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Job scheduling and management of wearing tools with stochastic tool lifetimes

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Abstract The problem of scheduling jobs using wearing tools is studied. Tool wearing is assumed to be stochastic and the jobs are processed in one machining centre provided with a limited capacity tool magazine. The aim is to minimize the expected average completion time of the jobs by choosing their processing order and tool management decisions wisely. All jobs are available at the beginning of the planning period. This kind of situation is met in production planning of CNC-machines. Previous studies concerning this problem have either assumed deterministic wearing for the tools or omitted the wearing completely. In our formulation of the problem, tool wearing is stochastic and the problem becomes very hard to solve analytically. A heuristic based on genetic algorithms is therefore given for the joint problem of job scheduling and tool management. The algorithm searches the most beneficial job sequence when the tool management decisions are made by a removal rule taking into account the future planned usage of the tools. The cost of each job sequence is evaluated by simulating the job processing. Empirical tests with heuristics indicate that by taking the stochastic information into account, one can reduce the average job processing time considerably.

Keywords CNC manufacturing · Job scheduling · Tool wear · Stochastic tool lifetimes · Heuristic algorithms

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1 Introduction

This paper studies *Job Scheduling with Stochastic Tool Lifetimes* (JSSTL). The JSSTL-problem arises in an environment which consists of a single *machining centre* (see Gibbs and Crandell 1991) to process *jobs* (also called *parts*) with a set of tools that are subject to wearing as a result of their usage. The job set is known in advance and the problem is to select their processing order and make such a tool management decisions that the expected average processing time of the jobs is minimal. While the processing order is free, it is supposed that the order of using different tools on a given job is specific to the particular job. It is further assumed that the manufacturing process is stochastic in the sense that the wearing of the tools follows some known probability distributions which are specific to the processing steps. These steps are atomic processing phases of a job and the processing time of each step is deterministic because the so-called machining parameters (see for example Turkcan et al. 2003; Billatos and Kendall 1991; Iakovou et al. 1996) are assumed to be fixed. The capacity of the machining centre's *tool magazine* is limited and it can be reconfigured between the jobs, only. Tools which are not in the magazine are kept in a secondary storage in the vicinity of the machine. A time cost is associated with each *tool change* (called also *tool switch*) which is a load (unload) operation of a tool to (from) the magazine. Tool changes made during the processing of a job are not allowed in our problem formulation. When a tool breaks down during the processing of a job, the unfinished job must be discarded and processed again from the beginning. It is assumed that each time a tool is loaded into the tool magazine it is "new," i.e. no partially worn tools are reloaded. No magazine reorganization (see Hirvikorpi et al. 2006) is needed because the tools use uniform amount of space from the magazine.

In scheduling and tool management research the variables are usually divided to *machining* and *non-machining* parameters. The machining parameters, for example cut speed and depth, affect indirectly the non-machining parameters by changing the rate of tool wear. As said before we assume that the machining parameters are fixed and therefore the processing times of the job steps are known in advance. The non-machining parameters in our problem setting are for example the processing order of the jobs and the tool management decisions.

Tool management literature considers mostly tool changing to occur due to the demands of the job mix. The objective there is to minimize the *total number of tool changes* (for a fixed order of jobs) or to minimize the *number of change instants* by grouping the jobs, see Knuutila et al. (2001). For infinite tool lifetimes (wearing does not occur) the KTNS-rule developed by Tang and Denardo (1988) minimizes the number of tool changes when the order of the jobs is fixed and each tool uses the same amount of space from the tool magazine, see Crama et al. (1994) for proof. The study of Gray et al. (1993) reported, however, that tool changes occurring due to the *tool wear* may be ten times more frequent than those occurring due to the job mix.

Stochastic tool management was considered by Billatos and Kendall (1991) in a study which calculated the optimal machining parameters and tool replacement times when the processing order of the jobs is fixed. Their model does not consider the tool changes made between the secondary storage and the machine magazine, i.e. they assumed that all the tools needed by the jobs fit to the magazine simultaneously. Jianqiang and Keow (1997) also studied a similar problem of finding optimal tool replacement intervals and machining parameters when the tool wear is stochastic. Several other studies which consider the problem of finding optimal machining parameters exist, see for example Iawata et al. (1972).

The Bernstein reliability model (Ahmad and Sheikh 1984) has been used in many studies on stochastic tool management and scheduling problems. Ahmad and Sheikh (1984) studied the properties of this model from mathematical point of view. Our study, on the other hand, uses the well-known Weibull distribution (see Murthy et al. 2004) in modelling the tool wear. The Weibull distribution is more complex to handle in theoretical sense than the Bernstein model but it is flexible.

In scheduling research the tool changes have usually been omitted completely. The simplest form of scheduling is the case where a set of jobs with certain fixed processing times is given and the problem is to choose such a processing order that the average completion time of the jobs is minimal. This problem can be solved exactly for the single machine case (with uniform tool capacity demands) by using the *shortest-processing-time-order* (SPT-rule). Adiri et al. (1989) studied a more complex scheduling model including machine breakdowns with stochastic repair times. They proved that the SPT-rule minimizes the expected total flow time if the breakdown times are negative exponentially distributed. Lee and Liman (1992) studied this same kind of model with deterministic repair times and proved that the SPT-order lies always within the factor of 9/7 of the optimal flow time.

Lee (1996, 1997) studies also a two-machine model in which it was assumed that a breakdown or maintenance period occurs only once during the processing of the jobs. Turkcan et al. (2003) considered a problem where multiple non-identical CNC machines are working in parallel. Their objective was to minimize the manufacturing costs and total weighted tardiness by choosing the machining parameters and part scheduling intelligently. Their study assumed deterministic lifetimes for the tools.

