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Estimation of Part Waiting Time and Fleet Sizing in AGV Systems*

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Abstract. As manufacturing systems have grown in size and complexity, material flow control has become one of the key issues for system efficiency, and determination of the number of vehicles required is an important issue in the design of the AGV (automatic guided vehicle) systems for automated material flow control. In an AGV system, a part issues a delivery request for its transportation, and then an empty vehicle is assigned based on a pre-determined vehicle selection rule and provides delivery service.

This research presents a fleet sizing procedure for an AGV system with multiple pickup and delivery stations. A queueing model is used to estimate part waiting times. The fleet sizing procedure estimates the minimum number of vehicles needed to ensure a predefined part waiting time limit. While most stochastic models assume first-come-first-served or random vehicle selection rules for the selection of an empty vehicle, this model considers such additional rules as the nearest vehicle selection rule, which is the most popular among all vehicle selection rules. The performance of the proposed model is examined through computational experiments.

Key Words: AGV systems, fleet sizing, queueing, stochastic approximation

1. Introduction

Automated guided vehicles (AGVs) are currently widely used in manufacturing and service industries, including flexible manufacturing systems (FMSs), semiconductor fabrication shops, automobile production plants, and seaport container terminals. AGVs offer many advantages over traditional transporters in terms of flexibility, better space utilization, shop floor safety, and easier interface with other automated systems. Because of the randomness and the large number of variables involved, the design and evaluation of AGV systems is very complex, typically including determination of the vehicle guidepath layout, the traffic flow pattern, the number of vehicles required, the buffer capacity for the vehicles, and the location of pickup/delivery stations (Mahadevan and Narendran, 1990; Ganesharajah, Hall, and Sriskandarajah, 1998). Among these issues, the number of vehicles required is a fundamental decision to be made after a traffic network is determined.

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This paper presents an analytical procedure to determine the number of vehicles required, or fleet size, in AGV systems. In the AGV systems under consideration, the specific times and places of the delivery requests are not known in advance, while the total delivery requirements during a planning horizon are known. When a load calls for a vehicle for delivery service, one of the available vehicles is selected in real time based on a pre-defined vehicle dispatching policy. This research considers such dynamic vehicle selection rules as the nearest vehicle selection, which considers the current state of a system. If there is no empty vehicle at the time of delivery requests, the delivery service is provided when a vehicle becomes idle. The determination of the fleet size starts with estimating the total vehicle travel time required in the planning horizon. The total vehicle travel time includes empty vehicle travel time, loading time, loaded vehicle travel time, and unloading time. Among these, loading time, loaded vehicle travel time, and unloading time are often easily estimated when total delivery requirements are known in advance. However, empty vehicle travel time is often difficult to estimate because it is a function of the vehicle dispatching rules. Vehicle dispatching decisions are concerned with assigning vehicles and delivery requests to each other in real time based on the state of the system. Such rules as random vehicle selection, longest idle vehicle selection, least utilized vehicle selection, and nearest vehicle selection are a few examples.

From the viewpoint of a queueing system, AGVs can be considered as resources and the delivery requests (from the parts which need to be delivered from a station to another) as customer arrivals. The number of servers required (fleet size) depends on the parameters related to customer arrivals (delivery requests) and service time (vehicle travel time). In Figure 1, the part waiting time consists of assignment waiting time and empty vehicle travel time. The assignment waiting time is the time delay that a part has to wait until a vehicle becomes available before its delivery request can be assigned to this vehicle. If there is an idle vehicle at the time of the delivery request, the assignment waiting time is zero. The assignment waiting time is estimated based on queueing theory. Two parameters, mean and variance, of the vehicle travel time are used to estimate the expected assignment waiting time. For estimating empty vehicle travel time for various vehicle dispatching rules, an analytical model presented in Koo and Jang (2002) is adopted. Once assignment waiting

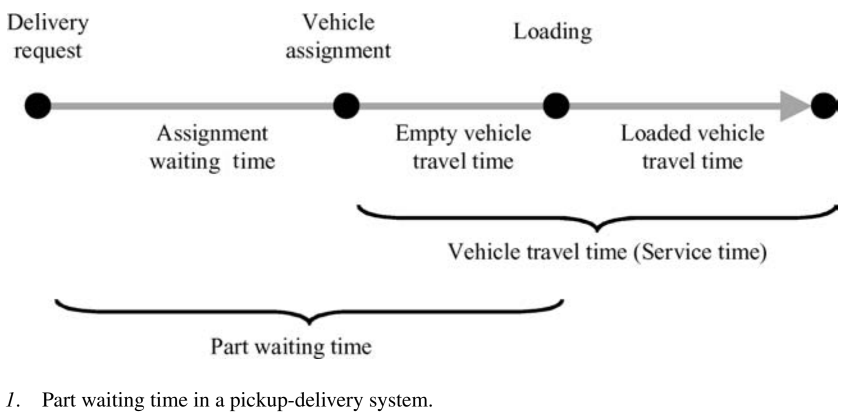

Figure 1. Part waiting time in a pickup-delivery system.

