Int J Flex Manuf Syst (2005) 17: 5–21
DOI 10.1007/s10696-005-5991-7

A note on productivity gains in flexible robotic cells

H. Neil Geismar · Suresh P. Sethi · Jeffrey B. Sidney · Chelliah Sriskandarajah

Received: March 2004 / Accepted: September 2005

Abstract Flexible robotic cells combine the capabilities of robotic flow shops with those of flexible manufacturing systems. In an m-machine flexible cell, each part visits each machine in the same order. However, the m operations can be performed in any order, and each machine can be configured to perform any operation. We derive the maximum percentage increase in throughput that can be achieved by changing the assignment of operations to machines and then keeping that assignment constant throughout a lot's processing. We find that no increase can be gained in two-machine cells, and that the gain in three- and four-machine cells each is at most $14\frac{2}{7}\%$.

Keywords Flexible robotic cells · Robotic open shop · Scheduling · Sequencing

1. Introduction

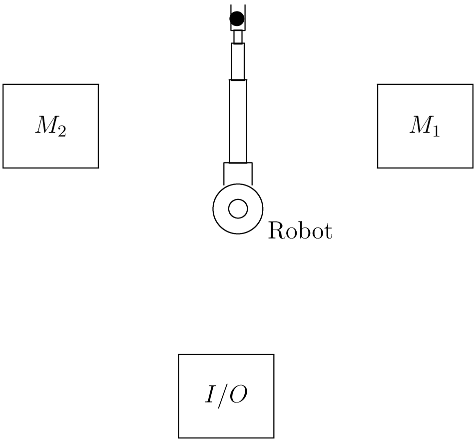
Robots are used for a wide range of applications in manufacturing companies (Asfahl 1985, Miller and Walker 1990). One important application of robots in manufacturing is their use for material handling in robotic cells. In such a cell, the robot is located at the approximate center of the workcell, and a number of machines (M_1, $M_2, \ldots,$ M_m) and an input/output (I/O) hopper are arranged around it. A real-world example of a three-machine robotic cell is given in Asfahl (1985). In this example, a robotic cell processes castings for truck differential

H. N. Geismar
Prairie View A&M University, USA
e-mail: hngeismar@pvamu.edu

S. P. Sethi · C. Sriskandarajah
University of Texas at Dallas, USA
e-mail: {sethi,chelliah}@utdallas.edu

J. B. Sidney
University of Ottawa, Canada
e-mail: sidney@admin.uottawa.ca

Fig. 1 A two machine robotic cell



assemblies. The cell includes a drilling machine (operation: cross pin drill), a boring machine (operation: bore pinion holes), another drilling machine (operation: ear hole drill), and an input/output hopper. Although these operations can be done in any order, for operational convenience and efficiency the cell operates as a flow shop in which the robot moves the parts from I/O to machines M_1, M_2, M_3, and finally to I/O.

Another example of the many manufacturing environments that can operate as an open shop (i.e., the order of the operations is immaterial) is at Xerox Corporation, Rochester, NY (Miller and Walker 1990, pp. 4–21). This system uses three independent robotic cells to produce a family of products called "duplicator fuser rollers." Each cell consists of two machine tools that are both CNC center-driver lathes. The entire system is under the supervision of a programmable controller. Multiple NC and robot programs facilitate flexible changeover from one product to another in the family of roller designs.

To generalize these examples, we consider a *circularly-configured robotic cell* in which each job (part) visits the machines in the following sequence: (I/O, M_1, $M_2, \ldots$, M_m, I/O). Each job requires m operations which can be performed in any order. Furthermore, an operation i requiring processing time of p_i can be assigned to any of the m machines by a relatively inexpensive (but not instantaneous) tool change. This capability, called *operation flexibility* (Sethi and Sethi 1990), arises from the use of flexible manufacturing machines which have quick tool-changing capability. These systems, called *flexible robotic cells*, can process differing lots with differing processing requirements (differing processing times or differing operations) efficiently by changing the assignment of operations to machines, i.e., reconfiguring machines between lots can improve throughput.

A two-machine robotic cell is illustrated in Figure 1. The machines are served by a central robot. The robot arm rotates to handle inter-machine movements of parts. A part is picked up at the hopper (I/O), processed once on each machine, and finally dropped at the hopper (I/O), i.e., after operations have been assigned to machines, the cell operates as a flow shop. The processing of any part on a machine is nonpreemptive. Each machine can process at most one part at a time and has neither an input nor an output buffer. Thus, any part in the cell is always either on one of the machines, at I/O, or being handled by the robot. Moreover, even if a part A has completed processing on a machine, no other part can be loaded onto that machine until the robot has removed part A from the machine (this is a *blocking* condition). After loading a part onto a machine, the robot either waits at that machine for the part to finish processing, moves to another machine to unload a different part (when that part finishes processing on that machine), or moves to the I/O hopper to pick up a new part.

The purpose of this paper is to investigate the productivity gains that can be achieved by using flexible robotic cells. Our goal is to aid practitioners who must decide whether the marginal benefits of using flexible cells outweigh the marginal costs. We show that for $m \leq 4$ there is an upper bound on the improvement in throughput that can be realized by using a flexible robotic cell. Our methodology is to compare flexible cells' throughputs for all possible assignments of operations to machines and all possible combinations of processing times for these operations. Since for a given lot the parts produced are identical, for a given assignment of operations to machines, we need only to examine different sequences of moves performed by the robot.

2. Literature review

Because detailed reviews of the literature on robotic cells can be found in surveys by Crama et al. (2000) and Dawande et al. (2005b), we give only a brief summary here. A thorough survey of the literature concerning flexibility in manufacturing can be found in Sethi and Sethi (1990).

Many analytic studies of robotic cell scheduling consider cells in which the machines are arranged linearly. In a seminal paper, Sethi et al. (1992) consider the problem of minimizing the cycle time in robotic flow shop cells with $m = 2$ and $m = 3$. For two-machine cells, the optimal solution is given by a simple cycle producing one unit. Crama and van de Klundert (1999) and Brauner and Finke (1999) show that the best one-unit cycle is optimal among the class of all cyclic solutions in the three-machine case. Brauner and Finke (1997, 2001) show that in m-machine cells (for $m \geq 4$), the best one-unit cycle is not necessarily optimal over the class of all cyclic solutions.

Circularly-configured cells are common in semiconductor manufacturing (Herrmann et al. 2000, Perkinson et al. 1994, Perkinson et al. 1996, Venkatesh et al. 1997, Wood 1996). Cycle times for these cells with $m = 2$ and $m = 3$ are derived in Sethi et al. (2001). Analysis of such cells with dual gripper robots can be found in Sriskandarajah et al. (2004) and Drobouchevitch et al. (2004).

3. Notation

The following notation used to describe a robotic cell is similar to that in Sethi et al. (1992):

$M_1, \ldots, M_m$: the machines in the robotic cell in the processing order.
I/O: the input/output hopper, also called M_0 or M_{m+1} (and referred to as a machine).
p_j: the processing time of operation j.
σ: permutation of operations $o_1, \ldots, o_m$.
vector $(\sigma(1), \ldots, \sigma(m))$: order of the processing operations, where σ is the permutation of operations, $\sigma(i)$ is the ith operation, and $\sigma(i)$ is performed on machine M_i.
δ: the time taken by a rotational robot movement when traveling between two consecutive machines M_{j-1} and M_j, $1 \leq j \leq m+1$, where both M_0 and M_{m+1} mean I/O.
ϵ: the time taken by the robot to pick up or drop off a part at I/O, or the time taken by the robot to load or unload a part at any machine.

