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A cost assignment policy for home care patients

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Abstract The patient assignment problem in Home care (HC) consists of allocating each newly admitted patient to his/her reference operator, chosen among a set of possible operators. The continuity of care, where pursued, imposes that the assignment is not changed for a long period. The main goal of the assignment is to balance the workload among the operators. In the literature, the problem is usually solved with numerical approaches based on mathematical programming that do not consider the stochastic aspects of the problem. We derive a structural policy to assign a newly admitted patient while balancing the workload among the operators, by minimizing the expected value of a cost function that penalizes the overtime of operators. The workloads already loaded to the operators are assumed to be random variables as they are in the practice, while the demand related to the new patient is considered both deterministic and stochastic. Results show that the variability of the new patient's demand is negligible with respect to the variability of the already assigned workloads and that similar assignments are obtained both in the presence or in the absence of this demand variability. A numerical comparison with the current practice of assigning the new patient to the operator with the highest expected available capacity shows that better balancings and cost savings can be reached by implementing the proposed policy.

Keywords Home care · Resource management · Patient assignment

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1 Introduction

Home Care (HC) service consists of providing medical, paramedical and social services to patients at their domicile. The main benefit of HC is the reduction of the hospitalization rate, that leads to significant quality of life increase for the assisted patients and to relevant cost savings for the entire health care system (Comondore et al. 2009).

In the US, about 3.3 million patients received HC services from more than 11,400 agencies, and Medicare spent 19 billion dollars on HC services in 2009 (Medicare Payment Advisory Commission MEDPAC 2011). HC is a relevant aspect of the healthcare domain also in Europe, Canada and Australia (Chevreul et al. 2005). This is a growing sector in western countries, due to the ageing of the population, the increase in chronic pathologies, the introduction of innovative technologies and the continuous pressure of governments to contain healthcare costs.

Several types of resources are involved in the service delivery, including different categories of operators (nurses, physicians, physiotherapists, social assistants and psychologists), support staff and material resources. Managing these HC resources is a difficult task that is still executed without appropriate methodologies and tools (Chahed et al. 2006; Asquer et al. 2007). In fact, resource management in HC requires to assign and schedule the human and material resources for delivering the care services to a high number of patients that live in different locations of a geographical area. In addition, the amount of service required by each patient is intrinsically random, thus making the planning problem more complex (DeAngelis 1998; Lanzarone et al. 2010). Finally, the existence of some constraints, such as the continuity of care (Haggerty et al. 2003) and the risk of incurring in burn-out of operators (Cordes and Dougherty 1993; Borsani et al. 2006), makes the HC resource planning peculiar and different from the planning problems encountered in other production and service systems.

The main issues in HC human resource planning are the human resource dimension, the partitioning of a territory into a given number of districts, the assignment of visits to operators and the routing problem (Chahed et al. 2006). In particular, the operator assignment is a critical issue for maintaining a high quality of the provided service (Lanzarone and Matta 2009) when the HC provider wants to preserve the continuity of care, i.e., when the service is delivered by the same actors. Working with continuity of care should preserve a dimension of the service quality perceived by the customer, because the patient receives the care from the same operator within each category (named *reference operator*) for a long period (usually a semester), and thus he/she does not have to continuously change his/her relationships with a new operator (Haggerty et al. 2003).

In the literature, there are several works dealing with the human resource planning in HC services without continuity of care or with a partial continuity of care (Eveborn et al. 2006, 2009; Bertels and Fahle 2006; Thomsen 2006; Akjiratikarl et al. 2007; Chahed et al. 2009; Bennett and Erera 2011). However, there are not many studies dealing with the continuity of care in HC providers, yet. Among them, Borsani et al. (2006) study the scheduling of visits in two HC organizations and propose an assignment model coupled to a scheduling model.

Hertz and Lahrichi (2009) propose two mixed programming models for allocating operators to patients in the Cotes-des-Neiges local community health clinic in Montreal, Canada. Ben Bachouch et al. (2008) develop a mixed integer linear programming model to minimize the total distance traveled by nurses. Lanzarone and Matta (2009) propose a set of linear programming models to balance the workloads of the operators of different categories for different configurations of HC providers.

An important research stream deals with the assignment and scheduling of nurses and medical staff in hospitals and a certain number of papers testify this research activity. Cheang et al. (2003) report a bibliographic survey of the models and methodologies available to solve the nurse rostering problem. Burke et al. (2004) discuss on the strengths and weaknesses of the literature on nurse rostering problem and outline the key issues to be addressed in the research. Bard and Purnomo (2005, 2007) and Purnomo and Bard (2007) propose integer programming models for scheduling the nurses in the presence of a fixed nursing staff, that vary in the objective pursued (i.e., taking into account the preferences of nurses or minimizing the number of uncovered shifts) and in the solution procedure: solved with a branch-and-price algorithm, a column generation approach, or decomposed using Lagrangian relaxation. Belien and Demeulemeester (2008) integrate the nurse and the operating room scheduling processes adopting the column generation method. Punnakitikashem et al. (2008) develop a stochastic integer programming model for the nurse assignment in hospital. The model takes into account the variability of the demand and is solved with a decomposition technique. DeGrano et al. (2009) propose a scheduling that considers both nurse preferences and hospital constraints. Sundaramoorthi et al. (2010) develop two numerical policies to make nurse-to-patient assignments when new patients are admitted during a shift. A recent review on nurse scheduling models is also reported in the chapter of Bard (2010). Finally, Brunner et al. (2009, 2011) solve the flexible scheduling of physicians to shifts in hospital with the goal of minimizing the overtime to be extra paid.

This analysis shows that the operator assignment in the healthcare domain is solved with numerical approaches, as the mathematical programming, while no attempts of formalizing structural policies can be found. The knowledge about the structure of the optimal assignment policies could be helpful mainly for three reasons. HC providers could easily apply simplified policies without requiring expensive software applications, and this aspect can be particularly helpful for small providers that does not have ERP-like platforms and a complex organization. Furthermore, analytical policies can solve the problem with a limited computational effort, allowing to easily include the high variability of the demand. Finally, the structure of the optimal policy could be used to help research the optimum in the heuristic-based algorithms that have to be adopted for large scale problems faced by large HC providers.

This paper deals with the problem of assigning one newly admitted patient to a HC operator (e.g. nurse, physician, physiotherapist) under uncertain workload (related to the amount of care needed by the patients) and respecting the continuity of care. Specifically, the paper proposes a structural policy for solving the assignment problem, choosing the reference operator among a set of possible operators compatible with the newly admitted patient. In general, this set refers to all of the operators belonging to patient's district and, for this reason, compatible in terms of skill and territory; however, in the presence of specific patient's requests (e.g., specific treatments related to the patient's pathology or the gender compatibility between the patient and the operator) the set can exclude some operators in the district.

The reference operator of the new patient is chosen to reduce the expected value of a penalty cost that is function of the amount of the time for visits that each operator supplies in excess of his/her capacity (in accordance with the working contract). Hence, the proposed policy allows planners to assign the new patient with the minimum increase of the expected cost sustained by the HC provider under stochastic patient demand.

In this paper we focus on nurses, who provide the largest amount of visits to the HC patients and manage the emergencies and the fluctuations of the patient demand.

The paper is structured as follows: the detailed problem description and the assumptions introduced are reported in Sect. 2. Then, in Sect. 3, the policy is formally presented. In Sect. 4, the proposed policy is compared to the real practice of HC providers that assign patients based on the expected workloads of their operators. Finally, a real case analysis is reported in Sect. 5 and the conclusions of the work are drawn in Sect. 6.

2 The model

This paper analyzes the case in which a new patient is admitted into the HC service and has to be assigned to his/her reference operator. We consider the assignment of the reference operator independent of the districting of the provider. It means that the provider has already been divided into districts and that the human resources have been already assigned to each district. Moreover, the problem is solved in a configuration where the assignment is also independent of the routing of operators. This is valid for several HC providers, where the territorial extension of the districts is limited and the routes are drawn after the assignments are defined.

Each operator has been already loaded with a certain number of patients that, all together, give an initial stochastic workload X_0, expressed as the amount of time spent for visits in the planning period (variable from a week to a month). To keep simple the notation, we refer to a generic operator that is under consideration for the assignment of a new patient; therefore, we omit the index denoting the specific operator. The initial workload X_0 is assumed distributed with a triangular probability density function $\Phi_0(x)$ characterized by parameters a, b and c; parameter a is the minimum workload, b the modal workload and c the maximum workload.

We consider a cost function for each operator that depends on the amount of time for visits that he/she supplies in surplus to his/her capacity v in the planning period (in accordance with the working contract). The analysis considers differential costs, because only the visits above the contract threshold v are significant for the assignment problem. The fixed costs for the HC provider are not considered (i.e. costs associated with visits below v), because they are always sustained by the provider independent of the assignments of patients to their reference operator. An implicit assumption is that nurses are employees of the HC provider.

The cost related to an operator is assumed to be a quadratic function of time for visits above the capacity. Hence, an operator who works two hours of overtime is more costly than two operators who work one hour of overtime. Therefore, the expected cost C related to an operator under a stochastic workload $X \sim \Phi(x)$ is defined as:

$$C = \int_{v}^{\infty} (x - v)^2 \Phi(x) \mathrm{d}x \qquad (1)$$

Given a set of n compatible operators, the assignment of the reference operator to the new patient is chosen to minimize the increase of the expected cost. This leads to balance, in some way, the workload among the operators. Workload balancing among operators is a goal pursued in practice by most of HC providers.

