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Leibniz's Theory of Space

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Abstract In this paper I offer a fresh interpretation of Leibniz's theory of space, in which I explain the connection of his relational theory to both his mathematical theory of analysis situs and his theory of substance. I argue that the elements of his mature theory are not bare bodies (as on a standard relationalist view) nor bare points (as on an absolutist view), but *situations*. Regarded as an accident of an individual body, a situation is the complex of its angles and distances to other co-existing bodies, founded in the representation or state of the substance or substances contained in the body. The complex of all such mutually compatible situations of co-existing bodies constitutes an *order of situations*, or instantaneous space. Because these relations of situation change from one instant to another, space is an accidental whole that is continuously changing and becoming something different, and therefore a phenomenon. As Leibniz explains to Clarke, it can be represented mathematically by supposing some set of existents hypothetically (and counterfactually) to remain in a fixed mutual relation of situation, and gauging all subsequent situations in terms of transformations with respect to this initial set. Space conceived in terms of such allowable transformations is the subject of Analysis Situs. Finally, insofar as space is conceived in abstraction from any bodies that might individuate the situations, it encompasses all possible relations of situation. This *abstract space*, the *order of all possible situations*, is an abstract entity, and therefore ideal.

Keywords Leibniz · Space · Analysis situs · Relational theory of space · Situation · Fixed existents · Clarke

This essay draws on earlier published work, including the introductory essay in my (Leibniz 2001), as well as several papers based on joint work with Graham Solomon, including my (Arthur 1987, 1988). Graham's tragic early death prevented any further collaboration between us. I publish this in his memory. I am deeply indebted to Ed Slowik for his detailed comments and suggestions on an earlier draft, and also to Steve Savitt for his comments on the penultimate one. All translations of Leibniz's Latin or French are my own, except where noted, and many of them from Leibniz (2001). I have used the notation 'ôr' for the 'or' that translates *seu* or *sive*, the 'or of equivalence', to distinguish it from disjunction or alternation. It has the sense of "that is to say".

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> I don't say that space is an order or situation, but an *order of situations*, or an order according to which situations are disposed, and that abstract space is that order of situations when they are conceived as being possible. (To Clarke, 5th Letter, §104)

> Only *atoms of substance*, that is to say real unities absolutely devoid of parts, can be the sources of actions, and the first absolute principles of the composition of things, and as it were the ultimate elements in the analysis of substantial things. They might be called *metaphysical points*; they have a kind of *vitality* and a sort of *perception*; *mathematical points* are their *points of view* for expressing the universe. (*New System*, §11)

If Leibniz's contribution to the theory of space is measured by his influence on subsequent developments in mathematics and physics, it is a tale of brilliant innovation, incisive criticisms and inspired suggestions. First and foremost, Leibniz is celebrated as the most powerful and influential protagonist of the relational theory of space, according to which space consists solely in the relations among bodies, and is not (as Newton claimed) an entity existing in its own right. His criticisms of Newton's absolutism and defence of a relationalist alternative have reverberated down three centuries, helping to inspire Mach's philosophy of space and time, Einstein's theories of relativity, and now a new generation of researchers in quantum gravity.[1] In mathematics, his idea of *Analysis Situs*, a generalized geometry of situation not involving co-ordinates, is celebrated as having inspired Graßman's invention of vector algebra as well as the modern mathematical science of topology. If we judge Leibniz's contribution by what he himself achieved, however, it is a tale of largely unfulfilled promise and enigma, concerning which there is next to no interpretive consensus. His manuscripts on *Analysis Situs* are episodic attempts in an ongoing enterprise that he never managed to bring to completion, and most historians of mathematics have rejected any close relation between this project and modern topology. It is, moreover, not at all clear whether or how this mathematical work connects with his relational theory of space, nor how the latter can be made adequate to the description of motion necessary for Leibnizian dynamics. These difficulties are accentuated by the standard interpretation of Leibniz's metaphysics, according to which his doctrine of the ideality of space is supposed to follow from his "denial" of relations. Thus on the one hand, it is widely held that fundamental reality for Leibniz consists only in monads (simple substances), which are not located in space. This is problematic, since phenomenal bodies, which Leibniz describes as aggregates of simple substances, *are* located in space. On the other hand, the idea that Leibniz excluded relations from his fundamental ontology sits very uneasily with his relational theories of space, time and motion, and would appear to undermine the very foundations on which they are based.

In this essay I shall try to resolve some of these enigmas. Although the incomplete nature of Leibniz's theory of space cannot be remedied, I will attempt to show that there is considerably more coherence to his views than is usually credited. The connection between the relational theory and mathematical analysis situs requires a recognition that the elements of his mature theory are not bare bodies (as on a standard relationalist view) nor bare points (as on an absolutist view), but *situations*, concrete situations being conceived as individual, relational accidents of bodies. These situations of all the bodies in the actual world make up an *order*, whose foundation in reality is God's immensity. Monads are situated in this instantaneous space through their bodies, whose relations of situation mark their boundaries from instant to instant; a monad's *point of view* comprises the situation of its body within each

[1] See for instance, Julian Barbour's account in Barbour (1999, 40) of the influence of Leibniz on his own and Lee Smolin's work on the theory of time and space. See also Smolin (1997).

instantaneous order of situations. Such a space is therefore a three-dimensional partition, or system of boundaries or figures, corresponding to any one of the actually infinite divisions or foldings of matter at any given instant. Insofar as space is regarded as perduring, on the other hand, it is a *phenomenon*, in that it is an accidental whole that is continuously changing and becoming something different. It can be represented mathematically by supposing some set of existents hypothetically (and counterfactually) to remain in a fixed mutual relation of situation, and gauging all subsequent situations in terms of transformations with respect to this initial set. Finally, insofar as space is conceived as encompassing all possible relations of situation, it is an abstract entity, and therefore ideal. *Abstract space* is the *order of all possible situations*. These possible situations, in abstraction from any bodies that might individuate them, are the subject of the mathematical theory of space.

Let me now turn to an exposition of these claims.

1 Relative Space and the Order of Situations

On Leibniz's relational theory, space consists solely in the relations among bodies, and is not an independently existing entity. Leibniz's clear statement of this position, particularly in his controversy with Newton and Clarke, is seen as a vital link in the chain of relationalist thought stretching before him from Aristotle to Descartes, and after him through Mach to Einstein. But although his place in this tradition is undeniable, the brand of relationalism he advocates is, I shall argue, crucially different in certain respects from that of Aristotle and Mach, resulting in a highly original construction of space. His innovations, I shall argue, are intimately related both to his founding work in generalized geometry (*analysis situs*) and to his peculiar characterization of continuity.

According to the traditional relationalist view advocated by Aristotle, Descartes and Mach (I shall ignore the idiosyncrasies of their views and any differences that exist among them), a place or spatial location is not a position in some absolute or container space, but rather a position *relative to some body or bodies*. For Aristotle, "the place of a thing is the innermost boundary of what contains it";[2] for Descartes, the terms 'place' and 'space' "do not signify anything different from the body which is said to be in a place; they merely refer to its size, shape and position relative to other bodies" (Descartes 1644, II, §13; Cottingham et al. 1985, 228); for Mach, "the motion of a body K can only be estimated by reference to other bodies, $A, B, C \ldots$ When accordingly we say that a body preserves unchanged its direction and velocity *in space*, our assertion is nothing more or less than an abbreviated reference to *the entire universe*."[3] Space, on this perspective, has no reality apart from the things existing in it, and consists precisely in the system of spatial relations of all other bodies to a given body or set of reference bodies. I shall call this kind of relational theory of space *body-relativism*. Crucial to it is the idea that space is defined in terms of relations to a reference body, so that the motion of a body consists in the changing of its spatial relations with respect to that body, taken as immobile. On the basis of such statements as that "space, taken apart from things, has nothing in itself to distinguish it, and indeed has nothing actual about it",[4] Leibniz has usually been taken as one of the chief representatives of this position.

[2] Aristotle, *Physics,* IV, 232a 4–5; quoted from Huggett (1999, 59).

[3] Ernst Mach, *The Science of Mechanics*, transl. T. J. McCormack (1960); quoted from Huggett (1999, 175) 177).

[4] 5th Letter to Clarke, §67; (Leibniz 1995b, 235).

Also, this view has traditionally had a close association with the principle of the *universal relativity of motion*, or as it was called in the seventeenth century when it was much in vogue, the "equivalence of hypotheses".[5] According to this principle, *all motion is relative to which body or system of bodies is hypothesized to be at rest*, and the phenomena (although not their description) and laws of physics are invariant under change of hypothesis. The snug way in which body-relativism and universal relativity fit together can be seen in the writings of Descartes:

> To determine the position, we have to look at various other bodies which we regard as immobile; and in relation to different bodies we may say that the same thing is both changing and not changing its place at the same time. (Descartes 1644, II, §13; Cottingham et al. 1985, 228)

Naturally this tends to lead to a questioning of the absoluteness of place and space. In the continuation of this passage, Descartes acknowledged that one might well determine the motions of terrestrial bodies by reference to "certain fixed points in the heavens". But, he argued, "if we suppose that there are no such genuinely fixed points to be found in the universe (a supposition which will be shown below to be probable), we shall conclude that nothing has a permanent place, except as determined by our thought" (*ibid.*).

Now it is well known that Leibniz himself subscribed to this principle of the "equivalence of hypotheses". He claimed that it is a consequence of this principle that "not even an angel could discern, in mathematical rigour, which of several bodies [in mutual relative motion] is at rest, and is the centre of motion of the others" ([1689], Leibniz 1966, 590). Thus it is natural to suppose that for him, as for Descartes before him, this relativity demonstrated the ideality or thought-determinedness of permanent place. For if an object's being at rest depends only on an arbitrary choice of hypothesis, then the concept of absolute place—a place that remains the same place over time—is also determined solely by our own choice. Following this line, we may be tempted to see the origin of Leibniz's rejection of absolute space in consideration about relative motion and the ideality of place.

But there is a second, less historical, motivation for seeing Leibniz as a relationalist of the traditional kind, which I think is much more influential. This is that it provides exactly the expected contrast with the views of Newton. For a body-relative space is one determined "from the positions and distances of things from any body considered as immovable", and this is precisely how Newton defines what he calls *relative space* in his *Principia*. Thus according to this almost irresistible contrast, we have Newton the absolutist, who holds that a class of absolute motions can be physically distinguished, and that these define a preferred frame of reference for all motion, *absolute space*, and Leibniz the relativist, who holds all motion to be relative to which body is taken to be at rest, places to be relative in the same way, and space to be any of the *relative spaces* so determined.[6]

Neat though it may be, however, there are some reasons for questioning whether this is the best reading of Leibniz's mature position. There are three main considerations to give us pause. In the first place, according to Leibniz's considered definition of space as "the

[5] In what follows I shall ignore the discussions in the recent literature concerning Leibniz's commitment to the Equivalence of Hypotheses and its alleged tension with his use of mv^2 as the measure of force. My line is essentially that for Leibniz it is the most intelligible hypothesis that determines which bodies are in absolute motion (this pertains to the causes of motion), but that this will still be consistent with the Equivalence of Hypotheses (which pertains to the phenomenal effects).