Qi et al. (1999) considered a single machine scheduling problem with multiple maintenance intervals of variable duration. Their model is equivalent to the one considered by Akturk et al. (2002) who seem to be the first authors to study the tool wear in scheduling. The model of Akturk et al. allows the use of one tool type which has a deterministic lifetime. Although quite simple, this kind of formulation is relevant because it is not unusual that only one tool is used to process several different jobs, for example in a cutting application.

Hirvikorpi et al. (2006) gave a general formulation of the wearing tools' problem by modelling the use of several tool types and supposing that the capacity of the tool magazine is limited. This formulation assumes deterministic tool wear and uniform tool sizes. The present paper extends the model of Hirvikorpi et al. (2006) by relaxing the assumption on deterministic tool wear but still keeping the sizes uniform. This leads to a situation where the tool wear is stochastic and specific to each processing step. A natural question then arises—how to plan the tool management and the processing order in such a way that the processing of the jobs is as efficient as possible (in the sense of the expected average completion time of the jobs).

There are several approaches to solving stochastic optimization problems. Birge and Louveaux (1997) considered at least three different approaches. The scenario approach categorizes the possible outcome of the random events and an integer programming formulation is created from this categorization. For the JSSTL-problem this approach is somewhat limited because there are only weak dependencies between the wear of different tools. There would be an exhaustive number of scenarios if one would like the solution to be at least somewhat accurate. Analytical approach is something that works for simple problems, i.e. solving the objective function in closed form. The JSSTL-problem contains a large number of NP-hard problems within it and therefore it is hard to believe that the problem can be solved analytically. The third approach would be to define the JSSTL-problem as a multi-stage stochastic optimization problem in which case one could use the nested L-shape method (see Birge and Louveaux 1997). However, the JSSTL-problem does not have a fixed number of stages and the number of possible outcomes after each stage is very large, thus leading to an exhaustively large integer programming formulation. The approach of the present paper is to formulate the problem mathematically to define its theoretical properties like the optimal solution and feasibility. However, the actual solution algorithms rely on simulation when calculating an approximate value for the objective function.

The rest of the paper is organized as follows. Section 2 defines the JSSTL-problem. Section 3 introduces a heuristic cost evaluation algorithm when the order of jobs is predefined. This algorithm is used to guide the search process in the genetic algorithm (GA) designed for solving the joint problem of scheduling and tool management, as presented later in the same section. Section 4 presents a summary of empirical results computed using the proposed GA and the well-known SPT-rule. The paper is concluded in Sect. 5.

2 Job scheduling with stochastic tool lifetimes-problem

Consider a manufacturing situation where there are several jobs to be scheduled for one machining centre (machine from now on). The jobs have no fixed deadlines but the customers are waiting for their completion so we need to process all of them as quickly as possible. The machine can process one job at a time and each job requires several consecutive processing steps from the same machine. Once all steps have been completed successfully for a single job it can be delivered to the customer. To maximize the customer satisfaction, the Job Scheduling with Stochastic Tool Lifetime (JSSTL) problem calls for minimizing the average waiting time of the customers which is the sum of the completion times (=total time) of the jobs divided by the number of jobs. The contingency in the machine operations forces us to minimize the *expected* average processing time.

The contingency of the machine operation is caused by the tools which the machine uses to process the jobs. The machine holds the tools in a magazine and one tool is associated with each processing step of each job. The tools wear out and we do not know exactly when a certain tool is going to break down, thus we must make pre-emptive tool loads/unloads (changes) to the magazine to avoid interruptions in the processing. Tool changes can be made only between the jobs because the machine operation cannot be interrupted between the processing steps of a single job.

The only case where the machine is interrupted during the processing of a job is when some tool breaks down. In this case, the unfinished job must be discarded and restarted from the beginning with a blank job. The other type of tool change is caused by the limited capacity of the tool magazine which cannot hold all the tools needed by the jobs simultaneously. These changes (due to job mix) are made between jobs along with the changes of the worn-out tools. The following subsections formalize the problem and concepts presented above.

2.1 Tool modelling

It is important to make a separation between a tool and tool type. We say that a tool is a physical instantiation of a tool type. A tool type on the other hand describes the general properties for all tools of the same type. The amount that a tool wears during a processing step of a job is stochastic and the parameters of the probability distribution are specific to each processing step. The used probability distribution on the other hand depends on the tool type. The distribution parameters are derived from the machining parameter values. We assume that the machining parameters are decided before solving the JSSTL-problem. For studies considering the optimal selection of the machining parameter values with respect to the cost minimization, see for example Turkcan et al. (2003), Billatos and Kendall (1991).

The parameters of the probability distribution of tool wearing depend on several factors some of which are deterministic and others stochastic. For example, in cutting application the minor differences and inequalities in the processed material and in the tool quality change the tool-wearing rate. These are clearly stochastic factors. The machining parameters change the shape of the step-specific probability distribution and possibly the processing time of the step. In Fig. 1 we have demonstrated the tool wear during several processing steps which have varying material and machining parameters.

The total tool wear value is calculated as the cumulative sum of the wear values of the previous steps and the realized wear value on the current step. There is a fixed tool type specific level of total tool wear after which the tool cannot be used anymore, see Fig. 1.

