Flex Serv Manuf J (2016) 28:644–668
DOI 10.1007/s10696-016-9245-7

Pricing storage of outbound containers in container terminals

Youn Ju Woo[1] · Jang-Ho Song[2] · Kap Hwan Kim[1]

Published online: 18 June 2016
© Springer Science+Business Media New York 2016

Abstract In container yard of container terminals, a storage charge is imposed to encourage customers to limit the space required for their containers. This study addresses the storage price scheduling problem for the storage of outbound containers. The price schedule consists of the free-time limit, which is the maximum duration for a container to stay in the yard without any charge, and storage charge per day for storing a container past the free-time-limit. Empirical data suggests that the efficiency of loading operations significantly depends on the space utilized by a vessel's outbound containers. Mathematical models are developed to determine the optimal storage price schedule in such a manner that the terminal's total profit is maximized or the total system's cost is minimized. Both single and multi-vessel cases are considered. Properties of the optimal solution are derived from the mathematical models and numerical experiments are conducted to validate solutions.

Keywords Container terminal · Pricing · Storage space · Outbound containers

✉ Kap Hwan Kim
kapkim@pusan.ac.kr

Youn Ju Woo
yjwoo@pusan.ac.kr

Jang-Ho Song
remysong@pncport.com

[1] Department of Industrial Engineering, Pusan National University, Jangjeon-dong, Gumjeong-gu, Busan 609-735, Korea

[2] Pusan Newport Co., Seongbuk-dong, Gangseo-gu, Busan 618-821, Korea

1 Introduction

Storage spaces in container terminals provide a place for inbound and outbound containers in a container terminal to be stored. Such containers are temporarily stored at the yard before being loaded onto a vessel or carried out of the gate by trucks. The goal of this study is to determine the storage price for outbound containers by considering a terminal's cost and revenue and customer response to different price schedules.

In stowage plans, adjacent slots are dedicated to a group of outbound containers with the same attributes (e.g., port of destination and size), and in general, are consecutively filled. For speeding up the loading operation, outbound containers of the same group are usually stored in adjacent slots in the yard. A container yard is usually divided into several blocks, each of which consists of 20–30 yard-bays. In the yard-crane (YC) system, each yard-bay consists of between six and ten rows (stacks). Figure 1 illustrates a block with ten yard-bays and a yard-bay with six stacks of four tiers. Note that adjacent slots are allocated to the same group of containers to increase shipping operation efficiency. Adjacent empty slots are blocked or reserved for a group in advance to accommodate and easily locate randomly arriving containers. Note that in Fig. 1, some slots are already occupied by containers and others are reserved for selected groups of outbound containers.

Table 1 illustrates the proportion of the arrivals of outbound containers, which was collected from a real terminal in Busan during 1 year. Outbound containers start to arrive at the terminal before 9 days before the loading operation. It can be observed that the arrival rate of outbound containers increases as the loading operation time approaches. Figure 2 shows the inventory of containers for a vessel. The upper curve represents the cumulative reserved space in the twenty-foot-equivalent-unit (TEU), and the lower curve corresponds to a vessel's inventory of containers. Note that all the required spaces are not reserved at once but instead they are reserved in split amounts in multiple times to save the space. The difference between the cumulative reserved space and inventory corresponds to the reserved empty space. When the space requirement for outbound containers is estimated, both the inventory of containers and space for reservation must be considered. The average space utilization is area OAB divided by area OCDEFAB. Here, OAB and OCDEFAB are the vertices of polygons in Fig. 2.

In general, terminal operators impose storage charges to reduce the space requirement of containers. Beyond the free-time limit to store outbound containers, storage charges are collected to reduce the dwell time of outbound containers at the yard. Shippers store their cargo in or near the terminal. Containers arriving considerably early to the port are temporarily stored at off-dock-container-yards (ODCYs) or remote container yards. An ODCY generally charges lower storage rates than a port container terminal (PCT); however, additional transportation costs between an ODCY and PCT are incurred.

Outbound containers follow one of two possible routes prior to being delivered to a PCT. The first route is to directly move a container to the PCT, and the second route is to deliver a container to an ODCY for temporary storage. If the sum of

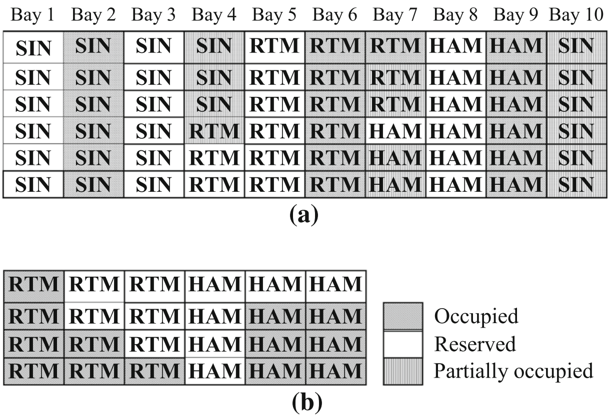


Fig. 1 Occupied, reserved, and partially occupied space (Woo and Kim 2011). **a** A block with containers of different container groups (*top view*). **b** Reserved and occupied space (front view of Bay 7)

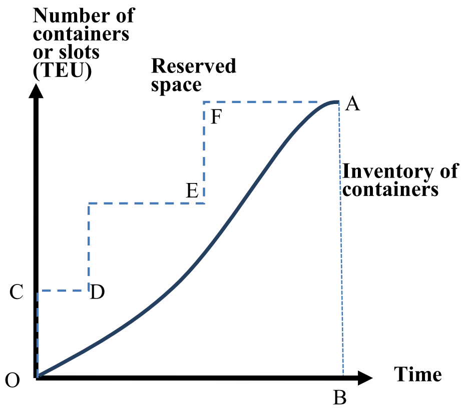
Table 1 Distribution of dwell times of outbound containers before loading

Range of dwell time (h)	%
0–24	21.92
24–48	34.92
48–72	17.28
72–96	10.24
96–120	7.03
120–144	4.11
144–168	2.03
168–192	1.34
192–216	0.76
216–240	0.36
Total	100.00

ODCY storage and transportation costs is greater than PCT storage costs, the container is moved directly from the shipper to the PCT; otherwise, it is temporarily stored at an ODCY and then transferred to the PCT after the free-time limit begins. The dwell time of a container at the port area and that in the yard of the container terminal are denoted as DTP and DTY, respectively, from now on. Figure 3 illustrates two possible flows of outbound containers.

Figure 4 illustrates a price schedule that is being used at a terminal for storing an outbound container beyond the free-time limit (4 days in this example); the storage charge is approximately proportional to the DTY exceeding the free-time limit. When the PCT offers a lower daily storage price and a longer free-time limit, the inventory of containers in the yard increases; similarly, when a higher daily storage price and shorter free-time limit are imposed, there is a lower density of containers in the yard. However, it is well-known that the density of containers in the yard

Fig. 2 Inventory of outbound containers and reserved space



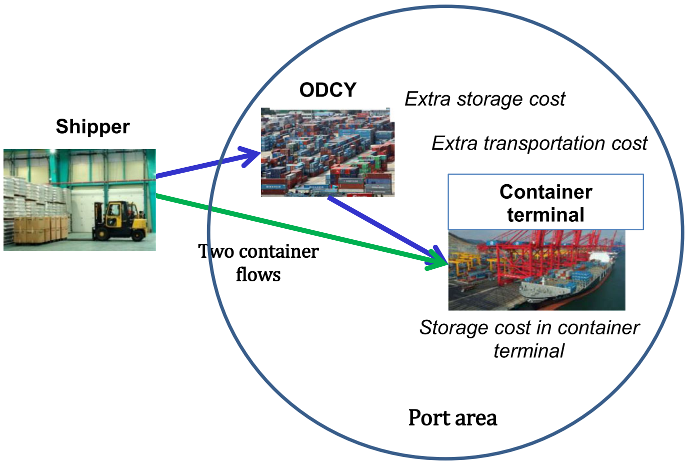


Fig. 3 Two possible flows of outbound containers

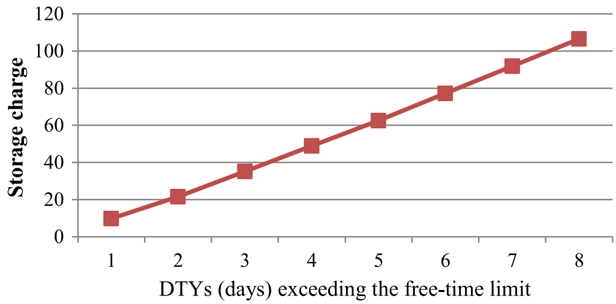


Fig. 4 Storage charge (US$) for various DTYs (days) of a container (TEU) exceeding the free-time limit

significantly affects the productivity of handling operations in a PCT. In the next section, we analyze the relationship between yard space utilization and productivity of vessel operations.