[6] See for example, Nick Huggett's characterization of Leibniz's position as involving "relative reference frames", each consisting in "the relative locations of all bodies from some reference body", so that "this view is the opposite of Newton's position that space is a substance separate from matter" (Huggett 1999, 160–161).

order of existence of states which are simultaneous" (*Initia rerum mathematicarum metaphysica* [April 1715]; Leibniz 1971 vii 17; Leibniz 1969, 666), it comes out as something *instantaneous*; that is, it is the ordering of all those states belonging to the same instant. Thus space for Leibniz, at least on this definition, is not an enduring space, as it would be if it were a Newtonian relative space, which is a space relative to some body or system of bodies considered as remaining at rest through some duration.[7]

A second reason for questioning the usual reading is that, although Leibniz explicitly approves of Aristotle's and Descartes' relational accounts of place and the relativity of motion consequent on this,[8] he does not argue from this relativity to the ideality of space. As we shall see, he has a different argument for the ideality of space on the grounds of the indistinguishability of its parts. But this pertains only to space considered independently of things, not to a space whose parts "are determined and distinguished by the things which are in them". Such a filled space, Leibniz tells Clarke, is "truly actual" (Fifth Letter to Clarke, §27; Leibniz 1995b, 225; Leibniz 1969, 700).

A third reason for pause, and one that is crucial for the connection of Leibniz's relational theory with his *analysis situs*, is that Leibniz explicitly rejects the pivotal assumption of traditional body-relativism, namely that space depends on some particular body or arrangement of bodies. In the third letter of his correspondence with Clarke, §41, Leibniz disclaims any direct dependence of space on bodies. He writes:

> The author contends that space does not depend on the situation of bodies. I answer: 'Tis true, it does not depend on such and such a situation of bodies, but it is that order which renders bodies capable of being situated, and by which they have a situation among themselves when they exist together. (Leibniz 1969, 690).

This reply also shows that we cannot interpret Leibniz's term *situation* as being synonymous with a *spatial ordering of bodies*, as it would be on a traditional body-relativist reading. For if 'order' and 'situation' are synonyms, then, as Clarke objected, Leibniz's assertion that space is "that order which renders bodies capable of being situated" would amount to the nonsensical claim "that situation is the cause of situation" (Clarke, 4th reply, §41, Leibniz 1969, 695). In response Leibniz agrees that to say that "space is an order or situation which makes things capable of being situated … would be nonsense", but denies that this is what he said:

> I don't say that space is an order or situation, but an *order of situations*, or an order according to which situations are disposed, and that abstract space is that order of situations when they are conceived as being possible (5th Letter, §104, Leibniz 1969, 713–714).

This should suffice to alert us to two highly original features of Leibniz's theory. The first is that the primitives of his theory are neither *points*, as they would be on an absolutist view, nor merely *bodies + spatial relations*, as they would be on a conventional relationalist view. Instead they are *situations*, conceived as accidents of bodies, and it is the ordering of these that constitutes space. Secondly, Leibniz distinguishes from space so conceived the further idea of *abstract space*, the order of all situations conceived as possible. It is this concept, as

[7] As we shall see, however, Leibniz does have a concept of space corresponding to Newton's relative space, a space taken as an order of situations relative to a system of bodies taken to be at rest. This phenomenal space, however, is a construction, rather than an actually enduring entity.

[8] See for instance his *A Specimen of Discoveries* (c. 1686), where he writes that "like place, motions too consist only in relation, as Descartes correctly recognized" (Leibniz 1923–, VI iv 1622; Leibniz 2001, 315).

he makes clear, that corresponds to Newton's continuous mathematical space. But it is also the case that it is this type of space that he regards as *ideal*.

In what follows I shall try to spell out the significance of these novel features for an understanding of Leibniz's relationalism. The situations in Leibniz's theory of space, I argue, are the *sitûs* of his *Analysis Situs*.[9] But in order to make good this connection, I need first to motivate it by considering the development of Leibniz's thinking about the nature of space.

2 The Genesis of Leibniz's Theory of Space

In Leibniz's earliest studies, space appears as something extended—an *extensum*[10]—which can be thought independently of body.[11] It is therefore, *contra* Descartes, distinct from the matter in it, whose essence "consists in antitypy ôr impenetrability".[12] But even though it contains the matter in it, it should not be thought of as something *existing* independently of matter. In these early studies, as indeed throughout his career, Leibniz assumes that the world is a plenum. Space can therefore be construed as the place of the whole world, just as in Aristotle, notwithstanding its being conceivable independently of matter. It is, however, regarded by Leibniz as being continuous and entirely homogenous. He writes: "although it involves magnitude and figure, it does not involve a determinate magnitude and figure" (Leibniz 1969, 111). In this respect it is like what Leibniz calls *primary matter*, about which he writes in 1669–1670:

> Now this continuous mass which fills the world is, as long as all its parts are at rest, primary matter; everything is produced out of it through motion, and everything is dissolved back into it through rest. For regarded in itself there is no diversity in it, but only homogeneity, except as a result of motion. … Matter has *quantity* too, but this is *interminate*, as the Averroists call it, or *indefinite*. For so long as matter is continuous, it is not cut into parts, and therefore does not actually have boundaries in it (I am not speaking of the inside boundaries of the world or whole mass, but those of its parts), although it does have extension or quantity in it. (Leibniz 1923–, VI ii, 435; Leibniz 2001, 337)

Here Leibniz distinguishes between primary matter as something indeterminate, and secondary matter as something actually divided into determinate parts, i.e. into bodies of various shapes and sizes, a distinction that remains basic to many of his later elucidations (including,

[9] I argued this connection between the theory of space and *analysis situs* in Arthur (1986, 1987, 1988). The connection was confirmed with the publication of Leibniz's *Characteristica geometrica* by Echeverría and Parmentier in Leibniz 1995a, and has been articulated in great detail by Vincenzo De Risi in his magisterial study (2007).

[10] Throughout this paper I will leave the term *extensum* untranslated, in recognition of its function as a technical term on a par with continuum. In Leibniz (2001), I translated it as "the extended", "extended thing", etc.

[11] Leibniz (1969, 143). Compare with Hobbes, in whose works Leibniz had been immersed at this time: "Space *is the phantasm of a thing existing without the mind simply*; that is to say, that phantasm, in which we consider no other accident, but only that it appears without us" (*De corpore*, chapter vii, Molesworth translation, p. 45).

[12] "On Primary Matter" (Leibniz 1923–, VI ii, 435; Leibniz 2001, 337). This point is reiterated in criticisms of Descartes penned in 1675: "He says the nature of body consists in extension, for everything else can be taken away from body except, so to speak, corporeality. To me it seems that there is a certain quality besides extension that cannot be taken away from body, namely impenetrability, i.e. what makes one body yield to another; and I do not see how this could be derived from extension." (Leibniz 1923–, Vi iii, 215, Leibniz 2001, 25).

for instance, the discussion of matter in the preface to his *Nouveaux Essais*). Now it is easy to see how this distinction works if secondary matter consists of atoms separated by the void. But, Leibniz claims, the parts of secondary matter may be regarded as discrete even if there is no void between them. In explanation of this he invokes Aristotle's distinction between continuity and contiguity: the parts of a thing whose adjacent extremities are together are *contiguous*, i.e. merely touching; whereas those whose adjacent extremities are one and the same boundary are *continuous*.[13] On this reading, even the Cartesian *plenum* is discrete insofar as it is comprised of contiguous parts, notwithstanding the continuity of extension conceived prior to any division.

The same goes for space. In fact, the *only* difference between space and primary matter conceived as something indeterminate is the *impenetrability* of matter, its resistance to being moved or compressed. Thus in December 1675 Leibniz argues

> If I imagine in space, instead of an *extensum*, a perfect fluid which is at rest, but which, when another body is floating in it, moves to keep the place filled, then I mean nothing other than empty space. It would be matter if the motion of the body were retarded by its motion. (Leibniz 1923–, VI iii 466; Leibniz 2001, 31)

Therefore space as an *extensum*, so long as it is conceived as continuous, is itself "interminate" in the sense that it "does not actually have boundaries in it". But by the same token, insofar as it contains bodies that are moving within it, it will be divided into parts along with the matter it contains. This means, although Leibniz does not spell this out in these early writings, that while space in the abstract includes all possible divisions into parts, space conceived through time is a constantly changing partition, whose ever-changing boundaries are determined by the motions of the matter within it. Leibniz does not see the need to argue this conception of the parts of space as determined by the parts of matter in it, perhaps because it is clearly consistent with an Aristotelian conception of the parts of space as places of bodies, and with Descartes' definition of external place as "the surface immediately surrounding what is in the place", the surface being "merely the boundary between the surrounding and the surrounded bodies" (*Principles* II, §15; Cottingham et al. 1985, I, 229).

Consistently with this, in March 1676, in his last year in Paris, we find Leibniz arguing:

> Supposing space to have parts—that is to say, so long as it is divided by bodies into empty and full parts of various shapes—it follows that space itself is a whole or entity accidentally, that it is continuously changing and becoming something different: namely, when its parts change, and are extinguished and supplanted by others. (Leibniz 1923–, VI iii, 391; Leibniz 2001, 53)

By the time Leibniz writes this, however, there have been substantial developments in his thinking. In particular, his engagement with the philosophy of Spinoza, perhaps refracted through the prism of his friend Tschirnhaus's understanding of it, is evident in some of the themes and terminology of the Paris notes in 1676. Indeed, with regard to Leibniz's theory of space, it does not seem too exaggerated to style this period as one of pene-Spinozism (for almost-Spinozism, by analogy with penevalid arguments). For in these manuscripts Leibniz distinguishes between the *extended* per se (translating *extensum* per se), which, although actually infinite, is not divisible into parts and exists eternally, and extension as it appears phenomenally, where it is divided into finite parts. This corresponds to Spinoza's distinction between God as a substance, one, indivisible, and *extended* per se, each of whose attributes

[13] See Aristotle's discussion in his *Physics*, Books 5 and 6 (i.e. E and Z), and my translations and discussion in Appendix 2a of Leibniz (2001, pp. 347–348).

"expresses an infinite and eternal essence and is thus immense (*immensum*, immeasurable)";[14] and *extension as conceived abstractly*, which has finite parts that can be enumerated in the imagination, and which is therefore measurable (Leibniz 2001, 113). Now although it is true that the idea of God as immense is theologically traditional, it is perhaps significant that the term *immensum* makes its first appearance in Leibniz's meditations on space just after he begins discussions of Spinoza's thought with Tschirnhaus in February 1676. In any case, the theme is taken up in earnest in the continuation of the above-quoted passage written in March:

> But there is something in space which remains through the changes, and this is eternal: it is nothing other than the immensity of God, namely an attribute that is one and indivisible, and at the same time immense. Space is only a consequence of this, as a property is of an essence. It can easily be demonstrated that matter itself is perpetually being extinguished, or becoming one thing after another. In the same way it can be demonstrated that mind also continuously changes, excepting that which is divine in us, or which comes from without. In a word, just as in space there is something divine, the immensity of God itself, so in mind there is something divine, which Aristotle used to call the active intellect, and this is the same as God's omniscience…. (Leibniz 1923–, VI iii, 391; Leibniz 2001, 55)

Expanding on this a month or so later, Leibniz again contrasts changeable space with its unchanging basis, identifying the latter as the indivisible "extended per se", "the *immensum*," which has modes but no parts:

> Space, by the very fact that it is dissected into parts, is changeable, and variously dissected; indeed, it is continuously one thing after another. But the basis of space, the extended per se, is indivisible, and remains during changes; it does not change, since it pervades everything. Therefore place is not its part, but a modification of it arising from the addition of matter…[I]t is the immensum which persists during continuous change of space. (Leibniz 1923–, VI iii 519; Leibniz 2001, 119–121).

