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High-Frequency Plate Flutter

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Abstract — Earlier, using the global instability method, the stability of a strip plate in a supersonic gas flow was investigated. In addition to the classical (low-frequency) flutter developing upon the interaction between the plate oscillation modes, a novel (high-frequency) flutter type in which the oscillations are unimodal was detected. In the present study, the effect on the high-frequency flutter of the plate width (earlier only an asymptotic analysis for a width tending to infinity was performed), its damping characteristics, and the presence of a gas at rest on the side opposite the flow is investigated.

Keywords: high-frequency flutter, plate flutter, panel flutter, stability of a plate, global instability.

In [1], the problem of the stability of a thin elastic plate in a unilateral homogeneous supersonic inviscid perfect gas flow was considered in the linear approximation. The gas-plate system behavior can be described by the five dimensionless parameters

$$\mathrm{M} = \frac{u\cos\theta}{a}, \quad \mathrm{M}_w = \frac{\sqrt{\sigma/\rho_m}}{a}, \quad D = \frac{D_w}{a^2\rho_m h^3}, \quad L = \frac{L_w}{h}, \quad \mu = \frac{\rho}{\rho_m}$$

Here, u, a, and ρ are the flow velocity, the speed of sound, and the gas density, σ is the tensile stress in the mid-plane of the plate, ρ_m, L_w, and h are the material density and the width and thickness of the plate, and $D = Eh^3/(12(1-\nu^2))$ is its flexural stiffness. The gas flows in the region $z > 0$, the plate occupies the region $z = 0$, $|x| \le L/2$, and the angle between the flow direction and the plate edges is equal to $\pi/2 - \theta$. We assume that $\mathrm{M} > 1$ and $\mu \ll 1$.

In [1], the problem was investigated asymptotically as $L \to \infty$. Two instability types, high- and low-frequency flutter, were detected. The classical low-frequency flutter developing from the interaction of two plate oscillation modes via an aerodynamic coupling has been investigated in detail in numerous studies of plate flutter. High-frequency flutter, where the oscillations are unimodal and arise as a result of negative aerodynamic damping, was discovered theoretically for the first time.

The condition $L \to \infty$ was used twice. Firstly, in solving an eigenvalue problem using the global instability method [2, 3, § 65], it was assumed that of the four traveling waves present in the plate-gas system at a given frequency only two are essential. Secondly, the pressure acting on the oscillating plate was not calculated from the exact formula but was assumed to be a superposition of the pressures acting on harmonic waves in a fictitious unbounded plate.

The present paper continues [1] and is devoted to estimating the width of the plates to which the results obtained for high-frequency flutter are applicable. Moreover, the effects of structural damping and energy dissipation in the plate material, as well as of the presence of a gas at rest in the region $z < 0$, will be studied.

It is assumed that the condition $\mathrm{M} > \mathrm{M}_w + 1$, which is a high-frequency flutter criterion for sufficiently wide plates, is satisfied.

1. EFFECT OF THE PLATE WIDTH ON THE PRESSURE DISTRIBUTION

The global instability criterion for one-dimensional systems of large but finite extent was applied in [1] to a gas-plate system in which the plate has a finite length, while the gas flows along the entire x axis. The natural plate oscillation modes were constructed as a superposition of waves traveling along an infinite plate, having the form

$$w(x,\ t) = e^{i(kx-\omega t)}, \quad |x| < \infty \tag{1.1}$$

and satisfying the boundary conditions at the edges $x = \pm L/2$, while the pressure was assumed to be a superposition of the pressures

$$p(x,\ t) = \mu \frac{(\omega - \mathrm{M}k)^2}{\sqrt{k^2 - (\omega - \mathrm{M}k)^2}} e^{i(kx-\omega t)} \tag{1.2}$$

acting on these waves. As a result, all the equations and boundary conditions are satisfied, except for the requirement that there be no perturbations at $x < -L/2$. To satisfy this requirement we would have to construct solutions from waves of the form

$$w(x,\ t) = W(x)e^{-i\omega t}, \qquad |x| \le L/2, \qquad w(x,\ t) = 0, \qquad x < -L/2$$

satisfying the equation of plate motion with account for the pressure acting on it. In the neighborhood of the leading edge of the plate ($x = -L/2$) this pressure may differ substantially from (1.2) but for unstable perturbations this difference tends to zero with distance from the leading edge. Since the flow is supersonic, the pressure difference generated by the trailing edge does not affect the plate oscillations.