We now introduce a set of further assumptions that focus the analysis on realistic cases and allow us to simplify the analytical derivation of the optimal policy. These assumptions are consistent with the real situation of several HC providers (Chahed et al. 2006).

The operator capacity v is assumed to be in the second part of the triangular density function Φ_0 (i.e., $b < v < c$) (see Fig. 1). The other possibilities for v (i.e., $v < b$ and $v > c$) are not of interest for a practical application of the assignment policy. Indeed, the case $v < b$ is the case in which the operator is highly overloaded; for this reason he/she should not be considered for the assignment of another patient. The case $v > c$ refers to a highly underloaded operator that is not frequent in real organizations.

The initial workload X_0 of each operator is assumed to be stationary in the planning period. This assumption might not be valid for some types of patients that

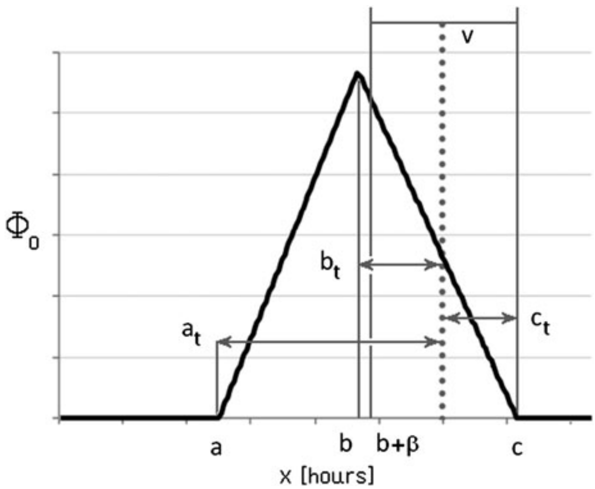


Fig. 1 Triangular probability density function $\Phi_0(x)$ with the related parameters. Parameter v is included between $b + \beta$ and c

may change their conditions during the care pathway. However, for chronic diseases and some other pathologies that are relevant in the HC, this assumption may hold.

2.1 Initial cost

The initial expected cost C_0 of an operator, according to Eq. (1) and the triangular density function $\Phi_0(x)$, is function of the current workload (i.e., a, b and c) and capacity v:

$$C_0 = \frac{(c - v)^4}{6(c - a)(c - b)} \tag{2}$$

The above expression is a convex function decreasing in v. When v is equal to the maximum workload value c, the expected cost is 0. This is simply because the probability of incurring in an extra time for visits is null.

2.2 Assignment cost

The new patient to assign has a stochastic service request Y (time demanded for visits in the planning period) with a probability density function $\Psi(y)$ that strictly depends on his/her characteristics at the beginning of the care pathway.

Two cases are considered in this paper. In the case the variability of the new patient's demand is neglected, a deterministic value μ is assumed for Y (with $\mu \geq 0$). In the case the variability of the new patient's demand is included in the model, it is difficult to go beyond the estimation of a minimum value α and a maximum value β of the time requested for visits in the planning period. In the absence of detailed information, the most convenient choice is to assume that the density Ψ is uniformly distributed within the interval $[\alpha, \beta]$.

The demand Y is assumed to be stationary in the planning period. Moreover, the maximum workload related to a single patient is always inferior to the smallest workload already assigned to an operator, i.e., $\beta < a$. This assumption is motivated because a patient usually requires at maximum one or two visits per day, while an operator provides a larger number of visits on each day.

Finally, the assumption regarding v of each operator is modified into $b + \beta < v < c$ to derive the assignment cost (see Fig. 1).

When the newly admitted patient is assigned to the reference operator, the initial workload of this operator is increased with the demand of the new patient. In the case of a deterministic patient demand, the new workload distribution $\tilde{\Phi}_1$ of the assigned operator is simply shifted of μ with respect to Φ_0. Therefore, the new expected cost (assignment cost) $\tilde{C}_1$ is:

$$\tilde{C}_1 = \int_v^{\infty} (x - v)^2 \tilde{\Phi}_1(x) \mathrm{d}x = \frac{(c + \mu - v)^4}{6(c - a)(c - b)} \tag{3}$$

Alternatively, $\tilde{C}_1$ can be directly obtained from Eq. (2) by adding μ to the parameters of the triangular distribution, or by subtracting μ from v.

With a uniform patient demand distribution Ψ, the new workload distribution Φ_1 is derived with the convolution product between the initial workload distribution Φ_0 and the uniform distribution Ψ:

$$\begin{aligned}\Phi_1(x) &= \int_{-\infty}^{\infty} \Phi_0(x-n)\Psi(n)\mathrm{d}n \\ &= 2\left[\frac{g(x-a)}{(c-a)(b-a)} - \frac{g(x-b)}{(b-a)(c-b)} + \frac{g(x-c)}{(c-a)(c-b)}\right]\end{aligned} \tag{4}$$

where

$$g(x) = \int_{-\infty}^{x} \left[\int_{-\infty}^{\tau} \Psi(\xi)\mathrm{d}\xi\right] \mathrm{d}\tau \tag{5}$$

Using a triangular density function for Φ_0, Φ_1 is equal to:

$$\Phi_1(x) = \int_{x-\beta}^{x-\alpha} \frac{1}{\beta-\alpha}\Phi_0(n)\mathrm{d}n \tag{6}$$

The resulting expression (limiting to the part $x > b + \beta$ that is used in Eq. (1) for calculating the cost) is:

$$\Phi_1(x) = \begin{cases} -\frac{2x-\alpha-\beta-2c}{(c-a)(c-b)} & b+\beta < x \leq c+\alpha \\ \frac{(x-\beta-c)^2}{(\beta-\alpha)(c-a)(c-b)} & c+\alpha < x \leq c+\beta \\ 0 & x > c+\beta \end{cases} \tag{7}$$

Then, the new expected cost C_1 is obtained considering the distribution Φ_1:

$$C_1 = \int_{v_i}^{\infty} (x_i - v_i)^2 \Phi_1(x)\mathrm{d}x \tag{8}$$

The analytical expression of C_1, whose form is not simple, is reported in Appendix 1. In detail, the analytical expression of the difference between $\tilde{C}_1$ and C_1, leading to the expression of C_1, is derived. This difference measures the accuracy of the approximation derived from neglecting higher moments in patient demand Y.

2.3 Objective function

The goal of the assignment is to allocate the newly admitted patient to only one reference operator, among the n compatible ones, characterized by the lowest increment of the expected cost (i.e., the difference between the assignment cost in the case of patient assignment and the initial cost).

In the case of deterministic patient demand, the objective is to minimize the increment $\tilde{\delta}$:

$$\tilde{\delta} = \tilde{C}_1 - C_0 \tag{9}$$

where the expression of $\tilde{\delta}$ has a simple analytical form:

$$\tilde{\delta} = \frac{(c+\mu-v)^4-(c-v)^4}{6(c-a)(c-b)} \tag{10}$$

In the case of uniform patient distribution Ψ, the objective is to minimize the increment δ:

$$\delta = C_1 - C_0 \tag{11}$$

which has a more complex analytical expression (see Appendix 1).

In Sect. 2.4, the comparison between the two cases and the appropriateness of considering the minimization of $\tilde{\delta}$ instead of δ (with a simpler analytical form) are discussed.

2.4 Comparison between deterministic and stochastic patient demand

The two objective functions introduced in Sect. 2.3 are compared with a geometrical approach. For this purpose, we refer to the time for visits that can be loaded to the operator below the capacity v (i.e., the *assignable workload*) or the extra time for visits exceeding v (i.e., the *excess workload*). Therefore, the parameters of the distribution Φ_0 are expressed in relative terms with respect to the operator capacity v:

$$\begin{cases} a_t = a - v \\ b_t = b - v \\ c_t = c - v \end{cases} \tag{12}$$

c_t can be interpreted as the maximum workload in excess related to the analyzed operator and the absolute value of a_t as the maximum assignable workload (Fig. 1). The domain of these parameters is determined according to the relationships imposed among the variables:

$$\begin{cases} a_t \leq b_t \leq 0 \\ c_t \geq 0 \end{cases} \tag{13}$$

Let us now define the parameter r_b, related to the asymmetry of the distribution Φ_0, as the ratio:

$$r_b = \frac{b-a}{c-a} = \frac{b_t - a_t}{c_t - a_t} \tag{14}$$

with $0 < r_b < 1$.

The analytical expressions of $C_0, \tilde{C}_1$ and $\tilde{\delta}$ are modified as follows:

$$C_0 = \frac{c_t^4}{6(1-r_b)(c_t-a_t)^2} \tag{15}$$

$$\tilde{C}_1 = \frac{(c_t+\mu)^4}{6(1-r_b)(c_t-a_t)^2} \tag{16}$$

$$\tilde{\delta} = \frac{(c_t + \mu)^4 - c_t^4}{6(1 - r_b)(c_t - a_t)^2} \tag{17}$$

and the constraints of Eq. (13) are modified into:

$$\begin{cases} c_t \geq 0 \\ a_t \leq -\frac{r_b}{1-r_b} c_t \end{cases} \tag{18}$$

The isolevel curves of C_0, δ and $\tilde{\delta}$ can be reported in the plane c_t, a_t. Each point in the admissible zone of the plane c_t, a_t (included in the IV quadrant according to Eq. (18)) represents a possible operator, and the contour line passing through the point his/her associated expected initial cost or cost increase. Figure 2 reports the contour plots of δ (in two cases, with different variability of the uniform patient demand Ψ) and $\tilde{\delta}$, while the contour plot of C_0 is reported in Fig. 3.