$S_{i,m}$: robot move cycle i for a cell having m machines.
T_i: the cycle time for robot move cycle $S_{i,m}$ (it will be apparent from context how many machines are in the cell being discussed).

When travelling between two non-adjacent machines, the robot passes each intervening machine. It may travel in either direction around the cell. Hence, the travel time between machines M_i and M_j is $\min\{|j-i|, m+1-|j-i|\}\delta$.

The standard classification scheme for scheduling problems (Graham et al., 1979) as updated for robotic cells by Dawande et al. (2005b) denotes the robotic flow shop scheduling problem by $RF_m|(blocking,A,cyclic\text{-}1)|\mu$. The three fields indicate the scheduling environment (RF_m: m-machine robotic flow shop), restrictive requirements (*blocking,A,cyclic*-1: the cell has blocking, additive travel-time, and we seek one-unit cyclic solutions), and the objective function to be minimized (μ : the throughput). The scheduling problem for flexible robotic cells with circular layouts is denoted by $FRC_m^{\circ}|(blocking,A,cyclic\text{-}1)|\mu$.

4. Cyclic production

The concept of *activity* is very useful in the study of robotic cells. Activity A_i, $i = 0, \ldots, m$, consists of the following sequence:

1. The robot unloads a part from M_i
2. The robot travels from M_i to M_{i+1}
3. The robot loads this part onto M_{i+1}.

The activity sequence (A_i, A_k) implies that after completing activity A_i by loading machine M_{i+1}, the robot travels to machine k to begin activity A_k.

The study of cyclic production is motivated by its prevalence in industrial implementations. Additionally, Dawande et al. (2005a) show that it is sufficient to consider only cyclic solutions in order to maximize throughput. Cyclic production employs a repeatable sequence of activities:

Definition. A *k-unit cycle* is the performance of a feasible sequence of robot moves which loads and unloads each machine exactly k times in a way which leaves the cell in exactly the same state as its state at the beginning of those moves.

To be *feasible*, a sequence of activities must satisfy two criteria:

- The robot cannot be instructed to load an occupied machine.
- The robot cannot be instructed to unload an unoccupied machine.

All one-unit cycles are feasible.

Let the function $F(A_i, t)$ represent the time of completion of the tth execution of any activity A_i, for fixed i.

Definitions (Crama and van de Klundert 1997). A robotic cell repeatedly executing a k-unit cycle π of robot moves is operating in *steady state* if there exist constants $T(\pi)$ and N such that for every A_i, $i = 0, \ldots, m$, and for every $t \in \mathbb{Z}^+$ such that $t > N$, $F(A_i, t+k) - F(A_i, t) = T(\pi)$. $T(\pi)$ is called the *cycle time* of π.

The per unit cycle time of a k-unit cycle π is $T(\pi)/k$. This is the reciprocal of the throughput and is easier to calculate directly. Therefore, rather than maximizing throughput, we minimize per unit cycle time. In this study we consider only one-unit cycles, so the cycle time equals the per unit cycle time.

Brauner and Finke (2001) show that repeating a k-unit activity sequence will enable the robotic cell to reach a steady state (or cyclic solution) in finite time. Therefore, since we are maximizing the long-run average throughput, i.e., assuming that the cells operate in steady state for an infinite time, there is no contribution from the initial transient phase. Hence, there is no loss of generality by studying only the steady state behavior.

4.1. Two-machine robotic cells

It has been proven (Sethi et al. 1992) that in an m machine robotic cell there are $m!$ one-unit cycles, corresponding to the $m!$ permutations of $\{A_1, \ldots, A_m\}$. Thus, the two robot move cycles in a two-machine robotic cell are $S_{1,2} = (A_0, A_1, A_2)$ and $S_{2,2} = (A_0, A_2, A_1)$. Sethi et al. (2001) show that the cycle times in a circularly-configured cell are

$$T_1 = 3\delta + 6\epsilon + p_1 + p_2 \tag{1}$$

$$T_2 = \max\{6\delta + 6\epsilon,\ p_1 + 3\delta + 4\epsilon,\ p_2 + 3\delta + 4\epsilon\}. \tag{2}$$

The derivation of these formulas can be found in the appendix. They lead to the following result.

Lemma 1. *In* $RF_2^\circ|(blocking,A,cyclic\text{-}1)|\mu$, *cycle* $S_{1,2}$ *is optimal if* $\delta \geq (p_1 + p_2)/3$, *whereas cycle* $S_{2,2}$ *is optimal if* $\delta \leq (p_1 + p_2)/3$.

Proof: Follows from equations (1) and (2). See also Sethi et al. (1992). □

4.2. Three-machine robotic cells

In a three-machine robotic cell, the six one-unit cycles are

$$S_{1,3} = (A_0, A_1, A_2, A_3),\quad S_{2,3} = (A_0, A_2, A_1, A_3),$$
$$S_{3,3} = (A_0, A_1, A_3, A_2),\quad S_{4,3} = (A_0, A_3, A_1, A_2),$$
$$S_{5,3} = (A_0, A_2, A_3, A_1),\quad S_{6,3} = (A_0, A_3, A_2, A_1).$$

Their cycle times are presented in the following lemma.

Lemma 2. *For problem* $RF_3^\circ|(blocking,A,cyclic\text{-}1)|\mu$, *the cycle times of the six one-unit cycles* ($S_{1,3}, \ldots, S_{6,3}$) *are given by:*

$$T_1 = 4\delta + 8\epsilon + p_1 + p_2 + p_3,$$
$$T_2 = \max\left\{8\delta + 8\epsilon, 4\delta + 6\epsilon + p_1, 4\delta + 4\epsilon + p_2, 4\delta + 6\epsilon + p_3, 2\delta + 4\epsilon + \tfrac{p_1 + p_2 + p_3}{2}\right\},$$
$$T_3 = \max\{8\delta + 8\epsilon + p_1, 4\delta + 6\epsilon + p_1 + p_2, 4\delta + 4\epsilon + p_3\},$$
$$T_4 = \max\{8\delta + 8\epsilon + p_2, 4\delta + 6\epsilon + p_2 + p_3, 4\delta + 6\epsilon + p_1 + p_2\},$$

$T_5 = \max\{8\delta + 8\epsilon + p_3, 4\delta + 6\epsilon + p_2 + p_3, 4\delta + 4\epsilon + p_1\}$,

$T_6 = \max\{12\delta + 8\epsilon, 4\delta + 4\epsilon + p_1, 4\delta + 4\epsilon + p_2, 4\delta + 4\epsilon + p_3\}$.