Places are not parts but modifications of the immensum, arising from the addition of matter, i.e. "bulk, ôr mass." When this is added, "there result spaces, places and intervals, whose aggregates give Universal Space" (Leibniz 1923–, VI iii, 519). Thus universal space is the aggregate of places, as the Republic of Minds is the aggregate of individual minds,[15] and the divine mind is to ours as the immensum, or "real space," is to universal space (519).

Now it might be thought that this commitment to "real space" is something that would evaporate as soon as Leibniz had formulated his relational theories of space and time, where space is the order of situations, and time the order of successives. The relational nature of space and motion is already explicitly asserted by Leibniz as early as 1677, and time is described as a "relation of things with each other" (namely before, after or simultaneous with) in a manuscript of 1678–1680.[16] In this regard, it is very interesting to compare the

[14] See Leibniz's copy of the excerpts from Spinoza's *Ethics* sent him by Schuller (Leibniz 1923–, VI iii, 275–282; Leibniz 2001, 101–117). Intriguingly, the clause "and is thus immense" is missing from Definition 6 in the canonical version. See Leibniz (2001, n. 6, p. 399).

[15] The one disanalogy between aggregates of minds and aggregates of places, Leibniz notes, is that the minds endure whilst the places, being continually changed by the motions of matter, are continually destroyed and recreated.

[16] See "Space and Motion are Really Relations" (Leibniz 1923–, VI iv 1968; Leibniz 2001, 225); and "And this relation of things with each other is called *time*, which is also [like space] generic, and comprises all there

above quoted manuscript from the spring of 1676 with another written in Hanover about ten years later, "On Time and Place, Duration and Space".[17] The latter begins by taking for granted the relational theory, time and space being "real relations, or orders of existing." As real relations, however, they must have a foundation in reality,[18] and Leibniz declares, just as he had in the papers of 1676, that this foundation of time and space is divine magnitude, that is, eternity and immensity, respectively:

> Time and place, or duration and space, are real relations, i.e. orders of existing. Their foundation in reality is divine magnitude, to wit, eternity and immensity. For if to space or magnitude is added appetite, or, what comes to the same thing, endeavour, and consequently action too, already something substantial is introduced, which is in nothing other than God or the primary unity. That is to say, real space in itself is something that is one, indivisible, immutable; and it contains not only existences but also possibilities, since in itself, with appetite removed, it is indifferent to different ways of being dissected. But if appetite is added to space, it makes existing substances, and thus matter, ôr the aggregate of infinite unities. ([c. 1686]; Leibniz 1923–, VI iv, 1641; Leibniz 2001, 335).

Here we see that, just as before, immensity or real space is characterized as "one, indivisible, immutable": it is only divided into parts by the addition of something substantial. Only now what has to be added is not matter directly, but endeavour, equated with appetite: when this is added to space, "it makes existing substances, and thus matter, ôr the aggregate of infinite unities". Space, considered in itself, "contains not only existences but possibilities"; it is only a particular order of existing things when it has been divided by the motions or endeavours within it. This should be compared with what Leibniz publicly replied to Bayle in 1702: "But space and time taken together constitute the order of possibilities of the one entire universe, so that these orders—space and time, that is—relate not only to what is actually is but also to anything that could be put in its place, just a numbers are indifferent to the things numerated." (Leibniz 1978 iv 568; Leibniz 1969, 583).[19]

But it is not so much what the view of space expressed in this manuscript has in common with earlier or later views that is striking here, but the change in ontology since 1676. In Paris the "spaces, places and intervals" are modifications or modes of the *immensum* that result from the addition of mass directly. The aggregate of these places constitute Universal Space, which "is an entity by aggregation, and continuously variable", "like a net which continuously receives a different form, and thus changes". But what persists through these changes "is the immensum itself, which is God insofar as he is thought to be everywhere"

Foonote 16 continued
is, for nothing can occur which is not before, after or simultaneous with any other given thing." ("Metaphysical Definitions and Reflections" [Summer 1678–Winter 1680/1]; Leibniz 1923–, VI iv 1397; Leibniz 2001, 243).

[17] As i explain in Leibniz (2001), my dating for this piece as c. 1686 is somewhat speculative. But the doctrine of space and time as real relations is also found in *A Specimen of Discoveries* written at around this time (Leibniz 1923–, VI iv, 1621; Leibniz 2001, 313), and in a piece that can confidently be dated as mid-1685, we find Leibniz affirming that "whatever is real in space and time consists in God comprising everything" (Leibniz 1923–, VI iv, 629, Leibniz 2001, 275).

[18] For a comprehensive review and interpretation of Leibniz's highly ramified position on relations, see Mugnai's (1992), and my footnote to this (1994b).

[19] Cf. his earlier reply Foucher in 1695: "Extension or space, and the surfaces, lines and points which can be conceived within it, are only systems of relations or relations of co-existence, as regards both the actual existent and the merely possible existent which could be put in its place." (*Remarques sur les Objections de M. Foucher*; 1978 iv 490; Leibniz 1998, 184); and also "Space, like time, is a certain order…, which embraces not only actuals but also possibles." (1978 ii 379).

(Leibniz 1923–, VI iii, 519; Leibniz 2001, 121). In the Hanover manuscript, by contrast, what is added to the immutable real space is not matter directly but endeavour ôr appetite, which, when added to space, "makes existing substances, and thus matter, ôr the aggregate of infinite unities". Real space, that is, is like primary matter, lacking all forms, "indifferent to different ways of being dissected". It is only the addition of the endeavours ôr appetitions belonging to each of the infinite individual substances that gives form to space and constitutes it as an order of existents. In short, between the composition of the Paris manuscript and the Hanover one, Leibniz has rehabilitated substantial forms, and these comprise perception, or "a kind of cognition, that is, an expression or representation of external things in a certain individual thing", together with a principle of action, "i.e. an endeavour or appetite in accordance with this cognition of acting". (Leibniz 2001, 287)

To see what prompted this change, we need to take a step back and examine what led Leibniz to be so receptive to a pene-Spinozan conception of space in 1676. We have seen that Leibniz distinguished primary matter, which possesses a mere potentiality for, or receptivity to, being divided into parts, from secondary matter, in which the parts are definite, with determined boundaries; and that correspondingly, he conceived extension too both as an abstract potential for having figures or shapes and magnitudes, and as being divided at each instant into parts with determinate figures and magnitudes. This much is more or less orthodox Aristotelianism, tweaked into modern form by identifying the Aristotelian forms with the shapes or figures of the parts of matter and space.[20] These parts, however, are determined by motion, a part of matter being (according to this Cartesian conception) an extended region that is co-moving.[21] Since the parts change from one instant to the next, however, it is the *motion at an instant* that individuates the parts: in a word, it is the *conatus* or endeavour. Leibniz's studies of Hobbes in 1670–1671 had convinced him of the importance of this notion, and his first published theory of motion is built on the idea of a motion being comprised of an actual infinity of infinitely small endeavours, conceived as actual indivisibles. Now although Leibniz abandoned the interpretation of the continuum as composed of indivisibles soon after arriving in Paris in 1672, he did not at all abandon the idea of its containing an actual infinity of infinitely small entities; indeed, he was led to put even more stress on endeavour as the foundation of motion and thus of body. In a piece dating from the winter of 1672/1673 (*De minimo et maximo*),[22] he argued that in order to *avoid* bodies being composed of indivisibles, the "beginning of a body" must be defined as "the beginning of motion itself, i.e. endeavour".

By the Spring of 1676, however, the situation has undergone a dramatic change. In a paper written in early April, Leibniz refers to a "very recent demonstration" that endeavours are not, after all, infinitely small motions:

> But on the other hand there is the great difficulty that endeavours are along tangents, so that motions will be too. For I have demonstrated elsewhere very recently that endeavours are true motions, not infinitely small ones. (Leibniz 1923–, VI iii, 492; Leibniz 2001, 75)

[20] Cf. Leibniz's letter to Thomasius, 30th April 1669: "Here if we suppose *form* to be nothing other than shape, again everything is in wonderful accord [between the Scholastics and the Moderns]. For since shape is the boundary of the body, boundary will be needed for introducing shapes into matter." (Leibniz 1923–, II i, 16–17; Leibniz 1923–, VI ii, 435; Leibniz 2001, 337).

[21] "Aristotle's primary matter is the same as Descartes's subtle matter. … Each lacks form and motion in itself, each acquires forms through motion." (Leibniz 1923–, VI ii, 279; Leibniz 2001, 343).

[22] "*De minimo et maximo. De corporibus et mentibus* (*On Minimum and Maximum; on Bodies and Minds)* (Leibniz 1923–, VI iii, N5; Leibniz 2001, 8–19).

It is not known what demonstration this refers to, but it appears to be intimately connected with Leibniz's increasingly sophisticated understanding of the infinitesimals of his calculus, which he now characterizes as fictions. In any case, as a consequence of this change of their status, endeavours can no longer serve as the foundations of motion, and Leibniz has to go back to the drawing board to find new foundations for his natural philosophy. There is no indication that Leibniz is unduly upset by this turn of events, but the profundity of the change can be seen by comparing the earlier endeavour-based theories with the treatment of the continuity of motion in the dialogue *Pacidius Philalethi*, written by him on his journey from England to the Netherlands in November 1676, on his way to meet with Spinoza. After long detours through the Sceptics' paradoxes concerning motion and change, and without the word "endeavour (*conatus*)" appearing once, Leibniz concludes by reverting to a version of Occasionalism he calls *transcreationism*. According to this theory, the apparently continuous motion of a body occurs by its being annihilated at one point and then resuscitated at a contiguous point, by means of an action that "does not belong to the very body being transferred" (Leibniz 1923–, VI iii, 566; Leibniz 2001). What moves and transfers the body is rather "a superior cause which by acting does not change, which we call God. Whence it is clear that a body cannot even continue its motion of its own accord, but stands in continual need of the impulse of God, who, however, acts constantly and by certain laws in keeping with his supreme wisdom" (567; Leibniz 2001,).