Two operators can be considered in the same contour plot if their workload density functions Φ_0 have the same value of r_b (same asymmetry of the distribution). As regards the nurses, basing on several real cases, this condition is often verified among the n nurses compatible with a new patient to assign. This probably holds because the assignment splits the different types of patients among the operators, with consequent similar mixes, and the mode of the distribution is in the same intermediate position between the minimum value and the maximum value.

Points in the upper left region represent operators with a low variability of the workload (low difference $c_t - a_t$), while points in the lower right region represent

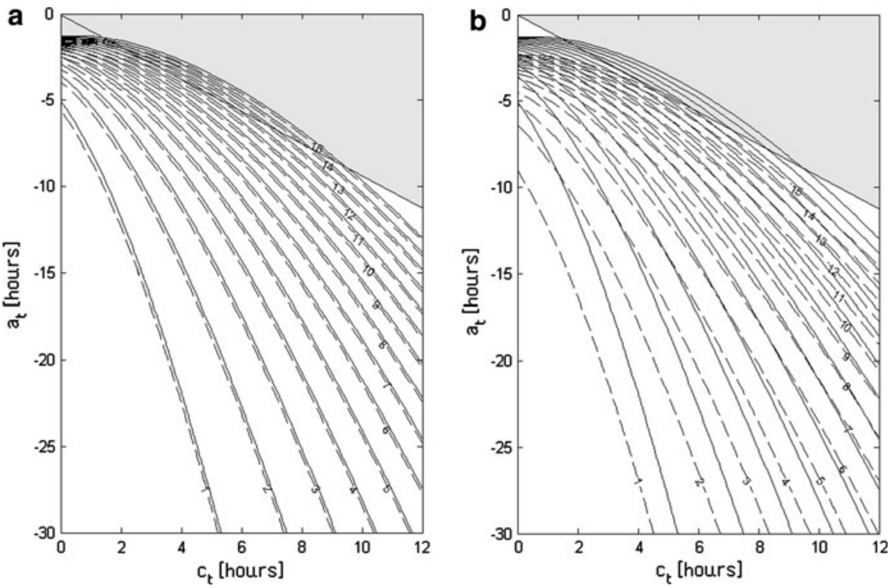


Fig. 2 Contour plot of δ (*dotted lines*) and $\tilde{\delta}$ (*continuous lines*) with $r_b = 0.5$ and $\mu = 3$: **a** case with medium patient variability ($\alpha = 2$ and $\beta = 4$), and **b** case with high patient variability ($\alpha = 0$ and $\beta = 6$). The non admissible region, given by Eq. (18), is *grey* colored

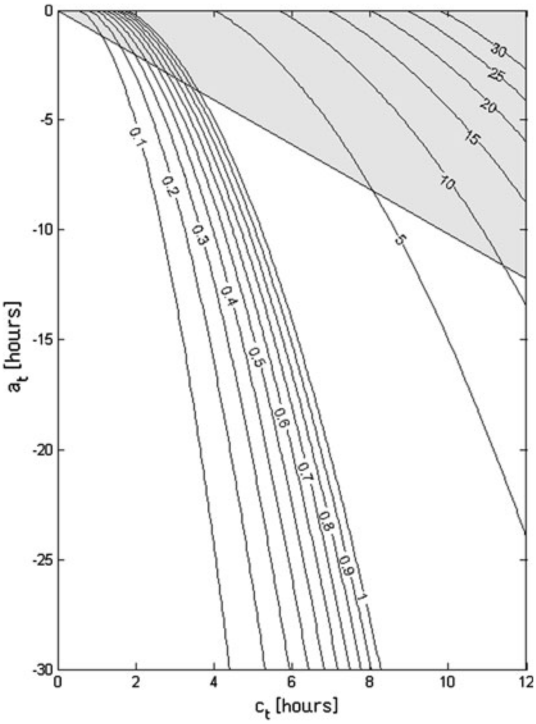


Fig. 3 Contour plot of C_0 (with $r_b = 0.5$). The non admissible region is *grey* colored

operators with a high variability of the workload (high difference $c_t - a_t$). Moreover, the more a point is close to the line $a_t = -\frac{r_b}{1-r_b} c_t$ that limits the admissible region, the more the operator has an expected workload close to his/her capacity.

The expected cost increases δ and $\tilde{\delta}$ are compared in terms of the direction of the gradient of their isolevel curves in the plane c_t, a_t, where the direction D_k of a curve k is defined as the ratio between the derivative of k with respect to a_t and the derivative of k with respect to c_t. D_k represents the tangent of the angle between the gradient vector of k and the axis c_t.

It is possible to notice a good superposition between δ and $\tilde{\delta}$ for a low patient demand variability, with a difference that limitedly grows when the variability increases (Fig. 2). In detail, directions $D_{\tilde{\delta}}$ and D_{δ} are similar in each point, even in the presence of a large patient demand variability. As a consequence, in the majority of cases, the operator with the lowest cost increase remains the same including or neglecting the variability of the new patient's demand.

The motivation for this similarity is that the patient demand variability is negligible with respect to the variability already assigned to the operators. When an operator takes care of a certain number of patients, the variability of the already assigned workload is prevailing on the variability of the new patient in the determination of the cost increase. Therefore, it can be reasonable to consider $\tilde{\delta}$ rather than δ.

Also considering other shapes for the probability density function Ψ, the variability of the new patient's demand is negligible with respect to the variability already assigned and the assumption of considering $\tilde{\delta}$ remains valid.

After the assignment of a patient to an operator, the point relative to the operator moves in the contour plot as a consequence of the workload increase: both the mean value and the standard deviation of the operator workload increase, while the skewness does not significantly change because a uniform distribution for the new patient's demand is assumed (we can therefore suppose that r_b remains the same). Following the geometrical approach, the operator point movement in the plane c_t, a_t after the assignment is derived in Appendix 2.

3 Assignment policy

The policy minimizes the expected cost increase related to the patient-operator assignment. The cost increase $\tilde{\delta}$ is considered in the analytical derivation of the policy, as discussed in Sect. 2.4.

The following Lemmas will be useful for deriving the policy.

Lemma 1 *The direction D_{C_0} is always positive $(D_{C_0} > 0)$ and C_0 increases with* c_t and a_t.

Proof The direction D_{C_0} is calculated as the ratio between the derivative of C_0 with respect to a_t and the derivative of C_0 with respect to c_t:

$$D_{C_0} = \left[\frac{2(c_t - a_t)}{c_t} - 1\right]^{-1} \tag{19}$$

This expression is greater than zero if $a_t < \frac{c_t}{2}$, that is always valid in the admissible region because $a_t < 0$ and $c_t > 0$.

Moreover, the derivative of C_0 with respect to a_t is equal to:

$$\frac{\partial C_0}{\partial a_t} = \frac{c_t^4}{3(1 - r_b)(c_t - a_t)^3} \tag{20}$$

that is always positive in the admissible region; therefore, C_0 increases with a_t.

Also the derivative of C_0 with respect to c_t is always positive, because of $D_{C_0} > 0$ and $\frac{\partial C_0}{\partial a_t} > 0$. Therefore, C_0 increases with c_t. $\square$

Lemma 2 *For any assignment patient-operator characterized by an initial operator workload distribution Φ_0 and a patient demand μ, the directions $D_{\tilde{\delta}}$ and D_{C_0} are related in the following way: $D_{\tilde{\delta}} > D_{C_0}$.*

Proof The direction $D_{\tilde{\delta}}$ is calculated as the ratio between the derivative of $\tilde{\delta}$ with respect to a_t and the derivative of $\tilde{\delta}$ with respect to c_t:

$$D_{\tilde{\delta}} = \left[\frac{2(c_t - a_t)\left(3c_t^2 + 3\mu c_t + \mu^2\right)}{4c_t^3 + 6\mu c_t^2 + 4\mu^2 c_t + \mu^3} - 1\right]^{-1} \tag{21}$$

Considering Eqs. (19) and (21), the condition $D_{\tilde{\delta}} > D_{C_0}$ is expressed as follows:

$$\left[\frac{2(c_t - a_t)\left(3c_t^2 + 3\mu c_t + \mu^2\right)}{4c_t^3 + 6\mu c_t^2 + 4\mu^2 c_t + \mu^3} - 1\right]^{-1} > \left[\frac{2(c_t - a_t)}{c_t} - 1\right]^{-1} \tag{22}$$

After some manipulations it results in:

$$\frac{3c_t^2 + 3\mu c_t + \mu^2}{4c_t^3 + 6\mu c_t^2 + 4\mu^2 c_t + \mu^3} < \frac{1}{c_t} \tag{23}$$

and then in $(c_t + \mu)^3 > 0$. Therefore, inequality (22) is satisfied for all of the admissible c_t and μ (both positive values). □

Lemma 3 *The direction $D_{\tilde{\delta}}$ is always positive $(D_{\tilde{\delta}} > 0)$ and $\tilde{\delta}$ increases with c_t and a_t.*

Proof Inequalities $D_{\tilde{\delta}} > D_{C_0}$ (Lemma 2) and $D_{C_0} > 0$ (Lemma 1) result in $D_{\tilde{\delta}} > 0$. Moreover, the derivative of $\tilde{\delta}$ with respect to a_t is equal to:

$$\frac{\partial \tilde{\delta}}{\partial a_t} = \frac{(c_t + \mu)^4 - c_t^4}{3(1 - r_b)(c_t - a_t)^3} \tag{24}$$

that is always positive in the admissible region; therefore, $\tilde{\delta}$ increases with a_t.

