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Leibniz and Newton on Space

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Abstract This paper reexamines the historical debate between Leibniz and Newton on the nature of space. According to the traditional reading, Leibniz (in his correspondence with Clarke) produced metaphysical arguments (relying on the Principle of Sufficient Reason and the Principle of Identity of Indiscernibles) in favor of a relational account of space. Newton, according to the traditional account, refuted the metaphysical arguments with the help of an empirical argument based on the bucket experiment. The paper claims that Leibniz's and Newton's arguments cannot be understood apart from the distinct dialectics of their respective positions vis-à-vis Descartes' theory of space and physics. Against the traditional reading, the paper argues that Leibniz and Newton are operating within a different metaphysics and different conceptions of "place," and that their respective arguments can largely remain intact without undermining the other philosopher's conception of space. The paper also takes up the task of clarifying the distinction between true and absolute motion, and of explaining the relativity of motion implied by Leibniz's account. The paper finally argues that the two philosophers have different conceptions of the relation between metaphysics and science, and that Leibniz's attempt to base physical theory on an underlying metaphysical account of forces renders his account of physics unstable.

Keywords Leibniz · Newton · Absolute space · Relationalism · Absolute motion · Laws of motion

1 Introduction

The origins of modern debates on the nature of space and time are located in two important texts; the Scholium to the definitions in Newton's *Principia* (1999) and the *Leibniz-Clarke Correspondence* (Alexander 1956). Newton introduced absolute space in the Scholium because he believed that such an entity is necessary for articulating a viable physical

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theory. Without absolute space, the science of motion is not powerful enough to analyze phenomena. In the correspondence with Clarke, Leibniz presented a forceful criticism of absolute space and articulated a compelling relational account. By drawing heavily from his metaphysical system, Leibniz—with the unwitting aide of Clarke—was shifting the debate away from the arguments Newton presented in the Scholium. While Newton was focused on grounding his scientific method, Leibniz was interested in demonstrating how physical theory ought to mesh appropriately with a theistic worldview. The Leibniz-Clarke Correspondence obscured for generations later the significance of the Scholium and Newton's reasons for believing in absolute space. The Correspondence left a false impression suggesting that Newton's arguments *for* absolute space are meant to destroy Leibniz's relational account or to argue against the general relativity of motion. The Correspondence also left the impression that Leibniz's metaphysical arguments *against* absolute space provide a definite blow to Newton's account. But the truth of the matter is that there is not much direct contact between Newton's and Leibniz's respective accounts of space, and both can live (almost) intact given the empirical state of things. While Newton's reasons for believing in absolute space are mired in a particular scientific orientation, Leibniz's reasons for disbelieving in absolute space are placed within the context of an elaborate, sophisticated metaphysical system. Despite the viability of both accounts, an interesting lesson is that the two thinkers employ different styles of evaluating physical theory.

One of the aims of this paper is to expose the style of evaluating physical theory implicit in Newton's arguments for absolute space. This Newtonian style of critiquing physical concepts was overshadowed by the arguments in the Correspondence. The main purpose is to show that, while Newton's style of philosophical evaluation is crucial for securing the foundations of his science, it only has indirect relevance to Leibniz's account. Newton's arguments for absolute space do not show that Leibniz's relationalism is false. However, they do demonstrate that Leibniz was less careful than Newton in laying a secure foundation for his physics.

In Sect. 2 I briefly review Leibniz's metaphysical arguments against absolute space and show how Clarke's responses helped obscure the significance of Newton's Scholium. In Sect. 3 I argue that each philosopher begins his respective analysis from a different conception of "place," necessarily coloring his argument with a different system of spatial concepts, rendering the arguments incommensurable. In Sect. 4 I argue, contrary to a common misinterpretation of Leibniz's assertions, that both Newton and Leibniz distinguish between true and apparent motion; although Leibniz believes this distinction occurs mostly on a metaphysical level of description, and that true motion does not necessarily involve the existence of absolute space. This section attempts to consolidate Leibniz's various assertions on true motion. On the one hand Leibniz believes that there is no phenomenal criterion for true motion, committing Leibniz to the general relativity of motion. On the other hand, true motion can be attributed to bodies once a complete dynamic explanation is found. I argue that Leibniz's commitment to the existence of true motion is consistent with his commitment to the general relativity of motion on the phenomenal level, and that his assertions can be defended against Newton's famous bucket experiment. In Sect. 5 I reconstruct Newton's arguments for absolute space in the Scholium, focusing on the relation between these arguments and Newton's attempt to find a secure foundation for a science of motion. This section pays close attention to the argument from the properties of motion for distinguishing between absolute and relative motion, arguments that are woefully neglected and misunderstood by most commentators on the Scholium. I demonstrate that Newton does not have at his disposal a direct refutation of Leibniz's relationalism, and to a large extent his arguments are not relevant to Leibniz's project. Although, in Sect. 6, I distill from Newton's Scholium some relevant argument demonstrating that Leibniz's theory of motion is unstable, and that his various commitments do seem to create genuine tensions in his theory. This weakness in Leibniz's physics stems from a lack of proper foundation for his science of motion.

2 Leibniz's Metaphysical Arguments Against Absolute Space

The Leibniz-Clarke Correspondence provides the impression that two contradicting philosophical conceptions of space are engaged in a great battle of wits. While Leibniz manages to raise doubts about the existence of absolute space, using the Principle of Identity of Indiscernibles to great rhetorical effect, Newton has the bucket argument at his disposal, with which he is able to undermine Leibniz's sophisticated philosophical arsenal. Initially, Clarke engages Leibniz and tries to defend Newton against Leibniz's metaphysical arguments. Later Clarke shifts his strategy and claims that Newton provided in the *Principia* a conclusive physical argument for absolute space:

> It is largely insisted by Sir Isaac Newton in his *Mathematical Principles*, (Definit. 8) where, from the considerations of properties, causes and effects of motion, he shows the difference between real motion, or a body's being carried from one part of space to another; and relative motion, which is merely change of the order or situation of bodies with respect to each other. This argument is a mathematical one; showing, from real effects, that there may be real motion where there is none relative; and is not to be answered, by barely asserting the contrary. (Alexander 1956, C.IV.13)

Here Clarke is referring to the Scholium to the definitions in Newton's *Principia*, where Newton argued that he could distinguish between relative and absolute motion. Clarke emphasizes the argument from the effects of absolute motion, wherein Newton utilizes the famous bucket experiment to show that he could distinguish between absolute and relative accelerations. Newton describes a bucket filled with water that is given a push so that it begins to rotate. Once the water in the bucket begins to rotate due to friction with the sides of the bucket, a concavity along its surface is generated, suggesting a tendency of the water to recede from the center of rotation. Newton argues that this tendency exists in the water irrespective of the relative motion between the water and the walls of the bucket. This demonstrates that absolute motion can be distinguished from relative motion.

By citing the Scholium only briefly and superficially, Clarke created the impression that Newton's Scholium is the place where a conclusive answer to Leibniz's relationalism could be found, and that the primary place to look is in the bucket experiment. The legacy for later generations is the expectation that Newton's argument would refute every possible brand of relationalism. But the imagined debate between Newton and Leibniz stems from a false interpretation of the history of the debate and the significance of the Scholium. Newton published the first edition of the *Principia* (together with the Scholium on space and time) in 1687. The Scholium was primarily concerned with showing that Descartes' physics was inconsistent, and that absolute space is indispensable for a science of motion that analyzes motions via exchanges of quantity of motion.[1] In 1715, 28 years after the first edition of the *Principia* was published, Leibniz sent a letter to Caroline, Princess of Wales, in which he strongly criticized key elements in Newtonian doctrine. Clarke took up the task of defending Newton's philosophy of nature against Leibniz's attack. In the intervening years, Leibniz was

[1] The Scholium's implicit reference to Descartes was first insisted on by Koyré (1965) and discussed in detail by Stein (1970). In Belkind (2007) I expanded on the conceptual relation between absolute space and the quantity of motion.

engaged in acrimonious disputes with the Newtonians over the priority in inventing infinitesimal calculus. Leibniz also made public his criticism of Newton's theory of gravitation, accusing him in the *Theodicy* of introducing an occult force that acts at a distance, a criticism he repeated in a letter to Hartsoeker, published first in 1712 and later reprinted in the *Journal des Savants*. Thus Leibniz and Clarke were preoccupied with evaluating whether Newtonian theory can accommodate the cultural and scientific sensibilities of the day. This task was more urgent than the task of determining the precise merits of the arguments in the Scholium and their relevance to Leibniz's philosophy.

Leibniz's letter to Caroline, Princess of Wales, begins:

> Natural religion itself, seems to decay (in England) very much. Many will have human souls to be material: others make God himself a corporeal being…Sir Isaac Newton says, that space is an organ, which God makes use of to perceive things by. But if God stands in need of any organ to perceive things by, it will follow, that they do not depend altogether upon him, nor were produced by him. (Alexander 1956, L.I.1 & 2)

These lines are the starting point of what later became the correspondence with Clarke. Leibniz here is referring to Newton's *Opticks* where Newton seems to be asserting that space is God's Sensorium. According to Leibniz, this view of space denigrates God and makes him corporeal. Clarke responds that by describing space as the Sensorium of God, Newton only meant this as a metaphor (Alexander 1956, C.I.3).[2] Thus, in evaluating the Correspondence in relation to the Scholium, we should keep in mind that Leibniz aims to show that Newton's conception of absolute space does not fit within a theistic worldview. According to Leibniz, a properly theistic metaphysical system conceives of God as a supreme being that is clearly distinguished from created material bodies. In intimating that space somehow belongs to God's nature, and in failing to find the ontological place of space within a theistic world, Newton committed a metaphysical blunder.

It is within the context of Leibniz's theistic metaphysics in which the direct argument against absolute space is placed. Leibniz's main objection to absolute space is based on the Principle of Sufficient Reason (PSR) (Alexander 1956, L.III.5). Since space is something completely uniform, whether an object is placed in one point in space or another makes no genuine difference. However, our world was created by an infinitely wise and benevolent being, and so there cannot be any fact about which God is indifferent. Leibniz therefore asserts the PSR, according to which "nothing happens without a reason why it should be so, rather than otherwise" (Alexander 1956, L.II.1). If we translate all material bodies in the universe to a different position in absolute space while keeping the spatial relations intact, or if we transpose East into West, we would get a new universe which seems exactly the same as the previous universe. Thus, given that God does not have sufficient reason to create one universe over the other, an independent featureless space must be a redundant structure. We should instead construct spatial descriptions from relations of co-existence and temporal descriptions from relations of succession that exist between material substances.