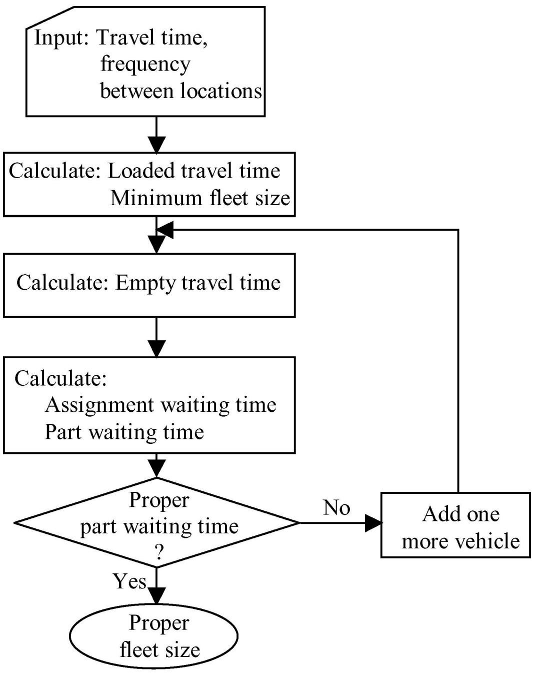

Figure 2. Fleet sizing procedure.

time and empty travel time are calculated, the expected part waiting time for vehicle service can be estimated, which is then used for fleet sizing. Figure 2 shows the overall fleet sizing procedure.

Section 2 reviews the literature and Section 3 introduces the new fleet sizing procedure. Section 4 shows a numerical example, and Section 5 tests the accuracy of the model by simulation experiments. Section 6 gives a summary.

2. Previous work and problem description

The work on fleet size estimation can be classified into deterministic and stochastic studies (Ganesharajah et al., 1998). In a deterministic system, all parts to be moved are known in advance and ready for delivery at the beginning of a planning time horizon. In a stochastic system, delivery requests are issued at random points of time and served by vehicles selected in real-time.

For deterministic systems, Maxwell and Muckstadt (1982) propose a mathematical model to determine the minimum number of AGVs for a given number of delivery requests during a time window. Each location is associated with a net flow of vehicles that is defined as the difference between the numbers of incoming and outgoing deliveries. The net flow represents the trip frequencies of empty vehicles into or out of the station. Flow balances of locations have to be achieved by empty vehicle movements: the stations with positive net flows have empty vehicles available to be assigned to other stations with negative net flows.

The model gives the lower bound on the number of vehicles needed in the system. This work is extended by Leung, Khator, and Kimbler (1997) who additionally consider vehicle types of different load carrying capacity and speed, and Rajotia, Shanker, and Batra (1998) who impose one more constraint that only a small portion of delivery requests from a location can be served by vehicles idle at the same location due to the randomness of the delivery request. Egbelu (1987) proposes an analytical procedure to estimate vehicle requirements when vehicle requests are served by randomly selected vehicles. Sinriech and Tanchoco (1992a) develop a multi-criteria optimization model considering cost and throughput to determine the AGV fleet size. Egbelu (1993) presents a model to simultaneously specify unit load sizes, vehicle size, and fleet size in an AGV-based production system. The model solves the combined problem by using a combination of numerical search, computer simulation, and statistical approaches. Malmborg (1990) provides a scheme for computing empty vehicle travel time which is the opposite of the approach taken by Maxwell and Muckstadt (1982). The frequency of empty travels is based on the total number of delivery loads rather than net flow. Each vehicle, after having completed a load transfer, is routed to the farthest station. The model provides the upper bound on the total empty vehicle travel time, which may be later used in determining fleet size. He argues that the actual empty travel will be a weighted average of the upper bound and the lower bound of Maxwell and Muckstadt (1982). Malmborg (1991) later extends his work by developing tightened analytical bounds on the total volume of empty vehicle travel.

Beaujon and Turnquist (1991) present a nonlinear mathematical model to optimize the fleet size and vehicle allocation under a multi-period transportation planning environment. The model is transformed to a minimum cost network flow problem with a nonlinear objective function that can be solved using yet another proposed solution procedure based on the Frank-Wolfe algorithm. Du and Hall (1997) address fleet sizing and empty vehicle redistribution for systems with a one-to-many (or hub-and-spoke) transportation structure. Terminals are classified into surplus and shortage terminals based on the balance of the incoming and outgoing transportation requirements. A proper fleet size is determined based on inventory control theory with operating costs for excessive number of vehicles and shortage costs for insufficient number of vehicles. Mahadevan and Narendran (1990) present an analytical model to estimate the fleet size in a flexible manufacturing system with alternative part routing. The model does not consider the timing of transportation requirement and empty vehicle travel. Mahadevan and Narendran (1993) extend their previous work by additionally considering limited buffer capacity and central buffer. Vis, de Koster, Roodbergen, and Peeters (2001) present an algorithm to determine the necessary number of AGVs at an automated container terminal. A network flow based model and a polynomial time algorithm are developed to solve the problem in which containers are available for transport at known time instants. Koo, Lee, and Jang (2004) present a fleet management procedure in a container transportation system. The two-phase procedure is aimed at simultaneously finding the minimum fleet size and travel route for each vehicle to satisfy all the transportation requirements within a planning horizon. In phase one, an optimization model is developed to obtain a fleet planning with minimum vehicle travel time and to provide a lower bound on the fleet size, and in phase two, a tabu search procedure is used to construct a vehicle routing with the least number of vehicles. In the deterministic systems, the number of vehicles required is estimated without consideration of the waiting time for delivery requests.