We will estimate the error using the exact integro-differential equation of motion for a plate in a gas flow [4, § 4.7]

$$D\frac{\partial^4 W}{\partial x^4} - \mathrm{M}_w^2 \frac{\partial^2 W}{\partial x^2} - \omega^2 W + \frac{\mu}{\sqrt{\mathrm{M}^2 - 1}}\left(-i\omega + \mathrm{M}\frac{\partial}{\partial x}\right) \times$$
$$\int\limits_{-L/2}^{x} \left(-i\omega W(\xi) + \mathrm{M}\frac{\partial W(\xi)}{\partial \xi}\right) \exp\left(\frac{i\mathrm{M}\omega(x-\xi)}{\mathrm{M}^2 - 1}\right) J_0\left(\frac{-\omega(x-\xi)}{\mathrm{M}^2 - 1}\right) d\xi = 0 \tag{1.3}$$

In [5], a general solution of this equation was obtained. Its basis is formed by four solutions which in the notation used can be written as follows:

$$W_j(x) = \sum_{l=1}^{4} b_{jl} e^{ik_l(x+L/2)} + V_j(x), \qquad k_l = k_l(\omega,\ \mu) \tag{1.4}$$

$$b_{jl} = \frac{k_l(\omega,\ \mu) - \omega/\mathrm{M}}{k_l(\omega,\ \mu) - k_j(\omega,\ 0)} \frac{iD\sqrt{\mathrm{M}^2 - 1}}{\sqrt{k^2(\omega,\ \mu) - (\omega - \mathrm{M}k_j(\omega,\ \mu))^2}} \left(\left.\frac{\partial F}{\partial k}\right|_{k=k_j(\omega,\ \mu)} \right)^{-1}$$

$$V_j(x) = \mathrm{M} \int\limits_0^{2\mathrm{M}/(\mathrm{M}^2-1)} \frac{g(\rho,\ \omega)}{i\omega(\rho - \mathrm{M}/(\mathrm{M}-1))/\mathrm{M} + ik_j(\omega,\ 0)} e^{-i\omega(x+L/2)(\rho - \mathrm{M}/(\mathrm{M}-1))/\mathrm{M}} d\rho$$

$$g(\rho,\ \omega) = g_1(\rho,\ \omega)/g_2(\rho,\ \omega)$$

$$g_1(\rho,\ \omega) = -\frac{2i\omega}{\mathrm{M}}\left(\rho - \frac{1}{\mathrm{M}-1}\right)\left|\sqrt{\rho\left(\frac{2\mathrm{M}}{\mathrm{M}-1} - \rho\right)}\right| G(\rho,\ \omega)$$

$$g_2(\rho,\ \omega) = -\frac{\omega^2\rho}{\mathrm{M}^2}\left(\frac{2\mathrm{M}}{\mathrm{M}^2-1} - \rho\right)G^2(\rho,\ \omega) - \frac{\mu^2\mathrm{M}^4}{\mathrm{M}^2-1}\frac{1}{D^2}\left(\rho - \frac{1}{\mathrm{M}-1}\right)^4$$

$$G(\rho,\omega) = -\left(\rho - \frac{\mathrm{M}}{\mathrm{M}-1}\right)^4 \frac{\omega^2}{\mathrm{M}^2} - \frac{\mathrm{M}_w^2}{D}\left(\rho - \frac{\mathrm{M}}{\mathrm{M}-1}\right)^2 + \frac{\mathrm{M}^2}{D}$$

The wave numbers $k_j(\omega,\ \mu)$ are solutions of the dispersion equation for the unbounded plate-gas system

$$F(k,\ \omega) = (Dk^4\ +\ \mathrm{M}_w^2 k^2\ -\ \omega^2)\ -\ \mu\frac{(\omega\ -\ \mathrm{M}k)^2}{\sqrt{k^2\ -\ (\omega\ -\ \mathrm{M}k)^2}} = 0 \tag{1.5}$$

the case $\mu = 0$ corresponding to the absence of a gas, that is, to a plate oscillating in a vacuum. In (1.5), for $\mathrm{Im}\,\omega \gg 1$, the branches of the square root are so chosen that its real part is positive. This condition selects four branches of the solution $k_j(\omega)$ that are continued analytically onto the entire complex plane ω from which a neighborhood of the line $\mathrm{Re}\,\omega = 0$, $\mathrm{Im}\,\omega \le \mathrm{M}_w^2/(2\sqrt{D})$ of width $\sim \mu^{2/3}$ containing all the solution branch points is cut off. The branches $k_j(\omega)$ are so numbered that for $\mathrm{Im}\,\omega \gg 1$ an increase in the number j corresponds to a decrease in the imaginary part $\mathrm{Im}\,k_j$.