This conception of a transcendent God acting by divine impulse is clearly much closer to then-contemporary Occasionalist theories than anything that Spinoza would tolerate. Nevertheless, another development in Leibniz's thought at this time brings his views into closer alignment with Spinoza's. This is the consideration of the implications of the relativity of motion. For if the quantity of motion is conserved in a collision between two bodies, it will be conserved no matter which of the two bodies is regarded as being at rest when the collision occurs. In other words, the conservation of the quantity of motion applies to *relative motion*. Leibniz had already made a note at the head of his dialogue *Pacidius Philalethi* that one of the things that remain to be treated is "the subject of motion, so that it may be clear which of two bodies changing their mutual situation motion should be ascribed to." He returns to this theme in February 1677, soon after arriving in Hanover from Holland, announcing the "remarkable fact" that "motion is something relative, and one cannot distinguish exactly which of the bodies is moving. Thus if motion is an affection, its subject will not be any one individual body, but the whole world…." (Leibniz 1923–, VI iv 360; Leibniz 2001, 229).

Evidently this leaves Leibniz's position precariously close to Spinoza's. Bodies, he agrees with the Cartesians (including Spinoza), are individuated by their motions. But motions are not affections of individual bodies, but rather of the whole world. The "subject of motion" is thus very much like Spinoza's unique first cause. With endeavour reduced to a mere motion, body no longer contains a principle of action, and all action is reserved for the first cause alone. Thus it is that in 1676–1677 Leibniz has little option but to identify bodies and their places too as modes of the *immensum* or extended per se, insofar as it contains matter. This predicament, I believe, explains why the discovery in early 1678 of the conservation of force is so crucial to the emergence of Leibniz's mature philosophy. For, it will be remembered, force is conserved for Leibniz not just overall and in individual collisions, but also in each individual substance. This enables him to interpret it as a principle of activity that is conserved within bodies, allowing a redefinition of substance in terms of force later that year. For with the rehabilitation of substantial form, understood as "endeavour or appetite" (Leibniz 2001, 287), there are again individual substances everywhere, whose bodies are modifications of them directly. This helps to explain the remark in the Hanover manuscript we were

considering above: "But if appetite is added to space, it makes existing substances, and thus matter, ôr the aggregate of infinite unities."

The problem of the "subject of motion" is now also deftly dealt with: each body is the subject of its own motions, conceived as its changing relations of situation to all the other bodies in the universe. That is, relative to itself it is stationary, but from that point of view its changing relations of situation to all the other bodies in the universe are encoded in its states and their changes, and it is in this that its motion consists: motion is a real relation. As Leibniz explains in a revealing passage in 1681,

> Insofar as God relates the universe to some particular body, and regards the whole of it as if from this body or, what is the same thing, thinks of all the appearances or relations of things to this body considered as immobile, there results from this the substantial form or soul of this body, which is completed by a certain sensation and appetite. For there is in all things a certain sensation and a natural appetite which does not at all detract from the laws of mechanism; for to God that appetite is not so much a cause as an occasion for acting. (Leibniz 1923–, VI iv, 1460; Leibniz 2001, 261)

There is thus an intimate connection between the relativity of motion and substantial form. From its own point of view, a body's representations of the rest of the universe change from one instant to the next. Provided the body is organic, and thus possesses some organs of sense, no matter how rudimentary, these representations are what Leibniz will term the *perceptions* of the substance whose body it is. The "relations of things to this body considered as immobile"—that is, its relations of situation—change from one instant to the next, in accordance with its "natural appetite", so that they form a series of representations or states governed by a law specific to that individual substance. The substantial form consists in this law-governed appetite together with the perception or representation of the rest of the universe at that moment.[23] But these changes of representation occur in each organic body in such a way that they all harmonize with one another: they must do so, since with respect to situation, the series of changing representations of each body are all mutually compatible descriptions of the same motions in the universe. Indeed, Leibniz claims, it is not intelligible how each body could act according to the laws of conservation of relative motion unless it expressed all the others:

> In fact, each substance is a kind of force of acting, i.e. an endeavour to change itself with respect to all the others according to certain laws of its own nature. Whence any substance whatever expresses the whole universe, according to its own point of view. And in the phenomena of motions this fact is especially apparent, for there every single body must be supposed to have a motion in common with any other, as if they were in the same ship, as well as its own motion, reciprocal to its bulk; how this could be so could not be imagined if motions were absolute and each body did not express all others. ("Motion is not Something Absolute" [c. 1686]; Leibniz 1923–, VI iv, 1638; Leibniz 2001, 333)

So much for the development of Leibniz's theory of substance away from pene-Spinozism. But before I return from this digression to his theory of space, there is one further development that must be discussed explicitly, and that is his view on the composition of the continuum. In 1676 there is substantial agreement between his views on the infinite and

[23] Cf. this passage from the *Discourse on Metaphysics*: "the result of each view of the universe, looked at from a certain position, is, if God finds it good to actualize his thoughts and to produce it, a substance which expresses the universe in conformity with that view. And as God's view is always correct, so too are our perceptions…" (WFPT66).

Spinoza's, as Leibniz realizes when (through the offices of Schuller) he gets his hands on Spinoza's "Letter on the Infinite" (Leibniz 2001, 100–117). For they agree that substance is indivisible, and this automatically precludes Descartes's divisible extension from counting as a substance; this accounts for their agreement that the basis of space, the extended per se, is indivisible, even while the extension of our experience is so divisible. But they also agree about the importance of the argument that Descartes uses in the *Principles* to show that at least some portion of matter in the plenum must be divided into parts that "will exceed every number"—at least, if this means "any finite number".[24] In order for continuous matter to flow through unequal spaces in the plenum, portions of it will have to move progressively faster so that no gaps could form. But since the parts of matter are individuated by their differing motions, this means there must be some parts smaller than others to infinity. At this point, however, the conclusions they draw from this state of affairs diverge. Spinoza inferred that not only the idea of extended substance having parts, but even number itself, are just facets of the imagination, aspects of extension-as-imagined but not extension per se.[25] Leibniz's position was more subtle. For him the actual division of bodies by their motions results in actual parts, and an actually infinite division is one that results in actually infinitely many parts. Commenting on Spinoza's inference (in his "Letter on the Infinite") from the Cartesian unequal spaces argument to the absence of parts, Leibniz comments,

> Now it is evident that what is really to be concluded from this is that matter, which is divisible to infinity, is in fact so divided into all the parts into which it can be divided. The same consideration applies in every case of a solid moving in a perfect liquid plenum. (Leibniz 1923–, VI iii, 281; Leibniz 2001, 113)

Similarly, in this *Specimen of Discoveries* (c. 1686) he writes:

> No body is so very small that it is not in turn actually divided into parts excited by different motions; and therefore in every body there are actually infinitely many bodies. (Leibniz 1923–, VI iv, 1626; Leibniz 2001, 323)

But there is no actually infinite number: according to Leibniz, for there to be actually infinitely many parts is for there to be so many that, however many (finite) parts are supposed, there will be more. As he wrote in an unpublished manuscript from the late 70s or early 80s,

> For any body whatever is actually divided into several parts, since any body whatever is acted upon by other bodies. And any part whatever of a body is a body by the very definition of body. So bodies are actually infinite, i.e. more bodies can be found than there are unities in any given number (Leibniz 1923–, VI iv 1393; Leibniz 2001, 235).

This is the *syncategorematic infinite*, upheld by nominalists such as Ockham. Leibniz's differential calculus is explicitly constructed on the basis of this construal of the infinite, with a parallel syncategorematic interpretation of infinitesimals as standing for the fact that parts

[24] Leibniz corrects Spinoza's claim that the parts "cannot be equated with any number" to "any finite number", noting that "if you employ infinite numbers (i.e. more than an assignable quantity of them), even irrationals can be expounded by a ratio of numbers to numbers" (Leibniz 1923–, VI iii, 280; Leibniz 2001, 111). This alludes to his expression of π as the sum of an infinite series.

[25] "So those who hold extended substance to be made up of parts or bodies really distinct from each other are, in a word, talking nonsense, not to say going insane …For all these arguments suppose corporeal substance to be made up of parts." (107).

can always be further divided. Leibniz rejected the existence of a *categorematic infinite*, one consisting in an infinite number of parts. As a consequence, a whole consisting in an actual infinity of parts cannot on his account be a collection of its parts: if it were a collection, it would have a corresponding infinite (cardinal) number, a categorematic infinity of parts, and this Leibniz denies. Thus the aggregate of the parts of matter or space at any instant is not a *real whole*, i.e. a completed totality of parts. The parts are nonetheless actual, made actual by the different motions exciting them, and are therefore, in Leibniz's favourite way of expressing this, *prior to the whole* which they constitute. In this they differ from a truly continuous whole, which is prior to any of the parts into which it can be divided. This reflects the nominalist axiom central to Leibniz's thought, that "any being by aggregation presupposes beings endowed with true unity, because it derives its reality only from that of the things which make it up." (to Arnauld, 30 April 1687; G.ii.96; Leibniz 1998, 123). Thus where for Spinoza the phenomenal world, the extended world appearing to the senses, is an imagination, emanating, as it were, from the *immensum* as the source of its reality, for Leibniz extended matter presupposes unities from which it is aggregated, and from which it derives its reality.

In 1676 Leibniz had written that the modifications of the *immensum* "do not occur by any change in it, but by the superaddition of something else, namely bulk ôr mass. From the addition of bulk and mass there result spaces, places and intervals, whose aggregates give Universal Space. But this universal space is an entity by aggregation, and is continuously variable." (Leibniz 1923–, Vi iii, 519; Leibniz 2001, 121) Space in this sense is compared to a net which "continually receives a different form, and thus changes" (Leibniz 2001, 121). The vehicle for such changes is the matter within each part or cell of this net, which Leibniz conceives as being inherently elastic. Although matter is, at this point in the development of his views, an entity by aggregation, its antitypy or resistance to being changed is conceived as an irreducible property. This changes with the adoption of substantial forms and Leibniz's new philosophy of substance: now bodies may be thought of as aggregated from prior unities—i.e. perduring substances—whose passive force accounts for their elasticity and resistance to penetration, and whose active force accounts for their continuance through time. But this does not affect the conception of space as a continuously changing *ens per aggregationem*: it still consists in the aggregation of the cells containing the continuously changing substances.

Thus according to Leibniz's mature view space is a kind of net or partition of cells, each one corresponding to the space occupied by a body. The instantaneous motion of each body is founded in the appetitions of the substances within it; an organic body's position relative to the other bodies in the universe at each instant corresponds to the representation or perception of that body from the point of view of its perceiving organs. Each body is itself actually divided by the differing motions within it, so that the whole of space at an instant is an infinitely divided net, but one that is divided in such a way that there is no limit to the division. Thus the particular motions of the infinity of parts of matter at a given instant result in a particular infinite division (partition) of matter, and therefore of space. At a subsequent instant, different instantaneous motions result in a different infinite partition. The various parts or cells of matter are conceived as being inherently elastic: a body has "a certain unequal resistance to bending" in its various parts, which accounts for its tendency to undergo elastic deformations which take it from one partition to another. This is the "physical continuum", which Leibniz describes eloquently in the dialogue *Pacidius Philalethi*:

> Accordingly the division of the continuum must not be considered to be like the division of sand into grains, but like that of a sheet of paper or tunic into folds; and so although there occur infinitely many folds, some smaller than others, a body is never

> thereby dissolved into points or minima. On the contrary, every liquid has some tenacity, so that although it is torn into parts, not all the parts of the parts are so torn in their turn; instead they merely take shape for some time, and are transformed; and yet in this way there is no dissolution all the way down into points, even though every point is distinguished from every other by its motion.… although some folds are smaller than others to infinity, bodies are always extended and points never become parts, but always remain mere extrema. (*Pacidius Philalethi*, Leibniz 1923–, VI iii, 555; Leibniz 2001, 185–187.)