Proof: Similar to the derivation of equations (1) and (2) and Sethi et al. (1992). □

4.3. Four-machine robotic cells

In a four-machine robotic cell, the twenty-four one-unit cycles are

$$\begin{aligned}
S_{1,4} &= (A_0, A_1, A_2, A_3, A_4),\ S_{2,4} = (A_0, A_1, A_3, A_2, A_4),\\
S_{3,4} &= (A_0, A_2, A_1, A_3, A_4),\ S_{4,4} = (A_0, A_2, A_3, A_1, A_4),\\
S_{5,4} &= (A_0, A_3, A_1, A_2, A_4),\ S_{6,4} = (A_0, A_3, A_2, A_1, A_4),\\
S_{7,4} &= (A_0, A_2, A_3, A_4, A_1),\ S_{8,4} = (A_0, A_3, A_2, A_4, A_1),\\
S_{9,4} &= (A_0, A_3, A_4, A_1, A_2),\ S_{10,4} = (A_0, A_4, A_1, A_2, A_3),\\
S_{11,4} &= (A_0, A_2, A_4, A_1, A_3),\ S_{12,4} = (A_0, A_4, A_1, A_3, A_2),\\
S_{13,4} &= (A_0, A_1, A_3, A_4, A_2),\ S_{14,4} = (A_0, A_3, A_1, A_4, A_2),\\
S_{15,4} &= (A_0, A_3, A_4, A_2, A_1),\ S_{16,4} = (A_0, A_4, A_2, A_1, A_3),\\
S_{17,4} &= (A_0, A_1, A_4, A_2, A_3),\ S_{18,4} = (A_0, A_4, A_2, A_3, A_1),\\
S_{19,4} &= (A_0, A_1, A_2, A_4, A_3),\ S_{20,4} = (A_0, A_2, A_1, A_4, A_3),\\
S_{21,4} &= (A_0, A_2, A_4, A_3, A_1),\ S_{22,4} = (A_0, A_4, A_3, A_1, A_2),\\
S_{23,4} &= (A_0, A_1, A_4, A_3, A_2),\ S_{24,4} = (A_0, A_4, A_3, A_2, A_1).
\end{aligned}$$

The cycle times for these cycles are presented in the following lemma. Note that in a four-machine cell, when the robot moves from M_4 to M_1 (or vice versa), it travels via I/O (requiring time 2δ) rather than via M_2 and M_3 (which would require time 3δ).

Lemma 3. *For problem $RF_4^{\circ}|(blocking,A,cyclic\text{-}1)|\mu$, the cycle times of the twenty-four one-unit cycles are given by:*

$$\begin{aligned}
T_1 &= 5\delta + 10\epsilon + p_1 + p_2 + p_3 + p_4\\
T_2 &= \max\{9\delta + 10\epsilon + p_1, 4\delta + 4\epsilon + p_3, 5\delta + 8\epsilon + p_2 + p_1, 5\delta + 8\epsilon + p_4 + p_1,\\
&\quad (5\delta + 10\epsilon + p_1 + p_2 + p_3 + p_4)/2\}\\
T_3 &= \max\{9\delta + 10\epsilon + p_4, 4\delta + 4\epsilon + p_2, 5\delta + 8\epsilon + p_1 + p_4, 5\delta + 8\epsilon + p_3 + p_4,\\
&\quad (5\delta + 10\epsilon + p_1 + p_2 + p_3 + p_4)/2\}\\
T_4 &= \max\{10\delta + 10\epsilon + p_3, 5\delta + 6\epsilon + p_2 + p_3, 5\delta + 6\epsilon + p_1, 5\delta + 8\epsilon + p_4 + p_3,\\
&\quad (5\delta + 10\epsilon + p_1 + p_2 + p_3 + p_4)/2\}\\
T_5 &= \max\{10\delta + 10\epsilon + p_2, 5\delta + 6\epsilon + p_3 + p_2, 5\delta + 8\epsilon + p_1 + p_2, 5\delta + 6\epsilon + p_4,\\
&\quad (5\delta + 10\epsilon + p_1 + p_2 + p_3 + p_4)/2\}
\end{aligned}$$

$$T_6 = \max\{13\delta + 10\epsilon, 4\delta + 4\epsilon + p_3, 4\delta + 4\epsilon + p_2, 5\delta + 6\epsilon + p_1, 5\delta + 6\epsilon + p_4, (5\delta + 10\epsilon + p_1 + p_2 + p_3 + p_4)/3\}$$

$$T_7 = \max\{9\delta + 10\epsilon + p_3 + p_4, 5\delta + 8\epsilon + p_2 + p_3 + p_4, 4\delta + 4\epsilon + p_1\}$$

$$T_8 = \max\{13\delta + 10\epsilon, 4\delta + 4\epsilon + p_3, 5\delta + 6\epsilon + p_2, 9\delta + 8\epsilon + p_4, 4\delta + 4\epsilon + p_1\}$$

$$T_9 = \max\{10\delta + 10\epsilon + p_4 + p_2, 5\delta + 8\epsilon + p_3 + p_4 + p_2, 5\delta + 6\epsilon + p_1 + p_2\}$$

$$T_{10} = \max\{9\delta + 10\epsilon + p_2 + p_3, 5\delta + 8\epsilon + p_4 + p_2 + p_3, 5\delta + 8\epsilon + p_1 + p_2 + p_3\}$$

$$T_{11} = \max\{10\delta + 10\epsilon, 5\delta + 6\epsilon + p_2, 5\delta + 6\epsilon + p_4, 5\delta + 6\epsilon + p_1, 5\delta + 6\epsilon + p_3\}$$

$$T_{12} = \max\{13\delta + 10\epsilon, 5\delta + 6\epsilon + p_4, 9\delta + 8\epsilon + p_1, 4\delta + 4\epsilon + p_3, 9\delta + 8\epsilon + p_2, 5\delta + 6\epsilon + p_1 + p_2\}$$

$$T_{13} = \max\{10\delta + 10\epsilon + p_1 + p_4, 5\delta + 6\epsilon + p_3 + p_4, 5\delta + 6\epsilon + p_2 + p_1\}$$

$$T_{14} = \max\{15\delta + 10\epsilon, 10\delta + 8\epsilon + p_3, 10\delta + 8\epsilon + p_1, 10\delta + 8\epsilon + p_4, 10\delta + 8\epsilon + p_2, 5\delta + 6\epsilon + p_3 + p_4, 5\delta + 6\epsilon + p_3 + p_2, 5\delta + 6\epsilon + p_1 + p_2\}$$

$$T_{15} = \max\{13\delta + 10\epsilon + p_4, 5\delta + 6\epsilon + p_3 + p_4, 4\delta + 4\epsilon + p_2, 4\delta + 4\epsilon + p_1\}$$

$$T_{16} = \max\{13\delta + 10\epsilon, 9\delta + 8\epsilon + p_4, 4\delta + 4\epsilon + p_2, 5\delta + 6\epsilon + p_1, 9\delta + 8\epsilon + p_3, 5\delta + 6\epsilon + p_4 + p_3\}$$

$$T_{17} = \max\{10\delta + 10\epsilon + p_1 + p_3, 5\delta + 6\epsilon + p_4 + p_3, 5\delta + 8\epsilon + p_2 + p_1 + p_3\}$$

$$T_{18} = \max\{13\delta + 10\epsilon + p_3, 5\delta + 6\epsilon + p_4 + p_3, 5\delta + 6\epsilon + p_2 + p_3, 4\delta + 4\epsilon + p_1\}$$

$$T_{19} = \max\{9\delta + 10\epsilon + p_1 + p_2, 4\delta + 4\epsilon + p_4, 5\delta + 8\epsilon + p_3 + p_1 + p_2\}$$

$$T_{20} = \max\{13\delta + 10\epsilon, 4\delta + 4\epsilon + p_2, 9\delta + 8\epsilon + p_1, 4\delta + 4\epsilon + p_4, 5\delta + 6\epsilon + p_3\}$$

$$T_{21} = \max\{13\delta + 10\epsilon, 9\delta + 8\epsilon + p_2, 4\delta + 4\epsilon + p_4, 9\delta + 8\epsilon + p_3, 4\delta + 4\epsilon + p_1, 5\delta + 6\epsilon + p_2 + p_3\}$$

$$T_{22} = \max\{13\delta + 10\epsilon + p_2, 4\delta + 4\epsilon + p_4, 5\delta + 6\epsilon + p_3 + p_2, 5\delta + 6\epsilon + p_1 + p_2\}$$

$$T_{23} = \max\{13\delta + 10\epsilon + p_1, 4\delta + 4\epsilon + p_4, 4\delta + 4\epsilon + p_3, 5\delta + 6\epsilon + p_2 + p_1\}$$

$$T_{24} = \max\{15\delta + 10\epsilon, 4\delta + 4\epsilon + p_4, 4\delta + 4\epsilon + p_3, 4\delta + 4\epsilon + p_2, 4\delta + 4\epsilon + p_1\}$$

Proof: Similar to the derivation of equations (1) and (2) and Sethi et al. (1992). □

5. Flexible robotic cells: throughput comparison

We examine the throughput gains in flexible robotic cells for all possible assignments of operations to machines. The analysis of this problem ($FRC_m^\circ|(blocking,A,cyclic\text{-}1)|\mu$ for $m = 2, 3, 4$) is based on two assumptions:

1. Any of the $m!$ processing orders for the operations of a part in an m-machine cell is feasible.

2. The machines are converted to conform with to the specified order of operations. For example, if $m = 3$ and the operations are processed in the order (3, 1, 2), then operation 3 is processed for p_3 time on M_1, operation 1 is processed for p_1 time on M_2, and operation 2 is processed for p_2 time on M_3.