Assuming μ to be a small parameter, it is easy to see that in the matrix (b_{ij}) the diagonal elements are of the order of $1/\mu$ and the nondiagonal elements of the order of 1. Therefore, in the inverse matrix (c_{jl}) the diagonal elements are of the order of μ and the nondiagonal elements of the order of μ^2.

We will go over from the basis of solutions (1.4) to the basis

$$W_j(x) = e^{ik_j x}\ +\ e^{ik_j L/2}\sum_{l=1}^{4} c_{jl} V_l(x) \tag{1.6}$$

If $\mathrm{Im}\,\omega > 0$ (case of instability), the functions $V_l(x)$ decrease exponentially with distance from the leading edge. Substituting a linear combination of waves (1.6) in the boundary conditions at the plate edges, we obtain a homogeneous system of linear algebraic equations for the coefficients of this combination. From the condition of existence of nontrivial solutions, which reduces to the equality of the system determinant to zero, the eigenfrequencies ω can be found. Performing with this determinant a procedure similar to that described in [2] and using the fact that the equality $\mathrm{Im}\,k_2(\omega) = \mathrm{Im}\,k_3(\omega)$ holds for $\mathrm{Im}\,k_{2,3} < 0$, it is possible to show that the difference between the exact solution composed of waves (1.6) and the solution composed of waves (1.1) is of the order of μ and does not restrict the plate width.

This is physically obvious: the global eigenfunctions are formed on the reflection of waves whose amplitude at the leading edge is of the order of 1 and increases with distance from it, while the amplitude of the additional term in (1.6) at the leading edge is of the order of μ and decreases with distance from it. Thus, these additional terms are negligibly small and cannot interfere with the mechanism of formation of the global eigenfunctions described in [1].

If $\mathrm{Im}\,\omega < 0$ (case of stability), the functions $V_l(x)$ are unbounded and at a sufficient distance from the leading edge the exact solutions differ from the harmonic waves to an arbitrarily large extent. However, this difference does not lead to instability and is manifested only in a significant change in the plate oscillation modes in a gas as compared with those in a vacuum.

2. EFFECT OF THE PLATE WIDTH ON THE MECHANISM OF FORMATION OF THE GLOBAL EIGENFUNCTIONS

In general, the eigenfunctions of a plate in a gas are a superposition of four traveling waves: $w(x,\ t) = (C_1 e^{ik_1 x} + C_2 e^{ik_2 x} + C_3 e^{ik_3 x} + C_4 e^{ik_4 x})e^{-i\omega t}$ (here we neglect the correction obtained in the previous section). However, two of them, the first and fourth, are damped exponentially rapidly with distance from the edges and in a certain central region of the plate the eigenfunctions can be considered as a superposition of only two waves traveling in opposite directions. This is the source of the idea of constructing global eigenfunctions. We will estimate the width L sufficient to neglect the effect of the damped waves on the plate oscillations so that they cannot interfere with the mechanism of formation of growing high-frequency eigenfunctions [1].

Assuming that $\omega = \omega_{\max} = (\mathrm{M}\ -\ 1)\sqrt{((\mathrm{M}\ -\ 1)^2\ -\ \mathrm{M}_w^2)/D}$ is the real frequency at which the maximum eigenfunction growth increment $\delta_{\max}$ is reached, we will consider an almost harmonic wave $C_2 e^{ik_2 x}$ ($|\mathrm{Im}\,k_2| \ll 1$) traveling from the leading to the trailing edge, being reflected from the latter, and generating two waves traveling in the opposite direction: one almost harmonic $C_3 e^{ik_3 x}$ and one damped $C_4 e^{ik_4 x}$. Then, we can assume that only the wave $C_3 e^{ik_3 x}$ has returned to the leading edge if on approaching this edge the amplitude of the damped wave was negligible as compared with the amplitude of the wave $C_3 e^{ik_3 x}$, that is,

$$\frac{|C_4| e^{\mathrm{Im}\, k_4 L}}{|C_3| e^{\mathrm{Im}\, k_3 L}} \le \varepsilon$$

where ε is a given small number, from which

$$L \ge (\mathrm{Im}\, k_3 - \mathrm{Im}\, k_4)^{-1} \ln\left(\frac{1}{\varepsilon}\left|\frac{C_4}{C_3}\right|\right) \tag{2.1}$$