Clarke's response is to argue against Leibniz's interpretation of PSR: "…intelligent beings are agents; not passive, in being moved by the motives, as a balance by the weights; so that, when the weights are equal, there is nothing to move; but they have active powers and do move themselves …" (Alexander 1956, C.IV,1 & 2). Since God is an active agent, he could have created the world one way rather than another, without having good *external* reasons

[2] Koyré and Cohen (1961) have compared various printed texts of the 1706 Latin edition of the *Opticks* and have found a weakened formulation of the Sensorium attribution to God with the word *tanquam* added in only later prints.

to do so. The sufficient reason for the existence of a brute fact, such as the position of an object in absolute space, is the mere will of God. Leibniz's metaphysics precludes such brute acts of creation, given that for Leibniz it is important to show that God created the best of all possible worlds, in this way demonstrating his perfect wisdom and benevolence.

Leibniz's argument against absolute space boils down to the strength of the PSR and to whether brute facts reducible to an "arbitrary" will of the creator are acceptable. Thus, despite the apparent redundancy in giving absolute positions to bodies, Leibniz's appeal to the PSR is not conclusive. However, Leibniz develops his argument further and appeals to the Principle of Identity of Indiscernibles (PII). According to the PII, "To suppose two things indiscernible, is to suppose the same thing under two names" (Alexander 1956, L.IV.6). While Leibniz derives the PII from the PSR[3], the PII seems plausible even without relating it to the more fundamental principle. The PII seems to embody Occam's razor, according to which entities should not be multiplied unnecessarily. Newton himself endorses simplicity in scientific explanations in his Rule 1 for the Study of Natural Philosophy, where he claims that no more causes should be admitted than are both true and sufficient to explain the phenomena (Newton 1999, p. 794). Leibniz applies the PII to absolute space by arguing that a translation of all material bodies in space (and events in time) produces a universe that is indiscernible from the first. Thus, since they are indiscernible, the descriptions of these two worlds must be taken as referring to the same universe; the absolute position of bodies is therefore only a fiction.

While the PII makes intuitive sense, there are various formulations of it and many objections to its validity.[4] The various versions of the PII depend on what Leibniz means by "indiscernible." According to a weak interpretation of the PII, two objects are indiscernible if and only if they have all the same intrinsic and pure relational properties. (A pure relational property is one where the relation does not involve an individual substance. For example, "being two meters from the Eiffel Tower" is an impure relation.) However, even such weakened versions of the PII have counterexamples. Black (1952) for example argued that one can conceive of a symmetrical universe with two identical spheres. If such a universe is possible the Weak PII is invalid.[5] French and Redhead (1988), French (1989) also argued that Quantum Theory is committed to the existence of indiscernible individual particles, thus providing evidence that the weak version of the PII is not even contingently true. According to a stronger version of the PII, two bodies are indiscernible if and only if they have exactly the same intrinsic properties. This is probably how Leibniz interpreted the principle, given his treatment of relations as ideal structures grounded in inherent properties of substances. However, this formulation seems too strong. We do seem able to differentiate between two objects by locating them in relation to other bodies. For example, we might conceive of the exact same fundamental particles—such as electrons—possessing exactly the same inherent properties but positioned in different contexts.

It seems as if Leibniz's metaphysical principles are disputable, giving Clarke enough room to doubt his metaphysical arguments against absolute space. But the overall legacy of the Correspondence is that Leibniz brings powerful metaphysical arguments against absolute space only to face a conclusive empirical argument, in the form of the bucket experiment.

[3] Rodriguez-Pereyra (1999) argues that Leibniz's argument that the PII follow from the PSR is unsound.

[4] Kant famously argued that the PII is false in the *Critique of Pure Reason* by claiming that it is possible to imagine two identical drops of water.

[5] See Hacking (1975) for an argument that Black's imagined symmetrical universe can always be redescribed to avoid violation of the PII. While Hacking is right that counterexamples of these kind can always be imagined away, it is my view that these alternative descriptions are always more difficult and less natural to conceive than the symmetric cases which makes them unlikely rebuttals.

The metaphysical arguments are undone by a scientific argument; absolute space is necessary if we are to explain the observable effects of inertia. Thus we ought to believe in absolute space despite the very appealing arguments driven by the PII for dispensing with absolute space.

3 Descartes, Newton and Leibniz on "Place"

The Leibniz-Clarke Correspondence focused on the metaphysical consequences of positing absolute space. But the Correspondence also implicitly shifted the terms of the debate, which makes it difficult to understand the Scholium. One of the ways in which the Correspondence helped obscure Newton's intentions, is that Leibniz had used the Correspondence to frame the debate using his own spatial concepts. In important ways, Leibniz's thinking about space and time is more modern than Newton's, rendering his arguments more palpable for the modern reader. Newton assumed that "it is the essence of space to be places" (Newton 1999, p. 410). Leibniz agreed that we should call the collection of all places *space* (Alexander 1956, L.V.47).[6] But despite this apparent agreement, Newton and Leibniz did not mean the same thing by "place." Newton was influenced in his account of "place" by Descartes, and so to understand the differences between Newton and Leibniz on "place" we need to begin our discussion with Descartes.

Descartes has done much to revolutionize our understanding of space, doing away with the Aristotelian closed, finite universe and replacing it with infinite space. Nevertheless, his account of place is much closer to the Aristotelian one than one may realize. Thus, while Newton and Leibniz equated space with the collection of places, Descartes argues that place is somewhat more complex than space, and we should not think that the two concepts are identical. Descartes wanted to distinguish between the extension a body takes up, i.e., the amount of space it occupies, and its spatial relation to other bodies. Thus Descartes differentiates between the region a body occupies, which he terms "internal place," and the relation a body has to the boundary surface immediately surrounding it which is its "external place." It is this external place which determines the position a body has in relation to other bodies, and it is change of external place which determines a body's motion in the "strict," i.e., in the true, philosophical sense (Descartes 1985, p. 229). Unlike Descartes, Newton differentiates between space and matter, and as a result he does not distinguish between internal and external place. Nevertheless, in the Scholium Newton's definition of place still remains the relation between the region a body occupies and its surroundings:

> Place is the part of space that a body occupies, and it is, depending on the space, either absolute or relative. I say the part of space, not the position of a body or its outer surface. For the places of equal solids are always equal, while their surfaces are for the most part unequal because of the dissimilarity of shapes; and positions, properly speaking, do not have quantity and are not so much places as attributes of places. (Newton 1999, p. 409)

Newton claims that place is "a part of space that a body occupies." With this locution Newton doesn't mean to equate "place" with a point of space, but with an extended region

[6] Vailati (1997, p. 114) cautions us to be careful in interpreting what Leibniz meant by equating space with a collection of places. If he meant space to be the set of places, it is not clear why he needed to insist that places are ideal, since the notion of set is also ideal. On the other hand, if space is the aggregate of all places, then it is not clear whether space as a whole exists prior to the the places which are the parts, which is what we assume when we have an aggregate.

that is a portion of space. To understand these assertions we must place them within a conceptual dialectic vis-à-vis Descartes. Newton is concerned with rejecting Descartes' distinction between external and internal place; instead he takes "place" to be an extended region that is contained within a larger space. He argues specifically against the notion that "place" should be equated either with "outer surface" or with "position." Neither outer surface nor position carry the quantity of place, i.e., the volume we attribute to the place a body occupies. We can compare the sizes of objects based on the quantity that their places carry.[7] But if Newton rejects Descartes' distinction between external and internal place he still thinks of place as a particular part of space, i.e., the volume a body occupies within a larger space that contains it.

What is important to recognize is that Newton does not understand the distinction between absolute and relative place as the distinction between *position* relative to absolute space or relative to other distinct bodies. Important to Newton's conception of place is the containment relation between the place and the region of which the place is part. Whether a place is relative or absolute depends on whether the larger, containing region is movable or immovable. For example, if a sailor walks within the interior of a ship, the ship as a whole can be seen as containing the place which the sailor occupies. But the ship as a whole is movable relative to a larger place that contains it, and so the place of the sailor within the ship is only relative. Absolute place, in distinction from relative place, is the place of an object within a large enough containing space that contains all moveable things (material bodies) and is itself not movable.[8] It is this understanding of place that informs Newton when he uses the example of earth and the air that surrounds it to illustrate the notion of relative place. The earth is ordinarily used as a sensible standard for an immovable place. We measure the motion of things relative to the earth and take the earth to be at rest. But the earth's place is itself only relative:

> For example, if the earth moves, the space of our air, which in a relative sense and with respect to the earth always remains the same, will now be one part of the absolute space into which the air passes, now another part of it, and thus will be changing continually in an absolute sense. (Newton 1999, p. 409)

Because we measure the motion of the earth relative to air that contains it, we often think of it as being at rest. Relative to the space which contains our air and the earth, the earth's relative place is at rest. But relative to absolute space the air, together with the earth, moves. Thus the place of the earth relative to its surrounding is only relative. It is at rest only relative to its immediate surrounding.

Leibniz's understanding of "place" is much closer to the modern sensibility, as he understands space as a collection of *positions*, i.e., as a collection of points that hold distance relations to other points. In the Correspondence, Leibniz defines place as follows:

> …*place* is that, which we say is the same to A and, to B, when the relation of the co-existence of B, with C, E, F, G, etc. agrees perfectly with the relation of the coexistence, which A had with the same C, E, F, G, etc. It may be said also, without entering into any further particularity, that *place* is that, which is the same in different moments to different existent things, when their relation of co-existence with certain other existences, which are supposed to continue fixed from one of those moments to the other, agree entirely together. And *fixed existents* are those, in which there has been no cause of any change to the order of their co-existence with others; or (which is the same thing,) in which there has been no motion. (Alexander 1956, L.V.47)

7 See Belkind (2007, pp. 276–280) for further explanation.

8 See Arthur (1994, p. 223) for a similar reading of Newton's notion of relative place.

Thus for Leibniz, *place* is grounded on distance relations that are ultimately defined between point-like substances. A body occupies a place when it has a particular set of distance relations to other bodies that are not moving (in Leibniz's language bodies that are *fixed existents*). One problem that immediately comes to mind is whether or not this definition of place is circular given that motion is ordinarily understood as change of place. If distance relations are determined in relation to bodies in which there is no motion, and if place is defined using distance relations, then it is not clear whether place or motion is the more fundamental concept. We shall consider this problem in Sect. 4.

The most striking aspect of Leibniz's definition of place is that it no longer appeals to relations of containment; those were part of the definition of place since Aristotle, through Descartes and Newton. Leibniz was concerned to reconstruct geometric knowledge. For this purpose, he devised a mathematical approach to geometric relations in his *Analysis Situs* (Leibniz 1979). According to his approach, a geometric figure is reduced to distance relations between vertices of the figure and angles between the figure's sides. For example, a tetrahedron can be characterized by the distance between one vertex and another vertex selected as the origin, together with the ratios of other sides to the first side and angles between the sides (see Arthur 2012). The new theory was specifically devised as a universal account of places ("situs"), to parallel algebra as a science of magnitude.

