin by employing identity to formulate theories about an unobservable domain, about which we learn from experiment and higher-level theory. If those resources seem to indicate that the entities we are dealing with behave very differently from the everyday objects we are used to, that identity seems not to make sense for them, should we stick to our old concepts and revise the lessons we derive from our higher order theory, or should we go on and investigate what would reality be like without such resources? Well, our suggestion is that it is our common sense notions that must be revised when we are faced with such revolutions as quantum mechanics and general relativity. Of course, such revisions are tentative and we are always on shaky grounds, but that is no news for anyone. Science is a revisionary project, and metaphysics goes with it. We may employ some concepts as building blocks for our higher order theory, and we may even find them useful for our everyday thinking, but the lesson from our best science seems to be that this does not render those concepts sacred.

2.4 Quantification and Identity

The next claim by Bueno concerns identity and quantifiers (Bueno 2014, p. 329). According to him, in order for quantifiers to make sense, we must have identity applied to every element of the domain of quantification. Intuitively speaking, “for all” means “for each”, thus, if we say that for all even numbers some property holds, then it holds for 0, for 2, for 4, and so on. In this sense, we need to identify all elements of the domain, hence, they must ‘have identity’. Hence, in order to understand the rule of universal generalization, namely, that from Fa it follows $\forall xFx$, where a is arbitrary in Fa (that is, a is a ‘parameter’, not a proper name of an individual object), we must know in advance that “each *distinct* object in the domain is in the range of the universal quantifier”. Furthermore, we must know that there is no object \mathbf{o} in the domain distinct from a such that \mathbf{o} is not ‘F’. Identity is involved in such claims, and so, the intelligibility of quantifiers, it seems, would require identity.

However, things are not as drastic as they seem. In the first place, even if we grant that Bueno is right on his analysis of the semantics of quantifiers (a big “if”, but let us concede for the sake of argument) one could apply a proof-theoretic kind of semantics in which the meaning of the quantifiers is fixed by the axioms and rules we use for such logical constants, such as the standard ones in first-order or in higher-order logics, and nothing about the domain is said from this purely formal point of view. According to this approach, the way quantifiers work is determined by the axioms we use, and not by the intended interpretation we have for them on a Tarski-style semantics. So, the universal quantifier in particular gets its meaning independently of identity.

Now, taking into account the purely semantic approach to quantifiers, let us first consider the claim by Bueno that identity is involved in universal quantification because “for all” means the same as “for each”. That is supposed to establish the desired conclusion because “for each” requires identification of each separate element of the domain. However, that confuses and conflates “identity” with “identification”. The claim that

universal quantification means the same as “for each” requires that we add an additional very strong epistemic ingredient to quantification: that we are able to identify each item that is being quantified over. However, that is certainly much more than could be asked for: consider quantification over the set of real numbers. No one can identify each real number. So, the equivalence between “for all” and “for each” does not hold. Remember that it was through this alleged sameness of meaning that identity infiltrates in our understanding of quantification. Hence, it seems that identity gets involved in quantification only if a very strict condition of identifiability is required, and, as we have argued, that seems too much to be asked.

Let us now take care of the claim that quantification requires the weaker notion of identity (that is, not involving the additional epistemic ingredient of identification). If that is what Bueno’s argument is claiming, then we can provide for an alternative approach which does not involve identity. Consider again the rule that goes from Fa to $\forall xFx$, with the proviso that a is arbitrary (i.e. a parameter). The only sense Bueno sees in this is that *for each object* of the domain, it has F . However, even in classical semantics, one can have an alternative interpretation that goes without mentioning *each* object of the domain: it is related to the approach to generalized quantifiers. In a nutshell, call $|F|$ the class of objects of the domain that have F , and let D be the domain of the interpretation. The interpretation for $\forall xFx$ can now be stated simply as saying that D is a subset of $|F|$. For instance, we may say that $|F|$ is the class of all (just two) Oxygen atoms in a molecule of O_2 with no need of identifying them. In the same vein, the interpretation for $\exists xFx$ simply means that $|F|$ is not empty. For instance, we may say that in an Helium atom in the fundamental state, there exists *one* electron with spin UP in a given direction, with no need to identify it (in fact, this is impossible according to standard quantum mechanics). In neither of the mentioned cases the identity of the objects being quantified over is required. Furthermore, this interpretation has the advantage of being generalizable and also of taking seriously the idea that a quantifier acts as a higher-level predicate.