Also the derivative of $\tilde{\delta}$ with respect to c_t is always positive, because of $D_{\tilde{\delta}} > 0$ and $\frac{\partial \tilde{\delta}}{\partial a_t} > 0$. Therefore, $\tilde{\delta}$ increases with c_t. □

Assuming valid to consider $\tilde{\delta}$ rather than δ for the expected cost increment (as discussed in Sect. 4), the policy derived in the following is optimal. In the derivation, the particular case of operators with the same initial cost is firstly considered and, then, the theorem for the general assignment policy is formulated.

Theorem 1 *In the presence of operators with the same C_0 and r_b, the optimal choice is to assign the patient to the operator with the highest assignable workload (lowest a_t) independent of the new patient's demand.*

Proof Lemmas 1, 2 and 3 affirm that $D_{\tilde{\delta}} > D_{C_0} > 0$. Therefore, considering different points on the same C_0 isolevel line (meaning operators with the same C_0 and r_b), the point with the highest a_t value intersects the isolevel line of $\tilde{\delta}$ with the highest value and vice versa (Fig. 4).

Hence, given different operators with the same initial cost C_0, the lowest cost increase is obtained assigning the patient to the operator with the lowest a_t value (highest assignable workload). □

In the presence of different initial costs, the general theorem of the assignment policy is now stated.

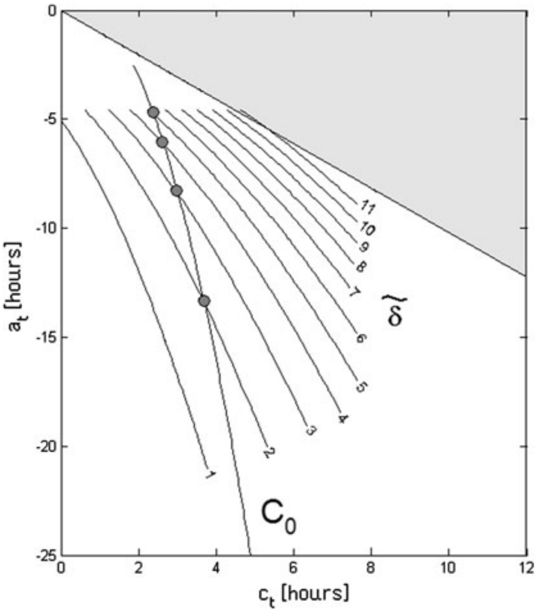


Fig. 4 Intersections of different operators points having the same C_0 value with the $\tilde{\delta}$ isolevel curves

Theorem 2 *In the presence of two operators i and j with the same r_b, if operator i has the lowest C_0 and the maximum assignable workload (lowest a_t), the optimal choice is to assign the new patient to operator i, independent of the new patient's demand. Otherwise, the optimal choice also depends on the new patient's demand and the optimal choice is to assign the new patient to the operator with the minimum expected cost increase.*

Proof The value of $\tilde{\delta}$ increases moving from the lower left part to the upper right part in the admissible zone of the plane c_t, a_t (Lemma 3). Consequently, considering the $\tilde{\delta}$ curve passing through the point of operator i, the region upper this line is characterized by higher cost increases (assuming operators with the same r_b value) and operator i is the best assignment with respect to all of the operators j whose point is in the region upper the $\tilde{\delta}$ curve through the point i.

Based on Lemma 2, the isolevel curve of C_0 passing through the point of operator i is on the right with respect to the isolevel curve of $\tilde{\delta}$ passing through the same point for a_t values greater than the value of operator i. On the contrary, the isolevel curve of C_0 is on the left with respect to the isolevel curve of $\tilde{\delta}$ for a_t values lower than the value of operator i (see Fig. 4).

Therefore, if operator j has higher C_0 and a_t values than operator i (with the same r_b), he/she consequently has a higher $\tilde{\delta}$ value and the optimal choice is to assign the patient to operator i. On the contrary, if operator j has lower C_0 and a_t values than operator i, he/she consequently has a lower $\tilde{\delta}$ value and the optimal choice is to assign the patient to operator j (Fig. 5).

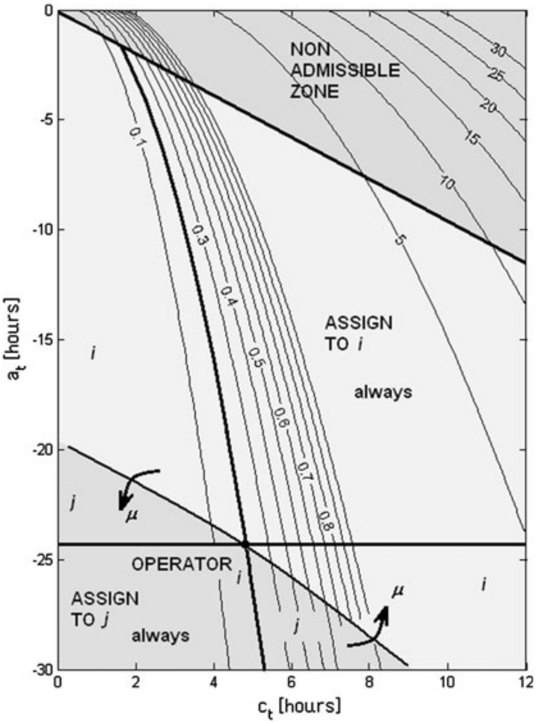


Fig. 5 Graphical representation of the assignment policy in the C_0 contour plot (with $r_b = 0.5$)

If operator j is in the remaining regions of the plane, the best assignment depends on the new patient's demand (i.e., μ) because the $\tilde{\delta}$ curve passing through the point of operator i is included in these regions, with a direction $D_{\tilde{\delta}}$ that depends on μ. Indeed, $D_{\tilde{\delta}}$ increases with μ where $\frac{\partial D_{\tilde{\delta}}}{\partial \mu} > 0$, with:

$$\frac{\partial D_{\tilde{\delta}}}{\partial \mu} = \frac{(c_t - a_t)\left(\mu^4 + 6\mu^3 c_t + 15\mu^2 c_t^2 + 16\mu c_t^3 + 6c_t^4\right)}{\left(4c_t^3 + 6\mu c_t^2 + 4\mu^2 c_t + \mu^3\right)^2 \left[\frac{2(c_t - a_t)(3c_t^2 + 3\mu c_t + \mu^2)}{4c_t^3 + 6\mu c_t^2 + 4\mu^2 c_t + \mu^3} - 1\right]^2} \tag{25}$$

Hence, because both μ and c_t are positive and a_t is negative, the condition $\frac{\partial D_{\tilde{\delta}}}{\partial \mu} > 0$ is always verified and $D_{\tilde{\delta}}$ always increases with μ. Therefore, if operator j has a lower a_t value and a higher C_0 than operator i, the optimal choice is to assign the patient to operator i for low μ values and to operator j for high μ values. On the contrary, if operator j has a higher a_t value and a lower C_0 than operator i, the optimal choice is to assign the patient to operator j for low μ values and to operator i for high μ values. □

More complex demonstrations can be developed for δ instead of $\tilde{\delta}$. We numerically verified that the same policy can be derived considering δ, as expected.

4 Comparison with the expected workload approach

In the current practice of the HC providers, the variability of patient demands, and consequently of future operator workloads, is not considered. The only information used for assigning the patient to his/her reference operator is the operator expected workload, often estimated as the sum of the standard times for visits of his/her patients in the charge (taken from existing clinical protocols). Consequently, the patient is assigned to the operator with the smallest utilization value scaled to his/her capacity.

To model this practice with the approach proposed in this paper, only the expected value is extracted from each triangular distribution Φ_0. Thus, the new patient is assigned to the operator with the highest *expected assignable workload* W that is given by the difference between the operator capacity v and his/her expected workload:

$$W = v - \frac{a+b+c}{3} = -\frac{a_t + b_t + c_t}{3} \tag{26}$$

Parameter r_b is introduced also in this case:

$$W = -\frac{a_t(2-r_b) + c_t(1+r_b)}{3} \tag{27}$$

An analogue contour plot is drawn for W, to compare the assignment policy with the expected workload approach (Fig. 6). When W assumes a negative value, it has to be interpreted as an expected workload in excess. Therefore, this region is not considered for the assignment of new patients, because an operator with an expected workload in excess than his/her capacity should not receive other patients. If $r_b = 0.5$, this region is equal to the region excluded from the hypotheses (Eq. (18)).

Similar directions $D_{\tilde{\delta}}$ and D_W in a region of the plane lead to similar assignments under both the assignment policy and the expected workload approach, while on the contrary, different directions indicate a solution of the assignment problem that depends on the approach.

The direction D_W is constant all over the admissible region and it is equal to:

$$D_W = \frac{2-r_b}{1+r_b} \tag{28}$$

D_W varies from 0.5 if $r_b = 1$ to 2 if $r_b = 0$, with a value equal to 1 in the case of a symmetric function Φ_0 with $r_b = 0.5$.

Because the assignment is for the highest W (while for the lowest δ in the case of the policy), the gradients to compare have opposite versa; however, the similarity between directions is not influenced by the different versa and the approaches can be compared by analyzing the direction.