Some research works provide stochastic models for fleet sizing. Tanchoco, Egbelu, and Taghaboni (1987) model a transportation system as a closed queueing network to analyze the performance of vehicles and determine a fleet size. They use CAN-Q (computerized analysis of network of queues) developed by Solberg (1979) to handle stochastic features. The model assumes Poisson arrivals of AGV requests, exponential loaded travel time between two locations, first-call-first-served dispatching rule, and no empty travel time. An iterative sequential search procedure is used in determining the economic fleet size: the number of vehicles is increased until no significant improvements in system performance (machine utilization, throughput rate, and flow time) are observed. Through simulation studies, they found that CAN-Q underestimates the number of vehicles required. They point out that the underestimation occurs because CAN-Q assumes a zero travel time for empty vehicle movements. Ozden (1988) conducts a simulation study to investigate the effect of several key factors related to AGVs such as traffic pattern, the number of vehicles, carrying capacity of each AGV, and input/output queue capacity of machines on the overall performance of a flexible manufacturing system. Bozer, Cho, and Shrinivasan (1994) present a queueing model to estimate the expected waiting time for delivery requests that occur in a manufacturing system with one vehicle, under the assumption that empty vehicles are dispatched according to a modified FCFS (first-come-first-served) policy. Kobza, Shen, and Reasor (1998) present a discrete-time Markov chain model to find moments and cumulative probabilities of the empty vehicle travel time. They ignore the vehicle-initiated dispatching condition and only consider two workcenter-initiated dispatching rules: the nearest vehicle selection rule and farthest vehicle selection rule. Since the model assumes that vehicles are always available when a load delivery request is, it tends to under-estimate empty travel time, especially when vehicles are heavily loaded. Under the same material handling environment as in Kobza et al. (1998) and Shen and Kobza (1998) provide a dispatching-rule-based algorithm (DRBA) to determine the minimum number of vehicles in an AGV system. The DRBA consists of three steps. Step one finds an upper bound for the required number of vehicles by assuming that the empty vehicle travel time is estimated to be twice the loaded travel time. This upper bound is used in the second step with a queueing model to determine the probability that a load must wait. If the probability is smaller than the specified value, the algorithm decreases the number of vehicles by one and repeats this step. Step three calculates the expected load waiting time by using an M/G/M queueing model. The DRBA considers only the workcenter-initiated dispatching conditions so the waiting time for delivery is underestimated. They argue that the DRBA can be used as a lower bound on the expected load waiting time. Chevalier, Pochet, and Talbot (2002) address a design problem for a two-station AGV system. The AGV system consists of a number of vehicles transporting products between two stations (i.e., workstation and warehouse). They propose a combined procedure to determine the dispatching rules and the minimal number of vehicles needed to guarantee certain product mean waiting time. Reorder point inventory policy is applied to determine the dispatching rules, while queueing theory and stochastic processes are used to estimate the minimum number of vehicles needed to guarantee predetermined product mean waiting time.

The stochastic delivery problems are likely to be more common in the future due to the advances in information and communication technologies (Psaraftis, 1995). The distinctive characteristics of this paper are that the stochastic aspect of the system and different AGV dispatching policies are take into account. While most stochastic models assume first-come-first-served or random vehicle selection rules for the selection of an empty vehicle, this model considers additional rules such as the nearest vehicle selection rule, which is the most popular among all vehicle selection rules. The fleet sizing model in this paper can be used for both highly and lightly loaded AGV systems, unlike the work of Kobza, Shen, and Reasor (1998). Different from the work of Chevalier et al. (2002) where only two stations are considered, this paper deals with an AGV system with multiple pickup and delivery stations. The assumptions made in this paper are as follows:

1. Travel time between pickup and delivery stations is unique and deterministic.
2. Average delivery request rates between locations are known. However, the exact time between delivery requests is known only probabilistically.
3. If there is no delivery request waiting for a vehicle, the vehicle stays at its current position waiting for the next request.
4. One vehicle can serve only one delivery request at a time.
5. If there are multiple idle vehicles at the time of a delivery request, one empty vehicle is selected by a predefined vehicle dispatching policy. If multiple delivery requests are waiting for an idle vehicle, the requests are serviced by a first-come-first-served basis.

Note that the times that an AGV travels for different delivery requests are different from each other because the source, destination, and the current location of AGVs for delivery requests are given randomly. In addition, the delivery request is issued at random points of time, and the AGV system is modeled in a stochastic way in the research.

3. Fleet sizing

This section introduces an approximated stochastic modeling of an AGV system for fleet sizing. First, means and variances of empty travel time and loaded travel time are estimated, and then the expected part waiting time for delivery service is estimated. The following notation is used in the model.

n: number of pick-up/drop-off locations
m: number of vehicles
ρ: vehicle utilization (= total AGV travel time/total vehicle time available)
f_{ij}: delivery request rate from location i to location j
t_{ij}: vehicle travel time from location i to location j
lu: sum of loading and unloading time
F: delivery request rate between all locations ($F = \sum_{i=1}^{n} \sum_{j=1}^{n} f_{ij}$)

3.1. Loaded travel time

Given transportation requirements and travel times between all pairs of locations, the mean and variance of loaded travel time including loading and unloading time (t_l) are calculated as follows:

$$E(t_l) = \sum_{i=1}^{n} \left[\sum_{j=1}^{n} \left\{ (f_{ij}/F)(t_{ij} + lu) \right\} \right]. \tag{1}$$

$$V(t_l) = \sum_{i=1}^{n} \left[\sum_{j=1}^{n} \left\{ (f_{ij}/F)(t_{ij} + lu)^2 \right\} \right] - E^2(t_l). \tag{2}$$

The expected value of t_l is a weighted average of the possible values that t_l can take on, each value being weighted by the possibility that t_l assumes that value. Equations (1) and (2) are independent of the empty vehicle selection rules to be discussed below.