The interpretation sketched in the last paragraph has another advantage: it can be employed to provide an interpretation for quantifiers by employing metalanguages without identity, like quasi-set theory. Given that this can be done, it seems to us that the claim that identity is required for us to make sense of quantifiers does not go through (for further discussions on this problem and development of quasi-set theoretical semantics, see Arenhart 2014). Even typical quantifiers which seem to involve identity, may be understood by this method; consider the uniqueness existential quantifier “there is only one object such that...”. By employing generalized quantifiers it may be defined by its extension, as the class of subsets of the domain having cardinality one (it may be generalized to deal with all finite numerical quantifiers). So, even if the traditional approach to such quantifiers comes from identity, they may be understood without it (again, see Arenhart 2014, Sect. 4.2 for details).

In such interpretations we also have an answer to another claim made by Bueno: that to make sense of universal generalization (to infer $\forall xFx$ from Fa), we must make sure that there is no object in the domain of interpretation that is *distinct from a* and that it is a ‘not- F ’. According to our proposal, all that is in need to be assured in order to grant that the inference works, besides the interpretation above, is that we make sure that there is nothing *discernible from a* that is a not- F . In fact, everything indiscernible from a will automatically be an F , otherwise they would not be indiscernible from a . So, all we need to take into account is discernibility, a relation we have already claimed to be strictly weaker than identity. So, to make sense of typical inferences of quantifiers, we need much less than identity.

This argument also works to solve a related problem posed by Bueno of the collapse of existential and universal quantifiers. According to Bueno, if we do not take into account that a is arbitrary in the inference from Fa to $\forall xFx$, and that a is not arbitrary in the inference from Fa to $\exists xFx$, both quantifiers end up collapsing. Identity is needed for that distinction, because a is said to be arbitrary in Fa , recall, when we are able to determine that no object distinct from a is not an F . However, with the interpretation sketched above, and taking into account only discernibility, and not identity, we are able to show that quantifiers do not collapse.

3 Identity and Quantum Mechanics

Bueno still makes a further point in connection with his claim that quantifiers do not make sense without identity. He relates such an issue with the consequent failure of an interpretation of quantum mechanics in which not every object has identity (Bueno 2014, pp. 329–330). According to Bueno, if his arguments are correct, this interpretation should not work.

On the other hand, if our above arguments are correct, then the relation of identity is not so precious that it cannot be left out of our “conceptual toolbox” in at least some domains of interpretation. Against such an attempt, Bueno advances another charge: that we cannot make sense of the cardinality of collections without identity (and here we recall and complement our discussion from Sect. 2.2). So, in the interpretations of quantum mechanics according to which objects do not have identity, we would not be able, according to Bueno, to attribute a cardinal number for collections of such entities (see French and Krause 2006, Chap. 4 for further discussions on the non-individuals in quantum mechanics).

One of the procedures used to establish a cardinal in quantum contexts that allegedly does not require identity is criticized by Bueno. According to such an approach, first presented by Domenech and Holik (2007), we may count the electrons in a Helium atom by putting it in a cloud chamber and using radiation to ionize it. We observe the track of an ion and the track of an electron. By repeating the procedure, we discover that only two electrons can be extracted by such a procedure. The whole point is that by employing this approach we don’t need to take into account the identity of the extracted electrons. All that matters is that we have two electrons.

Against this general strategy, Bueno states that in order to grant that we have two electrons, we must make sure that the extracted electrons are not the *same*, that each time we apply radiation we are extracting a *new* electron, that is, one that is not the same as the previous one (p. 330). Otherwise, we cannot make sure we are not counting the same thing twice.