Figure 7 reports the contour plot of $D_{\tilde{\delta}} - D_W$; very similar plots are obtained for $D_\delta - D_W$. In the admissible region, the differences $D_{\tilde{\delta}} - D_W$ and $D_\delta - D_W$ are almost everywhere negative and the directions are highly different. Therefore, different solutions of the assignment problem can derive from using W instead of the proposed policy. Similar directions are obtained in a limited region, where the

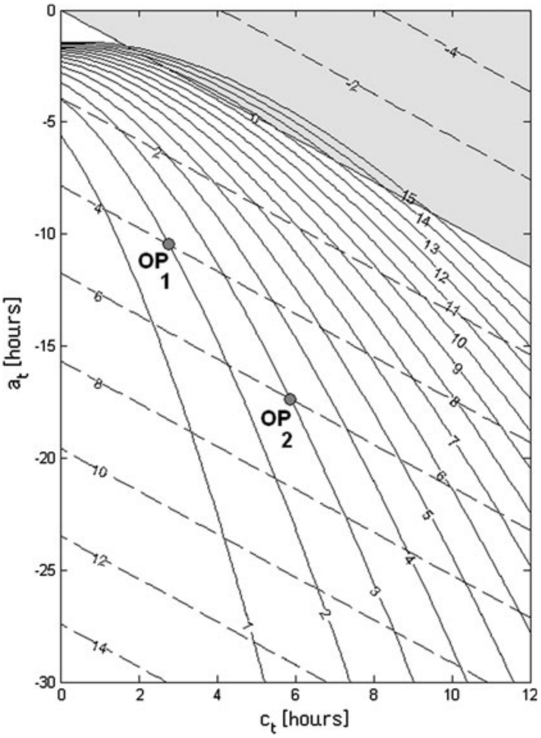


Fig. 6 Contour plot of δ (*continuous lines*) and W (*dotted lines*) with points relative to two operators (operator 1 with $a_t = -10.5$ and $c_t = 3.5$; operator 2 with $a_t = -17$ and $c_t = 6$). Parameters are the same as in Fig. 2a

differences are low (upper part of the admissible region, characterized by a quite null W and a low variability of the operator workload). This means that the expected workload approach is useful only in this zone, while in the other regions there is a potential added value in considering the service demand variability with its cost for the entire structure. Considering the real case analyzed in Sect. 5, the majority of operator points are in the lower region of the plane c_t, a_t, where the difference is high (Fig. 8).

As an example, comparing the two operators of Fig. 6, the approaches lead to opposite decisions: operator 2 is chosen with the expected workload approach, while operator 1 with the minimum cost increase policy. The corresponding triangular density functions of the two operators are shown in Fig. 9. Operator 1 has a lower W than operator 2 and the expected workload approach consequently assigns the new patient to operator 2. However, the part of the workload distribution Φ_0 greater than v suggests that operator 2 has a higher probability to exceed the capacity. This excess is translated into a higher cost for the structure, according to Eq. (1), and the minimum cost increase policy assigns the new patient to operator 1.

In detail, the assignment provided by the expected workload approach is different from the assignment provided by the minimum cost increase policy mainly if the assignments provided by the policy also depends on the new patient's demand (see Theorem 3), as shown in the following Corollary.

Fig. 7 Contour plot of $D_{\tilde{\delta}} - D_W$ with $\mu = 3$ and $r_b = 0.5$. The non admissible zone is *grey* colored

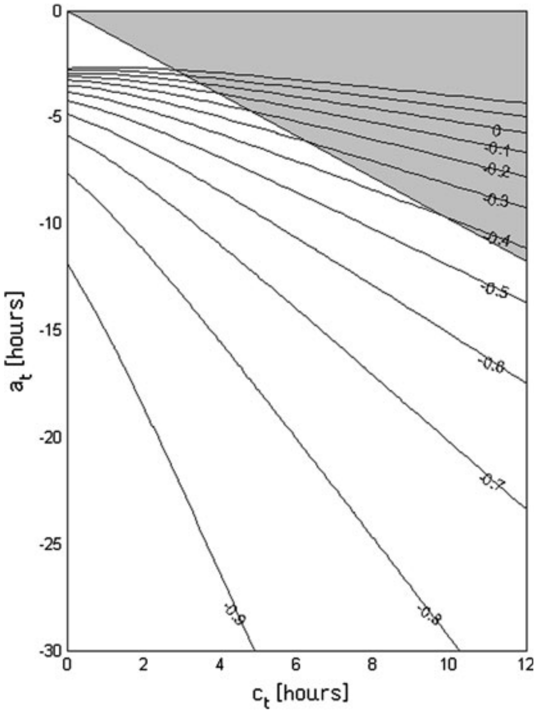
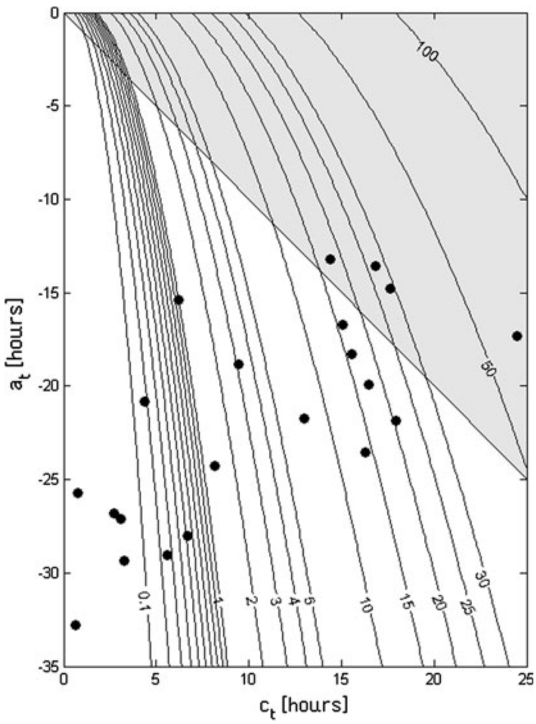


Fig. 8 Points relative to the nurses of the considered districts of the analyzed HC provider, reported in the contour plot of C_0 (the non admissible region is *grey* colored)



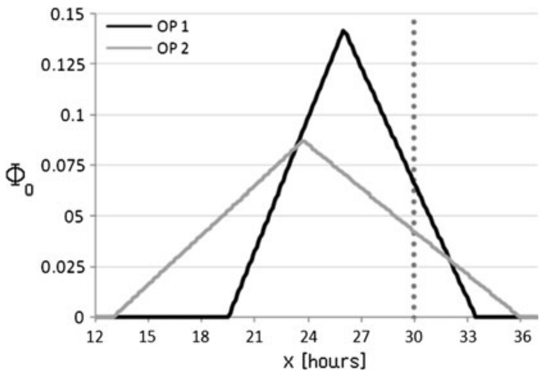


Fig. 9 Triangular probability density functions $\Phi_0(x)$ for the operators of Fig. 6

Corollary 1 *Considering two operators whose workloads are fitted with triangular density functions* Φ_0 *with the same* $r_b \geq \frac{1}{3}$, *a different assignment between the minimum cost increase policy and the expected workload approach is possible only if the assignment of the policy is influenced by the new patient's demand. If* $r_b < \frac{1}{3}$, a *different assignment between the minimum cost increase policy and the expected workload approach is also possible if the assignment of the policy is independent of the new patient's demand in the region where* $a_t < -\frac{c_t}{2}$.

Proof The direction D_W is always positive ($0.5 \leq D_W \leq 2$) and W grows moving to the lower left zone of the plane c_t, a_t.

Considering the point of an operator i, if $D_{C_0} < 0.5$ in the point, the W line through the point i is always included in the region where the best assignment of the policy also depends on the new patient's demand. This condition is verified for:

$$a_t < -\frac{c_t}{2} \tag{29}$$

When verified, the region where the policy assigns the new patient to operator i independently from the new patient's demand always refers to a higher W for operator i than for operator j and also the W approach assigns the new patient to operator i, and vice versa. Hence, opposite choices only occur where the best assignment of the policy also depends on the new patient's demand (see Fig. 5).

Considering the admissible region, given by Eq. (18), the condition of Eq. (29) is always verified if $r_b \geq \frac{1}{3}$. If $r_b < \frac{1}{3}$, Eq. (29) is not verified close to the boundary of the admissible region, where $\frac{r_b}{r_{b-1}} c_t < a_t < -\frac{c_t}{2}$ (operators with an expected workload close to the capacity). □

5 Real case analysis

5.1 Provider

The real case analysis is conducted considering the nurses of one of the largest Italian public HC providers. This provider operates in the north of Italy, covering a

Table 1 Districts of nurses in the considered division of the analyzed HC provider

District	Skill	Territory	Number of nurses
1	Non palliative	A	8
2	Palliative	A	3
3	Non palliative	B	4
4	Palliative	B	1
5	Non palliative	C	5
6	Palliative	C	1

region of about 800 km^2, with about 1,000 patients assisted at the same time and 50 nurses. It is divided into three divisions and the analysis refers to the largest one. In this division, six independent districts of nurses are present, divided by skill and territorial distribution. Table 1 reports the main data for the districts of the analyzed division; the districts with only one nurse are excluded from the analysis.

The capability of the proposed policy to balance the workload among the nurses and to reduce the variable costs is evaluated over a period of 26 weeks (data from April to September 2008).

5.2 Experimental set-up

An initial assignment of the reference nurse is carried out for the initial week (named week 0) for all of the patients in the charge, while the other assignments are carried out rolling at each week: at the beginning of each week, the new patients admitted in the service in the previous week are assigned.

The weekly arrivals of new patients are taken from the historical data of the HC provider, in terms of numerousness, characteristics and district, while the patient demands (demand expressed in terms of weekly hours for visits) are estimated with the patient stochastic model built with the procedure of Lanzarone et al. (2010).