3.2. Empty vehicle travel time

To keep the model manageable, let us assume that the probabilities for vehicles to be idle are mutually stochastically independent. (The effect of this assumption is examined in Section 5.) Then the number of idle vehicles, Z, at the time of a delivery request follows a binomial distribution,

$$b(m, 1-\rho), \text{ i.e., } P(z) = Pr(Z = z) = {}_mC_z(1-\rho)^z \rho^{m-z}, \quad \text{where } {}_mC_z = \frac{m!}{(m-z)!z!}.$$

Empty vehicle travel time depends on the locations of delivery requests and the empty vehicles assigned to the requests. If there are no idle vehicles at the time of delivery requests (case I), the request joins a queue for delivery service. This case happens with probability ρ^m, or $P(0)$. In this case, when a vehicle becomes idle, it takes a request from the queue based on first-call-first-served policy. If there is at least one idle vehicle at the time of a delivery request (case II), an idle vehicle is assigned to it based on an empty vehicle selection rule. The probability that a delivery request sees at least one idle vehicle is $(1 - \rho^m)$, or $\sum_{z=1}^{m} P(z)$.

Now, we estimate empty travel time of vehicles under various empty vehicle selection rules: random vehicle selection (RV), longest idle vehicle selection (LIV), least utilized vehicle selection (LUV), nearest vehicle selection (NV), and farthest vehicle selection (FV).

3.2.1. Random vehicle selection. Under this rule, the probability that a vehicle completes a delivery service at location k and is assigned to a delivery request from location i is a product of (1) the proportion of the delivery requirements to location $k(fd_k)$ and (2) the proportion of delivery requirements from location $i(fs_i)$. Let t_e be the empty travel time.

Then the mean and variance of t_e are as follows:

$$E(t_e) = \sum_{i=1}^{n}\left[fs_i \sum_{k=1}^{n}(fd_k t_{ki})\right]. \tag{3}$$

$$V(t_e) = \sum_{i=1}^{n}\left[fs_i \sum_{k=1}^{n}\left(fd_k t_{ki}^2\right)\right] - E^2(t_e), \tag{4}$$

where

$$fd_k = \frac{1}{F}\sum_{i=1}^{n} f_{ik}, \quad \text{and } fs_i = \frac{1}{F}\sum_{j=1}^{n} f_{ij}.$$

Equations (3) and (4) are valid for both cases I and II.

3.2.2. Longest idle vehicle selection and least utilized vehicle selection. In this case, vehicles are selected by vehicle idleness or vehicle utilization, not by location-related factors such as distances between delivery request locations and idle vehicles. Since the vehicle selection criteria are not related to the locations of delivery request and idle vehicles, the results from the random vehicle selection, equations (3) and (4), can be also used to estimate empty vehicle travel time for these cases. This assumption is evaluated by simulation in Section 5.

3.2.3. Nearest vehicle selection. When there are multiple idle vehicles, this rule assigns an idle vehicle closest to the delivery request station. This popular rule reduces vehicles' empty travel time. Let us suppose that there is a delivery request from location i to somewhere else. We arrange all loading and unloading locations in the system in ascending order of distance from location i as shown in Figure 3. Let $S(k, i)$ be the set of all locations that are closer to location i than location k is. Under the nearest vehicle selection rule, in order for an idle vehicle at location k to be selected for the request from location i, no idle vehicles should be at $S(k, i)$ and at least one idle vehicle should be at location k. Now, let $q(k, i)$ be the probability that an idle vehicle at location k is selected (that is, the nearest idle vehicle is at location k) when a delivery request is issued at location i. Then the probability that no idle vehicles are located at $S(k, i)$ is $1 - \sum_{r \in S(k,i)} q(r, i)$.

Now, let us first consider the cases in which we have z idle vehicles ($z > 0$) at the time of delivery request (case II). Let us assume that the idle vehicles and delivery requests see the

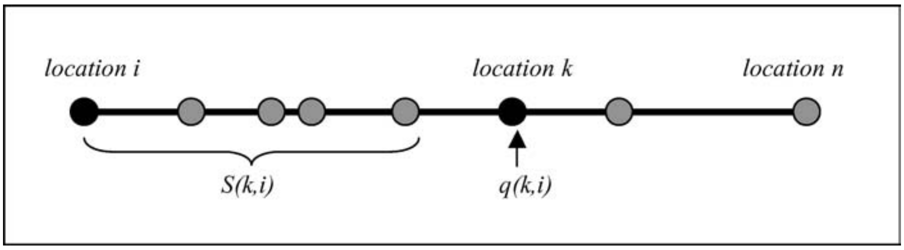

Figure 3. Arrangement of location in ascending order of distance from location i.

time average of the system, i.e., the PASTA (Poisson Arrivals See Time Averages) property (Wolff, 1982; Melamed and Whitt, 1990). Then we can approximate the probability that no idle vehicle is located at location k to be $(1 - fd_k)^z$. This approximation is true for random vehicle selection and is approximately true for other empty vehicle selection rules if the vehicles are utilized relatively evenly. Given that no idle vehicles are at $S(k, i)$, the probability that at least one idle vehicle is at location k, $A(k, i)$, is as follows:

$$A(k,i) = 1 - \left(1 - \frac{fd_k}{\sum_{r \notin S(k,i)} fd_r}\right)^z. \quad i, k = 1, 2, \ldots, n$$