Space management in container terminals has been widely researched. Regarding the space allocation of outbound containers, Taleb-Ibrahimi et al. (1993) proposed a space allocation strategy in which temporary storage areas are used for containers that arrive before a designated storage space is allocated (reserved). Kim and Kim (2002) proposed a method for determining the storage space for import containers. Kim and Park (2003) proposed a dynamic space allocation method for outbound containers in which the space in each block is allocated to each vessel for future container arrivals. Zhang et al. (2003) addressed a similar space allocation problem and attempted to balance the workload among different blocks to avoid possible congestions in parts of the yard. Lim and Xu (2006) proposed a method for locating reserved space for each group of containers in a block. This method attempted to schedule the allocation of empty spaces to each group so that in the final layout, adjacent spaces would be reserved for the same group. Lee et al. (2006) proposed yard-space allocation methods for a trans-shipment hub port. They suggested an algorithm for assigning a space unit (called sub-blocks) to containers that are to be loaded (discharged) onto (from) a vessel so as to minimize congestion during discharging and loading operations. Lee and Hsu (2007) proposed a method for pre-marshaling outbound containers to speed up loading operation. Dekker et al. (2006) proposed various algorithms for locating containers in automated container yards and compared their performance with each other. Cordeau et al. (2007) proposed a method for assigning vessel storage spaces that minimize container relocation in the yard of trans-shipment container terminals. Bazzazi et al. (2009) also attempted to minimize the variation in workload handling across various blocks during space allocation. Chen and Lu (2012) proposed a two-staged storage location assignment method for outbound containers. Zhen (2014) proposed a container yard template planning method considering varying numbers of loading containers for periodically visiting vessels. Martin et al. (2015) proposed statistical models for estimating space requirements for storing inbound and outbound containers in container terminals. Petering (2015) addressed the real-time storage location assignment problem at RTG-based transshipment container terminals. Li (2015) proposed a space planning method for export containers considering workload distribution and yard space utilization. Carlo et al. (2014) and Zhen et al. (2013) provide overviews on the storage yard operation studies.

Related to the pricing of general cargo storage, Castilho and Daganzo (1991) analyzed a pricing problem for temporary storage facilities like sheds at ports. They proposed a mathematical model representing shipper behaviors for temporarily storing their cargo responding to different price schedules. Holguin-Veras and Jara-Diaz (1999) derived formulae for determining the space allocation and prices for priority systems in CYs. They compared three different pricing rules: welfare maximization, welfare maximization subject to a breakeven constraint, and profit maximization. Holguin-Veras and Jara-Diaz (2006) extended this research by considering the case where the container arrival rate is sensitive to the terminal's storage charge. Holguin-Veras and Jara-Diaz (2010) generalized their previous studies to a more general capacity-constrained transportation facility pricing scheme that includes an entrance fee and a dwell charge. It was shown how the facility owner could control both the arrival rate and the dwell time of the user using this pricing scheme. However, these studies did not specifically assume the situation of container terminals but those of general cargo storage facilities.

Related to the pricing of container storage, Kim and Kim (2007) proposed a method for determining the free-time limit and storage price per unit time for inbound containers. The free-time limit was assumed to be a positive continuous value; in practice, however, a day is usually used to be the time unit. Kim and Kim (2010) discussed three cost models for the optimal pricing structure of inbound containers from the viewpoint of terminal operators. They used a discrete probability distribution for the dwell time. Saurí et al. (2011) studied the effects of imposing a yard storage tariff for inbound containers and used mathematical models to determine a price schedule for a profit-maximizing terminal. Lee and Yu (2012) suggested an inbound container storage price competition model between the container yard in a PCT and the remote container yard. Although previous studies handled storage pricing problems at container terminals, they were limited to the case of inbound containers. Woo and Kim (2012) proposed a mathematical model for pricing the storage of outbound containers. However, their model did not consider the effect of different price schedules on the congestion in the yard and thus on the productivity of the ship operations, which this paper considered in the formulation. Yu et al. (2015) analyzed two-level inbound container storage pricing problems involving a container terminal operator and an ocean carrier in two different inbound container storage contract systems: the free-time contract system and the free-space contract system.

Unlike most of previous studies, which addressed price scheduling problems of inbound containers, this study considers the storage pricing problem of outbound containers. From empirical data, it is shown that the efficiency of loading operations significantly depends on the space provided to each vessel. This relationship is used for analyzing the effects of the price schedule on terminal performance. Moreover, this study not only considers just a single vessel, but also considers multiple vessels.

In Sect. 2, we analyze the relationship between the storage price and the distribution of container arrivals and that between space utilization and a terminal's operational performance. Mathematical models are developed for determining the optimal storage price schedule in Sect. 3. Section 4 analyses the case of multiple vessels calling on a weekly basis. Useful properties are derived from our mathematical models and numerical experiments. Finally, we provide our concluding remarks in Sect. 5.

2 Storage charge, space utilization, and productivity of ship operations

As illustrated in Fig. 4, the price schedule is expressed by the following two parameters:

F = Free-time limit for outbound containers (days)
S = Daily storage price in the CY of the PCT for the storage before the free-time limit (F) (US$/TEU/day)

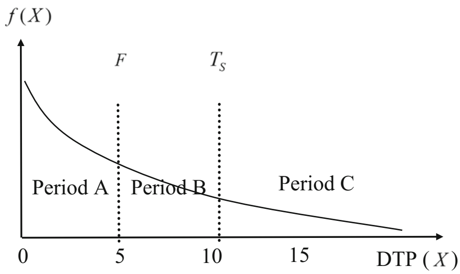


Fig. 5 pdf of DTPs

When a container is delivered through an ODCY, then the following cost terms need to be considered:

c_0 = Transportation charge for a container (TEU) between the PCT and an ODCY, and the handling cost at the ODCY (US$/TEU),
S_0 = Daily storage price of a container at an ODCY (US$/TEU/day). This study assumes that the daily storage price at an ODCY is lower than that at a PCT (S). This is why containers are sometimes temporarily stored in an ODCY.

This study assumes that the transportation cost between a PCT and each ODCY, the handling cost, and the storage price at the ODCY are the same for all outbound containers. Let X be the random variable representing the dwell time of an outbound container at the port (DTP). Note that after a container arrives at the port, it may be stored at an ODCY or directly delivered to the PCT. This is a continuous random variable.

Figure 5 shows the probability density function (pdf) of the dwell time at the port (DTP), f(X), of outbound containers. Let F(X) be the cumulative probability function of X. Note that X is a random variable from the viewpoint of the container terminal operator, while, from the viewpoint of each shipper, it is deterministic. That is, when a price schedule is announced by the terminal operator, each shipper determines the flow of the container (as a result, the DTP of the container becomes fixed) in the best manner to minimize his/her total cost. Previous studies assumed the exponential distribution (Watanabe 2001; Kim and Kim 2007), linearly decreasing distribution (Lee and Yu 2012; Yu et al. 2015), and the uniform distribution (Lee and Yu 2012; Yu et al. 2015) as a dwell time distribution at the Port, f(X). The real distribution in Table 1 is similar to the exponential distribution.

Let Y be the random variable representing the dwell time of a container in the yard of the PCT (DTY). Because a container may stay at an ODCY after it arrives at a port, $Y \leq X$. That is, X–Y represents the time for an outbound container stays at an ODCY. As a result, the pdf of the DTYs, r(Y), is derived as shown in PCT in Fig. 6. The pdf r(Y) is evaluated as follows: all containers for which movements to an ODCY are more economical than direct delivery to the CY are moved to an ODCY. They are then relocated to the CY of the PCT at the beginning of the free-time limit. All containers that are delivered to the port before a specific time, denoted by T_S in this study, are classified into this class. The other containers are

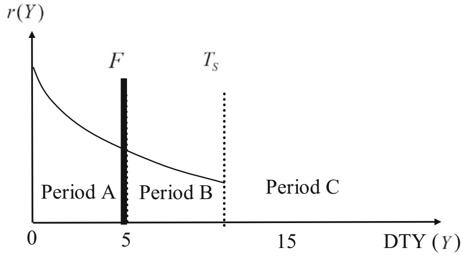


Fig. 6 pdf of DTYs of containers for a storage charge and free-time limit

directly delivered to the terminal. Note that T_S is a function of F and S and the value of DTP of a customer for whom the direct delivery of a container to PCT costs the same as delivering it through an ODCY, that is,

$$S(T_S - F) = c_0 + s_0(T_S - F). \tag{1}$$

Then, *r*(*Y*) is given by

$$r(y) = \begin{cases} f(y) & \text{for } 0 \leq y < F \\ \int_{T_S}^{\infty} f(y)dy & \text{for } y = F \\ f(y) & \text{for } F < y \leq T_S \\ 0 & \text{for } y > T_S \end{cases} \tag{2}$$

Note that r(y) in (2) is a mixed distribution, which is partially discrete and partially continuous. That is, y is a random variable taking a value of "F" or a value from $[0, F) \cup (F, \infty)$. However, the value of r(y) at y = F is not infinity but $\int_{T_S}^{\infty} f(y)dy$.