After Leibniz's rehabilitation of substantial forms, it is the endeavours or appetitions of the substances (monads) within matter that determine the momentary motions, and therefore the boundaries of the bodies which constitute the parts of space in each momentary partition.[26]

It will be worth pausing a moment to compare these early views of Leibniz with those of Newton and Clarke. The idea that God's immensity is the foundation of space has a clear resonance with Newton's contemporaneous view in *De gravitatione* that "space … is the emanative effect of an eternal and immutable being" (Newton 2004, 26), and with the view expressed by Clarke that space "is a consequence of a being infinite and eternal" (Clarke's Third Reply, §3).[27] As we have seen, Leibniz calls this foundation "real space" or "the *immensum*", and claims that "real space in itself is something that is one, indivisible, immutable"; this tallies with Newton's claim that "space is eternal in duration and immutable in nature" (Newton 2004, 26), as well as with Clarke's views that "Infinite space is immensity", that it "is one, absolutely and essentially indivisible" (Clarke's Third Reply, §3; Leibniz 1969, 685). For Leibniz, moreover, this *immensum* is the foundation or basis of space, and not the space that appears to the senses; again, this tallies with Newton's distinction of absolute space from *relative space*, which is a "measure or dimension … determined by our senses from the situation of the space with respect to bodies" (*Principia*, Scholium to the Definitions; Newton 2004, 64). Here, though, the resemblance ceases. In *De gravitatione* Newton describes space as possessing all possible forms, whereas for Leibniz the *immensum* is devoid of forms prior to the introduction of substances in it. For Newton identifies his immutable space with mathematical space, which is homogeneous, and contains all possible shapes in itself potentially. For Leibniz, space in this sense—the homogeneous, continuous, mathematical space—is space considered under the aspect of possibility. This, for him, is not the *immensum*, but abstract, mathematical space. No actual places are identifiable in it: it is "interminate", devoid of all actual determination, and therefore not an actual being. Moreover, whereas for Newton the places or parts of relative space may coincide with parts of absolute space, and move relative to them, for Leibniz, the *immensum* is the basis of space, not a container possessing parts: "place is not a part of [the *immensum*], but a modification of it arising from the addition of matter".[28] Places are found only in phenomenal space, where they are individuated by the bodies in them. Bodies may change place relative to one another from one instant to the next, but there is no sense on Leibniz's analysis in which a *space* may be said to move, nor in which—anticipating Clarke's shift arguments in the correspondence—

[26] "Endeavour cannot be conceived in mass by itself" ([1686]; Leibniz 1923–, VI iv, 1614; Leibniz 2001, 299).

[27] Compare with Leibniz's "Space is only a consequence of [the *immensum*], as a property is of an essence."

[28] Cf. also the *Nouveaux essais*: "The idea of the absolute, with reference to space, is just the idea of the immensity of God and thus of other things. But it would be a mistake to try to suppose an absolute space which is an infinite whole made up of parts." (Leibniz 1981, 158).

the whole material world could be said to move with respect to space. Phenomenal space is universal space, which consists in a different partition of places from one instant to another.

In fact, Leibniz's phenomenal space is phenomenal in as many as three distinct but complementary senses.[29] It is at any time an entity by aggregation consisting in an actual infinity of parts that do not make up a true whole, and is therefore an accidental whole and not a substance. (Insofar as its parts are actual, however, it is nonetheless an *actual phenomenon*). Second, viewed through time it is continuously changing and becoming something different, so that it is never the same thing from one instant to the next. As such an ephemeral being it is therefore phenomenal in Plato's sense of something always becoming and never being. Finally, after his reintroduction of substantial forms, it is also phenomenal in the sense that it corresponds to the content of the series of perceptions of each substance from a given point of view. The perceptions of each substance will agree with those of all other substances co-existing with it, in such a way that they each give a different spatial projection of the same world. Instantaneous space will be a kind of complex of such co-existing projections and, as we shall see, Leibniz will exploit this correspondence between the objective projections and the subjective perceptions to give a kind of phenomenological deduction of space. But taken through time, the perceptions of a given substance are its representations of the universe from the point of view of its body, considered as immobile.

It is in this last sense that phenomenal space is connected with the phenomenality of motion. In 1677 Leibniz had written "The absolute motion we imagine to ourselves, however, is nothing but an affection of our soul while we consider ourselves or other things as immobile, since we are able to understand everything more easily when these things are considered as immobile." In a piece written ten or so years later (perhaps after his reading of Newton's *Principia*),[30] he spells out the consequence of this for space:

> Absolute space is no more a thing than time is, even though it is pleasing to the imagination; indeed, it can be demonstrated that such entities are not things, but merely relations of a mind trying to refer everything to intelligible hypotheses—that is, to uniform motions and immovable places—and to values deduced on this basis. (Leibniz 1923–, VI iv 1638; Leibniz 2001, 333)

But it is not until his final years, under the stimulus of his exchange with Clarke, that Leibniz is moved to give a fuller account of how his theory of space is able to accommodate motion. In the intervening years he is occupied with his new science of Analysis Situs. Yet although his motivations for this project may appear to pertain to the generalization of geometry, and thus to be independent of the above metaphysical considerations concerning space and substance, it is an interesting fact that his first explicit attempts to develop this subject date from 1679, at the very time when he has just reintroduced substantial forms and has begun to articulate his mature metaphysics.

[29] This account should be contrasted with the interpretation of Hartz and Cover (1988), according to which the belief that space and time are well-founded phenomena was something Leibniz "entertained only briefly" (495), and then "abandoned …sometime between 1687 and 1696" (502); "nothing like it survives in the mature Leibniz" (495).

[30] In Leibniz (2001), I dated this piece as possibly as early as 1686 (Leibniz 2001, 426–427); but I now think the Akademie editors were probably correct in dating it as posterior to Leibniz's reading of Newton; for although the idea of absolute motion corresponding to the most intelligible hypothesis dates from Leibniz's work in the 1670s, it is only Newton who refers motion to "immovable places".

3 The Analysis of Situation

As part of an ongoing effort to gain admittance to the *Académie Française* in Paris, Leibniz had written to Huygens in September, 1679, informing him of a project for a new mathematical science he had conceived, attaching a specimen. The appended essay is a first sketch he had made of this "New Characteristic", which he also calls the Analysis of Situation (*Analysis Situs*). Describing it to his Dutch mentor, he writes:

> I am still not satisfied with algebra, because it does not give the shortest methods or the most beautiful constructions in geometry. This is why I believe that, so far as geometry is concerned, we need still another analysis which is distinctly geometrical or linear, and which will express *situation* [*situs*] directly as algebra expresses *magnitude* directly. And I believe I have found the way and that we can represent figures and even machines and movements by characters, as algebra represents numbers of magnitudes. (1971 ii 17–20, Leibniz 1969, 247–248)

Now, *situs* or situation occurs as a technical term even in Leibniz's earliest writings. Initially it denotes a disposition of smallest parts or unities in relation to the whole.[31] Five years later, in the *Theoria motus abstracti* of 1671, Leibniz denies the existence of Euclidean partless points on the grounds that "such a thing has no situation (*situs*), since whatever is situated somewhere can be touched by several things simultaneously that are not touching each other, and would thus have several faces." *Situs* is here already being used in the geometrical sense in which he would use it in his *Analysis Situs*, perhaps showing the influence of Hobbes, whom Leibniz was studying assiduously at this time.[32] At any rate, the geometrical character of situation is brought to the fore in the "New Characteristic" of 1679. Although Leibniz does not define the term there, a *situs* is to all intents and purposes a 1-, 2- or 3-dimensional *figure* of Euclidean geometry. As he had written in 1678: "The situation of the parts of a thing to each other is called *figure*. This is the source of *similars*, which cannot be discerned unless they are simultaneously perceived." (Leibniz 1923–, VI iv, 1987; Leibniz 1969, 278).[33] Thus a situation of bodies is a given geometrical arrangement of bodies, modelled on the arrangement of the vertices in a geometrical figure.

Consider, say, a tetrahedron. Anything at the apex of the figure has a determinate situation with respect to the other three vertices at its base, the situation being determined by the angles at each vertex, the proportions among its edges, and the distance between any two vertices. Again, the same situation between the four vertices could be represented by drawing lines from them to some arbitrary reference point, and then taking the angles between these lines, the proportions between them, and the distance from any one of them to the reference point. This is essentially Hobbes's definition of situation in *De Corpore*:

[31] Thus in his *Dissertatio de arte combinatoria* (1666) Leibniz writes concerning *complexions* that "the disposition of the smallest parts, or of the parts assumed to be the smallest (that is, the unities) in relation to each other and to the whole can itself also be varied. Such a disposition is called a *situs*". In his definitions, he says "*Situs* is the location of parts." Absolute situs is that of the parts with respect to the whole, relative situs that of parts to parts. (Leibniz 1969, 77). See also the quotation and discussion in (De Risi 2007, 42).

[32] De Risi (2007, 45) draws attention to another text written in the same period, where Leibniz writes: "A geometry must be written without motion, but only with situation, i.e. locus or distance." (*Elementa de Mente et Corpore* (1671), Leibniz 1923–, VI ii, 282). He also mentions the possibility that Leibniz was later inspired by his reading of the projective geometry of Desargues, La Hire and Pascal while in Paris (2007, 28).