The minimum cost increase policy is applied and evaluated in three different experiments:

1. Using triangular distributions Φ_0 for X and uniform distributions Ψ for Y, both computed to fit the empirical distributions from the patient stochastic model relative to the next week. Hence, the planning horizon is equal to one week.
2. Using triangular distributions Φ_0 for X and deterministic values μ for Y, both computed to fit the empirical distributions from the patient stochastic model relative to the next week, with a planning horizon equal to one week.
3. Using the empirical distributions for both X and Y, directly obtained from the patient stochastic model. In this case, the non stationarity of the patient demand is considered, using a planning period of eight weeks (with a different distribution for each patient demand at each week) and considering the cost function as the sum of the costs estimated at each one of the eight weeks.

The expected workload approach W is also evaluated, with a planning horizon equal to one week, using the expected values of the empirical distributions from the patient stochastic model for both the operators' workloads X and the new patient's demand Y.

In all of the cases, the initialization at week 0 is carried out using the minimum cost increase and the empirical distributions; then, in the other weeks, the different approaches are adopted.

The goal is to evaluate the impact of an inappropriate modeling of the initial workloads and the new patient's demand (Φ_0 and Ψ, respectively).

By the data collected from the analyzed division, r_b has a mean value of 0.467 and a standard deviation of 0.070 (estimations are made on the basis of a sample of 22 operators). Hence, each density function Φ_0 is estimated based on the expected value and the variance of the empirical distribution, assuming $r_b = 0.467$. Parameter a is imposed to be higher than 0; in the case $a < 0$, $a = 0$ is set, maintaining the same expected value and r_b and underestimating the variance.

For each uniform patient distribution Ψ, α and β are estimated based on the expected value and the variance of the empirical distribution. In the case $\alpha < 0$, $\alpha = 0$ is set, maintaining the expected value and underestimating the variance.

Because more than one patient has to be assigned at each week, a simple rule coherent with the assignment approach is adopted to choose the assignment order. For the cost increase approaches, the new patients are ranked by their expected cost increase (after having identified the best nurse for each patient). For the expected workload approach, the new patients are ranked by their expected demand. Then, the new patient with the highest cost increase or expected demand is the first assigned patient, and this ranking process is repeated for the other new patients, and so forth.

If an operator is underloaded with respect to the hypotheses (i.e., $c < v$), we consider a null cost increase so that he/she is the best assignment. On the contrary, if an operator is overloaded with respect to the hypotheses (i.e., $b + \beta > v$), he/she is the worst assignment. In the presence of two or more operators in one of these conditions, they are ranked starting from the patient with the highest difference $v - c$ (for underloaded operators) or the lowest difference $b - v$ (for overloaded operators).

The new assignments and the planned workload of each operator at each week are obtained in each experiment. The assignments are then executed in a set of 10 sample paths, generated with a mix between a Monte Carlo approach from the considered demand distributions and the real execution. The Monte Carlo approach is used for the majority of patients, while the demands of long stay non palliative patients (with a very low variability along with the time that does not represent an uncertainty source) are taken from their real execution. Finally, the assignments are evaluated with the performance indicators described in Sect. 5.3 These indicators allow to validate the hypotheses introduced and to compare the proposed policy with the expected workload approach.

5.3 Performance indicators

The mean utilization $\bar{u}$ along with the weeks is assumed to be the workload level indicator for each nurse. This indicator is calculated as the ratio between the weekly

Table 2 Results of the first cost increase approach (triangular distributions Φ_0 for X and uniform distributions Ψ for Y)

	District 1	District 2	District 3	District 5
$\bar{u}$				
Sample paths average	0.7997	0.6440	0.9635	0.9204
Z				
Planned	0.0589	0.0345	0.0460	0.0620
Sample path 1	0.1754	0.4337	0.3344	0.1803
Sample path 2	0.3934	0.2603	0.3127	0.2179
Sample path 3	0.2138	0.4677	0.3106	0.1814
Sample path 4	0.2244	0.4027	0.2654	0.1635
Sample path 5	0.3602	0.4402	0.3676	0.2011
Sample path 6	0.3334	0.3200	0.3168	0.1598
Sample path 7	0.3537	0.3175	0.3239	0.2477
Sample path 8	0.3228	0.4140	0.1927	0.1039
Sample path 9	0.2326	0.1593	0.4096	0.2929
Sample path 10	0.3446	0.5787	0.3220	0.3049
Sample paths average	0.2954	0.3794	0.3156	0.2053
Cost				
Planned	2.16*	0.90*	35.53*	12.56*
Sample path 1	5.66	4.71	42.07	15.17
Sample path 2	7.65	0.36	26.18	28.78
Sample path 3	6.61	13.05	44.06	9.47
Sample path 4	3.16	7.54	28.95	33.17
Sample path 5	9.11	5.24	26.13	34.68
Sample path 6	6.47	8.49	26.70	17.81
Sample path 7	8.68	6.73	40.89	29.54
Sample path 8	12.34	8.13	23.05	11.02
Sample path 9	11.45	5.77	49.43	7.76
Sample path 10	6.67	1.94	34.61	36.82
Sample paths average	7.78	6.20	34.21	22.42

* Planned costs for overloaded operators (i.e., $b > v$) are computed based on Eq. (15) after imposing $v = b$

time for visits spent by the nurse and his/her capacity v for each simulated week, averaged from week 1 through week 25.

Within each district, the range of $\bar{u}$ among the nurses (denoted with Z) is computed as the indicator of the workload balancing performance of the assignment in the district: the more a strict range is obtained, the more a higher workload balancing is performed.

Finally, the mean variable cost is computed for each district, as the average of costs among the nurses belonging to the district, averaged from week 1 through week 25.

Table 3 Results of the second cost increase approach (triangular distributions Φ_0 for X and deterministic values μ for Y)

	District 1	District 2	District 3	District 5
$\bar{u}$				
sample paths average	0.8010	0.6440	0.9636	0.9189
Z				
Planned	0.0669	0.0345	0.0419	0.0551
Sample path 1	0.2144	0.4337	0.3073	0.1903
Sample path 2	0.3992	0.2603	0.3200	0.2294
Sample path 3	0.2300	0.4677	0.3183	0.1714
Sample path 4	0.3072	0.4027	0.2713	0.2480
Sample path 5	0.3832	0.4402	0.3145	0.2031
Sample path 6	0.2876	0.3200	0.3266	0.2091
Sample path 7	0.3704	0.3175	0.3506	0.2496
Sample path 8	0.3570	0.4140	0.1881	0.1006
Sample path 9	0.1846	0.1593	0.4311	0.3009
Sample path 10	0.3340	0.5787	0.3385	0.2854
Sample paths average	0.3068	0.3794	0.3166	0.2188
Cost				
Planned	2.26*	0.90*	35.63*	12.61*
Sample path 1	6.61	4.71	45.94	13.61
Sample path 2	7.13	0.36	27.92	30.68
Sample path 3	7.46	13.05	45.15	9.09
Sample path 4	3.52	7.54	30.79	32.51
Sample path 5	9.46	5.24	21.53	33.65
Sample path 6	5.65	8.49	31.24	17.79
Sample path 7	9.35	6.73	43.05	28.02
Sample path 8	12.52	8.13	25.64	11.28
Sample path 9	9.43	5.77	49.78	7.66
Sample path 10	5.57	1.94	35.02	34.44
Sample paths average	7.67	6.20	35.61	21.87

* Planned costs for overloaded operators (i.e., $b > v$) are computed based on Eq. (15) after imposing $v = b$

6 Results and discussion

Results of the experiments are reported in Tables 2, 3, 4, and 5. Planned values refer to the expected demand predicted for assigning, while the executed values are reported for both each sample path and averaged on the paths.

The three experiments based on the minimum cost increase show very similar results. Thus, the adoption of the triangular approximation does not affect the results with respect to the use of the empirical distributions (comparison among Tables 2, 3 and 4). Moreover, the variability of the new patient's demand seems to be negligible with respect to the operator workload variability. Indeed, very

Table 4 Results of the third cost increase approach (empirical distributions from patient stochastic model)

	District 1	District 2	District 3	District 5
$\bar{u}$				
Sample paths average	0.8014	0.6304	0.9460	0.9199
Z				
Planned	0.0671	0.1108	0.0459	0.0413
Sample path 1	0.1948	0.3845	0.2463	0.0894
Sample path 2	0.4818	0.0760	0.3164	0.1377
Sample path 3	0.2817	0.3233	0.2549	0.1159
Sample path 4	0.2342	0.0793	0.2310	0.1388
Sample path 5	0.4106	0.3343	0.2589	0.1032
Sample path 6	0.3458	0.2895	0.1584	0.0877
Sample path 7	0.3176	0.1852	0.3219	0.1656
Sample path 8	0.3177	0.2398	0.2777	0.1213
Sample path 9	0.1492	0.1763	0.3201	0.2169
Sample path 10	0.3112	0.4240	0.3773	0.1628
Sample paths average	0.3045	0.2512	0.2763	0.1339
Cost				
Planned	0.78*	0.28*	16.60*	3.46*
Sample path 1	6.35	7.29	37.97	15.00
Sample path 2	9.87	0.35	28.51	21.65
Sample path 3	7.96	15.17	43.35	11.04
Sample path 4	5.27	4.10	23.91	29.96
Sample path 5	10.18	7.79	38.32	35.44
Sample path 6	5.34	2.81	20.20	19.15
Sample path 7	11.68	2.63	49.65	28.95
Sample path 8	10.28	10.63	32.46	16.85
Sample path 9	7.49	5.25	42.63	9.35
Sample path 10	4.74	6.81	36.96	33.18
Sample paths average	7.92	6.28	35.40	22.06

* Planned cost is the average over the adopted planning period, equal to 8 weeks; this is an underestimation of the cost related to the first week, because a decreasing trend of patient demands is present along with the weeks

similar results for districts 1, 3 and 5 and equal results for district 2 are obtained assuming a uniform distribution Ψ (Table 2) or a deterministic patient demand μ (Table 3).