Let $R(Y)$ be the cumulative probability distribution (cdf) of Y and $C_R(Y)$ be the complementary cdf of Y given by 1 − R(Y). Then, $C_R(Y)$ indicates the percentage of outbound containers, which are expected to arrive at the yard of the container terminal, when the remaining dwell time (DTY) is Y. In Fig. 7, the dashed line represents the complementary cdf of X and the solid line represents $C_R(Y)$. Figure 7b shows $C_R(Y)$ when the daily storage price becomes lower when compared with Fig. 7a, while Fig. 7c shows $C_R(Y)$ when the free-time limit becomes shorter. Note that Fig. 7b shows a larger amount of space requirement, while Fig. 7c shows a smaller amount of space requirement.

Reservations are done to realize an ideal layout of containers so that they can be efficiently loaded onto their corresponding vessel. The dotted line (HGIKA) in Fig. 7d represents the reserved space, which includes occupied space and reserved empty spaces. After initial reservations are made, spaces are progressively reserved as a result of the space planning, as the inventory of containers increases in the yard. The space availability and strategy of space planning affect the shape of the space reservation curve. Space utilization is given by the occupied space (area below the solid line) divided by the total space allocated to a vessel (area below the dotted line); that is, the area of OHDCAO divided by the area of OHGIKAO.

To analyze the relationship between space utilization and the cycle time of a quay crane (QC), data was collected from a container terminal in Busan, Korea from January 2011 to October 2012. The number of berths in the terminal was six and the

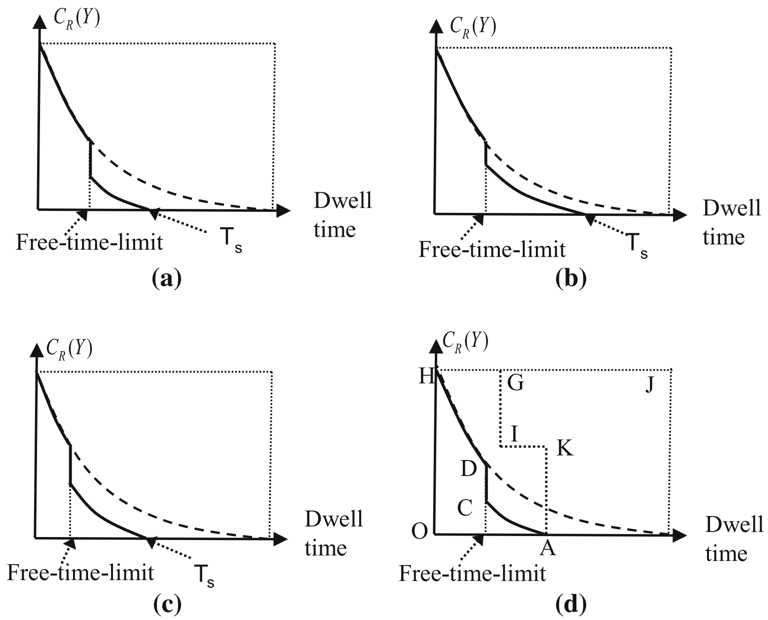


Fig. 7 Impact of storage charge and free-time-limit on inventory of containers. **a** Original price schedule. **b** Lower storage price. **c** Shorter free-time-limit. **d** Reservation

number of QCs was seventeen. The space utilization was defined as the number of slots (TEU) occupied by containers in the yard divided by the total number of slots.

For the ship operation, employees are usually organized in the unit of gang corresponding to a group of employees who work together with a particular QC to which a particular order of works are assigned. A typical example of a gang includes a QC driver, stevedores on the vessel for the lashing of containers, a foreman who coordinates the operations, stevedores on the apron, drivers for transport means and yard cranes.

When the number of gangs working at the same time was small, the productivity of QCs was high, even when the yard was congested. Thus, only the data collected during the shifts with more than 16 gangs deployed were used for the analysis. The number of shifts for which data was collected was 77.

Results of the regression analysis are provided in Fig. 8. We found that the cycle time of QCs increases as space utilization increases. The correlation coefficient between the space utilization and the average cycle time was 0.778. Let C and U represent the average cycle time of QC and the space utilization, respectively. The following regression functions are derived:

$$\text{(Linear function) } \mathrm{C} = 0.7547 + 1.795\mathrm{U};$$

$$\times \text{ (Quadratic function) } \mathrm{C} = 1.913 - 1.917\mathrm{U} + 2.942\mathrm{U}^2; \text{ and}$$

$$\times \text{ (Exponential function) } \mathrm{C} = 1.037\, e^{0.943\mathrm{U}}.$$

Three statistics for the three regression models were compared with each other. The summary of the statistical comparison among three models is as follows. The

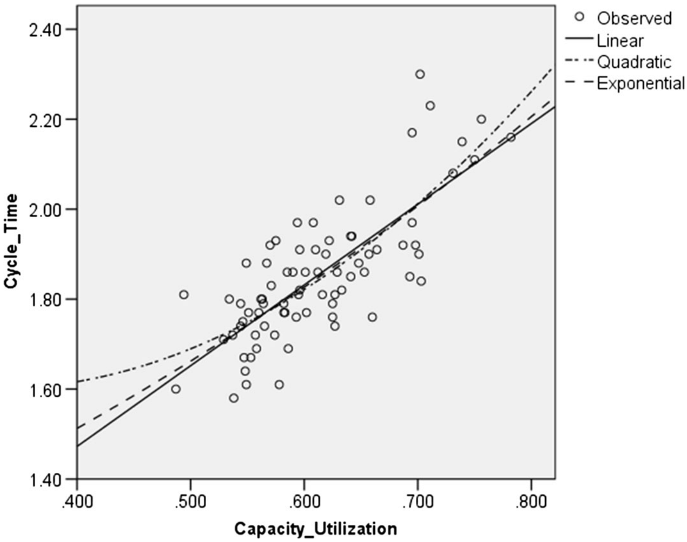


Fig. 8 QC cycle time versus space utilization

results show that the p-values for all the three models are very small which means the models significantly reduce variance significant. According to the statistics (R Square, overall F-test, and Mean Square Residual) which are used to assess the fit of regression model, it was found that three regression models fit well the data. Thus, the following linear regression model was selected for the simplicity of the analysis.

$$\text{Average cycle time (minutes)} = 1.795 \times (\text{Utilization of storage space}) + 0.7547. \quad (3)$$

About this simple linear regression model, the R-square value was 0.597, F value was 111.3 whose corresponding significance level is almost zero, and the mean square residual was 0.009. Results of this analysis will be used to describe the relationship between the space allocated to outbound containers of a vessel and the cycle time of QCs during the shipping operation for the vessel.

3 Pricing container storage of a vessel

In this section, we propose mathematical models for the price scheduling of container storage. Some terminal operators may consider the revenue from the storage charges their main source of income, while other operators may consider the storage charge as a mean of balancing congestion in the yard and customer convenience. If longer DTYs are allowed for outbound containers, then customer convenience may improve; however, high congestion of containers in the yard is expected. On the other hand, if shorter DTYs are allowed, then customers may be inconvenienced, but container congestion in the yard is generally lower. When the profit-making is not the main reason for terminal operators to impose the storage charge, the terminal operator will attempt to minimize the total cost of the terminal and customers, which can be considered to be public disutility.

This section introduces two different mathematical models. The goals of these models are to: (1) maximize terminal profits and (2) minimize the total cost of the system, including the terminal and customers, for outbound containers. The next section proposes a model in which the terminal operator attempts to minimize the total cost of the system resulting for weekly-calling multi-vessels.

Notation:

g = Average number of gangs assigned to a vessel.
c_t = Cost per minute for all the resources in a gang involved during the loading operation in the PCT (US$/min). It includes the cost for workers as well as for equipment.
c_v = Average cost per minute of vessels that are calling at the PCT, including operation and overhead costs (US$/min) (Jang 1992).
u = Total amount of containers to be loaded onto the vessel (TEU).
v = Total space available for the vessel under consideration (TEU × day).