Let T be the set of all tool types used by the jobs. It is assumed that there is an unlimited supply of tools of each type. For all tool types $t \in T$ the following properties and functions are defined:

(1) $f(t, L_s, k, a) \in \Re$ is the probability density function for tool type t that it will wear a units when it is used for time k with set L_s of probability distribution parameters,

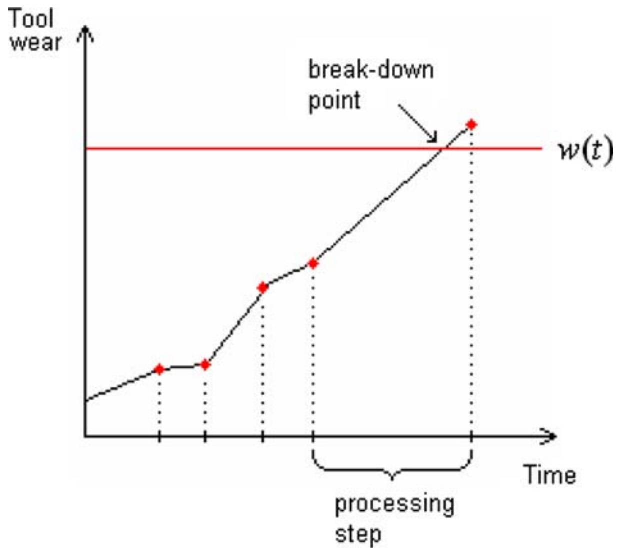


Fig. 1 Incurred tool wear as a piecewise linear function of processing time for a tool of type t; the effect of the machining parameters and the stochastic factors to the additive tool wear. Each of the five processing steps has its own machining parameters which affect the tool wear. For example in the second step the wearing rate has been quite slow which could be caused by shallow cutting depth (deterministic parameter). It could also be that material's temperature in the processed position is increased which we consider a stochastic factor. During the last step the tool breaks down because the threshold wear value $w(t)$ has been exceeded

(2) $P_{\geq}(t, L_s, k, a) \in \Re$ gives the probability that tool type t will wear more than a units when it is used for time k with set L_s of probability distribution parameters,
(3) $w(t)$ is the threshold wear value for the tool type t when it can no longer be used,
(4) $s(t)$ is the time required to load (unload) the tool type t to (from) the magazine. The time cost of changing the tool type t is assumed to be $2s(t)$.

2.2 Job modelling

The jobs are modelled as a sequence of (tool, operation time, set of density function parameters)-triplets which we call the processing steps. The operations are carried out by the machine in this order and it is not possible to interrupt the operation of the machine between the steps for two reasons. First, it may be that certain operations must be done very quickly one after another because otherwise the quality of the processed job deteriorates. Second, machine interruption is a time-consuming operation that may require reloading the machine NC-program if changes are made to the machine state.

Let us define the set J of N jobs formally. Each job $j \in J$ has the following properties:

(1) $n(j)$ is the number of processing steps,
(2) $t(j, i)$ is the tool type needed in the ith step,
(3) $p(j, i)$ is the processing time of the ith step and
(4) $L(j, i)$ is the set of parameters of the tool-type probability distribution for the tool type j of the ith step.

2.3 Machine model

The machine is composed of four major parts as shown in Fig. 2. The part (job) is fixated on the processing table, the processing head moves around the processing table and processes the job with the tool attached to the head. Once the current processing step is completed the processing head returns the current tool to the magazine and switches to another one. Finally, when the job is ready, i.e. all steps are finished, the machine is interrupted and the job is replaced with a new one. During this interruption the machine operator can make tool changes to the magazine. These changes (from an off-line storage to the magazine) should not be mixed with the tool changes the machine makes automatically (from the magazine to the processing head) during the processing of a job. Tools are changed in the magazine because different jobs may require different sets of tools and also due to tool wearing.

It is assumed that each tool type uses the same amount of space in the magazine. Because of this, the magazine is modelled as a single number, the capacity, which indicates the number of tools the magazine can hold simultaneously.

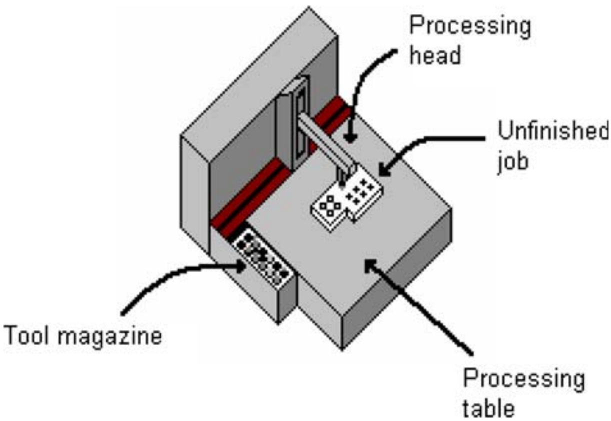


Fig. 2 The general working principle of a numerically controlled machining centre. The processing head moves in six directions and can make tool changes between processing steps. A tool change is performed as a two-phase operation by first returning the current tool to its position in the magazine and then fetching a new one. Tool changes are made automatically and they are not modelled in our problem setting. Tool changes to the magazine from outside are made only between jobs because they require a change of the machine's control program and the reconfiguration of the tool magazine

2.4 Processing plan

Because of the stochastic nature of the JSSTL-problem an optimal processing plan should be fully dynamic in the sense that the ordering of the jobs and the tool management decisions should depend on the history of the processing which may vary from case to case due to unexpected tool breaks. Because this kind of processing plan seems to be rather complicated to implement in practice, we simplify the solution space by supposing that the order of jobs remains fixed and independent from the previous tool breaks that are observed while executing the schedule. A benefit of this is that it becomes possible to give an estimate of the expected finishing time of each job in the schedule. Processing plan C is therefore defined to be a static permutation of the job set J.