Notice first that this goes straight against the idea that one can interpret quantum mechanics as comprised of entities without identity but with a definite cardinal, as shown in French and Krause (2006) (and as we have argued in Sect. 2.2). So, to grant the intelligibility of the project we must grant that this criticism does not go through. And, indeed, we believe it is not correct (in fact, this kind of objection is tied to an account of “counting” that is not correct even for middle-sized objects, according to Liebesman 2015; we shall restrict ourselves to the quantum case, however).

First of all, we grant that the experiment *can* be described as extracting two different electrons, the description of what is going on can be framed in a language with identity. We hold, however, that it *needs* not be so described. We can, for instance, absorb each

electron that is extracted from the atom, so that there is no doubt that an electron is not being counted twice. Furthermore, we may produce alternative counting procedures, such as weighting. According to such procedures, given that we know *the kind of particles* we have in a state, and given that we know the mass of each such element (remember that they are nomological objects, after all), we can determine how many objects there are. This procedure involves no extraction, and no claim of the identities of the elements needs to be made. So, in the end, cardinality may very clearly be seen as independent from identity. For a still simpler case, consider counting photons in a box. Suppose the frequency ν of energy in the box is $2h\nu$. By employing Planck's equation we easily discover that there are two photons (see French and Krause 2006; p. 157; for further discussion see also Arenhart and Krause 2014).

So, in the end, alternative counting procedures may be produced that do not require identity of the objects being counted. As we mentioned, one can in fact go on and attribute identity to such objects given only that there is a plurality of them, as Ladyman (2007) and Shapiro (2008) do, and as Bueno requires. However, as we mentioned, the fact that we have cardinality does not allow us to jump directly to identity: those facts, at least in the case of quantum mechanics (which we make clear here is not the case that worries Shapiro 2008 and in some measure; Ladyman 2007), are compatible with the presence of entities without identity, and we can make sense of the cardinality in such cases without introducing identity. So, non-individuals are plausible after all, and taking them as constituting pluralities or aggregates does not re-introduce identity by the back door.

4 Conclusion

We conclude that the arguments seeking to establish that identity is fundamental, according to Bueno, are unsuccessful. Almost every claim made to establish this thesis can be either shown not to achieve its goal or else to be amenable to be paraphrased in terms of discernibility or an alternative notion that does not involve identity. So, in the end, it seems that what we really need is at most a discernibility relation, which is in fact closer to our everyday necessities.

Of course, as a matter of our everyday linguistic practices identity seems to be widely present. However, one of the greatest lessons from our modern science in the twentieth century is that one should not take our linguistic practices at face value (recall our previous quotation of Feynman). The lessons about simultaneous events, about absolute space and time, about the strange behavior of matter, all come to our mind in this moment. Why should identity be forbidden to join the team of common sense notions uncrowned by science? Well, because it is fundamental, one could say. However, we hope to have argued that it is not fundamental in the sense advanced by Bueno, and that perhaps quantum mechanics is once again teaching us something important about reality that evades our common linguistic practices. Of course, that may be difficult to grasp and to understand, but every revolution is like that.

Furthermore, as we have mentioned in the beginning of the paper, Bueno does not strictly define what is meant by "identity" and by "fundamental". We hope to have shown that in the context that they are used by Bueno, the idea that identity is fundamental does not get established by his arguments. We would even go further in claiming that identity is, for practical purposes, unnecessary, but this is a matter for another work.

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Décio Krause is Professor of Logic and Philosophy of Science in the Department of Philosophy of the Federal University of Santa Catarina, Brazil. His works deal with the logical foundations of science, mainly involving the logical and metaphysical discussions about identity and individuality of quantum entities, and the applications of non-classical logics to sciences.

Jonas R. Becker Arenhart is Associate Professor at the Department of Philosophy of the Federal University of Santa Catarina, Brazil. His main research interests are logic and philosophy of logic, philosophy of science, the metaphysics of science, and the foundations of science.