3.1 Maximizing terminal profit

In the previous section, we showed that the expected cycle time of QCs can be expressed as a linear function of space utilization. Suppose v is the amount of the space allocated to a vessel; this quantity corresponds to the area under the dotted line (HGIKA) in Fig. 7d. Note that the units of v are TEU × day. This section assumes that the terminal operator maximizes terminal profits by optimizing the values of F and S. We define this problem (P1) as follows:

$$\max_{F,S,T_s} E(PF(F,S)) = \int_F^{T_s} S(x-F)f(x)dx - c_t\left(a\frac{u\int_0^{T_s} C_R(y)dy}{v} + b\right), \quad (4)$$

subject to

$$S(T_s - F) = c_0 + s_0(T_s - F), \quad (5)$$

$$u\int_0^{T_S} C_R(x)dx \le v \quad (6)$$

$$S, F, T_s \ge 0. \quad (7)$$

The first term in (4) is the revenue from container storage charges while the second term represents the operational costs for loading containers, which is proportional to the cycle time of quay cranes. Note that $u\int_0^{T_S} C_R(y)dy \Big/ v$ in (4) corresponds to the space utilization of storage space allocated to a vessel and thus $\left(a\frac{u\int_0^{T_s} C_R(y)dy}{v} + b\right)$ represents the estimated average cycle time of QC, which can be estimated as in (3) where a is the slope (=1.795) and b is the intercept (=0.7547) of the linear regression equation. The objective function can be simplified as follows:

$$\int_F^{T_S} S(x-F)f(x)dx - c_t a \frac{u\int_0^{T_s} C_R(y)dy}{v}$$

From Rohatgi (1976),

$$\int_0^{T_s} C_R(y)dy = \int_0^{\infty} yr(y)dy = \int_0^{T_s} xf(x)dx + F\int_{T_s}^{\infty} f(x)dx \quad (8)$$

Note that in (4), T_S is defined as the DTP of a container for which directly delivering to the PCT costs the same as temporarily storing at an ODCY. After being stored at the ODCY, the container is delivered to the PCT at the beginning of the free-time limit.

Property 1 *For (P1),* $F = 0$ *is an optimal solution. A proof of this property can be found in "Appendix 1".*

As a result of Property 1, E(PF(S)) replaces E(PF(F,S)). Using Property 1 and (8), the objective function can be revised as follows:

$$\max_S E(PF(S)) = S\int_0^{c_0/(S-s_0)} xf(x)dx - c_t a \frac{u\int_0^{c_0/(S-s_0)} xf(x)dx}{v}$$

subject to

$$u\int_0^{c_0/(S-s_0)} xf(x)dx \le v,$$

$$\mathrm{S, F, T_s} \ge 0.$$

3.1.1 When DTP follows a uniform distribution

This sub-section assumes f(x) follows a uniform distribution. The following useful property holds in this case.

Property 2 *Let* $f(x) = 1/b, 0 \le x \le b$. *E(PF(S)) is maximized at* $S = Max\{S_1, S_2, S_3\}$, *where* $S_1 = c_0/b + s_0, S_2 = 2ac_t u/v - s_0, and\ S_3 = c_0\sqrt{u/(2bv)} + s_0$. *A proof of this property can be found in "Appendix 2".*

A numerical experiment was conducted to determine meaningful properties of the solution. The following values of parameters were used for the experiment: $a = 1.795$, c_t = \$42.61, s_0 = \$13.70, c_0 = \$63.00, u = 922 TEU, v = 1919 TEU. Results are shown in Table 2. As shown in Table 1, total profit decreases as the upper bound of the storage period increases. Note that as the upper bound increases, the degree of uncertainty also increases.

Table 2 Results for various values of b with uniformly distributed DTP

b	1	2	3	4	5
S*	76.70	59.80	59.80	59.80	59.80
Total profit	19.98	10.76	7.18	5.38	4.31

Table 3 Results for various values of d with exponentially distributed DTP

d	1	1/2	1/3	1/4	1/5
S*	86.27	71.89	67.54	65.47	64.27
Total profit	10.69	7.24	5.44	4.35	3.62

3.1.2 When DTP follows an exponential distribution

Suppose $f(x)$ follows an exponential distribution; that is, $f(x) = de^{-dx}$ where 1/d is the average DTP. The objective function is modified as follows:

$$\max_{F,S,T_S} E(PF(F,S)) = d\left(S - \frac{auc_t}{v}\right)\int_0^{\frac{c_0}{S-s_0}} xe^{-dx}dx$$
$$= \left(S - \frac{auc_t}{v}\right)\left\{\frac{1}{d} - e^{-\frac{dc_0}{S-s_0}}\left(\frac{c_0}{S-s_0} + \frac{1}{d}\right)\right\}$$

The input values defined above were also used for this case. The solution was found by using a genetic algorithm. Results are shown in Table 3. As average DTP increases, the total profit and the optimal price decreases.

3.2 Minimizing total system cost

In many terminals, a storage charge is imposed to control the inventory level of containers in the yard rather than for profit making. In those terminals, the main source of terminal revenue comes from handling charges for containers. Since the main goal of the storage charge is to reduce congestion in the yard, the objective function for minimizing the total cost of the system may be used instead of maximizing profit. In particular, we define problem two (P2) as

$$\min_{F,S} E(TC(F,S)) = \min_{F,S,T_S}\left[\left(c_t + \frac{c_v}{g}\right)\frac{au\int_0^{T_S} C_R(y)dy}{v} + \int_{T_S}^{\infty}\{c_0 + s_0(x-F)\}f(x)dx\right] \tag{9}$$

subject to constraints (5)–(7). The first term of the objective function is the operation cost of loading a container, and c_v is the vessel cost per unit time including the overhead cost and operation cost. By dividing the vessel cost per unit time by the number of gangs assigned to the vessel, the vessel cost per unit time is allocated to each container. The second term represents the additional cost of temporarily storing a container in an ODCY and delivering it to the terminal.

3.2.1 When DTP follows a uniform distribution

This sub-section assumes f(x) follows a uniform distribution. In this case, the following property holds.

Property 3 *Suppose* $S, F, T_S \geq 0,\ f(x) = 1/b,\ 0 \leq x \leq b$ *follows a uniform distribution. E(TC(S,F)) is minimized when* $S = Max\{S_1,\ S_3,\ S_4\}$ *and* $F = 0$, *where* $S_1 = c_0/(b-F) + s_0, S_3 = s_0 + c_0\sqrt{u/(2bv)}$, *and* $S_4 = au(c_t + c_v/g)/v$. *If* $S^* = S_1$, *then all values of (S, F) satisfying* $c_0/(S - s_0) + F > b$ *are also optimal solutions. A proof of this property can be found in "Appendix 3".*

A numerical experiment was conducted to explore meaningful characteristics of the optimal solution. Additional parameter values used in the experiment are as follows: $g = 2$, $c_v = \$10.54$/min. The values of other parameters are the same as those used in the previous section. Table 4 lists the results for various values of *b*. Results indicate that total cost increases as the upper bound of the distribution, *b*, increases. In this study, the optimal free-time limit was 0; however, in practice, the free-time limit ranges from three to seven days.

As shown in Fig. 9, when the storage charge (s_0) in an ODCY increases, the total cost of the system also increases. Note that as s_0 becomes larger, S* remains the same until s_0 reaches 2.2 times of its initial value. When s_0 exceeds the value, S* begins to increase. The total cost steadily increases as the value of s_0 increases. Furthermore, when c_0 changes, patterns, similar to those observed when s_0 changes, can be observed for S* and the total cost.