As Arthur makes clear, Leibniz's account of space does not reduce them into relations between actual bodies. One should think of abstract space as the order according to which places are disposed, when and if bodies are actualized (Alexander 1956, L.V.104). Thus the geometric structure of abstract space can be discussed independently of any particular bodies, even if space does not exist independently of actual relations between bodies. When one considers space as a set of ideal relations, one should distinguish between space as a structure abstracted from any material, actualized bodies, and the set of spatial relations between actualized bodies. The former, abstract space, is ideal, i.e., it is constructed in the mind based on the potential set of relations between bodies; the latter, i.e., actualized space, is a set of "real" relations between material substances.

Places for Leibniz are therefore potential points that hold distance relations to other potential points describing the locations of fixed existences; lines to those other points also have angles between them. As such, places themselves are dimensionless. Clarke recognizes that Leibniz takes places to be dimensionless, and raises the objection (C.III.4, C.IV.16 & 17) that the order of places does not carry quantity. Leibniz's reply to Clarke's objection is that *relations* between places, not places themselves, do carry quantity: "I answer, that order also has its quantity; there is in it that which goes before, and that which follows; there is distance or interval. Relative things have their quantity, as well as absolute ones" (Alexander 1956, L.V.54). To this, Clarke responds by asking whether order and situation carry the *requisite* quantities:

> …going before, and following, constitutes situation or order: but the distance, interval, or quantity of time or space, wherein one thing follows another, is entirely a distinct thing from the situation or order, and does not constitute any quantity of situation or order: the situation or order may be the same, when the quantity of time or space intervening is very different. (Alexander 1956, L.V.54)

So Clarke's worry is whether the intervals of space and time, which carry a metric (and therefore size) and can be assigned a quantity, can derive their measure from relations of coexistence and succession between dimensionless points. The question is, to what do we reduce the spatial and temporal metric? This worry echoes Newton's argument from the Scholium that one must think of place as an extended region if we are to understand the size that bodies carry. It is not possible to "derive" this quantity from an underlying structure of points and positions, or the order which exists between them, and so we must begin with an account of place that is already extended. If a body occupies a set of points, it is not clear, from Leibniz's account, what allows this body to carry the quantity of extension that it carries, if extensions do not inhere somehow in places or in space itself.

The different meanings attributed to the notion of "place" by Newton and Leibniz reflect a difference in their philosophical orientations. Newton is primarily concerned with finding a robust set of bedrock concepts, which is enough to ground his scientific thinking. Newton introduces a notion of place that already carries a metric and a topological structure. He does so because he recognizes that without such structures, the practice of measuring the sizes of objects and comparing them seems unintelligible. But it is not his concern, at least in the *Principia*, to show what grounds these structures. Leibniz, on the other hand, is concerned with constructing a physical account from the sparsest of metaphysical concepts possible. When elaborating on the nature of space, Leibniz thinks one must provide an account of how to construct this structure from the most basic elements of reality. And if one is to make space intelligible, one ought to demonstrate what grounds this structure.

Spatial relations must for Leibniz be constructed from real entities, existing in their own right. In his search for a proper metaphysical grounding for spatial relations Leibniz is at first influenced by Spinoza's thinking. In this stage which is evident in his notes from 1676, he takes the space of actualized bodies to be underwritten by some immutable structure, which remains undivided and undifferentiated throughout the various changes which material parts undergo.[9] This immutable structure is a consequence of God's immensity (Leibniz 2001, p. 55). Later, when Leibniz introduces substantial forms, abstract space becomes an ideal structure, underlying the various *possible* relations between substances. In his mature writings, Leibniz reintroduces the notion of substantial forms, and conceives of substances as undergoing internal evolutions of perceptions or appetites. Each substance undergoes a separate process in which new appetites are constantly actualized. However, while the process of each substance is actualized independently of all the others, each substance has perceptions of the rest of the world. The different substantial perspectives form a coherent framework of perceptions, i.e., they amount to different representations of the same world. For example, if B is represented in A as if it is located 5 m from it, A must be represented in B as if it is 5 m from B. Therefore, while there is no genuine property relating A to B, as all appetites inhere in their substances, the perceptions in A and in B conform to the same spatial structure which is independent of the perspective of either substance. Thus spatial relations are not real relations, but ideal structures are abstracted from perspectival representations of the world. In order for these various perspectives to cohere, they must agree on the spatial structure underwriting each perspectival representation of other substances.[10]

Abstract space, for Leibniz, is an ideal structure which can be abstracted from actual relations, in analogy with lines of genealogy:

> But [place] can only be an ideal thing; containing a certain order, wherein the mind conceives the application of relations. In like manner, as the mind can fancy to itself an order made up of genealogical lines, whose bigness would consist only in the number of generations, wherein every person would have his place…And yet those genealogical places, lines, and spaces, though they should express real truth, would only be ideal things. (Alexander 1956, L.V.47)

[9] Here I am following Arthur (2012).

[10] See Northrop (1946) for the substantival grounding of spatial relations.

Strictly speaking spatial and temporal relations exist only in the mind; in reality objects never occupy the same place, hold the same relations to other bodies, or possess the quantity of extension. Spatial relations are somehow reducible or supervenient on monadic properties of substances in much the same way that "larger than" or "taller than" is reducible to the actual sizes of the bodies standing in the relation.[11]

Leibniz provides something of an explanation as to how extension is related to properties of substances in his letter to de Volder, written in 30 June 1704:

> …as I have often reminded you (although you seem not to have noticed), extension is an abstraction from the extended thing, and it is no more a substance than a number or multitude can be considered to be a substance; it represents only a certain nonsuccessive ([unlike] duration) and simultaneous diffusion or repetition of a certain nature, existing simultaneously, with a certain order among themselves. It is this nature, I say, that is said to be extended or diffused. And so the notion of extension is relative, that is, extension is the extension of something, just as we say that a multitude or duration is a multitude of something or a duration of something. Furthermore, the nature which is supposed to be diffused, repeated, continued, is that which constitutes the physical body; it cannot be found in anything but the principle of acting and being acted upon, since the phenomena provide us with nothing else. (Leibniz 1989, p. 179)

Thus the property of being extended, and presumably the magnitude that is carried by this property, is only a phenomenal measure of the power of bodies to act and to be acted upon (i.e., the power to resist being acted upon). The internal evolution of substances can be viewed as the activity of substances and the application of forces. The notion that bodies have inner forces that give rise to their extendedness is developed in *A Specimen of Dynamics*:

> …the *primitive force of being acted upon* or of *resisting* constitutes that which is called *primary matter* in the schools, if correctly interpreted. This force is that by virtue of which it happens that a body cannot be penetrated by another body, but presents an obstacle to it, and at the same time is endowed with a certain laziness, so to speak, that is, an opposition to motion …(Leibniz 1989, p. 119)

The metaphysical account of primitive forces of being acted upon therefore gives us the grounding on which extension and its magnitude are based. But this metaphysical grounding does not yield a physical theory of extension, since the phenomena themselves do not reveal the forces acting, only their phenomenal consequences. While Leibniz provides a metaphysical account of this grounding, he does not provide a mathematical theory that reduces the quantities of extension to other quantities. The metaphysical account of extension does not come hand in hand with a mathematical elaboration.

Regardless of whether Leibniz's account of extension is compelling, we should note an important transition from Descartes to Newton and Leibniz in the notion of "place," which results in an important shift in the concept of space. Newton conceived of place as directly carrying the quantity of extension, as part of its attributes. But Newton importantly thought of place as the part of space a body occupies. Essential to this account of place is the containment relation between a place and its surrounding region. Leibniz, unlike Descartes and Newton, no longer takes containment to be a primary relation that is involved in defining a body's place. Rather, place is now determined through distance relations between substances.

[11] The ideal nature of space and time also explains their continuous nature, which is distinct from the discrete nature of material bodies. See Vailati (1997, p. 118) for an argument that density (i.e., the property of space according to which between any two points there is another) entails continuity. See also Hartz and Cover (1988) for an account of the ideal realm, which for them exists over and above the phenomenal and the real.

Extension is not a genuine property of places, but a phenomenal measure of passive forces existing within substances. This transition from Newton to Leibniz in the meaning of "place" is primarily motivated by Leibniz's desire to reconstruct geometry from distance relations between points and angles between lines connecting points. Another motivation is Leibniz's attempt to ground the notion of extension in a theory of substances and substantial forms.

4 True Motion

The Correspondence, given its Leibnizian bent, obscures the irreconcilable differences between Newton's and Leibniz's conception of space. Clarke, by citing the bucket experiment as giving "mathematical" proof that relational accounts of space are false, is also responsible for rendering unintelligible the precise nature of Newton's arguments.

As we shall see in Sect. 5, most of the arguments in the Scholium are based on showing that there is a genuine distinction between absolute and relative motion. If such a distinction can be made, Newton argues, then absolute space is required for absolute motion to be defined. Some commentators have taken Leibniz's position to be that there is no such distinction, i.e., that all motion is relative *and* that there is no fact of the matter as to which bodies move and which do not. The dialectic seems to be that either we can show that some bodies possess motion as a matter of fact, in which case Newton is right, or that no distinction between rest and motion can be made, in which case Leibniz wins the day. However, this false opposition doesn't reflect the dialectic between Newton and Leibniz, since the distinction between absolute and relative motion is not the same as the distinction between true and apparent motion.

All 17th century natural philosophers agreed there is a genuine distinction between "true" and "apparent" motion.[12] True motion is the unique motion we attribute to the body and the one we can use to analyze causal relations. Apparent motions are the many relative motions a body has that do not genuinely belong to the body. For example, if we define the motion of a body relative to a fly, this motion will be apparent *not* because it is relative, but because it is too irregular and dependent on the reference body to reveal any causal relations in which the body is caught up. However, while all parties to the debate would agree that true motion has to be distinguished from apparent motion, true motion can still be either absolute or relative. The one true and genuine motion could be defined relative to absolute space or relative to an appropriately selected reference body.

Descartes introduces the distinction between true (or "proper") and apparent (or "common") motions in the following passage from the *Principles*:

> …if we consider what we should understand by motion, not in common usage but in accordance with the truth of the matter, and if our aim is to assign a determinate nature to it, we may say that *motion is the transfer of one piece of matter, or one body, from the vicinity of the other bodies which are in immediate contact with it, and which are regarded as being at rest, to the vicinity of other bodies*. (Descartes 1985, p. 233; emphasis in original)

Thus for Descartes true motion is defined as change of external place. This motion is both "true" and "relative." It is true because out of the many apparent motions a body has, the motion of the body relative to the surrounding bodies is the one motion defined according to the truth of the matter. This motion is the one we use in philosophical discourse to reveal

[12] See Rynasiewicz (1995) for the distinction between true and apparent motion.

the causal chains the body is involved in. It is relative motion since for Descartes there is no background space distinct from bodies that can serve as reference.