[33] Cf. also the remark in "On Analysis Situs" which equates situations with figures in geometry: "But *similarity* is seen best of all in the *situations* ôr figures of geometry". (Leibniz 1969, 255)

> 21. *Situation* is the relation of one place to another; and where there are many places, their situation is determined by four things: *by their distances from one another*; *by their individual distances from a given place*; *by the order of the straight lines drawn from a given place to the remaining places*; and *by the angles which are made by the lines so drawn*. …
> 22. Points, however many they may be, have a *similar situation* with an equal number of other points, when all the straight lines drawn from some one point to all of the latter have individually the same ratio to all those drawn at equal angles from the same single point to all of the former. (Hobbes 1839b; Part II, chapter 14, §20–21)

Leibniz's aim with his Analysis Situs is to represent the relations among geometrical figures directly, without recourse to the Cartesian co-ordinates and equations of ordinary analysis.[34] In explaining his name for this new type of calculus, he says he calls it this "because it explains situation directly and immediately, so that, even if the figures are not drawn, they are portrayed to the mind through symbols; and whatever the empirical imagination understands from the figures, this calculus derives by an exact calculation from the symbols" ("On Analysis Situs" [c. 1693]; Leibniz 1969, 257). The basic relations are those of *equality*, *similarity* and *congruence*, for each of which Leibniz provides its own symbol: $A = B$, $A \sim B$ and $A \simeq B$, resp. Most important of these is congruence, $\simeq$, since the relation of situation is based on it. Two figures are congruent if "one figure can be applied to or placed on the other without changing anything in the two figures except their place" (Appendix to the Letter to Huygens; Leibniz 1969, 251). This, of course, depends on the figures being embedded in Euclidean 2-space. Similar considerations apply to figures like the tetrahedron, which is embedded in 3-space. But the point is to abstract from the figures basic relationships that do not depend on anything concerning their composition, but only coincidence of their boundaries. Once these abstract relations are represented algebraically, they can be regarded as embodying these relationships independently of the Euclidean lines and surfaces in which they had been depicted, so that the background Euclidean space can be dispensed with. It acts, in other words, as a kind of scaffolding enabling the construction of the abstract theory, rather than a necessary presupposition of the theory's content. Proceeding in this way, Leibniz hoped eventually to be able to "extend the characteristic to things which are not subject to the sensory imagination." (Leibniz 1969, 253)

Basic to this project is the fundamental distinction between *quality*, which can be conceived in things individually, and *quantity*, a determination of which can only be made by comparison. As Leibniz explains in the 1693 essay "On Analysis Situs",

> In undertaking an explanation of quality or form, I have learned that the matter reduces to this: things are *similar* which cannot be distinguished when observed in isolation from each other. Quantity can be grasped only when the things are actually present together, or when some intervening thing can be applied to both. But quality presents something to the mind that can be known in a thing separately and can then be applied to the comparison of two things without actually bringing the two together, either immediately or through the mediation of a third object as a measure. (Leibniz 1969, 255)

[34] Together with the 1679 paper on the new characteristic that Leibniz sent to Huygens (Leibniz 1923–, III ii, N347; Leibniz 1969, 248–253), Loemker translates an undated paper that he ascribes to the same period "On Analysis Situs" (Leibniz 1971 v 178–183; Leibniz 1969, 254–258). As De Risi mentions, however, Aiton has dated it as belonging to 1693 (De Risi 2007, 85, 125).

Situation—or at least sameness of situation—is determined by congruence, and so involves distance in addition to similarity. It therefore involves quantity. This at any rate is true of situations in the sense of their representations as figures in Euclidean space. But although Leibniz is aiming at more general notions of similarity and of situation than those applicable just in geometry, what is fundamental here is that situation involves quantity, and this can only be determined by comparison, that is, when "things are actually present together", i.e. co-exist with other things at the same time:

> A *Situation* is nothing but that state of a thing by which it happens that it is understood to exist in a certain way with other *extensa* at the same time; that is, the mode of co-existing. (January 1680; Leibniz 1995a, 276–277)

Now it is far beyond the scope of this paper to describe in detail the developments of Leibniz's thinking on Analysis Situs. This has in any case already been achieved by De Risi in his comprehensive study (2007), on which I shall here depend heavily.[35] But I will attempt to sketch the main lines of the development of his thought on Analysis Situs and their connection with the metaphysics of space outlined above.

The first thing to note is that, as explained above, a situation may be regarded as determined either (i) by its own boundaries—a three-dimensional *extensum* by its surrounding surface, a plane figure by various lines, and a line by its terminating points—or (ii) by lines drawn from its vertices to a given point. In the first way of conceiving it, a situation is to all intents and purposes equivalent to a part of space, resulting from the actually infinite division of matter at a given instant by its interior motions. It is demarcated by the boundary of a figure in Euclidean space. But in the second way of conceiving a situation, it can be taken as a complex[36] of distances and angles of the vertices of such a figure to an arbitrary point; and indeed with respect to such a point one can take a whole order of situations of all the bodies co-existing with it. The situation of a body with respect to all other bodies simultaneous with it is thus given by all the angles and distances from the body (now thought of as a point) to all the other bodies co-existent with it. This is a representation of all those co-existents at such a point; God creates each substance as an actualization of such successive representations, as we saw above. The substances are *metaphysical points*, and the *mathematical points* from which the situations of all the other bodies are represented in the substances are their *points of view*.[37]

These considerations help explain the connection between Leibniz's Analysis Situs and his theory of space, and how both are related to his shift to an ontology of individual substances whose perceptions are representations from a given point of view. First, the notion of situation gives Leibniz a way of re-expressing his prior conception of space as a network of

[35] As De Risi notes, Leibniz's writings on the subject in one year alone—the *annus mirabilis* of 1679—take up much of a 350 page bilingual edition by Echeverría and Parmentier (EP).

[36] I use the term 'complex' advisedly. On the one hand, this is a technical term occurring in Combinatorial Topology, and so signals the close correspondence between that science and Leibniz's Analysis Situs, which I have explored elsewhere (Arthur 1987, 1988); and on the other, it is a term Leibniz himself uses in this connection. Thus in a passage on the meaning of 'being in', he says: "Thus, an accident is in no other place or time than in the subject, nor is a part in another thing different from the whole, nor is a thing placed anywhere than in place, and something ordered is not in an order different from the complex of the ordered things." (LH IV, 7 B, 3, Bl. 56v; quoted from an unpublished paper of Massimo Mugnai's).

[37] Cf. this passage from the *New System*: "It is only *atoms of substance*, that is to say real unities absolutely devoid of parts, that can be the sources of actions…They might be called *metaphysical points*…, and*mathematical points* are their *points of view* for expressing the universe. … Mathematical points really are indivisible, but they are only modalities." (Leibniz 1998, 149).

cells or spatial parts to a characterization of it directly in terms of situations. As he explains in a manuscript written probably in 1679,

> Hence now there arises the consideration of a certain generic *space* whenever certain particular situations are assigned to the phenomena, and one observes the distances and angles of things, which do not change without cause. … And this space is common to everything, and those very phenomena to which we can assign a situation, for example, stars, we call *bodies*; and there is no body which cannot be thought to exist in this generic space and to be at a distance from some other given body. (Leibniz 1923–, VI iv, 1397; Leibniz 2001, 241–243)

The basis for his new approach to space is to define an *extensum* (extended thing) in terms of situation. Already in 1678 Leibniz had established the definition "An *extensum* is what has magnitude and situation."[38] But with situation subsequently defined in terms of congruence, which involves (relative) magnitude and similarity, magnitude is already included in situation itself. Coupled with the result noted above that congruence presupposes co-existence, this leads Leibniz to a new definition of *extensum* as a whole with co-existing parts that have mutual situation. This makes for the following more sophisticated characterization of space in terms of extensum and situation, given in a manuscript dating from the early 1680s:

> We call *extension* whatever we observe as common to all simultaneous perceptions; and we call an *extensum* that by the perception of which we can perceive several things simultaneously; and this for some indefinite reason. Whence an extensum is a continuous whole whose parts are simultaneous and have a situation among themselves, and in the same way this whole behaves as a part with respect to another whole. A continuous whole is that whose parts are indefinite; space itself is such a thing, abstracting the soul from those things that are in it. Hence such a continuum is infinite, as are time and space. For since it is everywhere similar to itself, any whole whatsoever will be a part. Under "extensum" we consider being divisible into parts, being part of another, being bounded; having situation to another. A point is what has situation and does not have extension. ([1683–1685] Leibniz 1923–, VI iv, 565; Leibniz 2001, 271)

A second consequence of the relation between Leibniz's Analysis Situs and his deep metaphysics stems from the fact that situations are constituents of the points of view of monads from within their organic bodies, and therefore contained in the perceptions of those monads. As a result of this correspondence Leibniz is able to take a phenomenological approach to the construction of space. This is already indicated in the above passage, which begins with a consideration of "what is common to all simultaneous perceptions". But Leibniz goes much further, and in passages of the *Characteristica geometrica* of 1679 attempts a phenomenological deduction of both magnitude and path:

> If A and B are perceived simultaneously, and then C and D are perceived simultaneously, their situations can be distinguished. But what is perceived when A and B are co-perceived and what is perceived when C and D are co-perceived, are similar: for it is clear that nothing can be observed in them individually which will not be

[38] The passage continues: "Magnitude is a mode for determining all the parts of a thing, i.e. those [entities] by means of which the thing can be understood; *situation* is a mode for determining those [entities] by means of which a thing can be perceived." (*Conspectus for a Little Book on the Elements of Physics* (1678); Leibniz 1923–, VI iv, 1987; Leibniz 2001, 233). This definition is repeated verbatim in "Metaphysical Definitions and Reflections" [Summer 1678-Winter 1680/1681]: "*Extensum* est quod habet magnitudinem et situm" (Leibniz 1923–, VI iv, 1394; Leibniz 2001, 236).

> observed in both. Thus what is perceived when A and B are co-perceived and what is perceived when C and D are co-perceived, if they can be distinguished from each other when perceived separately, are distinguished solely by magnitude, so that when A and B are simultaneously perceived they are simultaneously perceived as having some magnitude.
>
> When two things are simultaneously perceived to exist in space, by that very fact a path is perceived from one to the other. And since they are congruent, by that very fact is conceived the path of one into the place of the other. But two points are congruent to one another. So what is perceived when two points are simultaneously perceived is thus a Line, that is, the path of a point. (10 August 1679; Leibniz 1995a, 228; modified after De Risi 2007, 411)

As De Risi observes, what Leibniz is doing here is giving a synthesis of elements occurring simultaneously in a given perception, that is, a synthesis of co-existents. But since space is the order of co-existence, and situation is the "mode of co-existing" (January 1680; Leibniz 1995a, 276–277), this is a spatial synthesis (De Risi 2007, 408). Here Leibniz is dealing with space in the abstract, that is, abstracting from the particular contents of any perception. Thus the only determination occurring is that of situation; and what "has situation and does not have extension" is a point. He is therefore able to define abstract space as the locus of all such abstract points congruent to a given point: "Universal Space is the locus of all points."[39] Thus, in his notation, if A is that arbitrary point, absolute space is all points Y for which $A \backsimeq Y$. Space is *constituted by points*. As De Risi comments,

> After a century of endless controversies between sorites and infinitesimals, limits and minima, the indivisible and the undivided, here comes Leibniz with this amazingly simple result—space is, somehow, *constituted by points* ("*constitui, dico, non componi*"). This apparently most anti-geometrical and anti-philosophical consideration comes to him from the more complex definition of a point he has used for some time now, that of a locus regarded as unextended but situated. … Space turns out for him to be a set of relations between unextended (but situated) elements. (De Risi 2007, 173–174)

The corresponding phenomenological deduction of space is as follows:

> When we conceive two points as simultaneously existing, and ask why we say they are simultaneously existing, we will think the reason is that they are simultaneously perceived, or at least that they can be simultaneously perceived. Whenever we perceive something as existing, we thereby perceive it to be in space, that is, we perceive that there can exist indefinitely many other things absolutely indiscernible from it. (Leibniz 1995a, 228–229)

Here the very fact that we are talking about *indiscernible* points being in the same relations of situation indicates that we are dealing with absolute or mathematical space. We are discussing what all perceptions must have in common, but not the specific contents of the perceptions of actual phenomena. The actual situations of phenomena, that is, are expressed more or less confusedly in the perception of each monad. Thus, although the representation of a plurality of co-existent objects regarded as indiscernible gives rise to the perception of a homogeneous extension (which is a purely extrinsic interrelation of positions), the relations