3.2.2 When DTP follows an exponential distribution

Suppose $f(x)$ follows an exponential distribution; that is $f(x) = de^{-dx}$ where 1/d is the average DTP. The objective function and constraints are modified as follows:

$$\min_S \left[\left(c_t + \frac{c_v}{g}\right)\frac{au}{v}\left(\int_0^{\frac{c_0}{S-s_0}+F} dxe^{-dx}dx + F\int_{\frac{c_0}{S-s_0}+F}^{\infty} de^{-dx}dx\right) + \int_{\frac{c_0}{S-s_0}+F}^{\infty} d\{c_0 + s_0(x-F)\}e^{-dx}dx \right]$$

subject to

$$\int_0^{\frac{c_0}{S-s_0}+F} dxe^{-dx}dx + F\int_{\frac{c_0}{S-s_0}+F}^{\infty} de^{-dx}dx \leq \frac{v}{u}.$$

Table 4 Minimizing the total cost of the system when DTP is uniformly distributed

B	1	2	3	4	5
S*	76.70	45.20	41.29	41.29	41.29
Total cost	20.65	41.29	59.58	72.42	82.87

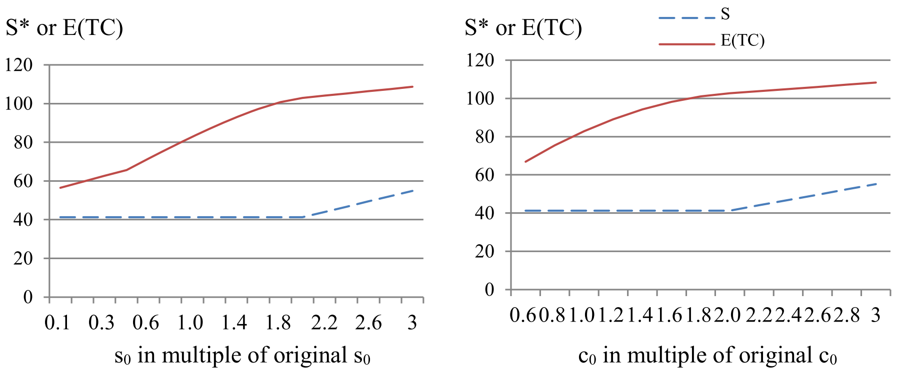


Fig. 9 Optimal storage charge and total cost for various multiples of original value of s_0 and c_0 for uniformly distributed DTP

Table 5 Minimizing the total cost of the system with an exponential distribution

d	1	1/2	1/3	1/4	1/5
S*	41.29	41.29	41.29	41.29	41.29
Total cost	38.48	64.96	85.21	102.80	119.08

For the numerical experiment, the solution was found by using a genetic algorithm. All input values remained the same. The optimal free-time limit was $F^* = 0$, and the optimal storage charge, S^*, was \$41.29, which was the same as in the case when $f(x)$ followed a uniform distribution. Results in Table 5 shows that total cost increases as the average DTP increases.

The average DTP was set to three days. Figure 10 shows that when the storage charge in an ODCY increases, the total cost of the system (E(TC)) also increases. Similar to the uniformly distributed DPT case, S* remains constant until s_0 exceeds 2.2 times of its initial value. When the transportation cost (c_0) increases, total cost increases as well; furthermore, S* remains the same until c_0 reaches 2.6 times of its initial value. When c_0 further increases, S* increases. As the values of s_0 and c_0 increase, the total cost monotonically increases. Figure 11 shows that as c_t increases, the optimal storage charge increases, and as c_v increases, the optimal storage charge increases as well. Overall, total cost increases.

4 Pricing container storage for weekly-calling multi-vessels

In the previous section, we assumed that some amount of space was pre-allocated to outbound containers of a vessel. In practice, a vessel usually calls at a terminal once a week. Because multiple vessels share a limited storage space in the yard, one important issue is how to allocate the storage space among them. It should be answered whether the space should be allocated proportionally to the total number

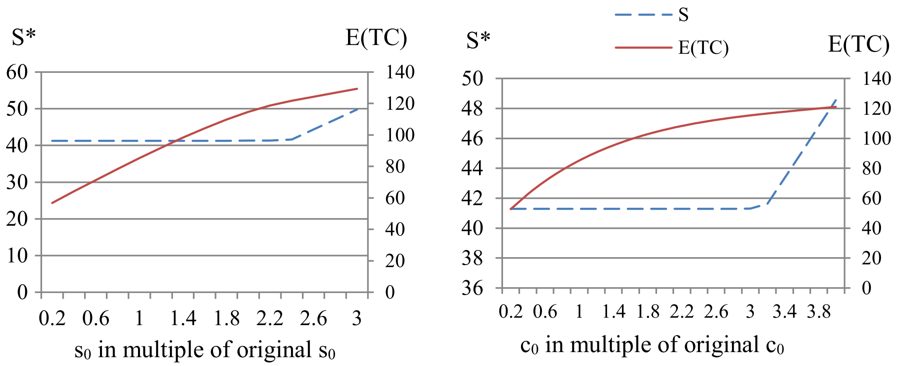


Fig. 10 Optimal storage charge and total cost for various multiples of original values of s_0 and c_0 for exponentially distributed DTP

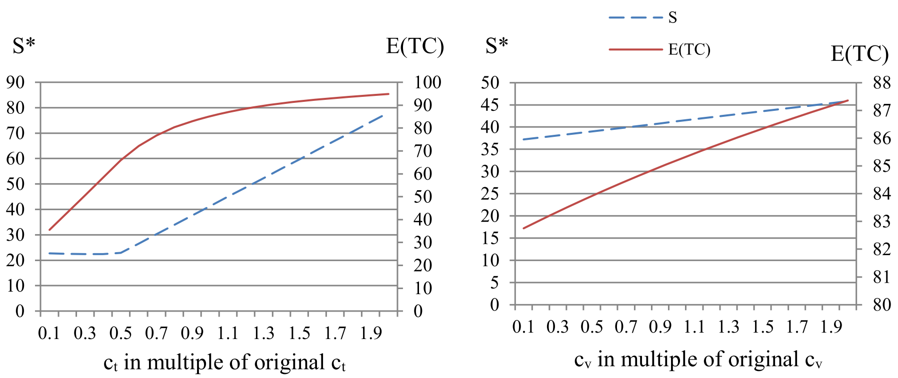


Fig. 11 Optimal storage charge and total cost for various multiples of original values of c_t and c_v

of containers of each vessel, or proportionally to the average dwell time of containers of each vessel. This section will propose a method to optimally allocate the storage space as well as to determine the free-time limit and the storage price.

Let the total available space to outbound containers to be $v_{TOT} \cdot V_i$, c_{oi}, s_{oi}, c_{vi}, and u_i represents the amount of space allocated to vessel i, the value of c_o, s_o, c_v, and u of vessel i, respectively.

This section assumes that the container arrival distribution differs among vessels. In this case, the cost minimization problem (P3) can be formulated as

(P3)

$$Min_{F,S,V_i} \sum_{i=1}^{n} E[TC_i(F,S,V_i)] \\ = \sum_{i=1}^{n} \left[\left(c_t + \frac{c_{vi}}{g_i} \right) \frac{au_i \int_0^{T_S} C_{R_i}(x)dx}{V_i} + \int_{T_S}^{\infty} \{c_0 + s_0(x - F)\} f_i(x)dx \right], \tag{10}$$

subject to

$$\sum_{i=1}^{n} V_i \leq v_{TOT}, \tag{11}$$

$$S(T_S - F) = c_0 + s_0(T_S - F), \tag{12}$$

$$u_i \int_0^{T_S} C_{R_i}(x)dx \leq V_i, \tag{13}$$

$$T_S, V_i \geq 0. \tag{14}$$

Constraint (11) implies that the total storage space is limited. Let $V = (V_1, V_2, \ldots, V_n)$. If constraint (11) is relaxed, (P3) can be modified as follows:

$$Min_{F,S,V} RC(S,F,V) = \sum_{i=1}^{n}\left[\left(c_t + \frac{c_{vi}}{g_i}\right)\frac{au_i\int_0^{T_S} C_{R_i}(x)dx}{V_i} + \int_{T_S}^{\infty}\{c_0 + s_0(x-F)\}f_i(x)dx\right] + \lambda\left(\sum_{i=1}^{n} V_i - v_{TOT}\right), \tag{15}$$

subject to constraints (12)–(14).

Let $w_i = au_i(c_t + \frac{c_{vi}}{g_i})\int_0^{T_S} C_{R_i}(x)dx$.

Then, $\partial RC(S,F,V)/\partial V_i = -w_i/V_i^2 + \lambda = 0$. Thus,

$$V_i = \sqrt{w_i/\lambda}, \quad \lambda = \left(\sum \sqrt{w_i}/V_{TOT}\right)^2 \tag{16}$$

After removing constant terms and replacing V_i with (16), The objective function (15) becomes

$$Min_{F,S,V} \widehat{\boldsymbol{RC}}(S,F,V) = \sum_{i=1}^{n}\left[2\sqrt{\lambda\left(c_t + \frac{c_{vi}}{g_i}\right)au_i\int_0^{T_S} C_{R_i}(x)dx} + \int_{T_S}^{\infty}\{c_0 + s_0(x-F)\}f_i(x)dx\right] \tag{17}$$

subject to constraints (12)–(14).