5.1. Comparison of performance: two-machine robotic cells

We now show that in problem $FRC_2^\circ|(blocking,A,cyclic\text{-}1)|\mu$ the flexibility to assign operations to machines does not allow for an increase in throughput.

Lemma 4. *The flexibility to assign operations to machines in a two-machine flexible robotic cell ($FRC_2^\circ|(blocking,A,cyclic\text{-}1)|\mu$) provides no increase in throughput.*

Proof: That the two permutations (i, j) and (j, i) will have the same time performance can be seen by interchanging values of p_1 and p_2 in equations (1) and (2). □

5.2. Comparison of performance: three-machine robotic cells

In linearly-configured three-machine cells, the optimal cyclic solution is given by a one-unit cycle (Brauner and Finke 1999). No such result has been proven for circularly-configured cells. However, we limit our analysis to only one-unit cycles ($FRC_3^\circ|(blocking,A,cyclic\text{-}1)|\mu$) because of their prevalence in practice (Dawande et al. 2002) and their simplicity. Of course, the methodology we describe here could be used to check whether multi-unit cycles could provide an improvement in cycle time. However, because the number of multi-unit cycles is large (e.g., there are 20 two-unit cycles for $m = 3$ and 260 two-unit cycles for $m = 4$ (Geismar et al. 2004)), such an analysis is beyond the scope of this note.

Additionally, in circularly-configured cells, pyramidal cycles do not dominate, as they do for linearly-configured cells (Crama and van de Klundert 1997). For example, if $\delta = 1.0$, $\epsilon = 0.5$, $p_1 = 5$, $p_2 = 3$, $p_3 = 5$, then non-pyramidal cycle $S_{2,3}$ is uniquely optimal among one-unit cycles: $T_1 = 21$, $T_2 = 12$, $T_3 = 17$, $T_4 = 15$, $T_5 = 17$, $T_6 = 16$.

As a result of the two assumptions stated at the start of Section 5, the formulas for T_i $(1 \le i \le 6)$ stated in Lemma 2 may be applied to any of the six permutations of the operations. For any order $\alpha = (i, j, k)$ of operations, let $T_i(\alpha)$, $1 \le i \le 6$, denote the corresponding value of the ith cycle time measure from Lemma 2 if the operations are processed in the order α. For example, if $\alpha = (2, 1, 3)$ then p_1 and p_2 are simply interchanged in each of the formulas, e.g., $T_3(\alpha) = \max\{8\delta + 8\epsilon + p_2, 4\delta + 6\epsilon + p_1 + p_2, 4\delta + 4\epsilon + p_3\}$. For the sake of efficiency in the remainder of this section, T_i will denote the cycle time value when the order of operations is (1, 2, 3).

Let $OPT(\alpha) = \min\{T_i(\alpha) \mid 1 \le i \le 6\}$. Let $T_{i,j}(\alpha)$ be the jth term in the maximization expression which gives the value of $T_i(\alpha)$, where the terms are ordered as in the formulas for T_i given in Lemma 2. For example, $T_{2,4}(1, 2, 3) = 4\delta + 6\epsilon + p_3$ and $T_{2,4}(2, 3, 1) = 4\delta + 6\epsilon + p_1$. The following theorem states that at most a $14\frac{2}{7}\%$ decrease in OPT can be obtained by changing the order of operations.

Theorem 1. *Let α and β be two different orders of the operations for a three-machine flexible robotic cell. Then $OPT(\beta) \le (7/6)OPT(\alpha)$, and this bound is tight.*

Proof : The proof will be presented in the form of six lemmas. The basic approach is to assume that an overall optimal solution (over all orders of operations (i, j, k)) is obtained using the order $(1, 2, 3)$. This leaves six possibilities as to which of the six expressions T_i, $1 \le i \le 6$, yields the optimal value OPT. For each $i = 1, \ldots, 6$, we assume $T_i = OPT$ and then show that for any order of operations $\beta \ne (1, 2, 3)$, there exists at least one j such that $T_j(\beta) \le (7/6)OPT = (7/6)T_i$.

Lemma 5. $T_2 \le T_4$.

Proof : Clearly, $T_{2,1} \le T_{4,1}$, $T_{2,2} \le T_{4,3}$, $T_{2,3} \le T_{4,3}$, and $T_{2,4} \le T_{4,2}$. Finally,

$$T_{2,5} = \ 2\delta + 4\epsilon + \sum \frac{p_i}{2} \le \ \frac{1}{2}(T_{4,2} + T_{4,3}),$$

Proof: which implies that either $T_{2,5} \le T_{4,2}$ or $T_{2,5} \le T_{4,3}$. Hence, for $j = 1, \ldots, 5$, $T_{2,j}$ is less than or equal to at least one of the $T_{4,k}$'s. □

As a result of Lemma 5, we need not analyze the case in which $T_4 = OPT$. Theorem 1 can be easily proven for two other cycles:

Lemma 6. *If $T_1 = OPT$ or $T_6 = OPT$, then Theorem* 1 *holds.*

$T_1(\alpha)$ is the same for all permutations α, and $T_6(\beta)$ is the same for all permutations β. Therefore, if $T_1 = OPT$, then $OPT(\alpha) \le T_1(\alpha) = T_1 = OPT$; if $T_6 = OPT$, then $OPT(\beta) \le T_6(\beta) = T_6 = OPT$.

Lemma 7. $T_1(\alpha) = OPT, \forall\alpha$, *if* $p_1 + p_2 + p_3 \le 4\delta$. $T_6(\alpha) = OPT, \forall\alpha$, *if* $\delta = 0$.

Proof: Easy and omitted. □

Hence, we may assume for the remainder of the proof that $0 < 4\delta < p_1 + p_2 + p_3$.

Lemma 8. *If* $0 < 4\delta < p_1 + p_2 + p_3$*, then* $T_2 = OPT$ *implies that Theorem* 1 *holds.*

Proof:

$$T_2 = \max\left\{8\delta + 8\epsilon, 4\delta + 6\epsilon + p_1, 4\delta + 4\epsilon + p_2, 4\delta + 6\epsilon + p_3, 2\delta + 4\epsilon + \left(\sum p_i/2\right)\right\}.$$

Suppose that for some permutation β and some number $\omega > 1$, we have

$$\begin{aligned} OPT(\beta) &= \min\{T_1(\beta), T_2(\beta), T_3(\beta), T_5(\beta), T_6(\beta)\} \\ &\ge \omega \cdot OPT = \omega T_2. \end{aligned} \tag{3}$$

First, note that relation (3) implies $T_2(\beta) > T_2$. Necessary conditions for $T_2(\beta) > T_2$ are $p_2 > \max\{p_1, p_3\}$ and that operation 2 is processed first or last among the operations of β, in which case $T_2(\beta) = 4\delta + 6\epsilon + p_2$.