2.5 Feasibility of a problem instance

Assume that a set of jobs J, tool-type set T and magazine capacity K are given. This problem instance is feasible if two conditions hold:

(1) For each job $j \in J$ the number of different tool types needed in processing is less than the capacity K;
(2) For each job the probability that it can be processed with the tools is greater than 0.

The first condition is trivial to check. The second condition is hard to verify in a general case. The reason for this is that each individual processing step has its own set of probability distribution parameters for tool wear. Thus, to check whether some particular tool type is able to process a job so that the tool does not wear more than its allowed limit would require solving a sum of random variables in which each variable has its own set of distribution parameters (one random variable originates from one processing step). There are methods for doing this for some distributions when the steps have equal parameters. However, in a general case it seems to be difficult to check the condition 2. In the special case where each tool type is used at most once for each job it suffices to know the mass function of each tool type's probability distribution to check the feasibility. To simplify the discussion, condition 2 is therefore replaced from now on with a more stringent condition.

(2′) For each job $j \in J$ and each tool type t used in j, the expected wear-out value caused by j does not exceed the wear-out limit $w(t)$. This condition means that for a newly changed tool one has a "good change" that the next operation can be finished successfully.

2.6 Problem definition

Consider a particular permutation π of the jobs J and rename the jobs in order of this permutation. Without loss of generality we can therefore consider an execution sequence $1, 2, \ldots, N$ of the N jobs. For this sequence the total cost C_{total} of processing can be expressed as a sum of the time costs C_j for different jobs $j \in J$. As the first approximation, the main time factor here is the sum $\sum_{i=1}^{n(j)} p(j,\ i)$ of the actual tool usage steps for each job j. A complicating factor here is that, in addition to this sum, C_j depends on the setting of the tools in the magazine originating from the previous job of the processing sequence, the wear-out state of each tool used by the current job and the possible unplanned tool breakdowns caused by unexpected fast wearing of the tools.

To formulate the JSSTL-problem, let $Tools_j = \{(t_{j,1},\ w_{j,1}),\ (t_{j,2},\ w_{j,2}),\ \ldots,\ (t_{j,n(j)},\ w_{j,n(j)})\}$ give at the moment of finishing the job j the cumulative wear-out status of the tools used by j. Here $t_{j,i}$ stands for the tool used at the ith step of j and $w_{j,i}$ gives the amount of wear out of the ith tool at finishing the job j.

As described before the wear-out states of the tools depend strongly on the previous processing steps so that the usage of a tool increases and replacement decreases the wear-out value.

One can then express the total processing time of the permutation π as

$$\begin{aligned} C_{\text{total}} =& C_1(Tools_1|Tools_0) + C_2(Tools_2|Tools'_1,\ Tools_0) \\ &+ C_3(Tools_3|Tools'_2,\ Tools'_1,\ Tools_0) + \cdots \\ &C_N(Tools_N|Tools'_{N-1}, \ldots,\ Tools'_1,\ Tools_0) \end{aligned}$$

The job processing time is therefore a function of the initial status of the tools used by it. Some of them are ready in the magazine and their status depends on the previous jobs. The remaining tools should be inserted into the magazine (Either they have never been used or they have been removed to make room for some new tools). It seems to be rather difficult to write a mathematical form of the expected value of C_{total} for most density functions of tool wearing.

One possibility to deal with the JSSTL-problem is to state the solution by the means of a job sequencing and tool management policy R. The idea is then to fix the tool management policy (in several alternative ways) and search for the most profitable order of jobs for this policy. While this method yields suboptimal manufacturing processes the solutions are realizable in practice. To the deterministic case without any tool breaks one can calculate the total tool change cost C_{change} by the formula

$$C_{\text{change}} = \sum_{j=1}^{|J|} change(j-1,\ j), \tag{1}$$

where $change(j-1,\ j)$ stands for the costs when moving from $j-1$ to j. These terms are obtained by counting the number of required tool change operations and charging each operation by a proper multiple of change costs $s(t)$. While the above looks straightforward there are still complicating factors in it. First, even in the deterministic tool management, the optimal politics of changing tools between jobs are hard to find and stochastic tool wearing complicates the situation essentially. Finding a proper rule for avoiding tool breaks is a hard problem. One way here is to base the tool change decisions on the worst case scenarios. Let, therefore $g(t, p)$ be a function that upperbounds (with a great probability) the stochastic tool wear of an operation of the length p and $w_{\text{curr}}(t)$ the cumulated tool wear obtained from previous usages of tool t by the upperbound function g. A tool break of t is then very unprobable during its next usage of length $p(j, t)$ on job j if

$$w_{\text{curr}}(t) < w(t) - g(t, p(j, t)). \tag{2}$$

The problem in this approach is the selection of an efficient g for tailed tool-wear distributions.

The concept of machine state is useful in the calculation of the tool change costs (1). Let $S_j = \{(t_{j,1}, w_{j,1}), (t_{j,2}, w_{j,2}),..., (t_{j,k}, w_{j,k})\}$ be the set of tools and their wear-out values that are stored in the magazine at the *beginning* of the jth job, and $T(S_j)$ be the set of tools in the magazine at the beginning of job j. One can operate possibly with several different settings for $T(S_j)$ as far as the tools in $Tools_j$ are included in them.

The cost of switching from machine state $j-1$ to j is with the above notations

$$change(j-1, j) = \sum_{(t,w(t)) \in S_{j-1} \otimes S_j} s(t) + \sum_{t \in S_{j-1} \cap S_j \text{ and } w_{curr}(t) > w(t) - g(t,p(j,t))} 2s(t) \tag{3}$$

One should remember that using (3) in formula (1) is an over simplification of the problem while it trusts on the crude condition (2) in tool changes.