Now let the wave $C_3 e^{ik_3 x}$ travel from the trailing to leading edge. After reflection it will transform into two waves: one almost harmonic $C_2 e^{ik_2 x}$ and one damped $C_1 e^{ik_1 x}$. Analogous considerations lead to the condition on which on the trailing edge the damped wave can be neglected:

$$L \ge (\mathrm{Im}\, k_1 - \mathrm{Im}\, k_2)^{-1} \ln\left(\frac{1}{\varepsilon}\left|\frac{C_1}{C_2}\right|\right) \tag{2.2}$$

With increase in L the amplitude ratios $|C_4/C_3|$ and $|C_1/C_2|$ decrease exponentially; therefore, for a fixed ε there exists a minimum plate width $L_{\min}$ such that inequalities (2.1) and (2.2) are satisfied for all $L \ge L_{\min}$.

Since correct to small quantities of the order of μ

$$\mathrm{Im}\, k_1(\omega,\ \mu) = -\mathrm{Im}\, k_4(\omega,\ \mu) = \mathrm{Im}\, k_1(\omega,\ 0)$$
$$\mathrm{Im}\, k_2(\omega,\ \mu) = \mathrm{Im}\, k_3(\omega,\ \mu) = 0$$

the simultaneous satisfaction of conditions (2.1) and (2.2) is equivalent to the inequality

$$L \ge \frac{1}{\mathrm{Im}\, k_1(\omega,\ 0)} \ln\left(\frac{1}{\varepsilon} \max\left(\left|\frac{C_4}{C_3}\right|,\ \left|\frac{C_1}{C_2}\right|\right)\right) \tag{2.3}$$

We will give examples of calculating $L_{\min}$ for the parameters

$$\mathrm{M} = 1.5, \qquad \mathrm{M}_w = 0, \qquad D = 23.8, \qquad \mu = 1.2 \cdot 10^{-4} \tag{2.4}$$

corresponding to a steel plate in an air flow under normal conditions for $\varepsilon = 0.01$. From dispersion equation (1.5), for $\omega = \omega_{\max} \approx 0.051$, we have $k_1(\omega,\ 0) \approx 0.1i$. Then, for both edges hinged

$$|C_1/C_2| = |C_4/C_3| \sim \mathrm{Im}\, k_{2,3}(\omega,\ \mu) < 10^{-3} < \varepsilon$$

from which $L_{\min} = 0$.

For both edges clamped

$$\left|\frac{C_1}{C_2}\right| = \left|\frac{C_4}{C_3}\right| \approx \frac{|\cos(Lk_2(\omega,\ 0)/2)|}{\cosh(Lk_2(\omega,\ 0)/2)}$$

and

$$\left|\frac{C_1}{C_2}\right| = \left|\frac{C_4}{C_3}\right| \approx \frac{|\sin(Lk_2(\omega,\ 0)/2)|}{\sinh(Lk_2(\omega,\ 0)/2)}$$

for the symmetrical and antisymmetrical oscillation modes. Substituting these expressions in (2.3), we find $L_{\min} \approx 35.4$.

It can be seen that the width for which the "subordinate" damped waves are important only near the edges is very small (for example, for a clamped plate 1 mm thick this width is equal to 3.54 cm). The same situation can be observed for other realistic boundary conditions and parameter values. It arises due to the high damping rate of the "subordinate" waves with distance from the edges and is well-known in the theory of plates as a "dynamic boundary effect" [6, § 34].

3. EFFECT OF PLATE DAMPING ON THE GROWTH OF THE HIGH-FREQUENCY EIGENFUNCTIONS

In order to estimate the damping effect, we must know the eigenfunction growth increment in an ideal system. By comparing this value with the damping constant we can establish the presence or absence of flutter.