Relation (3) also implies many other inequalities, some of which will be used to prove that $\omega \le 7/6$. First, we need to prove that $\omega < 5/4$. Suppose $\omega \ge 5/4$. Then,

$$\begin{aligned} T_2(\beta) = 4\delta + 6\epsilon + p_2 &\ge \frac{5}{4}T_2 \\ &\ge \frac{5}{4}\max\{T_{2,1}, T_{2,3}\}. \end{aligned}$$

Thus,

$$\begin{aligned} 4\delta + 6\epsilon + p_2 &\geq \frac{5}{4}T_{2,1} = 10\delta + 10\epsilon \\ &\iff p_2 \geq 6\delta + 4\epsilon, \end{aligned} \tag{4}$$

and

$$\begin{aligned} 4\delta + 6\epsilon + p_2 &\geq \frac{5}{4}T_{2,3} = 5\delta + 5\epsilon + \frac{5}{4}p_2 \\ &\iff p_2 \leq -4\delta + 4\epsilon. \end{aligned} \tag{5}$$

Since $\delta > 0$, (4) and (5) contradict each other, and we may conclude that $\omega < 5/4$. We now apply three other inequalities:

$$\begin{aligned} T_2(\beta) &\geq \omega T_2 \geq \omega T_{2,1} \\ 4\delta + 6\epsilon + p_2 &\geq \omega(8\delta + 8\epsilon) \\ &\iff p_2 \geq (8\omega - 4)\delta + (8\omega - 6)\epsilon \\ &\iff (\omega - 1)p_2 \geq (8\omega^2 - 12\omega + 4)\delta + (8\omega^2 - 14\omega + 6)\epsilon, \end{aligned} \tag{6}$$

and

$$\begin{aligned} T_2(\beta) &\geq \omega T_{2,3} \\ 4\delta + 6\epsilon + p_2 &\geq \omega(4\delta + 4\epsilon + p_2) \\ &\iff (\omega - 1)p_2 \leq (4 - 4\omega)\delta + (6 - 4\omega)\epsilon. \end{aligned} \tag{7}$$

Combining (6) and (7) yields

$$\begin{aligned} (8\omega^2 - 12\omega + 4)\delta + (8\omega^2 - 14\omega + 6)\epsilon &\leq (\omega - 1)p_2 \\ &\leq (4 - 4\omega)\delta + (6 - 4\omega)\epsilon \\ &\iff (8\omega - 8)\delta + (8\omega - 10)\epsilon \leq 0. \end{aligned} \tag{8}$$

$$\text{Since } 1 < \omega, \text{ we have } (8\omega - 8) > 0 \quad \text{and} \quad \frac{\delta}{\epsilon} \leq \frac{10 - 8\omega}{8\omega - 8}.$$

Note that

$$\begin{aligned} T_6 \geq \omega T_{2,3} &\Rightarrow T_6 = 12\delta + 8\epsilon. \quad \text{Thus,} \\ T_6(\beta) &= T_6 \geq \omega T_{2,1} \quad \text{implies} \\ 12\delta + 8\epsilon &\geq \omega(8\delta + 8\epsilon) \\ &\iff (12 - 8\omega)\delta + (8 - 8\omega)\epsilon \geq 0. \end{aligned} \tag{9}$$

Since $\omega < 5/4$, we have $(12 - 8\omega) > 0$ and $\dfrac{\delta}{\epsilon} \geq \dfrac{8\omega - 8}{12 - 8\omega}$.

Combining (8) with (9) yields

$$\frac{8\omega - 8}{12 - 8\omega} \leq \frac{10 - 8\omega}{8\omega - 8},$$

which when solved yields $\omega \leq 7/6$. □

Lemma 9. *The bound of* $(7/6)$ *in Lemma* 8 (*and therefore in Theorem* 1) *is tight.*

Proof: Use $\delta = 1, \epsilon = 2, p_1 = p_3 = 8$, and $p_2 = 12$ with $\beta = (2, 1, 3)$. □

Consider the two orders of operations $\alpha = (i, j, k)$ and $\beta = (k, j, i)$. It is easy to prove that $T_i(\alpha) = T_i(\beta)$ for $i = 1, 2, 4, 6$, and that

$$T_3(\alpha) = T_5(\beta) \text{ and } T_5(\alpha) = T_3(\beta).$$

Hence,

$$\begin{aligned} & OPT(\alpha) = T_3(\alpha) \Longleftrightarrow OPT(\beta) = T_5(\beta), \\ \text{and} \quad & OPT(\alpha) = T_5(\alpha) \Longleftrightarrow OPT(\beta) = T_3(\beta). \end{aligned}$$

From these symmetries, it follows that we need to consider only one of the two cases $T_3 = OPT$ and $T_5 = OPT$. We complete the proof of Theorem 1 by considering $T_5 = OPT$.

Lemma 10. *If* $0 < 4\delta < p_1 + p_2 + p_3$, *then* $T_5 = OPT$ *implies that Theorem* 1 *holds.*

Proof: Either T_2 is also optimal, i.e., $T_2 = T_5 = OPT$, in which case Lemma 8 applies, or $T_2 > T_5$. We therefore assume $T_2 > T_5$.

We use the same approach as in Lemma 8. First note that

$$\left.\begin{aligned} & T_{2,1} \leq T_{5,1}, \quad T_{2,3} \leq T_{5,2}, \quad T_{2,4} \leq T_{5,2}, \\ \text{and} \quad & T_{2,5} \leq \tfrac{1}{2}(T_{5,2} + T_{5,3}), \quad \text{which implies that} \\ & \text{either} \quad T_{2,5} \leq T_{5,2} \quad \text{or} \quad T_{2,5} \leq T_{5,3}. \end{aligned}\right\} \tag{10}$$

Since we are assuming that $T_2 > T_5$, relations (10) imply $T_{2,2} = T_2 > T_5 = OPT$. Thus,

$$T_{2,2} = 4\delta + 6\epsilon + p_1 > T_{5,2} = 4\delta + 6\epsilon + p_2 + p_3, \text{ so } p_1 > p_2 + p_3.$$

It follows that for any β,

$$T_2(\beta) \leq 4\delta + 6\epsilon + p_1 = T_2. \tag{11}$$

Assume that $\omega > 7/6$ is achievable using some permutation β. Then $T_6(\beta) = T_6 > (7/6)T_5$. If $T_6 = 4\delta + 4\epsilon + p_1$, then $4\delta + 4\epsilon + p_1 > (7/6)T_{5,3} = (7/6)(4\delta + 4\epsilon + p_1)$, which is a contradiction. If $T_6 = 12\delta + 8\epsilon$, then we have

$$\begin{aligned} T_6 &> \frac{7}{6}T_{5,1}, \\ 12\delta + 8\epsilon &> \frac{7}{6}(8\delta + 8\epsilon + p_3) \geq \frac{7}{6}(8\delta + 8\epsilon) \qquad (12) \\ &\Longleftrightarrow 2\delta > \epsilon, \end{aligned}$$

and

$$\begin{aligned} T_2 &\geq T_2(\beta) > \frac{7}{6}T_{5,3}, \\ 4\delta + 6\epsilon + p_1 &> \frac{7}{6}(4\delta + 4\epsilon + p_1) \\ &\iff -4\delta + 8\epsilon > p_1, \end{aligned} \tag{13}$$

and

$$\begin{aligned} T_2 \geq T_2(\beta) &> \frac{7}{6}T_{5,1}, \\ 4\delta + 6\epsilon + p_1 &> \frac{7}{6}(8\delta + 8\epsilon), \\ 6p_1 &> 32\delta + 20\epsilon. \end{aligned} \tag{14}$$

(13) and (14) imply

$$\begin{aligned} 32\delta + 20\epsilon &< -24\delta + 48\epsilon, \\ &\iff 2\delta < \epsilon. \end{aligned} \tag{15}$$

Since (12) contradicts (15), the assumption that $\omega > \frac{7}{6}$ is achievable must be wrong. This completes the proof of Lemma 10 and of Theorem 1. □

5.3. Comparison of performance: four-machine robotic cells

Although the optimal cycle may not be a one-unit cycle if $m = 4$, we limit our analysis to only one-unit cycles ($FRC_4^\circ|(blocking,A,cyclic\text{-}1)|\mu$) for the same reasons mentioned for $m = 3$ in Section 5.2.