The *job scheduling with stochastic tool lives* (JSSTL)-problem is stated as follows: When given job set J, tool-type set T, magazine capacity K and initial machine state S_0, define a processing order π^* and a tool replacement strategy R^* for which the total processing cost C_{total} is minimal. Note that R^* deals with two decisions: selection of the tools to be removed from the magazine when starting a new job and selection of the tools to be changed due to partial wear out when restarting a job. R^* has in this way an effect on machine states.

3 Heuristics for the JSSTL-problem

From the discussion of the previous section we find it hard to algorithmically determine simultaneously the optimal sequence of the jobs and the way of managing the tools of the JSSTL-problem. We therefore content ourselves to fix the tool management heuristics and for each of these to optimize the job processing ordering (job sequence) by the means of local search methods, see Reeves (1995). The solution presented here uses a genetic algorithm (GA) which searches an (sub)optimal processing order for the jobs. The goodness of the ordering (fitness of the individual in GA terms) is evaluated with stochastic cost evaluation algorithm (SCEA) that relies on simulation.

The general structure of the cost evaluation algorithm is given in Fig. 3. The algorithm applies a given tool management policy between the jobs by making loading, removal and replacement decisions for the tools. The simulation part of the algorithm updates the wear status of the tools when simulating the manufacturing process step by step. When a tool breakdown occurs the algorithm reapplies the tool management policy and processes the job again. The algorithm proceeds to the next job if the processing was successful.

```
SCEA(J, K, R, P) : average cost                 -- J is the sequence of jobs,
    let totaltime = 0;                          -- K   the capacity of the magazine,
    let lineartime = 0;                         -- R   the removal rule and
    let S = empty machine state;                -- P   the replacement rule
    For all jobs j in order of J do
        removals = 0
        For all tools t in T(j) - T(S) do
            if |T(S)| = K then
                t_r = R(j,J,S);
                S = r(S,t_r);
                lineartime = lineartime + s(t_r)
            S = S ∪ {(t,0)};
            lineartime = lineartime + s(t);
        -- simulate the processing of the job
        Repeat
            -- check the tools which need to be replaced
            For all tools t in T(S) do
                if P(j,t,J,S) then
                    lineartime = lineartime + 2s(t);
                    S = r(S,t) ∪ {(t,0)};

            -- simulate processing of the job
            success = true;
            For i = 1 to j(n) do
                lineartime = lineartime + p(j,i);
                -- the implementation of the random function depends on the
                -- used distribution
                r = random(l(j,i));
                -- u updates the machine state according to wearing occurs
                S = u(S,t,r);
                -- check if a tool break-down occurred
                if w(S,t) > w(t) then
                    success = false;
        Until success = true;
        totaltime = totaltime + lineartime;
    return totaltime / |J|
```

Fig. 3 Stochastic cost evaluation algorithm. The tool management policy is given as a parameter (removal rule R and replacement rule P) to this algorithm

3.1 Tool management policy

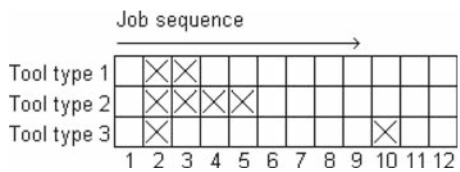
The tool management policy consists of three parts: loading rule, removal rule and replacement rule. The *loading rule* is simple because the tools are loaded just before they are needed in case they are not already in the magazine. When the magazine is full the loading rule applies a *removal rule* which selects one tool at a time to be removed from the magazine.

A *replacement rule* determines whether tool has worn out so much that it should be replaced to avoid breaking when used next time. In the deterministic case this is a simple check because tool replacements should be left to the last possible moment as shown by Hirvikorpi et al. (2006). For the stochastic case this moment is not known exactly and we therefore propose two new replacement rules.

3.1.1 Removal rules

Hirvikorpi et al. (2006) have found out that the traditional KTNS-rule performs quite poorly for the deterministic wearing tools problem. They proposed the Keep Tool which Wears out Last (KTWL)—rule (where instead of the next time of using, the moment of next future wear out is observed) and a hybrid of the KTNS- and KTWL-rule. They show that the KTWL-rule performs well on the deterministic wearing tools problem, but there are some issues it does not consider, see Fig. 4 where it is assumed that the remaining processing times of all tools are long. When ordering the three tools into a reversed order of keeping preferences by the KTWL-rule (giving the removal order of this rule) the tool to be *removed* first is tool 1 (it wears out soonest). This may be unfavourable because tool 1 is needed in two next jobs and after that not any more. The KTNS-rule might perform better in this case. The HYBRID-rule calculates therefore the sum of KTNS and KTWL rules but results with this rule have not been promising.

We propose here a new removal rule (SUM) which calculates for each tool a sum of terms $1/x_i$, where x_i stands for the distance (relative to the position of the current job in the sequence) to the ith future job using this same tool in the rest of the job sequence, see Fig. 4. This means that jobs closer in the sequence will get higher weights. The KTNS- and KTWL-rule make their decisions based on one factor, only. This can be problematic in situations where a tool is needed in the next job and in some other job a lot later. The KTNS- and KTWL-rule would keep this tool because it is needed very soon and it will last a long time. The SUM-rule gives benefit of both features and may thus favour some other tools.