For all $z > 0$ and for all pairs of locations i and k, we calculate $q(k, i)$ in ascending order of the distance from location i to location k as follows:

$$q(k,i) = \left[1 - \sum_{r \in S(k,i)} q(r,i)\right] A(k,i). \quad i, k = 1, 2, \ldots, n$$

In case of $z > 0$, the probability that a delivery request is issued from location i and is satisfied by an idle vehicle at location k is $fs_i \times q(k, i)$. On the other hand, if we have no idle vehicle at the time of delivery request (case I), then the probability that a vehicle at location k is assigned to the request is the same as the probability that the first idle vehicle is from location k. With the assumption of the PASTA property, the probability is fd_k.

Now we have the mean and variance of the empty vehicle travel time (t_e) under the nearest vehicle selection averaging cases I and II:

$$E(t_e) = \sum_{z=1}^{n} \left[P(z) \sum_{i=1}^{m} \left[fs_i \sum_{k=1}^{n} \{q(k,i) t_{ki}\} \right] \right] + P(0) \sum_{i=1}^{n} \left[fs_i \sum_{k=1}^{n} (fd_k t_{ki}) \right]. \quad (5)$$

$$V(t_e) = \sum_{z=1}^{n} \left[P(z) \sum_{i=1}^{m} \left[fs_i \sum_{k=1}^{n} \{q(k,i) t_{ki}^2\} \right] \right] + P(0) \sum_{i=1}^{n} \left[fs_i \sum_{k=1}^{n} (fd_k t_{ki}^2) \right] - E^2(t_e). \quad (6)$$

The empty vehicle travel times in equations (5) and (6) depend on $P(z)$, which is affected by the vehicle utilization, ρ, which in turn depends on the empty vehicle travel time. So we need to calculate the empty travel time iteratively. We first calculate the vehicle utilization using the empty vehicle travel time under the random vehicle selection rule, and use this utilization to calculate the empty vehicle travel time for the nearest vehicle selection case. With this new empty travel time, the vehicle utilization is recalculated. This iterative procedure converges because the vehicle utilization and empty vehicle travel time are bounded below and the values decrease after each iteration. This procedure is iterated until a very small change of vehicle utilization is observed over iterations. The procedure is summarized in Figure 4.

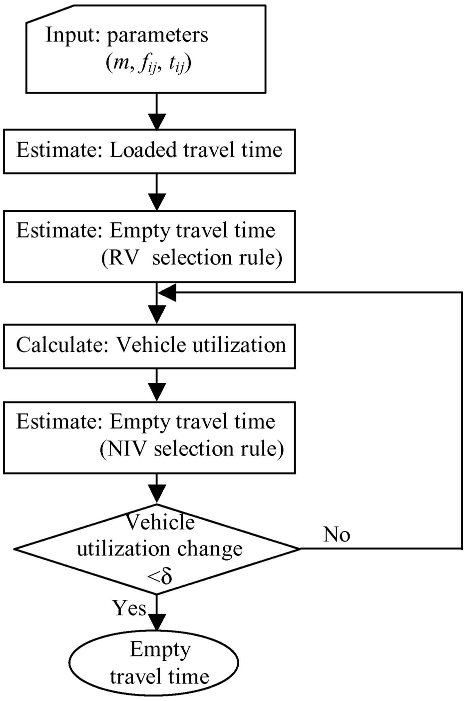

Figure 4. Calculation procedure for empty travel time under the nearest vehicle selection rule.

3.2.4. Farthest vehicle selection. Although this rule is not popular, the empty travel time for this selection rule can be obtained by a similar procedure as that of the nearest vehicle selection rule by redefining $S(k, i)$ as the set of locations that are farther from location i than location k is, and by recalculating $q(k, i)$ with the new definition of $S(k, i)$. Using equations (5) and (6) and a similar iterative procedure as that of nearest vehicle selection, the first and second moments of the empty travel time can be calculated for farthest vehicle selection.

3.3. *Total vehicle travel time*

Assuming that the loaded travel time and empty travel time are independent of each other, we obtain the mean and variance of total vehicle travel time (t_v) per vehicle request as follows:

$$E(t_v) = E(t_l) + E(t_e), \tag{7}$$

$$V(t_v) = V(t_l) + V(t_e). \tag{8}$$

3.4. Assignment waiting time

If there are idle vehicles at the time of service request, an idle vehicle is assigned to the request; otherwise, the request has to wait. By using the mean and variance of vehicle travel times obtained by equations (7) and (8), the assignment waiting time can be estimated based on queueing theory.