Table 6 reports the percentage differences for ranges and costs between the proposed policy (Table 2) and the expected workload approach (Table 5). Stricter ranges Z and lower costs are obtained with the minimum cost increase with respect to the expected workload approach W for the districts related to non palliative patients (districts 1, 3 and 5). In the case of palliative patients (district 2), no benefit is shown by the cost increase policy. This holds because

Table 5 Results of the expected value approach

	District 1	District 2	District 3	District 5
$\bar{u}$				
Sample paths average	0.8111	0.6487	0.9928	0.9440
Z				
Planned	0.0964	0.1502	0.0759	0.0598
Sample path 1	0.3504	0.4057	0.7676	0.3893
Sample path 2	0.4521	0.2290	0.5261	0.4566
Sample path 3	0.4865	0.3647	0.6210	0.3063
Sample path 4	0.4429	0.4177	0.5839	0.4132
Sample path 5	0.4332	0.4995	0.4117	0.3513
Sample path 6	0.4478	0.2468	0.5809	0.2955
Sample path 7	0.4447	0.2498	0.5197	0.4687
Sample path 8	0.4216	0.1917	0.4823	0.4019
Sample path 9	0.4813	0.1907	0.5189	0.3105
Sample path 10	0.4061	0.4423	0.6292	0.3967
Sample paths average	0.4367	0.3238	0.5641	0.3790
Cost				
Sample path 1	9.39	4.75	93.56	24.69
Sample path 2	9.72	0.30	42.57	30.53
Sample path 3	17.94	5.95	72.10	15.56
Sample path 4	7.85	13.68	78.22	32.97
Sample path 5	10.67	6.57	38.58	35.65
Sample path 6	11.42	4.22	67.22	30.63
Sample path 7	19.07	5.57	56.73	45.93
Sample path 8	13.81	10.38	49.10	28.69
Sample path 9	20.67	6.44	70.79	16.09
Sample path 10	9.59	3.78	80.09	45.44
Sample paths average	13.01	6.16	64.90	30.62

Table 6 Sample paths average values compared between the proposed policy (in the case of triangular distributions Φ_0 for X and uniform distributions Ψ for Y) and the W approach: value in the policy minus value in the W approach, divided by value in the W approach

	District 1 (%)	District 2 (%)	District 3 (%)	District 5 (%)
Z	−32.3	17.2	−44.1	−45.8
Cost	−40.2	0.5	−47.3	−26.8

of the lower variability of the demand of palliative patients (Table 7). As a matter of fact, the obtained cost is similar between the approaches and the range is minimized by the expected value approach that considers the expected available capacity rather than the probability of exceeding the capacity. The

Table 7 Mean value and standard deviation of the weekly patient demand in hours at the admission in the service for the different types of patients (data from the patient stochastic model of Lanzarone et al. (2010))

Type of care	Care profile	Mean	SD
Extemporary care	1	1.345	1.365
	15	0.921	1.188
Integrated	2	2.826	1.116
Home care	3	2.561	1.158
	4	2.755	1.382
	5	4.291	2.237
	9	1.935	1.099
	10	1.277	1.098
	12	2.347	1.108
	13	2.810	1.185
	14	2.707	1.388
Palliative care	6	6.509	3.013
	7	3.531	1.499
	8	1.482	1.093

lower variability can be explained considering that the palliative class includes patients with very similar pathologies and, consequently, similar care pathways.

7 Conclusions

The proposed policy allows resource planners to assign the reference operator to each newly admitted patient, with the main goal of reducing the expected costs for the HC provider under the constraint of respecting the continuity of care.

All of the hypotheses at the basis of the policy were evaluated considering the real data of the several HC providers (Chahed et al. 2006; Lanzarone and Matta 2009; Lanzarone et al. 2010). Therefore, the proposed policy can be considered general and applicable to a large number of structures. The comparison with the expected workload approach underlines that, for patients with a high variability of the demand, the proposed policy provides lower costs and higher balancing than the widespread approach, usually adopted by HC providers, based on the expected available capacity. Considering the typical classification of HC providers, this refers to non palliative patients.

Another result is that the demand variability of the newly admitted patient is negligible with respect to the variability already assigned to the operator workload (given by the variability of the already assigned patients). This result has been validated on one real case analysis.

Our future work will focus on the analysis of the entire probability density function of the cost increase, in order to develop a more general assignment policy based on stochastic orders. Moreover, the possibility of removing some assumptions made in this work will be evaluated. For instance, the assumption dealing with the stationarity of the workload distribution cannot be applied to some specific categories of patients.

Another extension will be the assignment of a set of newly admitted patients all together, instead of one patient at a time. Furthermore, the assignment decision framework could be more complex, since the HC provider could decide not to admit a new patient if the available capacity of the operators is not sufficient.

Appendix 1: Exact cost analysis

With a uniform distribution Ψ, the new expected cost C_1 is obtained by integrating Φ_1 (Eq. (7)) in the expression of cost (Eq. (8)). For the sake of simplicity, C_1 is alternatively derived as follows:

$$C_1 = \tilde{C}_1 - \Delta C_1 \tag{30}$$

where ΔC_1 is the difference between the new expected cost under deterministic patient demand μ and the new expected cost with a uniform distribution Ψ. Indeed, ΔC_1 is calculated as the difference of the integrals for cost between $\tilde{\Phi}_1$ and Φ_1, limiting the integral to the region where Φ_1 and $\tilde{\Phi}_1$ are different:

$$\begin{aligned}\Delta C_1 = & \int_{c+\alpha}^{c+\frac{\alpha}{2}+\frac{\beta}{2}} -(x-v)^2 \frac{2x-\alpha-\beta-2c}{(c-a)(c-b)} \mathrm{d}x \\ & - \int_{c+\alpha}^{c+\beta} (x-v)^2 \frac{(x-\beta-c)^2}{(\beta-\alpha)(c-a)(c-b)} \mathrm{d}x = \frac{2}{(c-a)(c-b)} H\end{aligned} \tag{31}$$

where:

$$\begin{aligned}H = & \, \omega\left[\omega^2\left(\frac{\omega}{4}-\frac{\lambda+2v}{3}\right)+\frac{\omega(2\lambda v+v^2)}{2}-\lambda v^2\right]+\frac{\lambda^4}{12}-\frac{\lambda^3 v}{3}+\frac{\lambda^2 v^2}{2} \\ & +\frac{\omega}{2(\beta-\alpha)}\left[\frac{\omega^4}{5}-\frac{\omega^3}{2}(\xi+v)+\frac{\omega^2}{3}\left(\xi^2+4\xi v+v^2\right)-v\xi\omega(\xi+v)+v^2\xi^2\right] \\ & -\frac{1}{2(\beta-\alpha)}\left[\frac{8\xi^5}{15}-\xi^4\left(\frac{\mu}{2}-\frac{v}{6}\right)+\frac{\xi^3 v^2}{3}\right]\end{aligned} \tag{32}$$

with $\omega = c+\alpha$, $\xi = c+\beta$ and $\lambda = c+\mu$.

In the same way, also the cost increase δ is determined considering ΔC_1:

$$\delta = C_1 - C_0 = \tilde{C}_1 - \Delta C_1 - C_0 = \tilde{\delta} - \Delta C_1 \tag{33}$$

Appendix 2: Operator point movement after the assignment

The analytical derivation of the operator point movement after the assignment starts from two opposite cases. In the derivation, it is assumed a triangular density function for the operator's workload also after the assignment.

- *Assignment of a patient with a null demand variability:* in this case, the variance of the workload remains constant after the assignment. It results in the same $a_t - c_t$ both before and after the assignment. Therefore, the operator point moves on a straight line parallel to the bisector of the I and III quadrant:

$$a_t = c_t + (a_{t0} - c_{t0}) \tag{34}$$

where the pedix 0 refers to the initial position of the operator.
If the demand of the new patient has a positive standard deviation, the variance of the operator workload increases and the new operator point is under this straight line. Therefore, the line is an upper limit to the possible operator movements in the plane c_t, a_t.
- *Assignment of a patient with a null expected demand and a positive standard deviation*. This is a non real condition because the demand Y does not admit negative values; however, this case is useful for analytically deriving the operator movement. In this case, the operator point moves with a constant expected value (same $a_t + b_t + c_t$ both before and after the assignment):

$$a_t = -\frac{1 + r_b}{2 - r_b} c_t + \left(a_{t0} + \frac{1 + r_b}{2 - r_b} c_{t0} \right) \tag{35}$$

This is a straight line with a negative slope, varying from -2 if $r_b = 1$ to -0.5 if $r_b = 0$. With a positive expected value of patient demand, the new operator point is over this straight line; therefore, this line is a lower limit to the possible operator movements in the plane c_t, a_t.

In the general case, the operator movement secondary to an assignment can be seen as the superposition of the assignments of a first patient with an expected demand $\mu_{patient}$ and a null standard deviation of the demand, and a second patient with a null expected demand and a positive standard deviation $\sigma_{patient}$ of the demand.

Starting from the initial operator position (characterized by μ_0 and σ_0), we define a new plane where the axes are the limits to the operator movement (Fig. 10).