The formulas for one-unit cycle times were developed in Lemma 3. For $m = 4$, we define $T_i(\alpha)$, $OPT(\alpha)$, $T_{i,j}(\alpha)$, and T_i, as natural extensions of their definitions for $m = 3$. We now show that for $m = 4$, at most a $14\frac{2}{7}\%$ increase in throughput can be obtained by changing the order of operations.

Theorem 2. *Let α and β be two different orders of the operations for a four-machine flexible robotic cell. Then $OPT(\beta) \leq (7/6)OPT(\alpha)$, and this bound is tight.*

Proof: The proof will be presented in a structure similar to that of Theorem 1: we use eight lemmas and consider cases that are defined by the value of i, where $T_i = OPT$. We first disqualify eight cycles because they are dominated by other cycles.

Lemma 11. *For a given assignment of operations to machines, we have the following dominance relationships: Cycles $S_{4,4}$, $S_{5,4}$, $S_{9,4}$, $S_{13,4}$, and $S_{17,4}$ are dominated by Cycle $S_{11,4}$. Cycles $S_{12,4}$, $S_{14,4}$, and $S_{16,4}$ are dominated by Cycle $S_{6,4}$.*

Proof: For cycles $S_{4,4}$ and $S_{11,4}$, $T_{11,1} \leq T_{4,1}$, $T_{11,2} \leq T_{4,2}$, $T_{11,3} \leq T_{4,4}$, $T_{11,4} \leq T_{4,3}$, and $T_{11,5} \leq T_{4,2}$. The proofs for the other pairs are similar. □

We now show symmetry for five pairs of cycles. These results imply that for each pair we need only to consider cases in which one of them is optimal.

Lemma 12. *Regarding Theorem* 2*, we have the following equivalences:*

- *Theorem* 2 *holds for* $T_2 = OPT$ *if and only if it holds for* $T_3 = OPT$.
- *Theorem* 2 *holds for* $T_7 = OPT$ *if and only if it holds for* $T_{19} = OPT$.
- *Theorem* 2 *holds for* $T_8 = OPT$ *if and only if it holds for* $T_{20} = OPT$.
- *Theorem* 2 *holds for* $T_{15} = OPT$ *if and only if it holds for* $T_{23} = OPT$.
- *Theorem* 2 *holds for* $T_{18} = OPT$ *if and only if it holds for* $T_{22} = OPT$.

Proof: It is straightforward to verify that $\min\{T_h(i, j, k, l)|1 \le h \le 24\} = \min\{T_h(l, k, j, i)|1 \le h \le 24\}$. That result along with the following equalities yields the result:

$$T_3(i, j, k, l) = T_2(l, k, j, i), T_{19}(i, j, k, l) = T_7(l, k, j, i),$$

$$T_{20}(i, j, k, l) = T_8(l, k, j, i), T_{22}(i, j, k, l) = T_{18}(l, k, j, i),$$

$$T_{23}(i, j, k, l) = T_{15}(l, k, j, i). \qquad \square$$

Hence, we need not consider cases in which cycles $S_{3,4}$, $S_{19,4}$, $S_{20,4}$, $S_{22,4}$, or $S_{23,4}$, are optimal.

Lemma 13. *Theorem* 2 *holds if* $T_1 = OPT$, $T_{11} = OPT$, *or* $T_{24} = OPT$.

Proof: T_1, T_{11}, and T_{24} are each independent of the assignment of operations to machines. Hence, $T_i(\beta) = T_i < (7/6)T_i$, $\forall\beta$, for $i \in \{1, 11, 24\}$. $\square$

Lemma 14. *Regarding Theorem* 2*, we have the following implications:*

- *If Theorem* 2 *holds for* $T_2 = OPT$*, then it holds for* $T_7 = OPT$.
- *If Theorem* 2 *holds for* $T_6 = OPT$*, then it holds for* $T_8 = OPT$.
- *If Theorem* 2 *holds for* $T_2 = OPT$*, then it holds for* $T_{10} = OPT$.
- *If Theorem* 2 *holds for* $T_6 = OPT$*, then it holds for* $T_{15} = OPT$.
- *If Theorem* 2 *holds for* $T_6 = OPT$*, then it holds for* $T_{18} = OPT$.
- *If Theorem* 2 *holds for* $T_6 = OPT$*, then it holds for* $T_{21} = OPT$.

Proof:

$$T_2(k, j, i, l) \le T_7(i, j, k, l), \quad T_6(j, i, k, l) \le T_8(i, j, k, l),$$

$$T_2(k, j, i, l) \le T_{10}(i, j, k, l), \quad T_6(k, j, i, l) \le T_{15}(i, j, k, l),$$

$$T_6(l, j, k, i) \le T_{18}(i, j, k, l), \quad T_6(j, i, l, k) \le T_{21}(i, j, k, l). \qquad \square$$

Lemma 15. $T_1(\alpha) = OPT, \forall\alpha$*, if* $p_1 + p_2 + p_3 + p_4 \le 4\delta$. $T_{24}(\alpha) = OPT, \forall\alpha$*, if* $\delta = 0$.

Proof: Trivial. $\square$

Our task now is to show that Theorem 2 holds for cells in which $0 < 4\delta < \sum p_i$ and either $OPT = T_2$ or $OPT = T_6$.

Lemma 16. *If* $0 < 4\delta < \sum p_i$*, then* $T_6 = OPT$ *implies that Theorem* 2 *holds.*

Proof:

Case 1. $T_{24}(\beta) = 15\delta + 10\epsilon, \forall\beta$. $T_6 \geq 13\delta + 10\epsilon, \forall\beta$, implies

$$\frac{T_{24}(\beta)}{T_6} \leq \frac{15\delta + 10\epsilon}{13\delta + 10\epsilon} \leq \frac{15}{13} < \frac{7}{6}.$$

Case 2. $T_{24}(\beta) = 4\delta + 4\epsilon + \max p_i, \forall\beta$. $T_6 \geq 4\delta + 4\epsilon + \max p_i, \forall\beta$, implies

$$\frac{T_{24}(\beta)}{T_6} \leq \frac{4\delta + 4\epsilon + \max p_i}{4\delta + 4\epsilon + \max p_i} \leq 1 < \frac{7}{6}.$$

□

Lemma 17. *If* $0 < 4\delta < \sum p_i$*, then* $T_2 = OPT$ *implies that Theorem* 2 *holds.*

Proof:

Case 1. $T_{11} = 10\delta + 10\epsilon$. $T_2 \geq p_1 + 9\delta + 10\epsilon$ implies that

$$\frac{T_{11}(\beta)}{T_2} \leq \frac{10\delta + 10\epsilon}{p_1 + 9\delta + 10\epsilon} \leq \frac{10}{9} < \frac{7}{6}.$$

Case 2. $T_{11} = 5\delta + 6\epsilon + \max p_i$. First, it is easy to see that $T_{11} > T_2$ implies that $T_{11}(\beta) = 5\delta + 6\epsilon + p_3,\ \forall\beta$. Furthermore, $T_6 > T_2$ implies that $T_6(\beta) = 13\delta + 10\epsilon,\ \forall\beta$.