For high-frequency eigenfunctions the growth increment has the form [1] (ω is the real plate oscillation frequency in the absence of a gas):

$$\delta(\omega) = -\frac{1}{2}\left(\frac{\partial k_2(\omega,\, 0)}{\partial \omega}\right)^{-1} \mathrm{Im}(k_2(\omega,\, \mu) - k_3(\omega,\, \mu)) \tag{3.1}$$

For $j = 2$, the leading term of the $k_j(\omega,\, \mu)$ expansion in the small parameter μ is linear for frequencies ω outside a neighborhood of $\omega_{\max}$ and for $j = 3$ for all ω (for $j = 2$ the expansion is also nonlinear for the frequency ω at which the phase velocity $c(\omega) = \omega/k_2(\omega,\, 0) \approx \mathrm{M} + 1$, but this case corresponds to damping of the eigenfunction and therefore will not be considered). From dispersion equation (1.5) we find

$$k_j(\omega,\, \mu) = k_j(\omega,\, 0) + \mu l(k_j(\omega,\, 0))$$
$$l(k_j) = \frac{(\omega - \mathrm{M}k_j)^2}{2k_j(\mathrm{M}_w^2 + 2Dk_j^2)\sqrt{k_j^2 - (\omega - \mathrm{M}k_j)^2}} \tag{3.2}$$

In a neighborhood of $\omega_{\max}$ the leading term of the expansion of $k_2(\omega,\, \mu)$ is of the order of $\mu^{2/3}$, except for the case $\mathrm{M} - 1 \sim \mathrm{M}_w \le \mu^{1/2}$ when in the leading term the μ exponent is less than 2/3. We will not consider this case due its lack of generality. Introducing in this neighborhood the local variable $p \in \mathbf{R}:\ \omega = \omega_{\max} + \mu^{2/3}p$ and substituting the expression $k_2(\omega,\, \mu) = k_2(\omega_{\max},\, 0) + \mu^{2/3}s(p)$ in (1.5), we obtain the relationship between s and p

$$\left(s\left(\frac{2(\mathrm{M}-1)^2 - \mathrm{M}_w^2}{\mathrm{M}-1}\right) - p\right)^2 (s(\mathrm{M}-1) - p) + \frac{\sqrt{(\mathrm{M}-1)^2 - \mathrm{M}_w^2}}{8\sqrt{D}(\mathrm{M}-1)^2} = 0 \tag{3.3}$$

Outside the neighborhood of $\omega_{\max}$, in order to match $k_2(\omega,\, \mu)$ with linear expansion (3.2), we must choose the $s(p)$ branch continuing the value

$$s(0) = \left(\frac{\sqrt{(\mathrm{M}-1)^2 - \mathrm{M}_w^2}}{8\sqrt{D}(\mathrm{M}-1)^3}\left(\frac{2(\mathrm{M}-1)^2 - \mathrm{M}_w^2}{\mathrm{M}-1}\right)^{-2}\right)^{1/3} e^{-i\pi/3}$$

Investigating Eq. (3.3) yields the following properties of the function $s(p)$:

(1) At $p > p^*(\mathrm{M},\, \mathrm{M}_w,\, D) > 0$, $s(p)$ is real and at $p = p^*$ the curve $s(p)$ in the complex plane s has a breakpoint with the angle $\pi/2$.

(2) At $0 < p < p^*$, $\mathrm{Im}\, s(p)$ is negative.

(3) At $p = 0$, $\mathrm{Im}\, s(p)$ reaches absolute minimum.

(4) As $p \to -\infty$, $\mathrm{Im}\, s(p)$ is negative and tends to zero as $1/\sqrt{|p|}$.

These properties make it possible to plot in the neighborhood of $\omega_{\max}$ a dependence $\delta(\omega)$ which outside that neighborhood matches with the dependence determined by formulas (3.1) and (3.2). It is qualitatively the same as in Fig. 1, where it is plotted numerically for parameters (2.4).

Using the fact that $s(p)$ reaches a minimum at $p = 0$, it is easy to calculate the maximum growth increment of the high-frequency eigenfunctions

$$\delta_{\max}(\mathrm{M},\, \mathrm{M}_w,\, D,\, \mu) = \delta(\omega_{\max}) = \mu^{2/3}\frac{\sqrt{3}}{8}\left(\frac{(\mathrm{M}-1)^2 - \mathrm{M}_w^2}{D}\right)^{1/6} \times$$
$$\frac{(2(\mathrm{M}-1)^2 - \mathrm{M}_w^2)^{1/3}}{(\mathrm{M}-1)^{4/3}} - \mu\frac{(2\mathrm{M}-1)^2}{4(\mathrm{M}-1)\sqrt{(2\mathrm{M}-1)^2 - 1}} \tag{3.4}$$

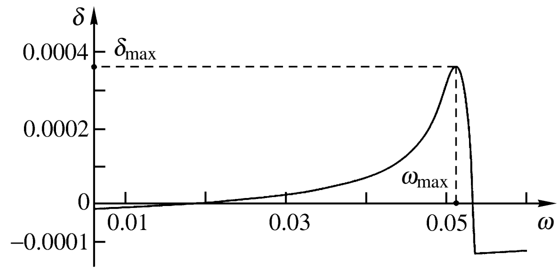

Fig. 1. The dependence $\delta(\omega)$ for parameters (2.4)

Let us now consider the model of a plate with damping. There are two damping sources: energy dissipation within the plate material and structural damping in the fixation sites [6, § 4]. We will study the plate width effect on damping on the assumption that the other problem parameters are fixed.