Since V_i is determined by (16), once the values of S and F are given, the decision variables of (P3) are S and F. This study uses the genetic algorithm to find optimal values for S and F.

The same approach may be applied to the model for maximizing the terminal profit. Let the profit maximizing problem for pricing container storage for weekly-calling multi-vessels be (P4). Then, the mathematical model may be derived as follows:

(P4)

$$Min_{F,S,V_i} \sum_{i=1}^{n} E[PF_i(F,S,V_i)] = \sum_{i=1}^{n} \left[\int_F^{T_S} S(x-F) f_i(x)dx - c_t \left\{ \frac{au_i \int_0^{T_S} C_{R_i}(x)dx}{V_i} \right\} \right] \tag{18}$$

subject to (11)–(14).

After relaxing constraint (11), (P4) can be modified as follows:

$$RP(S,F,V) = \sum_{i=1}^{n} \left[\int_F^{T_S} S(x-F) f_i(x)dx - c_t \left\{ \frac{au_i \int_0^{T_S} C_{R_i}(x)dx}{V_i} \right\} \right] + \lambda \left(v_{TOT} - \sum_{i=1}^{n} V_i\right)$$

subject to constraints (12)–(14).

Let $q_i = au_i c_t \int_0^{T_S} C_{R_i}(x)dx$.

Then, $\partial RP(S,F,V)/\partial V_i = q_i/V_i^2 - \lambda = 0$. Thus,

$$V_i = \sqrt{q_i/\lambda}, \lambda = \left(\sum_{i=1}^{n} \sqrt{q_i} \Big/ v_{TOT} \right)^2. \tag{19}$$

Because, once the values of S and F are given, V_i is determined by (19), the ultimate decision variables of (P4) are S and F as in the case of (P3). A search procedure like a genetic algorithm may be used to find optimal values for S and F.

A numerical experiment was conducted for (P3). In the experiment, common cost parameters were assumed to be the same except for the following: c_{vi} = \$10.54/min for all i, u_i = {81, 245, 521, 800, 1317}, and v_{TOT} = 10,745 (TEU × day). The distribution of DTPs for outbound containers varies from one vessel to another.

For the example of this study, the number of containers was fixed for five vessels (922 TEUs per vessel), and the mean DTP of each vessel was 1, 2, 3, 4, and 5 days. Figure 12 shows the percentage of space allocated to each vessel. Figure 12a

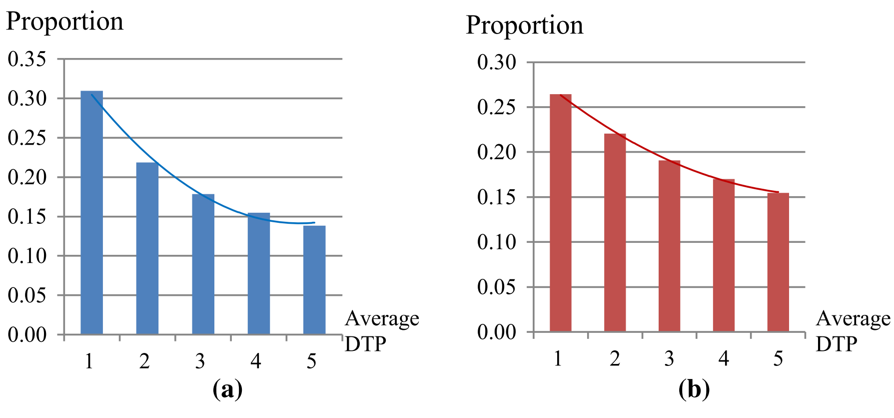


Fig. 12 Proportion of space allocated to each vessel for various average DTPs. **a** Uniformly distributed DTP. **b** Exponentially distributed DTP

assumes the DTP of outbound containers for each vessel follows a uniform distribution with different upper bounds; Fig. 12b assumes DTP follows an exponential distribution. In the case of uniformly distributed DTP, the optimal storage charge was \$52.20, and the total cost was \$384.36. Meanwhile, in the case of an exponentially distributed DTP, the optimal storage charge and total cost were \$72.60 and \$471.30, respectively. The optimal free-time limit was zero in both cases. This suggests that a container whose DTP is longer than T_s should be stored at an ODCY and moved to the PCT just before vessel operations begin. The optimal value of T_s was 1.64 and 1.07 days for the uniform and exponentially distributed cases, respectively. The optimal allocated space to a vessel with a longer DTP was smaller than that of a vessel with a shorter DTP when the number of containers was held constant.

A sensitivity analysis was conducted in which all the vessels contained the same number of containers (922 TEU) for exponentially distributed DTPs. The mean DTPs of the five containers were 1, 2, 3, 4, and 5 days. Various values of the total space, v_{tot} were investigated. Figure 13 shows the change in the proportion of space allocated to each vessel when the total amount of the space varies. Total space values tested were as follows: $1 \times 10{,}745$, $2 \times 10{,}745$, $3 \times 10{,}745$, …, $10 \times 10{,}745$ (TEU × Days). Results indicate that when total space is above a specific threshold, a greater proportion of space is allocated to vessels whose containers have a longer mean dwell time. However, when the total space is not enough, the vessel with the shorter mean DTP of containers has a larger portion of storage space.

The same sensitivity analysis was conducted for various values of the total space, v_{tot}, for vessels with the same mean, exponentially distributed, DTP of 3 days. In this numerical experiment, the number of containers differed from one vessel to another (81, 245, 521, 800, 1317 TEUs). Note that the proportion of space allocated to each vessel remained the same regardless of a vessel's total number of containers and changes to the total number of available spaces (Fig. 14).

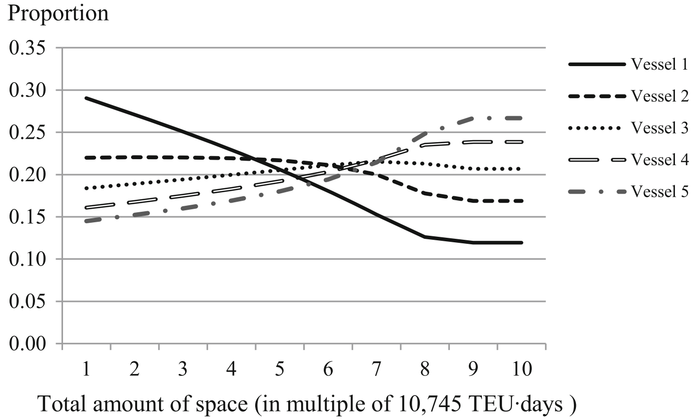


Fig. 13 Proportion of space allocated to vessels with various average DTPs for various total amounts of space

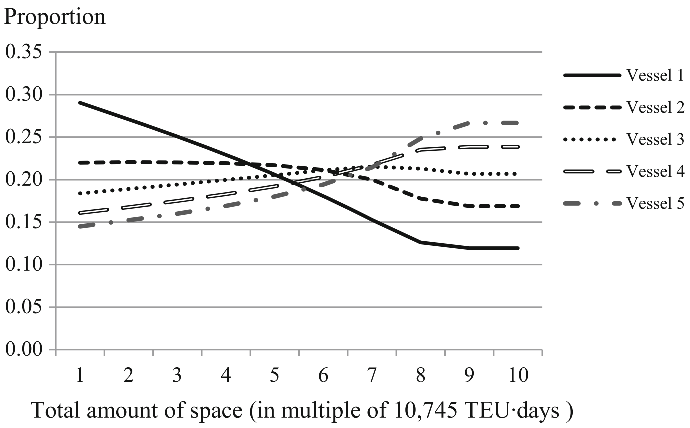


Fig. 14 Proportion of space allocated to vessels with different numbers of containers for various total amounts of space

5 Conclusions

This study addressed a method for scheduling storage prices by considering the storage price per day and free-time limit. A linear relationship was derived between the utilization of storage space and the cycle time of QC operations in a container terminal. Various mathematical models were developed to determine a price schedule; both uniform and exponential distributions were considered for the dwell time at port (DTP) of outbound containers. The first model maximized terminal profits from only a single vessel, and the second model minimized total system costs from outbound containers for a single vessel. The third model minimized the total cost of multiple weekly-calling vessels at a terminal.

Many conclusions can be drawn from our experimental results. First, the optimal value of the free-time limit was zero. For the single vessel problem, when the distribution of DTP was uniformly distributed, the optimal price was found by selecting one among several candidate values. When DTP was exponentially distributed, the optimal price was found by a simple line search. As the ODCY storage charge or transportation cost between the port terminal and ODCY increased, the optimal value of S increased. When multiple vessels were considered, the optimal value of space allocated to each vessel could be calculated using the values of S and F; thus, S and F were decision variables. The proportion of space allocated to each vessel was sensitive to the total available space when the mean DTP of containers differed from one vessel to another. However, when the mean DTP was held constant for all vessels, the proportion of space allocated remained the same, even when total available space changed.