(a) $T_2 = 9\delta + 10\epsilon + p_1$. This implies that $p_3 \leq 5\delta + 6\epsilon + p_1$, so

$$OPT(\beta) \leq \min\{T_6(\beta), T_{11}(\beta)\} \leq \min\{13\delta + 10\epsilon, 10\delta + 12\epsilon + p_1\}$$

Therefore,

$$\frac{OPT(\beta)}{T_2} \leq \min\left\{\frac{13\delta + 10\epsilon}{9\delta + 10\epsilon + p_1}, \frac{10\delta + 12\epsilon + p_1}{9\delta + 10\epsilon + p_1}\right\} \leq \min\left\{\frac{13\delta + 10\epsilon}{9\delta + 10\epsilon}, \frac{10\delta + 12\epsilon}{9\delta + 10\epsilon}\right\}.$$

Now assume that the result does not hold. This would imply that both terms in the minimization exceed $7/6$:

$$\frac{13\delta + 10\epsilon}{9\delta + 10\epsilon} > 7/6 \Rightarrow \epsilon < \frac{3}{2}\delta \quad \text{and} \quad \frac{10\delta + 12\epsilon}{9\delta + 10\epsilon} > 7/6 \Rightarrow \epsilon > \frac{3}{2}\delta.$$

This contradiction implies that at least one of these two terms must be less than $7/6$.

(b) $T_2 = 4\delta + 4\epsilon + p_3$. This implies that $p_3 \geq 5\delta + 6\epsilon$. Hence,

$$\frac{OPT(\beta)}{T_2} \leq \min\left\{\frac{13\delta + 10\epsilon}{4\delta + 4\epsilon + p_3}, \frac{5\delta + 6\epsilon + p_3}{4\delta + 4\epsilon + p_3}\right\} \leq \min\left\{\frac{13\delta + 10\epsilon}{9\delta + 10\epsilon}, \frac{10\delta + 12\epsilon}{9\delta + 10\epsilon}\right\} \leq \frac{7}{6},$$

as in a) above.

(c) $T_2 = 5\delta + 8\epsilon + p_1 + p_2$. This implies that $p_2 \geq 4\delta + 2\epsilon$ and that $p_3 \leq \delta + 4\epsilon + p_1 + p_2$, so $T_{11} \leq 6\delta + 10\epsilon + p_1 + p_2$. Hence,

$$\frac{OPT(\beta)}{T_2} \leq \min\left\{\frac{13\delta + 10\epsilon}{5\delta + 8\epsilon + p_1 + p_2}, \frac{6\delta + 10\epsilon + p_1 + p_2}{5\delta + 8\epsilon + p_1 + p_2}\right\}$$

$$\leq \min\left\{\frac{13\delta + 10\epsilon}{9\delta + 10\epsilon}, \frac{10\delta + 12\epsilon}{9\delta + 10\epsilon}\right\} \leq \frac{7}{6},$$

as in a) above. This argument applies to $T_2 = 5\delta + 8\epsilon + p_1 + p_4$, too.

(d) $T_2 = (5\delta + 10\epsilon + \sum p_i)/2$. This implies that $p_3 < -(3/2)\delta + \epsilon + \sum p_i/2$. Therefore,

$$T_{11} < \frac{7\delta + 14\epsilon + \sum p_i}{2}.$$

$T_2 > p_1 + 9\delta + 10\epsilon$ implies $\sum p_i > 2p_1 + 13\delta + 10\epsilon \geq 13\delta + 10\epsilon$. Hence,

$$\frac{OPT(\beta)}{T_2} \leq \min\left\{\frac{26\delta + 20\epsilon}{5\delta + 10\epsilon + \sum p_i}, \frac{7\delta + 14\epsilon + \sum p_i}{5\delta + 10\epsilon + \sum p_i}\right\}$$

$$\leq \min\left\{\frac{26\delta + 20\epsilon}{18\delta + 20\epsilon}, \frac{20\delta + 24\epsilon}{18\delta + 20\epsilon}\right\} = \min\left\{\frac{13\delta + 10\epsilon}{9\delta + 10\epsilon}, \frac{10\delta + 12\epsilon}{9\delta + 10\epsilon}\right\} \leq \frac{7}{6},$$

as in a) above. □

Lemma 18. *The bound of* (7/6) *in Lemma* 17 *(and therefore in Theorem* 2*) is tight.*

Proof: Let $\delta = 2, \epsilon = 3, p_1 = 0, p_2 = p_4 = 8, p_3 = 28$, and $\beta = (2, 3, 1, 4)$. $OPT = T_2 = 48$. $OPT(\beta) = T_3(\beta) = T_4(\beta) = T_6(\beta) = T_8(\beta) = T_{11}(\beta) = T_{16}(\beta) = T_{18}(\beta) = T_{20}(\beta) = 56$. Therefore, $OPT(\beta)/OPT = 7/6$.

This completes the proof of Theorem 2. □

6. Conclusions and recommendations for future study

We have examined the productivity gains that can be achieved in flexible robotic cells by changing the assignment of operations to machines. We found that flexibility provides no throughput increase for a two-machine cell. For both three- and four-machine cells, the maximum productivity increase is $14\frac{2}{7}\%$. These results should be very useful to those considering the purchase of a robotic cell, because flexible robotic cells are more expensive than robotic flow shops.

It is curious that the results of Section 5 show that the maximum throughput increase in a flexible robotic cell is $14\frac{2}{7}\%$ for both $m = 3$ and $m = 4$. It is not clear whether this trend would continue for $m \geq 5$. Examining this trend would be a challenging and useful question for future research. Another interesting line of inquiry would be to quantify the overall productivity gains for cells having $m \geq 3$, since the question of whether the best one-unit cycle is optimal among the class of all cyclic solutions remains open for circularly-configured cells. Furthermore, there is at present no algorithm to find an optimal cyclic solution in a circularly-configured cell.

Another direction for future research is towards finding similar results for cells that produce different part-types. Foundational work for such cells can be found in Hall et al. (1997), Hall et al. (1998), Kamoun et al. (1999), and Sriskandarajah et al. (1998). Dual gripper robotic cells, too, may prove to be a profitable field for the examination of flexible robotic cells. Cyclic solutions for dual gripper robotic cells have been studied in Su and Chen (1996), Sethi et al. (2001), Sriskandarajah et al. (2004), and Geismar et al. (2005).

A more sophisticated type of flexible robotic cell has *machine flexibility*: the change in a machine's operation can be done so quickly that a particular machine can perform different operations on successive parts without causing a delay (Browne et al. 1984, Sethi and Sethi 1990). Future research may quantify the productivity gains that can be realized by varying the order in which the processes are performed. As before, this would assist a manager deciding on which type of cell is most economical for his company.

Appendix

We derive the cycle times for producing parts in a circularly-configured robotic flow shop $(RF_2^{\circ}|(blocking,A,cyclic\text{-}1)|\mu)$ using the two cycles $S_{1,2} = (A_0, A_1, A_2)$ and $S_{2,2} = (A_0, A_2, A_1)$. This technique is directly applicable to larger cells and to flexible robotic cells.