The axis of movements corresponding to a null standard deviation of the new patient's demand is parameterized by t and the new expected workload of the operator after the assignment is linear with t:

$$\mu = \frac{a_t + b_t + c_t}{3} = \mu_0 + t \tag{36}$$

The axis of movements corresponding to a null expected patient demand is parametrized by u and the new variance of the operator after the assignment is quadratic with u:

$$\begin{aligned} \sigma^2 &= \frac{a_t^2 + b_t^2 + c_t^2 - a_t b_t - a_t c_t - b_t c_t}{18} \\ &= \sigma_0^2 + u^2 \frac{11 - 3r_b + 5r_b^2}{18(2 - r_b)^2} + \frac{u}{3}(c_{t0} - a_{t0}) \frac{r_b^2 - r_b + 1}{2 - r_b} \end{aligned} \tag{37}$$

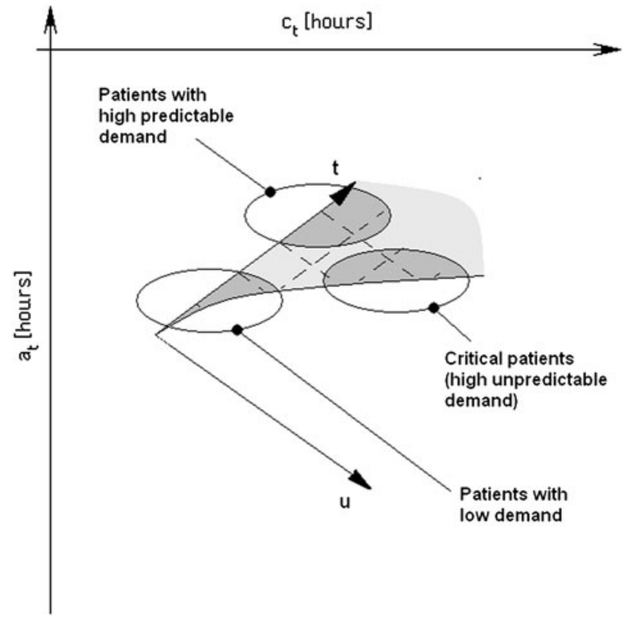


Fig. 10 Movement of an operator point after the assignment

Given the possible values of c_t, a_t and r_b, σ^2 grows with $u \geq 0$ with a positive concavity.

The operator movement is in the first quadrant of the plane t, u. However, a further constraint between $\mu_{patient}$ and $\sigma_{patient}$ has to be introduced for avoiding negative values of patient demand. This is $\mu_{patient} \geq H\ \sigma_{patient}$, where H is a coefficient depending on the distribution Ψ ($H = \sqrt{3}$ for the uniform distribution).

In the plane $t - u$, the curve $\mu_{patient} = \sqrt{3}\sigma_{patient}$ is determined with the condition $(\mu - \mu_0)^2 = 3\left(\sigma^2 - \sigma_0^2\right)$. After some manipulations, this results in the following equation:

$$\frac{(u + u^*)^2}{P^2} - \frac{t^2}{Q^2} = 1 \tag{38}$$

with:

$$\begin{aligned} u^* = P &= \frac{3(c_{t0} - a_{t0})(2 - r_b)\left(r_b^2 - r_b + 1\right)}{11 - 3r_b + 5r_b^2} \\ Q &= \frac{(c_{t0} - a_{t0})\left(r_b^2 - r_b + 1\right)}{\sqrt{2}\sqrt{11 - 3r_b + 5r_b^2}} \end{aligned} \tag{39}$$

This is an hyperbola (Fig. 10) with semiaxes P and Q, the foci on axis u, one vertex in the origin of the plane t, u and the other vertex for $u < 0$ (the distance between the vertices is proportional to $c_{t0} - a_{t0}$). The slope of the asymptotes is constant (Q/P is independent of the coordinates of the operator point).

The admissible movement is over this hyperbola. Hence, the movement area is a region on the right of the operator point, included between a straight line (Eq. (36)) and a hyperbola (Eq. (38)).

In this area, the coordinates t and u of the operator after the assignment are obtained from the expected value and the standard deviation of the new patient's demand Y, based on Eqs. (36) and (37). Different zones can be distinguished, depending on the new patient's demand (Fig. 10). Considering the data collected from the analyzed HC provider, these zones correspond to the different types of patients (Fig. 10): Extemporary Care patients with low level of demand, Integrated HC patients with a relevant number of visits with a certain variability, and Palliative Care patients with high level of demand associated with low variability (Table 7).

Reference

Akjiratikarl C, Yenradee P, Drake P (2007) PSO-based algorithm for home care worker scheduling in the UK. Comput Ind Eng 53:559–583

Asquer G, Borsani V, Matta A (2007) Analysis of the organizational structure of the home care providers (in Italian). Politiche Sanitarie 8:95–117

Bard JF (2010) Nurse scheduling models. In: Cochran JJ et al. (eds) Wiley encyclopedia of operations research and management science, Topic 4.3, Medicine and health care, vol 5. Wiley, New York, pp 3617–3627

Bard J, Purnomo H (2005) Hospital-wide reactive scheduling of nurses with preference considerations. IIE Trans 37:589–608

Bard J, Purnomo H (2007) Cyclic preference scheduling of nurses using a lagrangian-based heuristic. J Schedul 10:5–23

Belien J, Demeulemeester E (2008) A branch-and-price approach for integrating nurse and surgery scheduling. Eur J Oper Res 189:652–668

Ben Bachouch R, Fakhfakh M, Guinet A, Hajri-Gabouj S (2008) Planification de la tournée des infirmiers dans une structure de soins à domicile. In: Proceedings of the 4th conférence fancophone en gestion et ingénierie des systèmes hospitaliers (GISEH2008)

Bennett A, Erera A (2011) Dynamic periodic fixed appointment scheduling for home health. IIE Trans Healthcare Syst Eng 1:6–19

Bertels S, Fahle T (2006) A hybrid setup for a hybrid scenario: combining heuristics for the home health care problem. Comput Oper Res 33:2866–2890

Borsani V, Matta A, Beschi G, Sommaruga F (2006) A home care scheduling model for human resources. In: Proceedings of the international conference on service systems and service management (ICSSSM06), pp 449–454

Brunner J, Bard J, Kolisch R (2009) Flexible shift scheduling of physicians. Health Care Manag Sci 12:285–305

Brunner J, Bard J, Kolisch R (2011) Midterm scheduling of physicians with flexible shifts using branch and price. IIE Trans 43:84–109

Burke E, Causmaecker P, Berghe G, Landeghem H (2004) The state of the art of nurse rostering. J Schedul 7:441–499

Chahed S, Dallery Y, Matta A, Sahin E (2006) Operations management related activities in home health care structures. In: Proceedings of the 12th IFAC symposium on information control problems in manufacturing (INCOM06), vol 28, pp 641–646

Chahed S, Marcon E, Sahin E, Feillet D, Dallery Y (2009) Exploring new operational research opportunities within the home care context: the chemotherapy at home. Health Care Manag Sci 12:179–191

Cheang B, Li H, Lim A, Rodrigues B (2003) Nurse fostering problems-a bibliographic survey. Eur J Oper Res 151:447–460

Chevreul K, Com-Ruelle L, F, FM, Paris V (2005) The development of hospital care at home: an investigation of Australian, British and Canadian experiences. Technical Report 1610, Istitute de recherce et documentation en economie de la sante' (IRDES)

Comondore V, Devereaux P, Zhou Q, Stone S, Busse J, Ravindran N, Burns K, Haines T, Stringer B, Cook D, Walter S, Sullivan T, Berwanger O, Bhandari M, Banglawala S, Lavis J, Petrisor B, Schunemann H, Walsh K, Bhatnagar N, Guyatt G (2009) Quality of care in for-profit and not-for-profit nursing homes: systematic review and meta-analysis. British Med J 339:b2732

Cordes C, Dougherty T (1993) A review and an integration of research on job burnout. Acad Manag Rev 18:621–656

DeAngelis V (1998) Planning home assistance for aids patients in the city of rome. Interfaces 28:75–83

DeGrano M, Medeiros D, Eitel D (2009) Accommodating individual preferences in nurse scheduling via auctions and optimization. Health Care Manag Sci 12:228–242

Eveborn P, Flisberg P, Ronnqvist M (2006) LAPS CARE-an operational system for staff planning of home care. Eur J Oper Res 171:962–976

Eveborn P, Ronnqvist M, Einarsdottir H, Eklund M, Liden K, Almroth M (2009) Operations research improves quality and efficiency in home care. Interfaces 39:18–34

Haggerty J, Reid R, Freeman G, Starfield B, Adair C, McKendry R (2003) Continuity of care: a multidisciplinary review. British Med J 327:1219–1221

Hertz A, Lahrichi N (2009) A patient assignment algorithm for home care services. J Oper Res Soc 60:481–495

Lanzarone E, Matta A (2009) Value of perfect information in home care human resource planning with continuity of care. In: Proceedings of the 35th conference on operational research applied to health services (ORAHS 2009), number 17

Lanzarone E, Matta A, Scaccabarozzi G (2010) A patient stochastic model to support human resource panning in home care. Prod Planning Control 21:3–25

Medicare Payment Advisory Commission MEDPAC (2011) Home health services. In: Report to the congress: medicare payment policy (chapter 8)

Punnakitikashem P, Rosenberger J, Behan D (2008) Stochastic programming for nurse assignment. Comput Opt Appl 40:321–349

Purnomo H, Bard J (2007) Cyclic preference scheduling for nurses using branch and price. Naval Res Log 54:200–220

Sundaramoorthi D, Chen V, Rosenberger J, Kim S, Buckley-Behan D (2010) A data-integrated simulation-based optimization for assigning nurses to patient admissions. Health Care Manag Sci 13:210–221

Thomsen K (2006) Optimization on home care. Thesis in informatics and mathematical modeling, Technical University of Denmark

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