[39] "Scheda *on situation and extension*" [1695], definition. (LH XXXV, 1, 14, Bl. 90; De Risi 2007).

among the actual phenomena must be expressed in the perceptions.[40] As Leibniz writes in a piece written in about 1696,

> To be in a place seems, abstractly at any rate, to imply nothing but position. But in actuality, that which has a place must express place in itself; so that distance and the degree of distance involves also a degree of expressing in the thing itself a remote thing, either of affecting it or receiving an affection from it. So, in fact, situation really involves a degree of expressions. ("On the Principle of Indiscernibles", Leibniz 1995b, 133)

Leibniz gives this example as an illustration of his doctrine "that there are no purely extrinsic denominations." (133) Phenomena cannot exist in a relation of situation to one another as if this were purely extrinsic to them. Instead this relation of situation must be expressed as a particular quality in *A* representing the remoteness of B, and a quality in *B* representing the remoteness of A. As Leibniz explains, this is why quantity and position "are mere results, which do not constitute any intrinsic denomination per se, and so they are merely relations which demand a foundation from the category of quality, that is from an intrinsic accidental denomination." (134) Thus we falsely "conceive position as something extrinsic, which adds nothing to the thing posited, whereas in fact it adds the way in which that thing is affected by other things." (134)

It is upon this doctrine concerning extrinsic denominations, of course, that many commentators have founded their claim that Leibniz "denied relations". The idea is supposed to be that Leibniz accepted only monads and their accidents as real, and accordingly ruled out relations. Since they do not belong to the category of quality, they are precluded from belonging to this fundamental level of being. This is a vexed issue, of course, with many intricate ramifications, which I will not be able to do justice to here. But from what we have seen above, I would say the state of affairs regarding situations is rather as follows. Abstractly conceived, situations are indeed merely extrinsic. The situations of actually existing phenomena, on the other hand, must correspond to the degrees of expressivity in the substances which are representing these phenomena. Just as there is an intrinsic denomination corresponding to what is doing the expressing, so there is an extrinsic denomination corresponding to what is expressed. The first is the degree of expression within the monad; the second is the situation as it appears in the phenomena. That is, the actual situation is manifested as the lines and surfaces marking out the boundary of the phenomenal body of the monad. Monadic point of view is expressed or represented in the situation of its body. The abstract relations are ideal; but the situations of the actual phenomena will be marked out by lines and surfaces in phenomenal space, the shapes and figures which appear in perception.[41] These lines are relational accidents of the bodies in question, modes or modifications of these bodies. Relations in this sense are by no means precluded from Leibniz's ontology. Although they require a fundament in the monadic perceptions themselves, and are thus results of the harmony among these perceptions, they are nonetheless modes of actual phenomena, existing bodies. An explicit statement by

[40] Cf. De Risi: "from the mathematical point of view the concept of order of situations only leads to the representation of a *spatium absolutum*, which, being a uniform one, is inadequate to diversify among monads in perceptual terms. Therefore in order for a real expressivity of space to be possible, the addition of the phenomenal distinction of each monad's organic body is necessary." (2007, 492).

[41] Cf. Leibniz's "Remarks on M. Foucher's Objections" [1695]: "Properly speaking, the number 1/2 in the abstract is a simple relation, by no means formed by the composition of other fractions, even though in numbered things there is equality between two quarters and one half. And the same can be said of an *abstract* line, for there is composition only in *concrete* things, or *masses*, of which these lines mark the relations." (1978 iv 491: Leibniz 1998, 184; Russell 1900, 246) Until I understood Leibniz's analysis of situations, I was puzzled by Leibniz's claim here that lines "mark the relations" in concrete things.

Leibniz of the necessity of these relations among the phenomena for expressing the mutual correspondence of monads is this:

> But there would be no order among these simple substances, which lack the interchange of mutual influx, unless they at least corresponded to each other mutually. Hence it is necessary that there is between them a certain relation of perceptions or phenomena, through which it can be discerned how much their modifications differ from each other in space or time; for in these two, time and place, there consists the order of things which exist either successively or simultaneously. ("Metaphysical Consequences of the Principle of Reason" [undated; after 1695]; C11–16; Leibniz 1995b, 175–176)

As Leibniz goes on to say, it is on this foundation of the expression of the phenomena in the perceptions of the monads that the mutual distances of phenomena are based:

> From this it also follows that every simple substance represents an aggregate of external things, and that in those external things, represented in diverse ways, there consists both the diversity and the harmony of souls. Each soul will represent proximately the phenomena of its own organic body, but remotely those of others which act on its own body. (Leibniz 1995b, 176)

The distance between two organic bodies, that is, depends on a relation of perceptions represented in the entelechy (soul) of each one. It is on this very basis that Leibniz will later give his mature phenomenological definition of distance in terms of *path*, defined as "a certain order in the transition of our perceptions when it passes from one to another through intervening ones":

> But since this order can vary in infinite ways, there must necessarily be one simplest order, which would in fact be that order which proceeds according to the thing's own nature through determinate intermediate perceptions, i.e. through those which are related as simply as possible to the two extrema. For unless this were so, there would be no order, and no reason for distinguishing among co-existing things, since one could pass from one given thing to another by any path whatever. And this simplest order is the shortest path from one to the other, whose magnitude is called *distance*. (*Initia rerum mathematicarum metaphysica* [April 1715]; Leibniz 1971 VII 25; Leibniz 1969, 671; De Risi 2007, 423)

This corresponds with Leibniz's mature definition of *situation* in the same essay:[42] "Situation is a certain relationship of coexistence between a plurality of entities; it is known by going back to other coexisting things which serve as intermediaries, that is, which have simpler relation of coexistence to the original entities."

It must be noted, though, that these phenomenological constructions are premised on every simple substance "representing an aggregate of external things" (Leibniz 1995b, 176). This is of crucial relevance to a correct understanding of the phenomenological side of Leibniz's thought. For many commentators have assumed that a phenomenological construction commits Leibniz to a *phenomenalism* where bodies and space are constructed out of the sensations or perceptions of sentient beings, and have no reality beyond this constructed one. Thus a Leibnizian claim such as "when we say that bodies exist, we mean that there exist certain

[42] De Risi notes (2007, 99, n. 113) that the *Initia rerum* actually consists in two different essays, written on different sheets of paper, lumped together by Gerhardt and translated by Loemker as one piece of writing. The second essay begins with the above quoted definition of *situs*. In the first essay, Leibniz defines it as follows: "*Situs is a mode of co-existence*. Therefore it involves not only quantity but also quality." (1971 vii 18; Leibniz 1969, 667).

congruent sensations, having a particular constant cause" (Leibniz 1992, 67) is interpreted as a reduction of the existence of bodies solely to an agreement among perceptions.[43] It does consist in such an agreement, according to Leibniz, but this also presupposes a *constant cause that is responsible for the agreement*. That is, Leibniz does not deny, any more than did Hobbes, that there is a cause of the sensations lying outside the individual mind. Although he does not follow Hobbes in regarding material body as not further analysable, he will agree with him that "still, the object is one thing, the image or fancy is another" (*Leviathan*, ch. 1; Hobbes 1668, 7), since he agrees with Hobbes's statement that the representation or appearance is of something "without us, which is commonly called an object" (1668, 6). Thus in the late 1670s Leibniz writes, in opposition to the Academic skeptics, "The causes of the phenomena, too, must be outside you, and also outside other thinking beings, since they appear consonant with many things…"[44] Bodies, for Leibniz, insofar as they are the objects of representations, are aggregates of substances. Of course, where his philosophy differs radically from materialism is that each of these substances (as we have seen) is itself conceived as a representation of the bodies of all the other substances co-existent with it at a point, together with an appetition taking it through a series of such representations according to its inner law. Thus Leibnizian substances are not Kantian noumena, existing independently of the phenomena: they are instead "living mirrors", intrinsically representing (more or less confusedly) the phenomenal bodies of all the other substances with which they co-exist.[45] This mirroring is reflected in the mutuality of their relations of situation, whether concrete or abstract: "The proposition $A.B \backsimeq B.A$ is always true, that is to say, the situation of A to B is the same as that of B to A; for their situation is a mutual relation which does not involve any distinction between the points themselves…." ("Latin fragment of a letter to Huygens," September 1679; Leibniz 1995a, 236–237).

Thus Leibniz's space is, like Hobbes's, a "phantasm of the mind", since it is constructed by the mind as its way of representing co-existing things. These co-existing things nonetheless do exist externally to the thinking being, and do indeed stand in the real relations that the mind perceives.[46] In his mature rendering, space is a kind of complex of congruence relations, corresponding phenomenologically to our moving from one sensation (better, perception) to another in an order.

[43] It must be admitted that not all Leibniz's statements in his unpublished manuscripts on the matter of his attitude to phenomenalism are consistent with one another. Parkinson (Leibniz 1995b, intro, xxviii–xxxi) gives a nuanced discussion of Leibniz's apparent changes of position.

[44] "Metaphysical Definitions and Reflections"; a fuller quotation is "So it is beyond doubt that people who seem to be speaking to you today are people just as real as you, since there is just as much reason for them to be in doubt about you. The causes of the phenomena, too, must be outside you, and also outside other thinking beings, since they appear consonant with many things; and the reason for so many new appearances cohering among themselves cannot be provided from your nature thus limited." ([Summer 1678-Winter 1680/1681]; Leibniz 1923–, VI iv 1386–1397; Leibniz 2001, 241).

[45] Cf. "[Matter is] actually divided *ad infinitum*. Therefore, since every organic body is affected by the entire universe by relations which are determinate with respect to each part of the universe, it is not surprising that the soul, which represents to itself the rest in accordance with the relations of its body, is a kind of mirror of the universe, which represents the rest in accordance with (so to speak) its point of view—just as the same city presents, to a person who looks at it from various sides, projections which are quite different." ("Metaphysical Consequences of the Principle of Reason" [c. 1712], Leibniz 1995b, 176).

[46] Compare Hobbes's: "*Space is the phantasm of an existing thing insofar as it is existent*; that is, considering no other accident of the thing beyond that it appears outside the person imagining." (Hobbes 1839a, 94; Hobbes 1839b, 84).