In future studies, more complicated pricing schedules will be considered. One promising example of such new pricing schedules is the case where the terminal provides a free-space limit to each vessel liner and each vessel liner can provide a free-time limit to shippers for which the formulation for inbound containers (Yu et al. 2015) may be extended to the case of outbound containers. Some assumptions can be relaxed or generalized in the future studies. This paper assumed the cost parameters for shippers such as c_0 and s_0 to be the same for all the shippers. When there are shippers who have a large deviation among them in the values of the parameters, shippers may be classified into several groups for each which different values of the parameters are assumed. In this paper, two probability distributions for dwell time at the port (DTP) were used. For the application to other terminals, a new distribution function, which fits the data of the corresponding terminal, may be developed and analysed additionally. This study applied a simple search algorithm in some problems. By utilizing the unique characteristics of our problem, more efficient solution methods may be developed.

Appendix 1: Proof of Property 1

We will show that for a given value of T_S, $E(PF(F_1, S_1)) > E(PF(0, S_0))$ for an $F_1 > 0$, where $S_1 = s_0 + c_0/(T_S - F)$ and $S_0 = s_0 + c_0/T_S$, which are obtained from Eq. (5). The objective function $E(PF(F,S)) = E(PF(F)) = \int_F^{T_S} (s_0 + \frac{c_0}{T_S - F})(x - F)f(x)dx - c_t a \frac{u}{v}\left\{\int_0^{T_S} xf(x)dx + f\int_{T_S}^{\infty} f(x)dx\right\}$.

$\frac{dE(PF(F))}{dF} = \int_F^{T_S} \frac{c_0}{(T_S-F)^2}(x - F)f(x)dx - \int_F^{T_S}\left(s_0 + \frac{c_0}{T_S-F}\right)f(x)dx - c_t a\frac{u}{v}\int_{T_S}^{\infty} f(x)dx$.

Note that $\int_F^{T_S} \frac{c_0}{(T_S-F)^2}(x - F)f(x)dx \le \int_F^{T_S}\left(\frac{c_0}{T_S-F}\right)f(x)dx$ because $0 \le \frac{x-F}{T_S-F} \le 1$ for $F \le x \le T_S$. Thus, $\frac{dE(PF(F))}{dF} \le -\int_F^{T_S} s_0 f(x)dx - c_t a\frac{u}{v}\int_{T_S}^{\infty} f(x)dx \le 0$.

Hence, all these terms are non-positive. Namely, E(PF(F)) is a non-increasing function of F. Thus, F* = 0.

Appendix 2: Proof of Property 2

Let $f(x) = 1/b, \quad 0 \le x \le b$, which is a uniform distribution. Then, the objective function becomes $E(PF(S)) = S\int_0^{\min\left\{\frac{c_0}{S-s_0}, b\right\}} x\frac{1}{b}dx - c_t a\frac{u}{v}\int_0^{\min\left\{\frac{c_0}{S-s_0}, b\right\}} x\frac{1}{b}dx$.

Case 1 *$c_0/(S - s_0) \le b$, which can be rewritten as $S \ge c_0/b + s_0$.*

$$E(PF(S)) = \frac{1}{2b}\left(S - \frac{ac_t u}{v}\right)\left(\frac{c_0}{S - s_0}\right)^2$$

$$\frac{dE(PF(S))}{dS} = \frac{1}{2b}\left\{\left(\frac{c_0}{S - s_0}\right)^2 - 2\left(S - \frac{ac_t u}{v}\right)\frac{c_0^2}{(S - s_0)^3}\right\}$$
$$= \frac{c_0^2}{2b(S - s_0)^3}\left(-S - s_0 + \frac{2ac_t u}{v}\right)$$

Because $S \geq c_0/b + s_0$, which will be denoted as S_1, $c_0^2 \big/ (S - s_0)^3 \geq 0$. Thus, if $S \geq -s_0 + \frac{2ac_t u}{v}$, which we denote as S_2, then $\frac{dE(PF(S))}{dS} \leq 0$. Namely, E(PF(S)) is a decreasing function for $S \geq S_2$, and an increasing function for $S \leq S_2$.

(a) If $S_1 < S_2$, then the maximum objective value is obtained at $S = S_2$. (b) If $S_1 \geq S_2$, then in the range of $S \geq S_1$, E(PF(S)) decreases as the value of S increases. Thus, the maximum objective value is obtained at $S = S_1$. From constraint (6) and $\frac{u}{b}\int_0^{\frac{c_0}{S-s_0}} x dx = \frac{u}{2b}\left(\frac{c_0}{S-s_0}\right)^2 = \frac{uc_0^2}{2b(S-s_0)^2}$, we obtain $\frac{uc_0^2}{2b(S-s_0)^2} \leq v$, which is equivalent to $S \leq s_0 - c_0\sqrt{u/(2bv)}$ or $S \geq s_0 + c_0\sqrt{u/(2bv)}$. However, because $S \geq s_0, (c) S \geq s_0 + c_0\sqrt{u/(2bv)}$, which will be denoted as S_3. From (a), (b), and (c), (d) the maximum objective value is obtained at S = max {S_1, S_2, and S_3}.

Case 2 $S < c_0/b + s_0$.

In this case, $E(PF(S)) = S\int_0^b x\frac{1}{b}dx - c_t a\frac{u}{v}\int_0^b x\frac{1}{b}dx = \frac{b}{2}\left(S - \frac{ac_t u}{v}\right)$. Namely, $E(PF(S))$ is an increasing function of S. Now, consider the constraint in (6). Following the analysis in case 1, we know that $S \geq S_2$. If $S_3 \leq S_1$, then clearly E(PF(S)) is maximized at S_1, If $S_3 > S_1$, then there is no feasible solution. Thus, in case 2, if there is a feasible solution, then E(PF(S)) is maximized at S = S_1.

Considering cases 1 and 2 simultaneously, E(PF(S)) is maximized at S = max {S_1, S_2, and S_3}.

Appendix 3: Proof of Property 3

Suppose that $f(x) = 1/b,\ 0 \leq x \leq b$, Then, the objective function can be expressed as

$$E(TC(F,S)) = \left(c_t + \frac{c_v}{g}\right)\frac{au\int_0^{T_S} C_R(y)dy}{v} + \int_{T_S}^{\infty}\{c_0 + s_0(x-F)\}f(x)dx$$
$$= \left(c_t + \frac{c_v}{g}\right)\frac{au}{v}\left(\int_0^{\min\left\{\frac{c_0}{S-s_0}+F,b\right\}}\frac{x}{b}dx + F\int_{\min\left\{\frac{c_0}{S-s_0}+F,b\right\}}^{b}\frac{1}{b}dx\right)$$
$$+ \int_{\min\left\{\frac{c_0}{S-s_0}+F,b\right\}}^{b}(c_0 + s_0(x-F))\frac{1}{b}dx$$

Case 1 $c_0/(S - s_0) + F \geq b$. Then, the objective function becomes

$$E((TC(F,S)) = \left(c_t + \frac{c_v}{g}\right)\frac{au}{v}\left(\int_0^b \frac{x}{b}dx + F\int_b^b \frac{1}{b}dx\right) + \int_b^b\{c_0 + s_0(x-F)\}\frac{1}{b}dx$$
$$= \frac{abu}{2v}\left(c_t + \frac{c_v}{g}\right)$$

which remains constant for all values of F and S.