To estimate the assignment waiting time for general inter-arrival time (or inter-delivery-request time) and general service time (or vehicle travel time), the formula presented by Kimura (1991) for a $G/G/m$ queueing system can be used. According to Kimura, the assignment waiting time (W_q) is:

$$W_q = W_o\left(c_a^2 + c_v^2\right)gw/2, \tag{9}$$

where

$$g = \begin{cases} \text{Exp}\left[\dfrac{-2(1-\rho)\left(1-c_a^2\right)^2}{3\rho\left(c_a^2+c_s^2\right)}\right], & \text{if } c_a < 1; \\ 1, & \text{if } c_a \geq 1. \end{cases}$$

$$w = \begin{cases} \left[c_a^2 + c_v^2 - 1 + \left(1-c_a^2\right)/(1-4\gamma) + \left(1-c_v^2\right)/(1+\gamma)\right]^{-1}, \\ \quad \text{if } c_a < 1; \\ \left[2\left(c_a^2+c_v^2-1\right) + \left(1-c_a^2\right)(1-4\gamma) + \left(1-c_v^2\right)(1+\gamma)\right]\big/\left(c_a^2+c_v^2\right), \\ \quad \text{if } c_a \geq 1. \end{cases}$$

$$\gamma = \min[(1-\rho)(m-1)(\sqrt{4+5m}-2)/(16m\rho), 0.25(1-10^{-6})].$$

Here, W_0 is the waiting time for an exponential inter-arrival time and exponential service time with m servers ($M/M/m$ system), and is an exact value. Also, c_a and c_v are coefficients of variation of inter-arrival time and service time, respectively, e.g., $c_v^2 = V(t_v)/E^2(t_v)$. Here we assume that c_a is known or can be estimated by experience.

3.5. Determination of the number of vehicles

Now we determine the fleet size so as not to have excessively long part waiting time. The initial number of vehicles of the decision procedure can be determined by a static analysis: the initial number can be $(T_l/T_a)^+$, where T_l is total time required by vehicles for loaded travel, T_a is total effective time a vehicle is available, and $(x)^+$ is the smallest integer greater than or equal to x. Based on this initial number of vehicles, expected part waiting time is calculated by adding assignment waiting time and empty travel time. If this is larger than a certain value, the same procedure is repeated with one more vehicle until the part waiting time gets smaller than the desired value. Figure 2 shows the procedure.

Table 1. Delivery request rates between locations during 4800 minutes (f_{ij}).

Location	1	2	3	4	5	6	7	8	9	Pickup rate	fs_i
1	0	15	18	0	50	28	0	65	0	176	0.11
2	0	0	20	30	0	26	0	37	0	113	0.07
3	0	52	0	0	25	52	72	0	0	201	0.13
4	65	0	72	0	30	74	16	0	0	257	0.16
5	0	0	25	54	0	27	18	0	22	146	0.09
6	0	65	0	12	52	0	32	0	54	215	0.14
7	39	0	48	0	0	0	0	41	63	191	0.12
8	0	38	12	37	15	0	0	0	0	102	0.07
9	75	25	0	22	0	44	0	0	0	166	0.11
Drop-off Rate	179	195	195	155	172	251	138	143	139	1567	
fd_j	0.11	0.12	0.12	0.10	0.11	0.16	0.09	0.09	0.09		1.00

4. Numerical example

Tables 1 and 2 are request rates for delivery and travel time between locations presented in Mahadevan and Narendran (1993). The system has nine pick-up and drop-off locations and the total number of delivery requests (F) during a given time horizon (4,800 minutes) is 1567.

From equation (1), the average loaded travel time is obtained by dividing total loaded travel time by the number of deliveries: $E(t_l) = 15740/1567 = 10.045$ minutes. The variance is $V(t_l) = 172103/1567 - 10.045^2 = 8.921$ minutes^2. These values are the same for all empty vehicle selection rules and are not dependent on the number of vehicles in the system.

The mean and variance of empty travel time under random vehicle selection, longest idle vehicle selection, and least utilized vehicle selection are obtained by equations (3) and (4):

Table 2. Vehicle travel time between locations (minutes).

Location	1	2	3	4	5	6	7	8	9
1	0	4	8	10	12	8	8	6	6
2	8	0	4	6	8	8	8	6	6
3	10	8	0	2	8	8	12	6	10
4	10	6	6	0	6	6	10	4	8
5	12	8	8	2	0	8	12	6	10
6	8	8	8	6	4	0	4	6	6
7	4	8	12	14	16	12	0	10	2
8	6	6	6	8	6	6	6	0	4
9	2	6	10	12	14	10	10	8	0

$E(t_e) = 6.739$ minutes and $V(t_e) = 13.241$ minutes^2. The mean and variance of empty travel time under the nearest vehicle selection rule are obtained by equations (5) and (6), which need an iterative procedure. The vehicle utilization obtained under random vehicle selection is used as a starting value. If we assume seven vehicles in the system, the mean empty travel time converges to $6.739 > 5.621 > 5.221 > 5.079 > 5.028 > 5.010 > 5.004 > 5.002 > 5.001$ minutes. (The corresponding variance during the procedure also gets smaller, from 13.241 to 12.559 minutes^2.)

The mean and variance of vehicle travel time can be calculated by equations (7) and (8). The values for random vehicle selection are $E(t_v) = 10.045 + 6.739 = 16.784$ minutes and $V(t_v) = 8.921 + 13.241 = 22.163$ minutes^2, and the values for nearest vehicle selection are $E(t_v) = 10.045 + 5.001 = 15.046$ minutes and $V(t_v) = 8.921 + 12.559 = 21.480$ minutes^2. The vehicle utilization is 78.3% [$= 16.784 * 1567/(7 * 4800)$] under the random vehicle selection rule and is 70.2% under the nearest vehicle selection rule.