The implication of Descartes' definition of true motion is that there can be no common reference frame relative to which all true motions are defined. Each body requires its own reference frame determined by the bodies in the immediate surroundings.[13] Also, there is a sense in which true motions do not "add up." Assume that we have a sphere A located within another hollowed sphere B, which is located within another hollowed sphere C. According to Descartes, the true motion of A is determined relative to B, while B is regarded as being at rest. However, the motion of B is determined in relation to C, while C is regarded as being at rest. There is a sense in which B could be both moving and nonmoving. It is moving in the strict, philosophical sense if it is moving in relation to the body that contains it, i.e., relative to C. But it is regarded as nonmoving when we measure the true motion of A. Assume that A rotates with angular velocity ω relative to B, and B rotates with the same angular velocity ω relative to C. The true motion of A is ω, since we measure its motion relative to B, where B is regarded as being at rest. But it does not follow that A rotates twice as fast as B, even if B's true motion is ω, measured relative to C. *Relative* to body C, A rotates at a speed of 2ω, but this motion is only apparent and is not the true, philosophical motion we attribute to the body.

We are now ready to understand what Newton means when he says that "true and absolute motion cannot be determined by change of position from the vicinity of bodies that are regarded as being at rest" (Newton 1999, p. 411). The adjectives "true" and "absolute" carry different cognitive meanings, where "true" is taken to be the opposite of "apparent" and "absolute" the opposite of "relative". According to Newton, absolute motion, which is change in absolute place, is distinguished from relative motion, which is change in relative place. But we have to keep in mind that change in absolute or relative place is *not* identical to change in distance relations (defined either in relation to points in absolute space or in relation to other bodies). Change in absolute place is the change of place relative to an immovable containing space, while change in relative place is change of place relative to a movable container. Newton argues here that true motion must be absolute, i.e., he considers true motion to be change in absolute place, in contradistinction with Descartes who takes true motion to be relative, i.e., as the change of the position of a body relative to a vicinity of bodies that are regarded as being at rest.

The confusion between "true" and "absolute" has long misled commentators into thinking that true has the same meaning as absolute. This confusion left the false impression that Descartes was committed to the general relativity of motion. Part of the confusion stems from Leibniz's reaction to Descartes' relational definition of true motion, which is different from Newton's reaction. While Newton thought that problems with Descartes' physics suggest that one needs to define true motion relative to absolute space, Leibniz believed that the implication is that there is no phenomenal criterion of true motion. The different reactions probably stem from the thinkers' divergent theoretical approach to the notion of "place". We have seen that in the definition of place, Leibniz abandons the containment relation between a place and its surroundings, and reduces place to the distance a body has in relation to other reference bodies. This implicit shift in the meaning of "place" leads to an important consequence, namely, to Leibniz's commitment to the general relativity of phenomenal measures of motion. In his *Critical Thoughts on the General Part of the Principles of Descartes*, written in 1692, Leibniz argues that Descartes' definition of true motion implies that there is no phenomenally valid criterion of true motion:

[13] See Garber (1992) for a similar reading of Descartes' definition of true motion.

> If motion is nothing but change of contact or of immediate vicinity, it follows that we can never define which thing is moved. For just as the same phenomena may be interpreted by different hypotheses in astronomy, so it will always be possible to attribute real motion to one or the other of the two bodies which change their mutual vicinity or position. Hence, since one of them is arbitrarily chosen to be at rest or moving at a given rate in a given line, we may define geometrically what motion or rest is to be ascribed to the other, so as to produce the given phenomena. Hence if there is nothing more in motion than this reciprocal change, it follows that there is no reason in nature to ascribe motion to one thing rather than to others. The consequence of this will be that there is no real motion. (Leibniz 1969, p. 393)

Leibniz is in the peculiar position of deducing the relativity of motion from Descartes' definition of true motion. While Descartes intended his definition as a means of differentiating true from apparent (or "common") motion, Leibniz utilizes the very same definition to conclude that, if motion is nothing but change of contact, there is no reason to ascribe motion to one body rather than to another. The only way in which Leibniz could have misread Descartes in this way is to consider that Leibniz no longer takes the relation between a body and its immediate surroundings to involve a containment relation. For Leibniz, the change of contact or immediate vicinity is mere change in distance relations. And since motion is merely change in the distance relation between a body A and a reference body B, the hypothesis that A is moving and B is at rest is equivalent to the hypothesis that A is at rest and B is moving.

Leibniz repeats this objection to Descartes' definition of true motion in *A Specimen of Dynamics*, written in 1695:

> …it follows that motion taken apart from force, that is, motion insofar as it is taken to contain only geometrical notions (size, shape, and their change), is really nothing but the change of situation, and furthermore, that *as far as phenomena are concerned, motion is a pure relation*, something Descartes also recognized when he defined motion as the translation from the neighborhood of another. But in drawing consequences from this, he forgot his definition and set up the laws of motion as if motion were something real and absolute. (Leibniz 1989, p. 130)

Leibniz misrepresents Descartes, since Descartes explicitly claimed that the motion of a body from its immediate surroundings, taken to be at rest, was intended as the definition of true motion, and that this motion is distinguished from all apparent or common motions. Implicit to Descartes' definition was the assumption that the body surrounding the moving body contains it, and can therefore function as the reference for the definition of true motion. Leibniz takes his version of Descartes' definition of true motion as the only phenomenal way to determine the motion of bodies. But since change in distance relations is the phenomenal measure of motion, and since this change in distance relations is relative and symmetric, Leibniz concludes that there cannot be a phenomenal criterion for true motion. It is important to note, however, that the arguments from *Critical Thoughts* and *A Specimen of Dynamics* do not attempt to demonstrate that true motion does not exist. Rather, Leibniz argues that while Descartes' definition of motion is the one made available by the phenomena, this definition fails in giving us true motion, contrary to Descartes' claims. Current day commentators often and wrongly attribute to Descartes a commitment to the general relativity of motion, since like Leibniz, modern day commentators fail to notice that Descartes is committed to the containment relation between a body and the surrounding bodies.

Leibniz takes his conclusion one step further when he turns the relativity of motion into principle about that which can be gleaned from the phenomena. Following the previous quote from *A Specimen of Dynamics*, Leibniz explains that:

> Therefore, we must hold that however many bodies might be in motion, one cannot infer from the phenomena which of them really has absolute and determinate motion or rest. Rather, one can attribute rest to any one of them one may choose, and yet the same phenomena will result. From this follows something that Descartes did not notice, that *the equivalence of hypotheses is not changed even by the collision of bodies with one another*, and thus, that the laws of motion must be fixed in such a way that the relative nature of motion is preserved, so that one cannot tell, on the basis of the phenomena resulting from a collision, where there had to be rest or determinate motion in an absolute sense before the collision. (Leibniz 1989, p. 131)

The consequence of failing to articulate a phenomenal criterion of true motion is that laws of collision, as long as they are derived from the phenomena, must conform to the equivalence between rest and motion. As a result Leibniz finds fault in Descartes' laws of collision since they obviously do not obey this symmetry. A corrected set of laws should obey the general relativity of motion, as should all laws that merely describe the phenomena.

However, the lack of a phenomenal criterion for true motion does not imply that there is no true motion in nature. One has to to distinguish between that which can be directly measured and that which can be explained. In a letter to Huygens, dated June 22, 1694, Leibniz explains the important distinction between phenomenal measures of motion and their causes:

> As for the difference between absolute and relative motion, I believe that if the motion, or rather the moving force, of the body is something real, as it seems one must recognize, it is necessary that it have a subject. For *a* and *b* colliding, I hold that all the phenomena will come to the same no matter in which one of them one places the motion or the rest. And if there were 1,000 bodies, I would still agree that the phenomena could not provide us (nor even the angels) with an infallible reason for determining the subject of the motion or its degree; and that each could be conceived apart as being at rest. And this is also all that you do demand; but you will not deny, I believe, that each really has a certain degree of motion or, if you will, of force; despite the equivalence of hypotheses. It is true that I draw from this the consequence that there is in nature something other than that which Geometry can determine. And this is not the least among several reasons that I use to prove that, besides extension and its variations, which are something purely geometrical, it is necessary to recognize something superior, which is force. Mr. Newton recognizes the equivalence of hypotheses in the case of rectilinear motions, but in regard to circular motions he believes that the effort which rotating bodies make to recede from the center or axis of rotation makes known their absolute motion. But I have reasons which make me believe that nothing breaks the general law of equivalence. (Leibniz 1969, p. 418, quoted in Hartz 1984, p. 317)

The phenomena does not provide us with an "infallible reason" for determining the subject of motion, i.e., the phenomena does not indicate which object actually carries the motion. But a certain degree of motion is actually determined by the force contained in the body. Thus we should distinguish between that which can be learned from the phenomena and that which is attributed to bodies as a matter of fact. Since change of relative place is "infected" with the relativity of motion, all facts derived from observable motions are subject to the general relativity of motion. But phenomenal measures can be supplemented with a metaphysical account taking into consideration the inner forces of bodies and the causes of motion. Thus the conflict between the equivalence of hypotheses, which commits Leibniz to the general relativity of motion, and the view that there is a genuine, true motion is at first glance only apparent.[14] Phenomenal motion arises from determining the motion of a body through sensible measures, and so requires an arbitrary reference body relative to which it is defined. But this equivalence of hypotheses does not preclude presupposing that each body has a certain degree of motion, when the same bodies are considered from a metaphysical perspective.

In responding to Clarke's argument that in the Scholium Newton proved the existence of absolute space, Leibniz endorsed the existence of true motion:

> I grant there is a difference between an absolute true motion of a body and a mere relative change of its situation with respect to another body. For when the immediate cause of the change is in the body, that body is truly in motion; and then the situation of other bodies with respect to it will be changed consequently, though the cause of that change be not in them. It is true that, exactly speaking, there is not any one body that is perfectly and entirely at rest; but we frame an abstract notion of rest by considering the thing mathematically. (Alexander 1956, L.V.53)

Thus, Leibniz agrees that one has to distinguish between absolute true motion and a mere relative change of situation. Like Newton, Leibniz identifies true with absolute motion. However, Leibniz does not equate absolute motion with change in absolute place, but with the causes of motion.