4 Conclusion

In closing, let me now turn to Leibniz's treatment of space in the course of his correspondence with Samuel Clarke. Viewed through the lens of his Analysis of Situation expounded above, much of what he says there will appear in sharper focus. He tells Clarke:

> I have said more than once that I hold space to be something merely relative, as time is: that is, I hold it to be an order of co-existences as time is an order of successions. For space denotes, in terms of possibility, an order of things which exist at the same time, considered as existing together, without inquiring into their particular manner of existing. And when many things are seen together, one perceives that order of things among themselves. (Third Paper to Clarke, §4 [Feb 25, 1716]; Leibniz 1969, 682)

As discussed above, the "particular manner of existing" of co-existing things is their particular mode of co-existing, or *concrete situation*. If, on the other hand, we consider co-existing things purely in terms of their possibility of being mutually situated, we are considering space as an *order of situations in the abstract*: this is absolute or mathematical space. The latter kind of space is what is constituted in perception: when we perceive many things existing at the same time, without regard to the particular situations they actually have, we necessarily see them as existing in space.[47] As we have seen, such a space is homogeneous. It is "something absolutely uniform, and, without the things placed in it, one point of space does not absolutely differ in any respect whatsoever from another point of space" (Third Paper, §5, Leibniz 1969, 682): all the points of space are congruent to one another, and in themselves mutually indiscernible. It is different, of course, for all those points which are extrema or vertices of the figures of the actually existing bodies in actual space: these are distinguishable by means of the bodies. This is an order of actual situations. But this order cannot be differently situated in space as a whole, as it would be if God, while "preserving all the same situations of bodies among themselves," should have changed east into west. For

> if space is nothing else but that order or relation, and is nothing at all without bodies but the possibility of placing them, then those two states, the one such as it is now, and the other supposed to be the quite contrary way, would not at all differ from one another. (Third Paper, §5, Leibniz 1969, 682)

On similar grounds Leibniz is able to parry Clarke's objection that "if space were nothing but the order of things co-existing, it would follow that if God were to move the whole material world with any speed whatsoever, it would still always continue in the same place, and nothing would receive any shock upon the most sudden stopping of that motion" (Clarke, Third Reply, §4). For this supposes an absolute space with respect to which God could move the world; that is, places that can be identified without relation to the things in them. Clarke, of course, wishes to enlist God's will as a sufficient condition for choosing to create one of two things that are otherwise indiscernible. But this misses Leibniz's point that there is no criterion by which even God can single out one of two indiscernible states as opposed to another:

> For two indiscernible states are the same state, and consequently it is a change which changes nothing. … The uniformity of space means that there is neither *internal nor*

[47] Ironically, this is not so far from Newton's view, as expressed in his unpublished *De Gravitatione*: "And hence it follows that space is an emanative effect of the first existing being, since if any being whatsoever is posited, space is posited." (Newton 2004, 25).

> *external reason* for discerning its parts, and for choosing between them. For such external reason for discerning could only be founded in the internal one." (Fourth Paper, §13, §18).

Thus for Leibniz absolute space cannot have any actual parts. It is the complex of all points congruent to a given point. Insofar as these points could be points in particular relations of situation to one another, they can be conceived as the relata of such situational relations marking the boundaries of the parts of an actually divided space. But viewed in abstraction from the bodies whose motions divide the space, such parts all "perfectly resemble one other like two abstract units. But this is not the case with two concrete unities, nor with two actual times, nor with two occupied, that is to say, truly actual spaces." (Fifth Paper, §27; Leibniz 1969, 700).[48]

The connection with Analysis Situs also helps explain Leibniz's otherwise puzzling response to Clarke's (Newton's?) challenge that "Space and time are quantities, which situation and order are not" (Third Reply, §4; Leibniz 1969, 685; Fourth Reply, §16 and 17):

> I answer that order also has its quantity; there is that in it which goes before and that which follows; there is distance or interval. Relative things have their quantities as well as absolute ones. For instance, ratios or proportions in mathematics have their quantity and are measured by logarithms, and yet they are relations. (Fifth Paper §54; Leibniz 1969, 706).

The inadequacy of Leibniz's answer from a modern point of view has been widely remarked upon: ordering relations play an important role in topology, but a purely topological space is one without a metric. Also the remark about logarithms being the quantities of ratios seems both odd and irrelevant. It should, however, have alerted commentators to the fact that Leibniz is not using the words 'order' and 'relation' in the same sense that they are used today. As we have seen, for Leibniz lines (i.e. line segments) "mark the relations" between their endpoints; and he writes of "relations of situation" even though he believes that situations involve distance. Leibniz's mention of "order or situation" had already caused Clarke to regard them as synonyms, so that when Leibniz assured him that on his relational view space "does not depend on such and such a particular situation of bodies; but it is that order which renders bodies capable of being situated" (Fourth Letter, §41, Leibniz 1969, 690), Clarke replied that he could not understand. "It seems to me to amount to this," he wrote, "that situation is the cause of situation." (Fourth Reply, §41, Leibniz 1969, 695). I have already quoted Leibniz's response to this:

> I don't say, therefore, that space is an order or situation, but an *order of situations*, or an order according to which situations are disposed, and that abstract space is that order of situations when they are conceived as being possible. (Fifth Letter, §104, Leibniz 1969, 713–714)

But in fact by this point in the Fifth Paper Leibniz has already given a much fuller response in the long section 47, where he explains how it is that we come to form our notion of space. If we now turn to this, we will see that it amounts to a concise description of his Analysis of Situation; more importantly, we will also see how Leibniz intends his account to encompass

[48] Cf. "The parts of space are determined and distinguished only by the things which are in them, and the diversity of the things in space determines God to act differently on different parts of space. But space, taken apart from things, has nothing in itself to distinguish it, and indeed it has nothing actual about it." (Fifth Paper, §67, Leibniz 1995b, 235).

motion. He says there (Fifth Letter, §47, Leibniz 1969, 703–704) that when we consider several things *A, B, C, D, E*, etc. which exist at the same time, we "find them in a certain order of coexistence according to which the relation of one thing to another is more or less simple"; such an order of coexistence he calls "their situation or distance". That is, the situation of one to another involves, as we have seen, the path between them; and the simplest such path is the shortest, namely the straight line, whose magnitude is their mutual distance. The order according to which these situations are disposed is the figure with these bodies as its vertices. Now

> When it happens that one of those coexistent things changes its relation to a multitude of others which do not change their relations among themselves, and that another thing, newly come, acquires the same relation to the others as the former had, we then say it is come into the place of the former; and this change we call a motion in that body wherein is the immediate cause of change. (Fifth Letter to Clarke, §47, Leibniz 1969, 703.)

Or more precisely, he explains, we can suppose a motion or change in the relative situation of a system of bodies *A, B, C, E, F, G*, etc., such that the situation of *C, E, F, G*, etc. with respect to each other remains invariant (they are *fixed existents*), and such that the situations both of *A* with respect to *C, E, F, G*, etc., and *B* with respect to *C, E, F, G*, etc., change. And we may suppose that the "relation of coexistence" that *B* comes to have with *C, E, F, G*, etc., agrees perfectly with the situation *A* had with the same *C, E, F, G*, etc. prior to these displacements within the system. From the idea that *B* can acquire a relation of coexistence with the fixed bodies which agrees with the one *A* had, we acquire the notion of "sameness of place". From this we abstract the idea of *place* in a sense independent of the things we observe, and also the idea of a totality of all such places taken together, or *space*. But, Leibniz stresses, there is still a "difference between place and the relation of situation which is in the body that fills up the place" (Leibniz 1969, 703–704).

> For the place of *A* and *B* is the same, whereas the relation of *A* to the fixed bodies is not precisely and individually the same as the relation which *B* (that comes into its place) will have to the same fixed bodies; but these relations agree only. For two different subjects, as *A* and *B*, cannot have precisely the same individual affection, it being impossible that the same individual accident should be in two subjects or pass from one subject to another. But the mind, not contented with an agreement, looks for an identity, for something that should be truly the same, and conceives it as being extrinsic to the subject; and this is what we here call *place* or *space*.

Here we see that a relation of situation is characterized as an individual accident of the body possessing it; this is situation as a *concrete situation*, what is manifested as a phenomenon. This will correspond to a particular degree of expression in the substance. Once one represents these relations as a network or complex of lines and boundaries, one has abstracted away from the individual accidents, and represented the lines as if self-standing and external to the bodies, that is, as places. An order of these situations is space, namely, that "order which renders bodies capable of being situated".

Already—that is, prior to a consideration of *abstract space*, as this order of situations conceived as possible—this distinguishes Leibniz's position from that of Mach, who thought of fixed existents in terms of some actually situated bodies, the fixed stars. Indeed, for Leibniz strictly speaking there are no fixed existents in actuality, since every body, and even every part of every body, is constantly changing its situation with respect to everything else in the universe, even if by imperceptible degrees. Nevertheless,

> though many, or even all, the coexistent things should change according to certain known rules of direction and swiftness, yet one may always determine the relation of situation which every coexistent acquires with respect to every other coexistent, and even that relation which any other coexistent would have to any other, if it had not changed or if it had changed any otherwise.

Thus fixed existents are for Leibniz really fictions from which one builds the idea of place and space: one "supposes or feigns that among coexistents there is a sufficient number of them which have undergone no change" (*ibid.*). Thus to formulate Leibniz's conception of space, we need to drop the requirement that there actually is a set of bodies whose situation remains invariant, and replace it with the requirement that the situation of the original set of fixed existents be definable in the new space by some allowable transformations. In modern terms, the "known rules" of motion governing such transformations will be, for instance, the Galilean or Lorentz transformation groups.[49]

Obviously, the theory requires a good deal of further articulation in order to be adequate to a full-fledged dynamics. A promising step in this direction, it seems to me, is the recent work of Julian Barbour and his co-workers on explicitly Leibnizian premises. On Barbour's construction of space, only ratios of separations of bodies are physical. He takes the quotient of the whole Euclidean 3-space with respect to the equivalence relation of similarity, defined by its own transformation group, and calls this space *Shape Space*. He then uses his notion of "best matching" to define motions in this space: intuitively this involves shuffling the triangles that define situations (or other more complex figures) into a "best-matched" position, so that equilocality is established by a kind of "Principle of Least Incongruence". Clearly, it would take me too far afield to explore these developments further here. But I hope that these few words are enough to indicate that further development of Leibniz's theory into a theory of spacetime is still possible using modern concepts and methods.

In this essay I have tried to show that Leibniz's theory of space is a good deal more coherent than it is usually believed to be. His project of Analysis Situs is an attempt to give a formal rendering of the theory, and at the same time a phenomenological theory of space. It has a close relation to his metaphysics, the situations of bodies being included in their constituent substances' representations of the rest of the world from their own point of view. I have tried to indicate how this conception issues from his attempt to resolve the continuum problem, where the actual parts into which space is divided correspond to the endeavours of the constituent substances. The phenomenal space of Leibniz's mature phenomenological metaphysics therefore corresponds to the "universal" or "generic" space he described in the late 1670s and early 80s. It is constantly changing, a different partition of parts from one instant to another, and therefore a *unum per accidens*, always becoming, never enduring. Taking some set of existents as fixed, one can construct a relative space in the sense of a system of relations of situation to these fixed existents, and this will be sufficient to ground motion, so Leibniz believes. But this is neither abstract space, which comprises all possible relations of situation; nor is it instantaneous space, which is a particular order according to which the relations of situation of co-existing bodies are disposed at a given instant. It is more nearly what he had called "absolute space" in the piece quoted earlier from the 1680s, an entity "no more a thing than time is, even though it is pleasing to the imagination", but consisting merely in "relations of a mind trying to refer everything to intelligible hypotheses—that is, to uniform motions and immovable places—and to values deduced on this basis." (Leibniz 1923–, VI iv 1638; Leibniz 2001, 333)

[49] Cf. Arthur (1994a), where I suggest this approach to Leibnizian spacetime.

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