We will take into account the dissipation in the plate material on the basis of a viscoelastic model. Its simplest version reduces to the addition of two terms to the equation of plate motion: $-\gamma_1 \partial w/\partial t$ (viscous friction for vertical displacement of points on the plate) and $-\gamma_2 \partial^3 w/\partial^2 x \partial t$ (viscous friction in bending), where $\gamma_{1,2} > 0$ are the corresponding damping coefficients. This leads to the appearance of additional terms on the left side of dispersion equation (1.5): $-\gamma_1 i\omega$ and $-\gamma_2 ik^2\omega$. Assuming that $\gamma_{1,2} \ll 1$, it can be shown that these terms produce a correction to the eigenfunction growth increment $\delta_\gamma(\omega) = -(\gamma_1 + \gamma_2 k_2^2(\omega, 0))/2$. Then the condition of growth of the high-frequency eigenfunctions in the presence of energy dissipation in the plate material is given by the inequality

$$\delta_{\max} > -\delta_\gamma(\omega_{\max}) = \frac{1}{2}(\gamma_1 + \gamma_2 k_2^2(\omega_{\max}, 0)) \tag{3.5}$$

Criterion (3.5) is independent of the plate width. In particular, if in the material the energy dissipation is sufficiently intense, high-frequency flutter cannot develop no matter how wide the plate.

The structural damping is a result of friction associated with microdisplacements of the plate relative to the nonmoving parts of the fixing system. For a given fixation type these microdisplacements and, hence, the friction are determined by the oscillation wavelength and amplitude. Since for high-frequency flutter, irrespective of the plate width, the oscillation frequency and growth increment are close to $\omega_{\max}$ and $\delta_{\max} + \delta_\gamma(\omega_{\max})$, for plates of various widths the length and amplitude of the waves growing with time will be the same for the same parameters M, M_w, D, and μ. Thus, during flutter the friction at the fixation sites is independent of the plate width.

Macroscopically, the structural damping leads to a decrease in the coefficients of wave reflection from the plate edges. Let us again consider the wave reflection process that leads to the growth of the natural oscillation amplitude. On the leading edge let a traveling wave $C_2 e^{i(k_2 x - \omega t)}$ with unit amplitude be excited. On reaching the trailing edge, its amplitude will be equal to $C_2 e^{-\operatorname{Im} k_2 L}$. After reflection it will transform into the wave $A_{23} C_2 e^{-imk_2 L} e^{\operatorname{Im} k_3 L/2} e^{i(k_3 x - \omega t)}$, where the reflection coefficient $A_{23} = C_3/C_2 - A'_{23}$ and $A'_{23} > 0$ is a correction due to friction at the trailing edge fixation site. When this wave reaches the leading edge, its amplitude will be equal to $A_{23} C_2 e^{-\operatorname{Im} k_2 L} e^{3 \operatorname{Im} k_3 L/2}$. After reflection it will transform into the wave $A_{32} A_{23} C_2 e^{-\operatorname{Im} k_2 L} e^{3 \operatorname{Im} k_3 L/2} e^{-\operatorname{Im} k_2 L/2} e^{i(k_2 x - \omega t)}$, where similarly the reflection coefficient $A_{32} = C_2/C_3 - A'_{32}$ and $A'_{32} > 0$. Assuming A'_{ij} to be small and comparing the amplitude of the twice reflected wave with the initial value, we find that after double reflection the amplitude increases (i.e., the eigenfunction grows) if

$$\left(1 - \frac{C_3}{C_2}A'_{32} - \frac{C_2}{C_3}A'_{23}\right)e^{3(\operatorname{Im}k_3 - \operatorname{Im}k_2)L/2} > 1$$

which, in view of (3.1), correct to quantities of the order of A'_{ij}, is equivalent to

$$L > \frac{1}{3\delta_{\max}}\left(\frac{C_3}{C_2}A'_{32} + \frac{C_2}{C_3}A'_{23}\right)\left(\left.\frac{\partial k_2(\omega,0)}{\partial \omega}\right|_{\omega=\omega_{\max}}\right)^{-1} \tag{3.6}$$