The claim that true motion is correlated with internal forces also renders Leibniz's definition of place non-circular. The phenomenal meaning of "motion" is change of place, where place is defined as the distance relation the body has to other bodies that are "fixed existents." Leibniz then also explains that fixed existents are "those in which there has been no cause of any change to their coexistence with others, or (which is the same thing) in which there has been no motion." The latter use of "motion" is in its metaphysical sense, i.e., it is motion that is caused by inherent forces of the body.[15] Nevertheless, there is also a sense in which true motion does find its way into *physical*, or dynamical explanations. One is able to correlate part of the motion of the body with inherent forces when a full dynamic account is given to the phenomena:

> I understand here the *proper motion* of bodies (as I have called it) as opposed to the *common motion* that can be ascribed to the center of gravity. As a result, we should conceive of their proper motion (conceive of it in a hypothetical way, I say) as if the bodies are being carried on a boat which has the motion of their common center of gravity, and on the boat, we should imagine, they move in such a way that from the composite motion of the boat, which they have in common (that is, the motion of the center of gravity), and from their own proper motion, the phenomena can be saved. From what we have said we can also understand that *the action of bodies is never without reaction, and both are equal to one another, and directed in opposite directions*. (Leibniz 1989, p. 135)

[14] For those commentators accusing Leibniz of inconsistency, see Russell (1951, p. 121), Alexander (1956, p. xxvii), Earman (1989, p. 131). For claims that there is consistency because of complete separation between metaphysical and phenomenal motion, see Cook (1979, pp. 50–57), and for claims that Leibniz allows that dynamics would distinguish between absolute and relative motion see Roberts (2003) and Lodge (2003).

[15] According to Hartz (1984), there are theoretical reasons for supplanting the phenomenal description that includes space, time, motion and impenetrability or mass with a metaphysical account of forces. Hartz argues that Leibniz believes that geometric properties and mass or impenetrability are not sufficient in providing material explanations with a proper metaphysical foundation. None of these material properties are sufficient for individuating bodies.

Thus, despite the equivalence of hypotheses and the general relativity that infects our laws of motion, we can still correlate some motions with inherent forces. For example, in collisions, it is not possible to discern the true motions of the colliding bodies. But it is still possible to determine which part of the motion added to a body stems from forces in the body. To determine these forces, one may subtract the motion attributed to the center of gravity from the motions of the colliding bodies, and view the situation as if the center of gravity is at rest. Taking such a perspective reveals the part of motions that arise from inner forces, and thus they can be taken to be true, or proper motions.

Leibniz therefore allows that some parts of true motions could be revealed once a dynamic explanation is complete. When the causes of motion or the forces that give rise to motions are revealed, it is appropriate to take the motions that are caused by these forces as true. In a passage deleted from earlier versions of *A Specimen of Dynamics*, Leibniz writes on the process of using dynamic explanations to define the true motion of bodies:

> I also perceived the nature of motion. Furthermore, I also grasped that space is not something absolute or real, and that it neither undergoes changes, nor can we conceive absolute motion, but that the entire nature of motion is relative, so that from the phenomena one cannot determine with mathematical rigor what is at rest, or the amount of motion with which some body is moved. This holds even for circular motion, though it appeared otherwise to Isaac Newton, that distinguished gentleman, who is the greatest jewel that learned England ever had. Although he said many superb things about motion, he held that, with the help of circular motion, he could discern which subject contains motion from the centrifugal force, something with which I could not agree. But even if there may be no mathematical way of determining the true hypothesis, nevertheless, we can, with good reason, attribute true motion to that subject, which would result in the simplest hypothesis most suitable for explaining the phenomena. For the rest, it is enough for practical purposes for us to investigate not the subject of motion as much as the relative changes of things with respect to one another, since there is no fixed point in the universe. (Leibniz 1989, p. 125)

It is not clear why Leibniz erased this passage. Perhaps the notion of "simplest hypothesis" is too vague to be helpful. But it is clear from this passage that Leibniz does not believe that the bucket experiment demonstrates the true nature of circular motion. In this passage, Leibniz insists that any phenomenon in itself, cannot distinguish between true and apparent motion. Thus when one observes what appears to be a correlation between the rotation of the water in the bucket and the concavity along the water's surface, one may not yet deduce that the water truly rotates. But this is not the whole story, since one can with good reason attribute true motion to the subject, *which would result in the simplest hypothesis most suitable for explaining the phenomena*. Thus if the dynamic situation is fully known and understood, we have a means of finding the subject of motion.

It seems as if his assertions about true motion should give Leibniz some pause. On the one hand, the laws of motion are meant to describe the phenomena. And because true motion cannot be derived from the phenomena, the laws of motion are supposed to be infected by the general relativity of phenomenal measures of motion. On the other hand, the laws of motion help us construct a dynamic account at the end of which we can, with good reason, attribute true motions to bodies, at least to the extent it is possible to identify the forces giving rise to motions. Moreover, the laws of motion, as Leibniz must have realized, only obey Galilean relativity. The laws are invariant under boosts in rectilinear motions, but not under accelerated motions. Thus, it's not clear how the phenomena dictate the general relativity of motion, while laws of motion do not obey the general relativity of motion. Is Leibniz here simply inconsistent, allowing for the discernment of true motion when applying the laws of motion, but affirming the equivalence of hypotheses in general?

The solution to this apparent inconsistency is that Leibniz believes that forces can only give rise to rectilinear motions, and that all motions are constructed from rectilinear motions. For example, in collision cases, if *A* is moving towards *B* which is at rest, and gives it a push in the same direction in which it moves, then the new force created in *B* will give rise to rectilinear motion in the same direction. Thus, if Leibniz is correct in his conjecture that all forces give rise to rectilinear motions, then the Galilean relativity of laws of collision characterizes the forces, the true causes of motion, not the phenomenal measures of motion. If all preference for rectilinear motions is derived from the underlying causes, and not directly from the phenomena, then the Galilean relativity of laws of motion is not a refutation of the general relativity of motion, but a confirmation of it. The underlying causes cannot be inferred from the phenomena, but are revealed through a metaphysical analysis of forces inherent in substances. Once a dynamic explanation is constructed one might be able to notice some phenomenal consequences, but since forces are not inferred from the phenomena, it is not in conflict with the claim that there is no phenomenal criterion of true motion.

In *A Specimen of Dynamics*, Leibniz argues against the claim that inertial effects produced by rotation constitute an observable criterion of true motion:

> Also, since only force and the nisus arising from it exist at any moment (for motion never really exists, as we discussed above), and since every nisus tends in a straight line, it follows that *all motion is either rectilinear or composed of rectilinear motions*. From this it not only follows that what moves in a curved path always tries to proceed in a straight line tangent to it, but also—something utterly unexplained—that the true notion of solidity derives from this. (Nothing is really solid or fluid, absolutely speaking, and everything has a certain degree of solidity or fluidity; which term we apply to a thing derives from the dominant appearance it presents to our senses.) For if we assume something we call solid is rotating around its center, its parts will try to fly off in a tangent; indeed they will actually begin to fly off. But since this mutual separation disturbs the motion of the surrounding bodies, they are repelled back, that is, thrust back together again, as if the center contains a magic force for attracting them, or as if the parts themselves contained a centripetal force. Thus, the rotation arises from the composition of the rectilinear situs for receding on the tangent and the centripetal conatus among the parts. Thus, all curvilinear motion arises from rectilinear nisuses composed with one another, and at the same time, it is understood that all solidity is caused by surrounding bodies pushing a body together; if matters were otherwise, then it could not happen that all curvilinear motion is composed of pure rectilinear motions. (Leibniz 1989, p. 135)

Leibniz takes inherent force to be of one kind, i.e., the one that gives rise to rectilinear motion. To understand Leibniz's view we should avoid thinking of his laws of motion as equivalent to those of Newton. For Newton, the laws of motion only provide a general blueprint for analyzing the consequences of impressed forces. These forces could be of any kind. The acceleration that arises from impressed forces has to be combined with the inertial motion of bodies. Newton's scientific program is therefore more general and more flexible; in his case the strict Galilean relativity of laws of motion is in conflict with the general relativity of motion, since forces are revealed by measuring the acceleration of bodies. It is necessary for accelerations to be measured relative to inertial reference frames for them to reveal the nature of the impressed force. But in Leibniz's account, all forces give rise to rectilinear motion.

If motion is non-rectilinear, it must be constructed from an infinite number of rectilinear motions. Thus, acceleration, by definition, is never by itself the true motion of a body, given that it is a complex motion that should be reduced to an infinite number of rectilinear motions, each of which is correlated with a different force. For Leibniz, the Galilean relativity of laws of collision is equivalent to the general relativity of motion since for him the only existing forces are those giving rise to rectilinear motions.

Leibniz's account explains why he thinks Newton is wrong in inferring true motion from inertial effects. According to Newton, the rotation of the water in the bucket gives rise to inertial effects, thus it seems as if one can use inertial effects to determine the water is truly rotating. The tendency to recede from the axis of rotation perhaps stems from the true motion of the water's parts. However, the rotation of the water as a composite entity requires a dynamic explanation of the water's fluidity. When attributing rotation to the water as a whole, we also have to take into account the forces that pull the parts of the rotating body together and give it its semblance of relative solidity or fluidity. According to Leibniz, these forces involve the surrounding bodies that push on the rotating body in reaction to the tendency of the parts of the rotating bodies to pull apart. A full dynamic explanation of rotation and the inertial effects it produces would ultimately be explained by the laws of motion, which only admit forces that give rise to rectilinear motions. When the part is pulled away from the axis of rotation, there is also a force that pulls it in. Thus, the observed inertial effect is not indicative of the true rotational motion of the composite body, given that the water's motion is partially determined by some unknown combination of forces. In short, since the dynamic situation is not completely known, one cannot infer that the water's rotation is true (i.e., that its causes are known). Furthermore, if the water's rotation is reducible to many interactions between the water and its surroundings, each interaction arising from forces giving rise to infinitesimal rectilinear motions, it would be possible to replace the rotation of the water with an infinite number of rectilinear motions, each of which is true.

Leibniz's account of solidity or fluidity is far from satisfying. It is hard to imagine how the reactive forces of the surrounding fluids are coordinated just to produce the right amount of solidity or fluidity in a body whose parts tend to fly off on a tangent. But despite the implausibility of Leibniz's alternative explanation, it is still possible to appreciate the logic of his argument. Once Leibniz is able to raise doubts about the inference from inertial effects to the true motion of a composite entity, Newton's bucket experiment cannot be taken to be showing that a phenomenal criterion for true motion exists.

Leibniz's account of true motion is complex. According to him, there is no phenomenal criterion of true motion, and every fact derived from the phenomena must adhere to the general relativity of motion. However, when it becomes clear that a complete account can be given for a particular dynamic situation, true motions can be partially discovered when we correlate them with the inner forces of the interacting bodies. The fact that laws of collision only obey Galilean relativity, and not the general relativity of motion, is not inconsistent with Leibniz's assertion that no phenomenal criterion can be given for true motion. While deviations from common rectilinear motions, in complete dynamic descriptions, are associated with true motion, *it does not follow that accelerated motion directly causes inertial effects*. Only when such accelerations are incorporated within a full dynamic explanation can one take them to be true. Thus the bucket argument, given its incomplete account of the fluidity of water and the rigidity of the sides of the bucket, does not amount to proof that there is a phenomenal criterion of true motion.