	KTWL	KTNS	SUM
Tool type 1	3	1	1/2+1/3
Tool type 2	5	1	1/2+1/3+1/4+1/5
Tool type 3	10	1	1/2+1/10
Keep tool (1.)	3	any	3
Keep tool (2.)	2	any	1

Fig. 4 Operation of KTWL, KTNS and SUM rules before processing job 1. Job sequence is assumed to be given and "×" marks the jobs in which the tool type is needed. The example shows a situation where the tool to be removed should be chosen among the tools 1, 2 and 3; the current job (1) does not use any of them but it needs space in the magazine for a new fourth tool (for which the future usage is not shown in the figure). Numbers in the table indicate for KTWL the last moment (or wear-out time) of using the tool and the tool to be kept in the magazine (1. and 2. choice). The selection is purely random for KTNS. SUM-rule calculates the sum of the inverses of the distances to the future usage moments and tools with small sum values should be kept in the magazine

3.1.2 Replacement rules

In the deterministic case, a tool is replaced when it is known to be too worn out to process the next job in the sequence. In the stochastic case, there is no simple way to calculate even the probability for a tool breakdown during the next job. The problem is that each of the processing steps has its own probability distribution and we do not know an analytical approach to solve the probability that a tool will break out.

The simplest possible replacement rule is the Do Nothing (DN-rule), i.e. a tool is changed only if it has broken down. This will of course lead to the largest number of tool breakdowns. A little bit smarter rule uses the *expected wearing out values* of the processing steps (EV-rule), i.e. if the cumulative wear-out value is larger than the breakdown limit of the tool, the tool is changed before performing the next job in question.

Finding a balance between tool breakdowns and excessive tool changes is essential in seeking a good tool replacement policy. The number of tool breakdowns can be controlled by setting a factor r to the EV—rule. Let us assume that the expected tool wear for certain tool type t in the next job is e and that the tool in question has worn out w_0 units before the current step. Then, the replacement EV(r)-rule is as follows: if $e/(w(t) - w_0) > r$ then tool is replaced. By setting r to 1.0 this becomes the EV-rule. If r is very large then this rule is the same as DN-rule and when r is 0.0 this rule changes always all the tools needed by the next job to new ones.

3.1.3 A sample problem instance

Figure 5 demonstrates the operation of the cost evaluation algorithm SCEA for the EV(r) replacement rule. The problem instance consists of five tools and four jobs. For the case of illustration it is assumed that the processing time of each step is the η parameter (β is another parameter of Weibull) of the Weibull distribution (Murthy et al. 2004).

3.2 A genetic algorithm for job scheduling, SSA

The *stochastic scheduler algorithm* (SSA) described in this section is a modification of the genetic algorithm GAPSM developed by Hirvikorpi et al. (2006) for the deterministic version of the problem. For the description of the original GAPS algorithm, see Reeves (1995) and Akturk et al. (2002).

SSA includes a number of technical details which are not of general interest, therefore only an overall description of the algorithm is given here. The key components of a GA include coding an individual, fitness evaluation, crossover, mutation and selection. These components are discussed next and after that a brief description of the main phases of SSA is given.

The coding of an individual is based on the idea of the problem space search, see Storer et al. (1983). In the JSSTL-problem a set J of N jobs is given as an input. A single individual of SSA describes some permutation of the jobs in J. The individuals are therefore coded so that they can describe all the N! possible

Tool data:

Tools	Wear-out limit $w(t)$	Change time $s(t)$
Tool 1	64	7
Tool 2	60	12
Tool 3	96	13
Tool 4	56	12
Tool 5	98	7

Job data:

Steps	Job 1	Job 2	Job 3	Job 4
Step 1	*Tool 1*	*Tool 1*	*Tool 5*	*Tool 1*
β	1.073	1.028	1.026	1.074
η	20.15	19.36	41.22	20.15
Step 2	*Tool 3*	*Tool 3*	*Tool 2*	*Tool 2*
β	0.799	0.966	1.063	0.764
η	21.16	34.46	25.89	15.03
Step 3	*Tool 1*	*Tool 5*	-	*Tool 3*
β	0.896	1.128		0.799
η	18.90	15.65		21.16
Step 4	*Tool 4*	-	-	-
β	0.990			
η	20.24			

Processing of the jobs:
Job: 4
Add tool: 1
Add tool: 2
Add tool: 3
Step 1 cost(1) : 21.274 ; the amount of wear on the first step
Step 2 cost(2) : 18.362 ; “” on the second step
Step 3 cost(3) : 26.582
Job: 3
Add tool: 5
Step 1 cost(5) : 45.105
Step 2 cost(2) : 26.894 --> break-down -5.256725 ; tool wears more than five units over the wear-out limit
Step 1 cost(5) : 45.467
Step 2 cost(2) : 28.650
Job: 2
Replace tool: 5
Step 1 cost(1) : 22.448
Step 2 cost(3) : 40.556
Step 3 cost(5) : 19.349
Job: 1
Add tool: 3
Replace tool: 1
Step 1 cost(1) : 23.290
Step 2 cost(3) : 22.474
Step 3 cost(1) : 22.587
Step 4 cost(4) : 21.781

Fig. 5 A simple problem instance and the flow of a single simulation run with the stochastic cost evaluation algorithm (SCEA). Magazine size is 3 tools and the jobs are processed in SPT-order (job 4, job 3, job 2 and last job 1). In this flow the EV(1.2)-rule is used as the replacement rule and the removal rule is SUM. The tool wear is modelled with Weibull distribution and the parameters for each step are given in the job data. Symbols η and β are parameters of the Weibull distribution. The processing history includes one tool breakdown (tool 1, job 3) and in total 2 tool replacements

permutations; an individual of the GA population is here expressed as a vector of perturbation factors, which are real numbers from interval (0,1]. Each job in J is associated with its perturbation factor which is used as a multiplier to the processing time of the job in question. It is then possible to use a deterministic base heuristic to calculate an order for the jobs from these perturbed processing times. The heuristic must be deterministic in the sense that when it is given certain processing times, it produces always the same order. SSA uses the SPT-rule which sorts the jobs to an order of increasing processing times. It is now easy to see that by choosing the perturbation factors for the jobs properly, one can generate any of the permutations of the jobs by applying the SPT-rule to these perturbed processing times. In this way, an individual which is an ordering of the jobs is described fully by two components: a perturbation vector and the fixed base heuristic.