As shown above, A'_{ij} and, hence, for the given plate fixation type the entire right side of inequality (3.6) is independent of the plate width. The structural-damping effect on the high-frequency flutter thus differs fundamentally from the effect of energy dissipation in the material. Whereas a sufficiently high level of dissipation in the material suppresses the high-frequency flutter of plates of any width, structural damping prevents flutter development only if the width does not exceed the value (3.6); otherwise, flutter develops but the eigenfunctions will grow more slowly than in the absence of damping.

Taking both dissipation in the material and structural damping simultaneously into account leads to the following high-frequency flutter criterion:

$$(\delta_{\max} + \delta_\gamma(\omega_{\max}))L > \frac{1}{3}\left(\frac{C_3}{C_2}A'_{32} + \frac{C_2}{C_3}A'_{23}\right)\left(\left.\frac{\partial k_2(\omega,\, 0)}{\partial \omega}\right|_{\omega=\omega_{\max}}\right)^{-1}$$

In metals, as a rule, the energy dissipation is very weak and, in particular, an order smaller than the structural damping in real systems [6, § 4]. On a certain range of Mach numbers it is also small as compared with the eigenfunction growth increment in flutter $\delta_{\max}$. In fact, using experimental data on the logarithmic damping decrements for metals [7], we can calculate the values of the exponents δ_γ of the exponential curves enveloping the damped oscillation process. In particular, for specimens of aluminum, steel, and titanium alloys, for $\mathrm{M} = 1.5$, $\mathrm{M}_w = 0$ and purely flexural oscillations with a stress $\sigma < 2$ kG/mm^2 the values of $|\delta_\gamma(\omega_{\max})|$ do not exceed $1.7 \cdot 10^{-4}$, $5.7 \cdot 10^{-5}$, and $5.2 \cdot 10^{-6}$, respectively, whereas the $\delta_{\max}$ values for the same materials are several times higher and approximately equal to $6 \cdot 10^{-4}$, $3.5 \cdot 10^{-4}$, and $1 \cdot 10^{-3}$. Since for a sufficiently wide plate or for sufficiently reliable fixation the structural damping can be made arbitrarily small, for a metal plate we can select a width (and for a given width the clamping force) and a Mach number range such that the plate will be in the flutter region with account for the damping in it. We note that in high-molecular and, in particular, composite materials the energy dissipation may be significant [6, § 4] and lead to a substantial contraction of the flutter region or to its complete suppression for plates of any width.

4. EFFECT OF THE PRESENCE OF A COMPRESSIBLE GAS ON THE OTHER SIDE OF THE PLATE

The problem formulation considered describes both unilateral gas flow past a plate in the presence of a constant pressure on the other side and a bilateral flow of the same gas at the same velocity on both sides. We will consider the case of unilateral gas flow in the presence of an inviscid perfect gas at rest on the other side (in general, different from the flowing gas).

As before, the eigenfunction growth increment is determined by formula (3.1). The dispersion equation can be written in the form [8]:

$$F(k,\, \omega) = (Dk^4 + \mathrm{M}_w^2 k^2 - \omega^2) - \mu_1 \frac{(\omega - \mathrm{M}k)^2}{\sqrt{k^2 - (\omega - \mathrm{M}k)^2}} - \mu_2 \frac{\omega^2}{\sqrt{k^2 - (\omega/\chi)^2}} = 0 \tag{4.1}$$

Here, $\mu_j = \rho_j/\rho_m$, $\chi = a_2/a_1$, and ρ_j and a_j are the density and speed of sound of the j-gas (the subscripts "1" and "2" denote the flowing gas and the gas at rest, respectively). The radical branches are chosen in accordance with the same rule as in (1.5). Assuming that $\mu_j \ll 1$, we have the following expansion of the wave number in the parameters μ_j:

$$k(\omega,\ \mu_1,\ \mu_2) = k(\omega,\ 0,\ 0) + \left.\frac{\mu_1 l_1(k) + \mu_2 l_2(k)}{2k(\mathrm{M}_w^2 + 2Dk^2)}\right|_{k=k(\omega,0,0)}$$

$$l_1(k) = \frac{(\omega - \mathrm{M}k)^2}{\sqrt{k^2 - (\omega - \mathrm{M}k)^2}}, \qquad l_2(k) = \frac{\omega^2}{\sqrt{k^2 - (\omega/\chi)^2}} \tag{4.2}$$

valid for frequencies at which the phase velocity $c = \omega/k(\omega,\ 0,\ 0) \neq \mathrm{M} \pm 1$ ($c \neq \pm\chi$).