5 Newton's Scholium

The standard account of the Scholium points to the bucket experiment as providing "empirical" evidence that relational conceptions of space are false. The desire to showcase this argument over others is understandable, given the suspicious metaphysical nature of absolute space. By endorsing absolute space Newton seems to have outstepped the strictures of empiricist philosophy, while Leibniz, who does not eschew metaphysical obscurantism, is ironically on the side of parsimony. But the "empirical" proof provided by the bucket experiment is no proof that relational approaches to space are necessarily false. Moreover, Newton never intended the arguments in the Scholium as refutations of relational accounts such as the one provided by Leibniz, nor did he claim that the existence of absolute space somehow "explains" inertial effects. Newton divides his argument into various kinds when he asserts that "absolute and relative rest and motion can be distinguished from each other by their properties, causes and effect" (Newton 1999, p. 411). The bucket experiment is given in the section discussing the effects of absolute motion. But there are two arguments almost hidden in the paragraphs discussing the properties and the causes by which absolute motion is distinguished from relative motion. These arguments are more instructive than is ordinarily thought.

The first argument we'll consider assumes that rigid bodies amass their quantities of motion from their parts. The property that helps distinguish between absolute and relative motion is the part-whole relation governing the quantity of motion. We shall refer to this argument from properties as **API**:

> It is a property of motion that parts which keep given positions in relation to wholes participate in the motions of such wholes. For all the parts of bodies revolving in orbit endeavor to recede from the axis of motion, and the impetus of bodies moving forward arises from the joint impetus of the individual parts. Therefore, when bodies containing others move, whatever is relatively at rest within them also moves. And thus true and absolute motion cannot be determined by means of change of position from the vicinity of bodies that are regarded as being at rest. For the exterior bodies ought to be regarded not only as being at rest but also as being truly at rest. Otherwise all contained bodies, besides being subject to change of position from the vicinity of the containing bodies, will participate in the true motions of the containing bodies and, if there is not such change of position, will not be truly at rest but only be regarded as being at rest. For containing bodies are to those inside them as the outer part of the whole to the inner part or as the shell to the kernel. And when the shell moves, the kernel also, without being changed in position from the vicinity of the shell, moves as a part of the whole. (Newton 1999, p. 411)

It is clear from this passage that Newton is arguing against Descartes' relational definition of true motion. He is concerned with showing that true and absolute motion "cannot be determined by means of change of position from the vicinity of bodies that are regarded at rest." His argument here is directed at Cartesian physics, not relational definitions of true motion in general. The argument boils down to demonstrating that the quantity of motion of solid bodies cannot be defined if Descartes' definition of true motion is endorsed. On the one hand, there is the presupposition that the quantity of motion of composite bodies arises from the quantities of motion of the parts. "For all the parts of bodies revolving in orbit endeavor to recede from the axis of motion, and the impetus of bodies moving forward arises from the joint impetus of the individual parts." The notion of quantity of motion is governed by a part-whole relation—the impetus of the whole, or the tendency to continue moving in a straight line—arises from the tendencies of the individual parts. However, in a solid body, each part is at rest relative to its immediate surroundings given that all the parts move together. Thus, according to Descartes' definition of true motion, the parts are at rest. Nevertheless the whole still endeavors to move in a straight line, and this endeavor is supposedly derived from the endeavors of the parts. Thus, Descartes seems to be committed to all the parts being at rest, when his definition of true motion is in place. But at the same time he is committed to all the parts contributing quantity of motion to the composite bodies, since he asserts that the quantity of motion of the composite, solid body is proportional to the size of the body.

The upshot of Newton's argument is that it is absurd for Descartes to hold his definition of true motion and be committed to the additivity of quantity of motion. From this conflict, Newton implicitly concludes that Descartes' definition of true motion is inadequate. Since Descartes believes that the place of a body is always defined relative to the containing bodies, and that the containing bodies are themselves movable, the natural alternative to Descartes' definition of true motion is to posit the existence of a immovable container. Thus, absolute space is a large enough container relative to which all bodies can be placed and their absolute motions defined.

To fully understand how **API** distinguishes between absolute and relative motion, some of the assumptions implicit to the argument should be made explicit:

- Def1 Place is defined as the part of space a body occupies within the larger space that contains it.
- Def2 According to Descartes' definition, the true motion of a body is the change of place relative to the immediate surrounding bodies.
- Def3 A property of quantity of motion is that the quantity of motion of the whole is the sum of the quantities of motion of the parts.
- P1 Descartes' definition of true motion (Def2) is inconsistent with a property of quantity of motion (Def3).
  - (a) Since all parts of a rigid body do not change their place relative to their immediate surroundings in Descartes' definition (Def2), they are truly at rest.
  - (b) The quantity of motion of a rigid body is the sum of the motions of its parts, thus the quantity of motion of any rigid body is zero (Def3).
  - (c) If the quantity of motion of a rigid body is zero, it must be at rest. But it is clear that some rigid bodies are moving.
- P2 The quantity of motion is necessary for a science of motion.
- P3 True motion is defined either as the motion of a body relative to a movable container (Descartes) or relative to a immovable container (absolute space).
- P4 True motion exists.
- C True motion is defined as the motion of a body relative to a immovable container (absolute space).

The above quoted passage is mainly given in support of P1. But when the implicit premises are brought to the fore, one gets a clear sense of the argument. P1 claims that there is a conflict between Descartes' definition of true motion and the additive property of quantity of motion. Since quantity of motion is essential for a science of motion (P2), we conclude that true motion cannot be determined according to Descartes' definition. Since motion seems to be defined either relative to a movable or a immovable container, the alternative to Descartes' definition of true motion seems to be absolute motion, or the motion of a body relative to immovable container. And from the assumption given in P4 that true motion exists, we know that true motion is actually determined by motion relative to absolute space. P4 is necessary if we are to conclude that true motion is a property that bodies possess as a matter of fact. Without this premise, true motion may be defined but never actualized.

API is highly contextual, given that it is directed against Descartes' definition of true motion and presupposes that places are defined relative to containing spaces. The function of this argument is to expose the flaws of the paradigmatic science of the day. One is left with the possibility that perhaps other relational definitions of true motion would be consistent with the notion of quantity of motion, that is one can deny P3 if Def1 is given up. Given our presentation of Leibniz's views, it should be obvious that Leibniz would not deny the existence of true motion, as many commentators assume, but instead deny P3. His definition of true motion correlates the motion of bodies with internal forces that cause motion. Leibniz may also disagree with P2, given that he is committed to the conservation of living force rather than quantity of motion.

Since the bucket experiment is located in the argument from effects for distinguishing between absolute and relative motion, we shall refer to it as **AE**. Notice that **AE** runs in parallel lines to **API**. I shall not rehearse the familiar argument, but rephrase it as follows:

P1 The motion of the water relative to the bucket (Descartes' true motion) cannot be correlated with the water's tendency to recede from the axis of rotation.

- (a) Before the bucket is given a push, there is no relative motion and no tendency to recede from the axis of rotation.
- (b) Given an initial push, there is relative motion and no tendency to recede from the axis of rotation.
- (c) After the water rotates, there is no relative motion but there is tendency to recede from the axis of rotation.

P2 The water's tendency to recede from the axis of rotation is an observable consequence of the water's true motion.
P3 True motion is defined either as motion relative to the bucket (movable container) or relative to a absolute space (immovable container).
P4 True motion exists.
C True motion is defined relative to absolute space.

There is an obvious parallel between **AE** and **API**. In **AE** Newton is again pointing out a conflict between Descartes' definition of true motion and the empirical consequences of having impetus (i.e., the possession of quantity of motion). This conflict parallels the previous inconsistency in **API** between Descartes' definition of true motion and the property of quantity of motion. The similarity between the two arguments reinforces the view that Newton was concerned to show the same inconsistency in various ways. It demonstrates that Newton's aim was directed at a particular theory of motion. We now recognize that Leibniz would deny P2 and P3, but not P4. Leibniz's claim against P3 is clear, since again he denies that the only alternative to Descartes' definition is to define motion in relation to absolute space. We reconstructed his objection to P2 in the previous section. Given Leibniz's definition of true motion as correlated with the cause of motion, Leibniz denies this premise. Since Newton did not give a physical explanation for the water's fluidity and the bucket's solidity, Newton provided an incomplete dynamic account of the situation and the forces that could give rise to the water's rotation. These forces would ultimately ground the water's true motion, thus it is not true that the tendency to recede from the axis of rotation is an observable consequence of the water's true rotational motion.

Another argument from properties is perhaps more general than **API**, even though it is still primarily cached in Cartesian terms. We shall refer to this argument as **APII**:

> A property akin to the preceding one is that when a place moves, whatever is placed in it moves along with it, and therefore a body moving away from a place that moves participates also in the motion of its place. Therefore, all motions away from places that move are only parts of whole and absolute motions, and every whole motion is compounded of the motion of a body away from its initial place, and the motion of this place away from its place, and so on, until an unmoving place is reached as in the above-mentioned example of the sailor. Thus, whole and absolute motions can be determined only by means of unmoving places, and therefore in what has preceded I have referred such motions to unmoving places and relative motions to movable places. (Newton 1999, 411)

This argument presupposes the Cartesian definition of place and argues that quantity of motion cannot be defined unless we admit the existence of immovable places. The argument begins with Newton's account of place as the extended region a body occupies within the surrounding region that contains it. Thus, the sailor that moves within the interior of a ship occupies a particular place within a larger place, i.e., the ship that contains him. Newton begins his argument by claiming that "when a place moves, whatever is placed in it moves along with it." Thus, when we think of a moving ship, we think of it as occupying a place that moves relative to the ship's surroundings. When the place of the ship changes, everything that is placed within the ship, e.g., the sailor, moves along with it (or with the ship's place). But the ship is also a composite body *comprising* of everything within it, so that the quantity of motion of each of its parts contributes to the quantity of motion of the ship as a whole. But if the ship is moving in relation to a larger place that contains it, we also recognize that the ship's quantity of motion contributes to the quantity of motion of whatever it is that contains it.

Newton then attempts to show that these relations of containment between movable places and their contents, together with the assumption that bodies carry a determinate quantity of motion, leads to the distinction between movable and immovable places. Assume contrary to the conclusion that there is no distinction between movable and immovable places. Whenever we consider the quantity of motion of a body we recognize that it cannot be attributed to the body without examining its contribution to the larger place that contains it. The quantity of motion of the containing place must then be considered as contributing to the quantity of motion of the larger place that contains it. And if it contributes a quantity of motion to the containing place, then exchanges of quantities of motion of the containing place are part of the causal process that influences the part. But to determine the quantity of motion of the containing place we must consider the larger place that contains it, and so on. The consequence is that it is not possible to attribute a determinate quantity of motion to any part; each attribution of quantity of motion to a part must involve determining the quantity of motion in the containing place, and if there are no immovable places we will end up with an infinite series of containers.