Case 2 $c_0/(S-s_0)+F<b$ the objective function becomes

$$E(TC(F,S))=\left(c_t+\frac{c_v}{g}\right)\frac{au}{v}\left(\int_0^{\frac{c_0}{S-s_0}+F}\frac{x}{b}dx+F\int_{\frac{c_0}{S-s_0}+F}^{b}\frac{1}{b}dx\right)$$
$$+\int_{\frac{c_0}{S-s_0}+F}^{b}\{c_0+s_0(x-F)\}\frac{1}{b}dx$$

$$=\frac{au}{2bv}(c_t+\frac{c_v}{g})\left\{\frac{c_0^2}{(S-s_0)^2}-F^2+2Fb\right\}-\frac{s_0c_0^2}{2b(S-s_0)^2}-\frac{c_0^2}{b(S-s_0)}+\frac{s_0F^2}{2b}-\frac{c_0F}{b}$$
$$+\frac{s_0b}{2}+c_0-Fs_0.$$

$$\frac{\partial E(TC(S,F))}{\partial F}=F\left\{\frac{s_0}{b}-\frac{au}{bv}\left(c_t+\frac{c_v}{g}\right)\right\}+\frac{au}{v}\left(c_t+\frac{c_v}{g}\right)-\frac{c_0}{b}-s_0.$$

From $\frac{\partial E(TC(S,F))}{\partial F}=0$, $F=\frac{vg(c_0+bs_0)-abu(gc_t+c_v)}{vgs_0-au(gc_t+c_v)}$. Let this value of F be denoted as F_1. Note that the first term of E(TC(F,S)), $\left(c_t+\frac{c_v}{g}\right)\frac{au\int_0^{T_S}C_R(y)dy}{v}$, represents the cost at the PCT and $\int_0^{T_s}C_R(y)dy$ indicates the expected DTY of a container at the terminal. Thus, $\frac{au}{v}\left(c_t+\frac{c_v}{g}\right)$ represents the additional cost for a container to stay at the terminal one more unit time. In addition, s_0 represents the additional storage cost for a container to stay at an ODCY one more unit time. If s_0 is greater than $\frac{au}{v}\left(c_t+\frac{c_v}{g}\right)$, then no container may have to be stored at an ODCY (consider that c_0 needs to be additionally paid for the storage at an ODCY). Thus, $s_0<\frac{au}{v}\left(c_t+\frac{c_v}{g}\right)$. Thus, for a given value of S, E(TC(S,F)) monotonically increases when $F<F_1$ and monotonically decreases when $F\geq F_1$. Thus, $F^*=0$ or b. Note that $\min_S E(TC(S,b))=E(TC(0,0))$, because in both cases, no container visits an ODCY. However, in general, $E(TC(0,0))\geq \min_S E(TC(0,S))$. Thus, $F^*=0$.

Next, $\frac{\partial E(TC(S,F))}{\partial S}=\frac{c_0^2}{b(S-s_0)^3}\left\{S-\frac{au}{v}\left(c_t+\frac{c_v}{g}\right)\right\}$.

Considering $S>s_0$, E(TC(S,F)) decreases until S reaches $\frac{au}{v}\left(c_t+\frac{c_v}{g}\right)$ and then increases. Note also that this function is valid in the range satisfying $\frac{c_0}{S-s_0}+F(=0)\leq b$, which is equivalent to $S_1=c_0/b+s_0$. Let, $S_1=c_0/b+s_0$ and $S_4=au(c_t+c_v/g)/v$. From constraint (6), we obtain $\frac{uc_0^2}{2b(S-s_0)^2}\leq v$, which can be converted to $S\geq s_0+c_0\sqrt{\frac{u}{2bv}}$. Let $S_3=s_0+c_0\sqrt{u/(2bv)}$. Therefore, TC(S,F) is minimized at $S=Max\{S_1,\ S_3,S_4\}$. If $S^*=S_1$, then E(TC(S*,0)) is the same as the objective value of case 1, which is a constant for all the values of (S,F) and thus all the values of (S,F) satisfying $c_0/(S-s_0)+F\geq b$ become the optimal solutions.

References

Bazzazi M, Safaei N, Javadian N (2009) A genetic algorithm to solve the storage space allocation problem in a container terminal. Comput Ind Eng 56(1):44–52

Carlo HJ, Vis IFA, Roodbergen KJ (2014) Storage yard operations in container terminals: literature overview, trends, and research directions. Eur J Oper Res 235:412–430

Castilho BD, Daganzo CF (1991) Optimal pricing policies for temporary storage at ports. Transp Res Rec 1313:66–74

Chen L, Lu Z (2012) The storage location assignment problem for outbound containers in a maritime terminal. Int J Prod Econ 135:73–80

Cordeau J-F, Gaudioso M, Laporte G, Moccia L (2007) The service allocation problem at the Gioia Tauro Maritime Terminal. Eur J Oper Res 176:1167–1184

Dekker R, Voogd P, Van Asperen E (2006) Advanced methods for container stacking. OR Spectrum 28:563–586

Holguin-Veras J, Jara-Diaz S (1999) Optimal pricing for priority service and space allocation in container ports. Transp Res Part B 33(2):81–106

Holguin-Veras J, Jara-Diaz S (2006) Preliminary insights into optimal pricing and space allocation at intermodal terminals with elastic arrivals and capacity constraint. Netw Spat Econ 6:25–38

Holguin-Veras J, Jara-Diaz S (2010) Optimal two-part pricing and capacity allocation with multiple user classes and elastic arrivals at constrained transportation facilities. Netw Spat Econ 10(4):427–454

Jang YT (1992) A study on the cost of ship's time in port. Korea Institute of Ocean Science& Technology, BSPE00264-493-7

Kim KH, Kim HB (2002) The optimal sizing of the storage space and handling facilities for import containers. Transp Res Part B 36(9):821–835

Kim KH, Kim KY (2007) Optimal price schedules for storage of inbound containers. Transp Res Part B 41:892–905

Kim KY, Kim KH (2010) Pricing the storage of inbound containers with a discrete probability distribution of retrieval times. Int Eng Manag Sci 9(2):165–177

Kim KH, Park KT (2003) A note on a dynamic space-allocation method for outbound containers. Eur J Oper Res 148:92–101

Lee Y, Hsu N-Y (2007) An optimization model for the container pre-marshalling problem. Comput Oper Res 34:3295–3313

Lee CY, Yu M (2012) Inbound container storage price competition between the container terminal and a remote container yard. Flex Serv Manuf J 24:320–348

Lee LH, Chew EP, Tan KC, Han Y (2006) An optimization model for storage yard management in transshipment hubs. OR Spectrum 28:539–561

Li M (2015) Yard storage planning for minimizing handling time of export containers. Flex Serv Manuf J 27(2):285–299

Lim A, Xu Z (2006) A critical-shaking neighborhood search for the yard allocation problem. Eur J Oper Res 174:1247–1259

Martin E, Kim KH, Saurí S (2015) Optimal space for storage yard considering yard inventory forecasts and terminal performance. Transp Res Part E Logist Transp Rev 82:101–128

Petering MEH (2015) Real-time container storage location assignment at an RTG-based seaport container transshipment terminal: problem description, control system, simulation model, and penalty scheme experimentation. Flexible Service and Manufacturing Journal 27(2):351–381

Rohatgi VK (1976) An introduction to probability theory and mathematical statistics. Wiley, New York, p 84

Saurí S, Serra J, Martín E (2011) Evaluating pricing strategies for storage in import container terminals. Transp Res Rec 2238:1–7

Taleb-Ibrahimi M, Castilho BD, Daganzo CF (1993) Storage space vs handling work in container terminals. Transp Res Part B 12(1):13–32

Watanabe I (2001) Container terminal planning—a theoretical approach. WCN Publishing, Great Britain, p 59

Woo YJ, Kim KH (2011) Estimating the space requirement for outbound container inventories in port container terminals. Int J Prod Econ 133:293–301

Woo, Y.J. and Kim, K.H. (2012) ‘Analysis of storage charging and re-marshaling for outbound containers in terminals’, *Proceedings of the Asia Pacific Industrial Engineering & Management Systems Conference 2012,* V. Kachitvichyanukul, H.T. Luong, and R. Pitakaso Eds., pp.536-541

Yu M, Kim KH, Lee C-Y (2015) Inbound container storage pricing schemes. IIE Trans 47:1–19

Zhang C, Liu J, Wan YW, Murty KG, Linn RJ (2003) Storage space allocation in container terminals. Transp Res Part B 37:883–903

Zhen L (2014) Container yard template planning under uncertain maritime market. Transp Res Part E 69:199–217

Zhen L, Jiang X, Lee LH, Chew EP (2013) A review on yard management in container terminals. Ind Eng Manag Syst 12(4):289–305

Youn Ju Woo studied Industrial Engineering at Pusan National University (Bachelor and Master) and has had field experiences at Pusan Newport Co (container terminal). She is currently doing various researches on container terminals as a Ph.D student at Pusan National University.

Jang-Ho Song is an operations manager of Pusan Newport Co (container terminal). He has studied at Korea Maritime University (Bachelor & Master). He has carried out various activities related to new procedure development, process mining, and efficiency improvement at container terminals.

Kap Hwan Kim is a professor at the Department of Industrial Engineering of Pusan National University. He has studies at Seoul National University (Bachelor) and Korea Advanced Institute of Science and Technology (Master, Ph.D). He was a visiting scholar at Purdue University, Montreal University, and Shanghai Maritime University. His research has been focused on the design and operation problems of container terminals.