The assignment waiting time can be estimated by equation (9). Let us assume that the service request is Poisson, i.e., $c_a^2 = 1$. (In most cases in production, c_a^2 is expected to be less than unity, and this assumption gives a conservative result in determining fleet size.) Under the random vehicle selection rule, $c_v^2 = V(t_v)/E^2(t_v) = 22.163/16.784^2 = 0.079$, the expected assignment waiting time is 0.278 minutes, and the expected part waiting time is $6.739 + 0.278 = 7.017$ minutes. Under the nearest vehicle selection rule, $c_v^2 = V(t_v)/E^2(t_v) = 21.480/15.046^2 = 0.095$, the expected assignment waiting time is 0.184 minutes, and the expected part waiting time is $5.001 + 0.184 = 5.185$ minutes. If the expected part waiting time is larger than a predetermined value, this procedure is repeated with one larger number of vehicles until the part waiting time becomes smaller than the predetermined value.

5. Performance evaluation

Simulation is performed to test the performance of the proposed model by using Visual SLAM and AweSim based on the information given in Tables 1 and 2. Simulation time is 2500 hours including 500 hours of warm-up period. Table 3 compares the estimated values of the vehicle times obtained by the new model and simulation. The number of vehicles in this simulation is seven and the value of c_a^2 is unity.

Table 3 shows that the random vehicle (RV) selection rule overestimates the part waiting time by 0.9% and underestimates vehicle utilization by 0.5%, while nearest vehicle (NV) selection rule overestimates part waiting time by 10.4% and underestimates vehicle utilization by 1.7%. The simulation results for random vehicle (RV), least utilized vehicle (LUV), and longest idle vehicle (LIV) selection cases show that the same model can be used for these cases. These three rules provide almost the same results in terms of empty vehicle travel time, assignment waiting time, part waiting time, and vehicle utilization. The results also positively confirm the minimal effect of the assumption on the independency of idle vehicles introduced in Section 3.2.

Since the estimation for part waiting time under NV selection gives larger relative estimation error than the other selection rules, the accuracy of the new model has been compared with that of the other existing models. Figure 5 compares the results from simulation and

Table 3. Accuracy of the new model (seven vehicle cases).

Vehicle dispatching rule	Model	Empty travel time	Assignment waiting time	Part waiting time	Vehicle utilization (%)
Random vehicle	New model	6.739	3.111	9.852	78.3
	Simulation	6.737	3.024	9.761	78.7
	Error (%)	0.0	2.9	0.9	−0.5
Least utilized vehicle	New model	6.739	3.111	9.852	78.3
	Simulation	6.735	2.827	9.562	78.3
	Error (%)	0.1	10.0	3.0	0.0
Longest idle vehicle	New model	6.739	3.111	9.852	78.3
	Simulation	6.743	2.855	9.598	78.3
	Error (%)	−0.1	9.0	2.6	0.0
Nearest vehicle	New model	5.001	1.516	6.517	70.2
	Simulation	5.252	2.025	7.277	71.4
	Error (%)	−4.8	−25.1	10.4	−1.7
Farthest vehicle	New model	7.548	3.488	11.036	82.0
	Simulation	7.457	3.288	10.745	81.5
	Error (%)	1.2	6.1	2.7	0.6

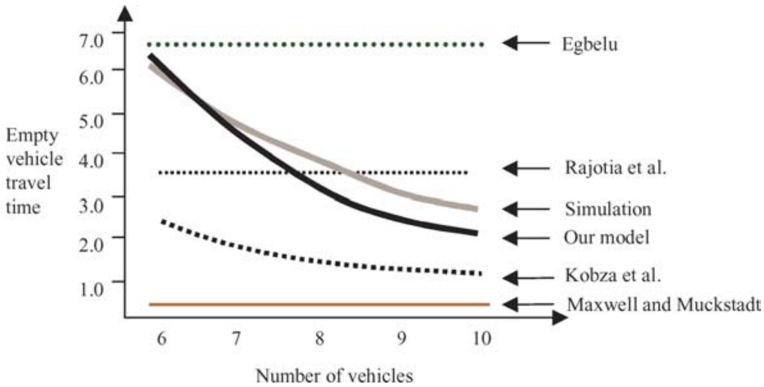

Figure 5. Estimation for empty vehicle travel time under the nearest vehicle selection rule.

estimates of several estimation models for empty vehicle travel time: (1) a mathematical model by Rajotia et al. (1998), (2) a Markov Chain model by Kobza et al. (1998), (3) a probabilistic model by Egbelu (1987), (4) an optimization model by Maxwell and Muckstadt (1982), and the new model proposed in this paper. In the literature, only Kobza et al. (1998) explicitly consider the nearest vehicle selection rule. Figure 5 shows that the new model keeps good track of the simulation results compared with other models. Egbelu's model shows high accuracy only when the system is heavily loaded. Maxwell and Muckstadt's model provides a lower bound of empty vehicle travel time.

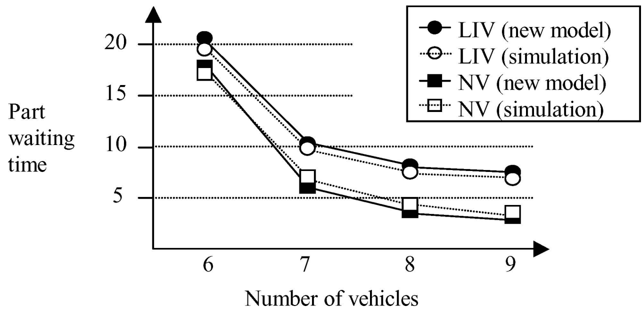

Figure 6. Estimation for part waiting time for various fleet sizes (LIV: longest idle vehicle selection rule, NV: nearest idle selection rule).