We will consider a pair of waves $k_{2,3}(\omega)$ traveling in opposite directions and forming a global eigenfunction. In [1], on the basis of a study of the phase shift between a wave traveling along the plate and the pressure exerted on it by the flowing gas, it was established that under the influence of the gas the upstream-traveling wave k_3 is always damped, while the downstream-traveling wave k_2 is amplified if $c < \mathrm{M} - 1$, conserves its amplitude if $\mathrm{M} - 1 < c < \mathrm{M} + 1$, and is damped if $c > \mathrm{M} + 1$; here, $c = \omega/k(\omega)$ is the phase velocity of the wave. Considering the phase shift between the wave and the pressure in the gas at rest, we can show that it affects the growth of these waves as follows: for $|c| < \chi$ the waves do not change amplitude and for $|c| > \chi$ they are damped.

From this it can be seen that the downstream-traveling wave (and hence the entire eigenfunction) can grow only under the influence of the moving gas, and the frequency corresponding to the maximum growth rate remains the same as without account for the gas at rest:

$$c(\omega_{\max}) = \mathrm{M} - 1 \implies \omega_{\max} = (\mathrm{M} - 1)\sqrt{((\mathrm{M} - 1)^2 - \mathrm{M}_w^2)/D}$$

At this frequency expansion (4.2) must be replaced by a nonlinear expansion as in Section 3.

Using the qualitative properties of the influence of the resting and flowing gases on the waves traveling along the plate it is easy to obtain the properties of the function $\delta_{\max}(\mathrm{M}) = \delta(\omega_{\max}(\mathrm{M}))$. If $c = \mathrm{M} - 1 < \chi$, the growth characteristics calculated with and without account for the gas at rest are the same, since the latter does not contribute to the change in the amplitudes of the traveling waves. If $c = \mathrm{M} - 1 > \chi$, the gas at rest damps the system: under its influence the growth increment decreases. Its damping effect is highest if $\mathrm{M} - 1 = \chi$, since in this case the phase velocity is equal to the speed of sound in the gas at rest and the latter has a maximum effect on the wave amplitudes. Depending on the relationship between μ_1, μ_2, and χ, the damping effect of the gas at rest either cannot suppress the flutter at $\mathrm{M} - 1 > \chi$, suppresses it completely, or suppresses it only at $\mathrm{M} - 1 \approx \chi$, whereas at higher M it develops again. As in Section 3, we can obtain an explicit expression for $\delta_{\max}(\mathrm{M})$ when $\mathrm{M} - 1 < \chi$, $\mathrm{M} - 1 \approx \chi$, and $\mathrm{M} - 1 > \chi$ and find the conditions of transition from one case to another, but these results are too cumbersome to be reproduced here.

Summary. The width of the plates to which the results obtained in [1] for high-frequency flutter are applicable is estimated. Four sources of inaccuracy are considered: inaccuracy in determining the pressure acting on the oscillating plate, the use of the global instability method for solving the eigenvalue problem for a plate of finite width, and neglecting the damping of the plate oscillations and the presence of a gas at rest on the other side of the flow surface.

The inaccuracy in determining the pressure acting on the oscillating plate is negligible and does not restrict its width.

An estimate is obtained for the plate width at which, in investigating the eigenvalue problem, the global instability theory can be used, that is, at which the eigenfunctions can be considered to be composed of two traveling waves and the damped waves neglected.

The effect of energy dissipation in the plate material and structural damping on the eigenfunctions is investigated and the condition for their growth is obtained. Sufficiently intense dissipation in the material prevents flutter in plates of any width, whereas for a sufficiently large width the structural damping can be made arbitrarily small and therefore cannot suppress flutter in sufficiently wide plates.

Taking into account the gas at rest on the side of the plate opposite to the flow surface does not affect the growth of the eigenfunctions at $\mathrm{M} - 1 < \chi$, where M is the Mach number and χ is the ratio of the speed of sound in the gas at rest to that in the moving gas, whereas at $\mathrm{M} - 1 > \chi$ it has a damping effect and can completely or partly suppress the flutter.

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