For $S_{1,2}$, if we start from the initial state in which the robot has just loaded part P_i onto M_2 and M_1 is free, the robot move cycle includes the following activities: wait until P_i is processed: (p_2), unload P_i from M_2: (ϵ) move to I/O: (δ), drop P_i at I/O: (ϵ), pick up P_{i+1} at I/O: (ϵ), move to M_1: (δ), load P_{i+1} on M_1: (ϵ), wait until P_{i+1} is processed: (p_1), unload P_{i+1} from M_1: (ϵ), move to M_2: (δ), and load P_{i+1} on M_2: (ϵ). Thus,

$$T_1 = 3\delta + 6\epsilon + p_1 + p_2.$$

For $S_{2,2}$, if we start from the initial state in which the robot has just loaded part P_{i+1} onto M_1 and M_2 is occupied by part P_i, the robot move sequence includes the following activities: move to M_2: (δ), if necessary wait until P_i is processed at M_2: (w_2), unload P_i from M_2: (ϵ), move to I/O: (δ), drop P_i at I/O: (ϵ), move to M_1: (δ), if necessary wait until P_{i+1} is processed at M_1: (w_1), unload P_{i+1} from M_1: (ϵ), move to M_2: (δ), load P_{i+1} on M_2: (ϵ), move to I/O: (δ), pick up part P_{i+2} at I/O: (ϵ), move to M_1: (δ), load P_{i+2} on M_1: (ϵ). Therefore,

$$\begin{aligned} T_2 &= 6\delta + 6\epsilon + w_1 + w_2, \text{ where} \\ w_1 &= \max\{0, p_1 - w_2 - 3\delta - 2\epsilon\}, \text{ and} \\ w_2 &= \max\{0, p_2 - 3\delta - 2\epsilon\}. \end{aligned}$$

By combining the expressions for w_1 and w_2, we obtain

$$T_2 = \max\{6\delta + 6\epsilon, p_1 + 3\delta + 4\epsilon, p_2 + 3\delta + 4\epsilon\}.$$

Acknowledgements The authors wish to thank the three anonymous referees and the associate editor, whose comments have significantly improved the content and the presentation of this paper. The research was supported in part by the Natural Sciences and Engineering Research Council of Canada (Grant number OGP0002507) and by the University of Texas at Dallas.

References

Asfahl CR (1985) Robots and manufacturing automation, John Wiley & Sons, New York John Wiley & Sons, New York (1995)

Brauner N, Finke G (1997) Final results on the one-cycle conjecture in robotic cells. Internal note, Laboratoire LEIBNIZ, Institut IMAG, Grenoble, France

Brauner N, Finke G (1999) On the conjecture in robotic cells: New simplified proof for the three-machine case. INFOR 37:20–36

Brauner N, Finke G (2001) Cycles and permutations in robotic cells. Mathematical and Computer Modeling 34:565–591

Browne J, Dubois D, Rathmill K, Sethi SP, Stecke KE (April 1984) Classification of flexible manufacturing systems. The FMS Magazine 2(2):114–117

Crama Y, Kats V, van de Klundert J, Levner E (2000) Cyclic scheduling in robotic flow shops. Annals of Operations Research: Mathematics of Industrial Systems 96(1):97–124

Crama Y, van de Klundert J (1997) Cyclic scheduling of identical parts in a robotic cell. Operations Research 45:952–965

Crama Y, van de Klundert J (1999) Cyclic scheduling in 3-machine robotic flow shops. Journal of Scheduling 2:35–54

Dawande M, Geismar HN, Sethi S (2005a) Dominance of cyclic solutions and challenges in the scheduling of robotic cells. SIAM Review 47(4):709–721

Dawande M, Geismar HN, Sethi S, Sriskandarajah C (2005b) Sequencing and scheduling in robotic cells: Recent developments. Journal of Scheduling 8(5):387–426

Dawande M, Sriskandarajah C, Sethi S (2002) On throughput maximization in constant travel-time robotic cells. Manufacturing and Service Operations Management 4(4):296–312

Drobouchevitch IG, Sethi S, Sidney J, Sriskandarajah C (2004) A note on scheduling multiple parts in two-machine dual gripper robotic cell: Heuristic algorithm and performance guarantee. International Journal of Operations and Quantitative Management 10(4):297–314

Geismar HN, Dawande M, Sriskandarajah C (2004) Robotic cells with parallel machines: Throughput maximization in constant travel-time cells. Journal of Scheduling 7(5):375–395

Geismar HN, Dawande M, Sriskandarajah C (2005) Scheduling constant travel-time dual gripper robotic cells with parallel machines. Production & Operations Management, to appear

Graham RL, Lawler EL, Lenstra JK, Rinnooy Kan AHG (1979) Optimization and approximation in deterministic sequencing and scheduling: a survey. Annals of Discrete Mathematics 5:287–326

Hall NG, Kamoun H, Sriskandarajah C (1997) Scheduling in robotic cells: Classification, two and three machine cells. Operations Research 45:421–439

Hall NG, Kamoun H, Sriskandarajah C (1998) Scheduling in robotic cells: Complexity and steady state analysis. European Journal of Operational Research 109:43–63

Herrmann J, Chandrasekaran N, Conaghan B, Nguyen M, Rubloff G, Shi R (2000) Evaluating the impact of process changes on cluster tool performance. IEEE Transactions on Semiconductor Manufacturing 13:181–192

Kamoun H, Hall NG, Sriskandarajah C (1999) Scheduling in robotic cells: Heuristics and cell design. Operations Research 47:821–835

Miller RK, Walker TC (1990) FMS/CIM systems integration handbook, The Fairmont Press, Lilburn, GA

Perkinson T, Gyurcsik R, McLarty P (1996) Single-wafer cluster tool performance: An analysis of the effects of redundant chambers and revisitation sequences on throughput. IEEE Transactions on Semiconductor Manufacturing 9:384–400

Perkinson T, McLarty P, Gyurcsik R, Cavin R (1994) Single-wafer cluster tool performance: An analysis of throughput. IEEE Transactions on Semiconductor Manufacturing 7:369–373

Sethi AK, Sethi SP (1990) Flexibility in manufacturing: A survey. International Journal of Flexible Manufacturing Systems 2:289–328

Sethi SP, Sidney JB, Sriskandarajah C (2001) Scheduling in dual gripper robotic cells for productivity gains. IEEE Transactions on Robotics and Automation 17(3):324–341

Sethi SP, Sriskandarajah C, Sorger G, Błażewicz J, Kubiak W (1992) Sequencing of parts and robot moves in a robotic cell. International Journal of Flexible Manufacturing Systems 4:331–358

Sriskandarajah C, Drobouchevitch IG, Sethi S, Chandrasekaran R (2004) Scheduling multiple parts in a robotic cell served by a dual gripper robot. Operations Research 52(1):65–82

Sriskandarajah C, Hall NG, Kamoun H (1998) Scheduling large robotic cells without buffers. Annals of Operations Research 76:287–321

Su Q, Chen F (1996) Optimal sequencing of double-gripper gantry robot moves in tightly-coupled serial production systems. IEEE Transactions on Robotics and Automation 12:22–30

Venkatesh S, Davenport R, Foxhoven P, Nulman J (1997) A steady-state throughput analysis of cluster tools: dual-blade versus single-blade robots. IEEE Transactions on Semiconductor Manufacturing 10:418–424

Wood S (1996) Simple performance models for integrated processing tools. IEEE Transactions on Semiconductor Manufacturing 9:320–328