Without determinable quantities of motion we cannot isolate exchanges of quantity of motion, which implies that we cannot track causal relations. A science of motion would then become impossible. The upshot is that to make causal relations intelligible, we must distinguish between movable and immovable places. And as soon as we admit this distinction we also admit the existence of absolute space, which is the collection of all immovable places.

APII assumes Def1 and Def2 from **API**. The argument may be summarized as follows:

Def1 Place is defined as the part of space a body occupies within the larger space that contains it.

Def2 A property of quantity of motion is that the quantity of motion of the whole is the sum of the quantities of motion of the parts.

P1 If all places are movable, then quantity of motion is indeterminable for individual bodies.

(a) To determine the quantity of motion of body A, one needs to determine the motion of a body relative to its (movable) place A'.

(b) But the quantity of motion of a body is only part of the quantity of motion attributed to the composite body B containing both the body A and its place A'.

(c) To determine the quantity of motion of the composite body B, one needs to determine its motion relative to its (movable) place B', and so on...

P2 A determinable quantity of motion is necessary for a science of motion.

P3 Either all places are movable or some are im-movable.

C There are immovable places, i.e., there is true motion defined relative to immovable places.

Most of the argument is focused on giving support to P1. Like the previous arguments, a hidden premise (P2) is that quantity of motion is required for a science of motion, since it is this quantity that reveals the causal chains in physical processes. But there are important respects in which **APII** is more general than **API**. The main difference is that P3 in **APII** is a necessary truth unlike P3 in **API** which can be denied. While the **API** is a negative argument against a specific conception of true motion, **APII** deals only with the distinction between movable and immovable places, without discussing which definition of true motion is at stake. However, this latter argument heavily depends on Def1. The relation of containment is taken as the primary relation that is involved in the definition of place. When one considers conceptions of space in which this definition of place is abandoned, one has to be extremely careful in weighing the relevance of this argument.

One final argument we should consider is Newton's argument from causes for distinguishing between relative and absolute motions:

> The causes which distinguish true motions from relative motions are the forces impressed upon bodies to generate motion. True motion is neither generated nor changed except by forces impressed upon the moving body itself, but relative motion can be generated and changed without the impression of forces upon this body. For the impression of forces solely on other bodies with which a given body has a relation is enough, when the other bodies yield, to produce a change in that relation which constitutes the relative rest or motion of this body. Again, true motion is always changed by forces impressed upon a moving body, but relative motion is not necessarily changed by such forces. For if the same forces are impressed upon a moving body and also upon other bodies with which it has a relation, in such a way that the relative position is maintained, the relation that constitutes the relative motion will also be maintained. Therefore, every relative motion can be changed while the true motion is preserved, and can be preserved while the true one is changed, and thus true motion certainly does not consist in relations of this sort. (Newton 1999, p. 412)

In this passage Newton asserts that true motion is correlated with the cause of motion, while relative motion is not. Changes in the relative motion between bodies may not be correlated with the impressed forces that cause changes in bodies. For example, the relation between two bodies may remain the same, while in reality, both bodies experience forces. As in previous arguments, there are again implicit premises. Thus **AC** can be summarized as follows:

P1 Changes in relative motion between bodies cannot be correlated with impressed forces operating on bodies in the relation.
P2 Impressed forces are necessary for a science of motion.
P3 Relational accounts take true motion to be defined as change of relative positions.
P4 True motion exists.
C True motion does not consist in change in relative positions.

This argument from causes, which we shall term **AC**, is not a positive argument for absolute motion, but a negative argument against relational definitions of true motion. It only claims that if true motion exists, it cannot be defined as change in relative positions. Notice the parallels between this argument and **API**. Since impressed forces (i.e., exchanges in quantity of motion) are necessary for the study of true motion, and since these cannot be correlated with changes in relative positions, we cannot define true motion in a relational way. Thus, the problem with Descartes' theory is that the Descartes' forces do not correlate with his relational definition of true motion. Notice, however, that other than disagreeing on the type of forces that exist, Newton and Leibniz are in complete agreement with regard to this argument. While Newton asserts that true motion correlates with impressed, or external forces, Leibniz argues that true motion correlates with forces inherent in a body. Leibniz's metaphysics of substance imply that all forces are inherent. Even in collisions where it seems as if one body impresses its force on another, the change is actually inherent to each body, and there is no genuine transfer between cause and effect. Substances experience their inherent changes that appear like causal interactions, because these inherent changes are determined by the principles of preestablished harmony. Thus **AC** does not undermine anything in Leibniz's account, but it does undermine Descartes' theory, since Descartes seems to believe that relational definitions of true motion are correlated with impressed forces.

We see that Leibniz's positions are not undermined by Newton's arguments in the Scholium. Leibniz agrees with Newton that we must distinguish between true and apparent motion, and he also agrees with him that Descartes' definition of true motion is untenable. Moreover, Newton's arguments presuppose a certain conception of "place," and the centrality of quantity of motion for a science of motion; presuppositions that Leibniz does not share. Thus, Newton's arguments are largely irrelevant to Leibniz's relationalism. For the argument that seems most relevant to Leibniz's account, namely **AE**, Leibniz has a legitimate response. Thus Leibniz is not being dense when he says:

> I find nothing in the Eighth Definition of the *Mathematical Principles of Nature*, nor in the Scholium belonging to it, that proves, or can prove, the reality of space in itself. (Alexander 1956, L.V.53)

Since Newton's argument is mainly focused on demonstrating that Descartes' definition of true motion is untenable, Leibniz is correct in claiming that it does not follow that space is real.

6 Potential Problems for Leibniz

Newton's arguments for absolute space in the Scholium are not directly relevant to Leibniz's theory of space. However, perhaps Newton's **APII**, which does not rely on a particular definition of true motion and appeals only to the distinction between movable and immovable places, can be adjusted to address Leibniz's thoughts about space. One way to understand Newton's argument is to consider the role of quantity of motion in analyzing causal relations. Since causal relations are analyzed through exchanges of quantity of motion, a determinate quantity of motion should be attributed to bodies if a science of motion is to be possible. A causal relation is analyzed by a transfer of quantity of motion from body A to body B. But for this transfer to be reflective of a causal relation, one must guarantee that the quantity of motion introduced into B originates from A. If it cannot be determined that an increase (decrease) in quantity of motion in B is correlated with a decrease (increase) of quantity of motion in A, then it is not possible to think of this equivalence in quantity of motion as a genuine "transfer" from cause to effect. Thus, to correlate changes in quantity of motion in A with changes in B one has to presuppose that the physical system containing A and B is causally isolated from the rest of the world, and that the total quantity of motion to be found in these systems is determined independently of anything else in the world. To track causal chains via exchanges of quantity of motion one therefore has to find a criterion for isolating physical systems. But if the motion of a system which is purported to be isolated is measured relative to a *movable* containing place, it is not clear whether the system is actually isolated. Some change in quantity of motion may be credited to a causal process influencing the movable containing place. Thus, it seems as if the only way to causally isolate a finite system, and to track transfers of quantities of motion among its parts, is to relate the motion of an isolated system to immovable places. The upshot is that absolute space is necessary for a science of motion, since immovable places are required if conserved quantity of motion is to correlate with isolated systems.

APII does not address Leibniz's theory of space. Phenomenal motion is defined by Leibniz as a change in distance relations. Since for Leibniz there is no containment relation between a particular place and its surrounding, Leibniz does not need to consider the quantity of motion of the containing place and the difficulty in separating out the quantity of motion of the containing place from the quantity of motion of the body under consideration. Moreover, Leibniz introduces the conservation of force and claims that this law of conservation is more fundamental than that of conservation of quantity of motion. Thus, despite the more general thrust of **APII** it is still irrelevant to Leibniz's theory of space, given that Newton's notion of place is different than Leibniz's.

But even if **APII** is irrelevant to Leibniz's theory of space, there is some analogous argument that can be distilled from it. What is relevant to Leibniz is whether he has given us a viable criterion for isolating physical systems. The problem lies with the central role of the conserved quantity of force in analyzing causal relations. For Leibniz, there are no genuine causal relations and no genuine transfers of force. By conservation of force Leibniz means that the inner force which increases for body A is correlated with a decrease in inner force in body B. The preestablished harmony with which God created substances allows us to analyze the change in A's living force mv^2 to be correlated with the change in B's living force. However, while there are no genuine causal relations, Leibniz argues that all natural phenomena are reducible to apparent chains of causes and effects, corresponding to God's preestablished harmony between substances.

Assume that body A interacts with body B to produce a certain effect in B. To analyze the effects produced in B, Leibniz attributes a living force mv_A^2 to A and mv_B^2 to B. The causal interaction between A and B is determined by how much living force was "transferred" from A to B. Thus a decrease in mv_A^2 must correspond to an increase in mv_B^2, and vice versa. However, to determine how much force was transferred, one must be certain that no force was transferred from A to another body and that the numerical difference in living force represents an actual transfer. That is, we must find a criterion for determining whether A (together with B) are causally isolated from the rest of the universe. In practice we verify that A and B are isolated by calculating the total force $mv_A^2 + mv_B^2$ and verifying that this total force is conserved. There is no criterion of isolation other than conservation of force. However, assume that true motion is defined as the motion of A relative to C that is a fixed existent. According to Leibniz, there is no phenomenal criterion that demonstrates C to be a fixed existent. It may be that the change in motion in A results not from having some of its force transferred to B, but from force transferring into C. But to estimate whether C is causally isolated, we have to estimate the living force in C by measuring its motion relative to some other body D that we take to be a fixed existent. Nevertheless, D may have living force, which implies that more and more bodies are implicated in the analysis ad infinitum.

Early in his writing, Leibniz intuitively recognizes that his attribution of real motion may implicate the whole world:

> A remarkable thing: motion is something relative, and one cannot distinguish which of the bodies is moving. And so, if motion is an affection, its subject will not be any individual body, but the whole world. (Leibniz 1923, Volume VI p. iv, quoted in Lodge 2003, p. 284)

Although the context of this passage does not exactly enable us to determine, Leibniz may have had in mind the problem that Newton was alluding to in **APII**. Lodge (2003, p. 285) argues that since Leibniz thought of the world as infinite, he later concluded that we cannot think of the world as a single, unified whole. Thus the whole world cannot be treated as a subject carrying a property. In later texts Leibniz no longer mentions the world as a whole as the sole possessor of real motion, perhaps in light of his conclusion that the world as a whole cannot be the subject of true motion.