The fitness of an individual is evaluated in two steps. In the first step SSA uses the SPT-heuristic (our base heuristic) to create an ordered sequence of the jobs. The processing times which have been perturbed (multiplied) with the perturbation vector of the individual are used in this step. The result of the first step is then used as an input to the cost evaluation algorithm SCEA. The result given by the cost evaluation algorithm, which is an approximate of the average processing cost for the given order, is the fitness of the individual (This is the major difference between GAPSM and SSA, since GAPSM uses a deterministic cost evaluation algorithm due to the different nature of the problem it solves).

In the crossover, parents are chosen by the tournament selection method. During each tournament round candidate parents are selected randomly from the population and compared against the current ones. The number of tournament rounds is selected randomly from an interval which is given as a parameter. For example, if the number of tournament rounds is 0 then the selection of the parents is purely random. After the selection of the parents, the algorithm performs one-point crossover by taking the first half of the perturbation factors from the other parent and the rest from the other. These halves are then concatenated to form an offspring.

Each individual in the population is allowed to mutate with a small probability during an iteration. When mutation event occurs, the algorithm selects one random position in the perturbation vector of the individual in question and replaces it with new random number from interval (0,1].

One offspring is created in a manner described earlier. This offspring then replaces the worst individual in the current population (i.e. the individual with the highest average processing cost). All the other individuals are directly transferred to the next iteration.

The SSA algorithm begins by creating an initial population of certain size (given as a parameter). This population is created purely random so that for each individual every perturbation factor is chosen from a uniform distribution of (0,1]. In the evolution phase SSA selects the individuals for the next generation through direct transfer and crossover. After the selection, each individual of the next generation is allowed to go through a mutation with a given probability (a parameter). The evolution phase is repeated a predefined number of iterations and the result of the SSA is the order described by the individual with the best fitness after the last iteration.

4 Empirical results

The number of tools and jobs varies greatly depending on the size of manufacturing facility and the type of parts processed. For example a manufacturer producing large engines may use a flexible machine for which the capacity of the tool magazine is several hundreds of tools. In contrast to this, small workshops may have machines for which the capacity of the tool magazine is less than 20 tools.

The data generation process first created the tool types. The maximum wear-out value for tools was selected from a uniform distribution as well as the change time. Once the tool types had been created, the process generated a large number of jobs. For a job the number of steps is selected from uniform distribution. After this, the process created a Weibull distribution for each tool-wearing step. The parameters of this Weibull distribution (β and η) were taken from normal distributions, and the processing step time was simply the expected value of the Weibull distribution. The problem instances were created finally by selecting random jobs without replacement from the large job set created earlier.

The selection of parameters for the data generation is a difficult task because there is a certain realm in which the algorithms we presented are useful. For example if the tool changes dominate over the job processing times then it is more useful to minimize the number of tool changes. On the other hand if job processing times dominate over the tool change times the SPT-rule will most likely be an effective solution method. In Fig. 6 we give the parameters used in the data generation.

4.1 Estimating the variance

In the first set of test runs we measured the standard deviation of the processing costs when evaluated by SCEA. The primary objective was to find out how many simulation runs are needed for small enough standard deviation (<1%). Another objective was to determine how the problem size and replacement rule affect the standard deviation. The results of these tests are summarized in Table 1 from which

Number of tool types: 50

- Change time: UD(5, 15)
- Wear out value: UD(40, 100)
- Distribution: Weibull

Number of jobs: 200

- Number of steps: UD(4,20)
- Steps:
  - Distribution: Weibull (depends on the tool type)
    - β ND(1,0.2)
    - η ND(0.3 * tool lifetime, 0.1)

Number of problem instances: 90

- Created with random sampling from the whole set of jobs:
  - 1) 30 problem instances with 20 jobs;
  - 2) 30 instances with 60 jobs;
  - 3) 30 instances with 100 jobs

Fig. 6 Parameters of the test data generation process. In feasibility checks some of the jobs were replaced because they did not fulfil the feasibility conditions 1 and 2′ set for the problem instances

Table 1 Standard deviation of the total processing costs using SCEA for DN and EV-replacement rules

Number of simulation runs	20 jobs, DN (%)	20 jobs, EV(0.5) (%)	100 jobs, DN (%)	100 jobs EV(0.5) (%)
1	54	0.61	69	0.28
10	15	0.39	20	0.18
50	8.4	0.19	7.8	0.09
100	6.2	0.12	5.9	0.07
200	4.1	0.08	4.0	0.02

Standard deviation of processing cost is calculated from 30 cost evaluations for single problem instance. The deviation is then normalized by the mean of the processing cost

one can see that the standard deviation diminishes rapidly as the number of simulation runs increases.

For small problem instances the number of simulation runs needed was greater than for larger problem instances. One reason for this is that a single tool breakdown in them causes larger proportional variation in the costs. Another observation is that without any replacement rule (i.e. "change when breaks") the variance is large because there are various tool breakdowns which makes the evaluated cost more dependent on the random events. This must be taken into account when comparing the replacement and removal rules.

4.2 Tuning the cost evaluation algorithm

In the second set of test runs the objective was to determine the best combination of the removal/replacement rules for tool management and the parameters of this combination. The best value for the r-parameter of the EV(r)-replacement rule was searched first and after this, five different removal rules KTNS, KTWL, HYBRID(0.5), SUM and TI were compared. The TI-rule has been proposed by Turkcan et al. (2003) and it calculates the number of operations the tool can still perform and multiplies it with the tool's remaining lifetime.

Figure 7 shows for the EV(r)-rule the effect of the r-parameter on the average job processing costs for the sample problems of Fig. 6. One can see that the optimal value for both small and large problem instances is 0.8 for the EV(r)-rule. However, one may not conclude that this value is best for all types of problem instances, instead of that it should always be tuned for the particular problem type in question. The shape of the curve in Fig. 7 can be explained as follows. Small r-values lead to excessive number of tool replacements and large r to large number of tool breakdowns. The optimal "risk" of tool breakdowns is somewhere between these values. The problem size affects on how large proportional effect the excessive tool changes have, i.e. for small problem instances the effect is larger as it can be seen from Fig. 7 ($r > 1.3$).

The combination of removal and replacement rules should work on all possible orderings of the jobs and therefore we used 30 random sequences when determining the best combination of these rules. From Table 2 one can see that the SUM-rule

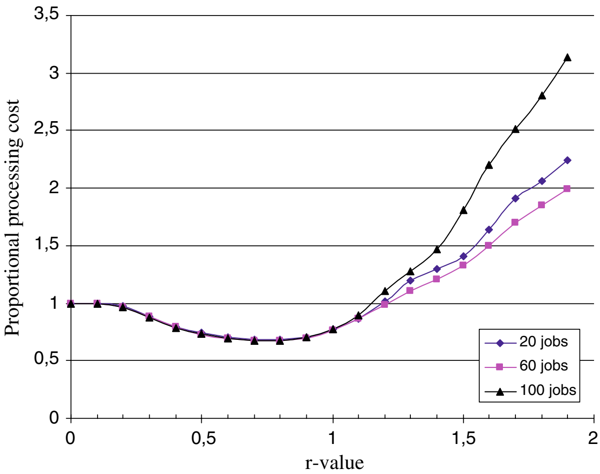


Fig. 7 The effect of parameter r to the average processing cost. The average cost was estimated from 30 random job sequences. Because these values are in different scale for different sized problems the values were calculated proportional to the EV(0.0)-value. The number of simulation runs was 200 and the tool removal rule was random selection

Table 2 The effect of removal rules on the average processing cost

	20 jobs		60 jobs		100 jobs	
	Best values	Average-%	Best values	Average-%	Best values	Average-%
KTNS	2	0.00	2	0.00	2	0.00
KTWL	0	1.81	0	1.91	0	1.94
RANDOM	0	4.21	0	3.72	0	4.15
TI	0	1.71	0	1.31	0	1.67
SUM	28	−0.74	28	−0.72	28	−0.63

The average processing cost was calculated for 30 random job permutations and their results were averaged. The "average-%" values are calculated as proportional difference to the KTNS-rule. "Best values" give the number of times out of 30 tests the rule has found the best solution among the five rules

was slightly better than the other removal rules. Surprisingly the KTWL-rule did not perform very well. Also the TI-rule was significantly worse than the new SUM-rule.

4.3 Comparison of SSA against traditional methods

The results of SSA with the proposed stochastic cost evaluation algorithm were compared against the SPT-rule. We also compared various rule combinations against each other. The results in Table 3 show that the SSA with SCEA using the new removal rules is significantly better than the previous methods.

Table 3 Comparison of the SPT-rule and the SSA-rule using different removal methods

	20 jobs		60 jobs		100 jobs	
	Best	Average	Best	Average	Best	Average
SPT(SUM, EV(0.8))	3	62,440	2	536,900	8	1452,000
SPT(TI, EV(0.8))	0	63,810	0	552,200	0	1512,000
SPT(KTWL, EV(0.8))	0	65,100	0	552,600	0	1508,000
SSA(SUM, EV(0.8))	21	61,490	21	526,500	21	1437,000
SSA(TI, EV(0.8))	5	61,850	4	534,400	0	1467,000
SSA(KTWL, EV(0.8))	1	63,240	3	534,700	1	1465,000

5 Conclusions

Stochastic scheduling and tool management were considered in this work. The complexity of the problem prohibited us from using standard stochastic optimization methods, like scenarios and analytical approaches. This complexity originated from the recursive nature of the problem which requires, in a case of tool breakdown, returning to the earlier stages of the processing. The feasibility and optimality of the solutions were also discussed. We concluded that an accurate feasibility check is difficult to do in a general case.

We proposed a cost evaluation algorithm which approximates the processing cost as an average of several hundreds of realizations of the simulated manufacturing process. The cost evaluation algorithm was parameterized for different removal and replacement rules. The new tool removal rule, EV(r), turned out to be superior in comparison to other rules tested here.

A genetic algorithm (SSA) was given for solving the stochastic scheduling and tool management problem. The empirical tests indicate that SSA gives considerably better results than processing with the traditional SPT-rule. The running times of SSA were acceptable for practical applications.

The mathematical formulation of the JSSTL-problem is complex and it cannot be solved with the existing optimization packages. Simplifying the problem to a form which enables one to use traditional (i.e. analytical or scenario) approaches requires more research. The cost function also includes only the time cost; material costs incurring from discarded jobs are not yet included to the model. Taking these into account requires the use of multi-objective optimization which would complicate the situation even further.

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