The problem with Leibniz's equivalence of hypothesis—i.e., his claim that a phenomenal measure of motion must always be relative—is that it is not clear how physical systems are isolated in practice. To put a stop before completing the infinite inclusion of reference bodies, one has to find a criterion for isolating individual physical systems. Bodies that appear to be at rest may still be moving or possessing a large amount of living force. And if large exchanges of living force is poured into the reference body, we cannot differentiate between change in relative place, which leads to a numerically different quantity of force, and actual variations in living force due to causal interactions.

On the other hand, if there is a safe inference from conservation of force to causal isolation, then the conserved quantity of force must be defined as such independently of any other bodies in the universe. A system of bodies with conserved quantity of force is taken to be isolated when the motion of system-parts are determined absolutely, independently of any other bodies. Now, it may be that we can isolate individual systems, or that we can only isolate the universe as a whole. If individual systems are isolable, then it seems as if we must suppose that parts of the system have absolute motions. If only the universe as a whole is isolable, then we run into the practical problem of determining the forces exchanged between individual systems. The result is that Leibniz seems to be facing the dilemma of accepting either the inference from conserved quantity of force to absolute motion, or that his dynamics are inapplicable to individual systems. While the first option is in conflict with the general relativity of motion, the second option makes it doubtful that his dynamics could ever be used in practice.

Leibniz's response may be that we can still isolate individual systems in practice even if we cannot be absolutely sure that they are, in fact, isolated. If we find bodies that are, relatively speaking, at rest, we can calculate the motion of bodies relative to these reference bodies. However, other than analyzing exchanges of forces, we have no means of verifying whether the reference body is in fact at rest and that there is no internal cause for motion. Since the conservation of force is the only measure of causal isolation, it is also the only measure by which we determine an individual body as being at rest. Thus, the practice is thoroughly enmeshed in the theoretical apparatus devised by Leibniz.

The difference between what this argument—the analogue of **APII**—forces on Leibniz, and what he grants, is subtle. Leibniz argued that we can attribute true motion to bodies when we have a complete dynamic explanation. After we have the complete dynamic account, we can take the hypothesis that renders the explanation simplest and take it to be true. Leibniz insists, nevertheless, that the phenomena do not enable us to distinguish between true and apparent motion. However, Leibniz does not make clear when and how dynamic explanations can be completed; he simply presupposed that some interactions can be examined in isolation from the rest of the world. But when a physical system is taken to be causally isolated, the motions of the parts are already thought of as true. The argument, it should be insisted, does not compel Leibniz to admit the existence of absolute space. Leibniz already grants the distinction between true and apparent motion, and provides an alternative definition of true motion. The analogue of **APII**, however, does suggest a stronger tension between the view that phenomenal motion must be relative and Leibniz's commitment to the idea that pseudo-causal chains are tracked through conservation of force. If conserved force functions as a criterion of isolation, it is not clear how Leibniz can assert that there is no phenomenal criterion of true motion.

7 Newton and Leibniz's Styles of Philosophizing

Newton's arguments in the Scholium suggest a certain style of philosophizing which sets him apart from contemporary natural philosophers, among them Leibniz. Newton emphasized his "experimental philosophy," which he took to be distinct from those of Descartes and Leibniz. The experimental method showcased the role of experience, observations and experiments in constructing theories about the natural world. Newton distinguished himself from those philosophers who hypothesize and construct explanations for the phenomena. In experimental philosophy, "propositions are deduced from the phenomena and are made general by induction" (Newton 1999, p. 793). While a full treatment of Newton's method is beyond the scope of the paper, a few words would help clarify the role of Newton's arguments about space in his experimental philosophy.[16]

Newton initially likened his experimental method to a standard 17th century account of mathematical method. According to this account, mathematics proceeds through analysis and synthesis. When certain mathematical constructions are given, the philosopher should analyze them until he or she reaches the ultimate entities, properties or principles which lie at the basis of the constructions. Then, by synthesis, one puts together from the basic materials new constructions or proofs that demonstrate the validity of certain propositions. Euclid's *Elements* only presents the synthetic part of geometry, since Euclid begins from five axioms and constructs the theorems of Euclidean geometry from them. But the proofs in Euclid's book must have followed the stage in which Euclid analyzed geometric figures, an analysis which has led him to his definitions and axioms.

[16] For a good historical survey of Newton's method, see Shapiro (2004); and for my account of Newton's application of his method, see Belkind (2012).

Newton erects an analogy between the analysis/synthesis account of the mathematical method, and the scientific method. Instead of complex mathematical constructions, the scientific method begins from given phenomena and analyzes them into their most basic ingredients and principles. Once the analytic stage is complete, the synthetic part of the method constructs from the basic materials the propositions that are taken to have scientific validity. Shapiro (2004, p. 192) argues that Newton dropped the explicit reference to analysis and synthesis in some parts of the *Principia* to avoid confusing his scientific method with the new method of mathematical analysis, or calculus, which he and Leibniz discovered independently and over which there was a bitter priority dispute between them. According to Shapiro, Newton emphasized the term experimental philosophy in the *Principia*, to defend his force of gravitation from Leibniz's accusation that Newton's gravitational force is occult, since it is a force of attraction that acts at a distance.

In one of the versions of his *Opticks*, Newton describes the method of natural philosophy as follows:

> As in Mathematicks so in Natural Philosophy the investigation of difficult things by the method of Analysis ought ever to precede the method of Composition. This Analysis consists in [arguing] <making experiments & observations & in arguing by them> from compositions to ingredients & from motions to the forces producing them & in general from [Phaenomena] <effects> to their causes & from particular causes to more general ones, till the Argument end in the most general: The Synthesis consists in assuming the causes discovered & established, as Principles; & by them explaining the Phaenomena proceeding from them, & proving the explanations. (Newton 1706, pp. 347–8)[17]

An important stage in the scientific method is to begin with the phenomena and provide an analysis of the phenomena into their basic ingredients and principles. Newton explains that what we observe are the effects or the phenomena, and what we try to infer are the causes. In the context of a science of motion, the effects are the observed motions, and the causes are the forces that give rise to these motions. Thus, a crucial tool of analysis is one that enables the scientist to infer the basic forces that give rise to the phenomena from the motions that are observed. Given that the change in a body's quantity of motion is essentially equivalent to the force that is impressed on it, the notion of quantity of motion is crucial for the analysis of motions. Without a viable notion of quantity of motion, one cannot carry out the process of analyzing complex phenomena into their most basic ingredients or principles.

Thus we should view the conceptual arguments for absolute space, in particular the arguments from properties **API** and **APII**, as demonstrating Newton's concern for grounding his *method* of experimental philosophy. Since he showed with a forceful argument that Descartes' definition of true motion conflicts with a central property of quantity of motion, Newton showed that an alternative view of space, one that appeals to the notion of immovable containers, would better ground the scientific method of analysis and synthesis.

Leibniz's style of philosophizing differs from Newton in that Leibniz seeks the most comprehensive philosophical system, where the physical *concepts* receive their proper metaphysical grounding. When developing the proper metaphysical foundation for knowledge, Leibniz accepts various demands, including the requirement that his theory of substance would satisfy the Principle of Sufficient Reason and that substances are created by an omniscient, omnibenevolent and omnipotent God. While Newton is seeking ways to guarantee

[17] Quoted in Shapiro (2004, p. 196).

his *inference* from motions to forces, or from the phenomena to their causes, Leibniz seeks to *ground* phenomenal measures in a metaphysical account of forces.

8 Conclusion

Newton's arguments in the Scholium primarily target Descartes' definition of true motion. Clarke's assertion that Newton "proved" the existence of absolute space using "mathematical" proofs is exaggerated. Newton presupposed that motion in relation to absolute space is the natural alternative to Descartes' definition of true motion. If one defines "place" as the volume a body takes up in relation to the containing place, then an argument demonstrating the need for immovable containers implies the existence of absolute space. But Leibniz defines true motion in relation to internal forces, and there is nothing in Newton to demonstrate that Leibniz's definition of true motion is invalid.

Leibniz's philosophy makes it difficult to falsify his account of true motion. According to Leibniz there is no observable criterion of true motion, given that a change in distance relation seems only to provide a symmetric definition of motion and the equivalence of hypotheses follows. If motion is defined as change in distance relations we cannot distinguish between bodies that move and the bodies that are at rest. Moreover, Leibniz argues that even if we can generate a clear association between rotation and inertial effects, such a correlation does not amount to an observable criterion of true motion. It is possible that the dynamic situation would explain the inertial effects using forces compounded from primitive forces leading to rectilinear motions. For example, the rotation of a solid body requires an account of solidity, and it may be that solidity (or fluidity) arises from interactions between the solid body and its surroundings. Thus, unless we take into account the full dynamic explanation that tells us what makes a body solid and how its motion is determined relative to the surroundings, it is not possible to take the inertial effects produced by rotation as a phenomenal criterion of true motion.

Leibniz is committed to the general relativity of motion as far as the phenomena are concerned (i.e., we cannot find any *phenomenal* criterion for distinguishing between true and apparent motion). Moreover, Leibniz concludes that laws derived from the phenomena must inherit the general relativity of motion from the phenomena. However, the distinction between true and apparent motion is still valid at the metaphysical level. Bodies do possess a certain degree of motion, which is correlated with inherent forces giving rise to motions. Once we have a dynamic account for a particular situation, we can attribute true motions to bodies if the account is complete. In the case of collisions, we can subtract the common motion of the colliding bodies by redescribing the collision in the reference frame of the center of gravity. In this reference frame, part of the body's motions are correlated with inner forces, and can be taken to be true. But to distinguish between true and apparent motion one must have a complete dynamic explanation at their disposal and find the hypothesis that simplifies the account. Thus, there is no phenomenal criterion of true motion, only a full dynamic account would help us differentiate true from apparent motion.

But Leibniz is on shaky ground with regards to his relativity of motion, and with the foundations for his dynamic theory, when he insists that conservation of force can be used to track causal relations. The implicit assumption is that a conserved quantity of force allows one to infer that a system is causally isolated from the rest of the world. There is an inference from conserved quantity of force to the causal isolation of a system. Without this inference from the conserved quantity to isolation it is not clear where the cut-off between the system and the environment is. A science of motion that has enough power to analyze causal relations between individual bodies requires treating conserved quantity of force as representing true motions.

This argument does not show the existence of absolute space. What it does is suggest that without presupposing that true motions are the foundation of conservation laws, the study of causal relations would not be possible. The strength of such an argument depends on the extent to which conservation laws are proven to be essential.

The Leibniz-Clarke Correspondence has masked the significance of Newton's Scholium, and reframed the debate in terms that obscure Newton's intentions. Newton's account of absolute space is based on the definition of place as involving the containment relation between the place and the surrounding space. The purpose of absolute space, according to Newton, is to ground the notion of quantity of motion, giving support to Newton's method of analyzing phenomena. Thus Newton's argument for absolute space amounts to an argument in support of his scientific method.

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