Figure 6 shows the estimation accuracy for part waiting time as the number of vehicles changes under two vehicle dispatching rules, longest idle vehicle (LIV) selection rule and nearest vehicle (NV) selection rule. As seen in Table 3, since the LIV selection rule provides almost the same part waiting time as the RV and LUV selection rules, only LIV and NV selection rules are examined. The figure positively confirms the estimation accuracy of the new model for different number of AGVs. In this sample system, to keep the average part waiting time less than nine minutes, we need to have eight vehicles under longest idle vehicle selection rule and seven vehicles under nearest vehicle selection rule. The part waiting time cannot be less than the empty travel time (i.e., 6.739 minutes) under random vehicle selection regardless of the number of vehicles; however, the part waiting time can be less than five minutes with eight vehicles in the nearest vehicle selection case. Figure 6 also shows that the impact of the vehicle utilization on the accuracy of the model is minimal: The vehicle utilization for the system with 6 AGVs is 88.7% under the NV selection rule and 91.5% under the LIV selection rule, while the vehicle utilization for the system with 9 AGVs is 49.6% under the NV selection rule and 60.9% under the LIV selection rule. The figure shows that the proposed model provides good estimation regardless of the vehicle utilization levels.

The proposed model also can be used for non-Poisson arrivals of the delivery requests. Simulation experiments have been performed with four different inter-arrival time distributions of $k\text{-}Erlang, k = 1, 2, 4, 8$ for a system with seven AGVs under two vehicle dispatching rules: the longest idle vehicle (LIV) selection rule and nearest vehicle (NV) selection rule. In *k-Erlang* distribution, the squared coefficient of variation (c_a^2) is $1/k$. Figure 7 compares the estimated part waiting time of the proposed model with that from simulation. The figure shows that the new model gives good estimation even when the arrivals of the delivery requests have small coefficient of variance up to 1/8.

This model assumes that the travel time between pickup and delivery stations is fixed and known. In reality, since the AGVs on their routes may encounter traffic congestion and other blockage, the travel times between two places are not usually deterministic. A set of simulations was performed to test the impact of the stochastic nature of travel times, where the travel time between two locations is assumed to follow such distributions as uniform

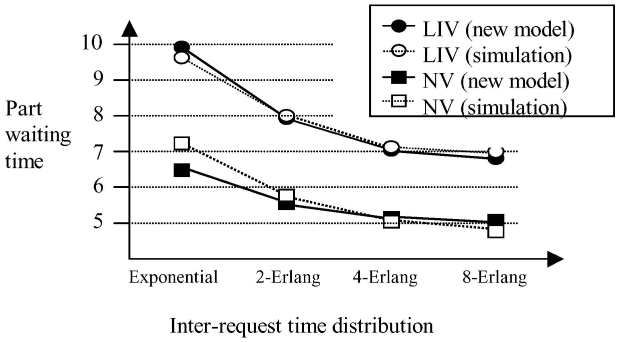

Figure 7. Estimation for part waiting time for various inter-arrival time distributions sizes (LIV: longest idle vehicle selection rule, NV: nearest idle selection rule).

distribution, triangular distribution, and normal distribution. The distributions have the same mean as what is given in Table 2. The range of the uniform distribution is from 80% of the mean to 120% of the mean. In the triangular distribution, the mode is the mean and the range is from 80% of the mean to 120% of the mean. In case of the normal distribution, the standard deviation is 10% of the mean ($cv^2 = 0.81$). Negative values from the distribution are truncated. Table 4 shows the estimation errors of the new model for each probabilistic travel time case. It is seen that the empty travel time is not sensitive to the travel time distribution. However, assignment waiting time is affected by the travel time distribution.

Table 4. Accuracy of the new model (probabilistic vehicle travel time cases).

Vehicle dispatching rule	Time	Estimation	Simulation results			
			Fixed	Uniform	Triangular	Normal
Longest idle vehicle	Empty travel	6.739	6.735	6.755	6.740	6.773
	(%error)		(0.06)	(−0.24)	(−0.01)	(−0.50)
	Assgn. waiting	3.111	2.827	2.814	2.994	3.076
	(%error)		(10.05)	(10.55)	(3.91)	(1.14)
	Part waiting	9.852	9.562	9.569	9.734	9.849
	(%error)		(3.03)	(2.96)	(1.21)	(0.03)
Nearest vehicle	Empty travel	5.001	5.252	5.205	5.252	5.221
	(%error)		(−4.78)	(−3.92)	(−4.78)	(−4.21)
	Assgn. waiting	1.516	2.025	1.981	2.059	2.233
	(%error)		(−25.14)	(−23.47)	(−26.37)	(−32.11)
	Part waiting	6.517	7.277	7.186	7.311	7.454
	(%error)		(−10.44)	(−9.31)	(−10.86)	(−12.57)

For example, the travel time with normal distribution gives an 8.8% (2.827 $\rightarrow$ 3.076) increase in assignment travel time compared with the case of fixed travel time under the longest idle vehicle selection rule, and 10.3% (2.025 $\rightarrow$ 2.233) increase under the nearest vehicle selection rule. For this reason, close attention should be made when travel times between two locations are not deterministic.

6. Conclusions

This paper presents a fleet sizing model where part waiting time is estimated for various vehicle dispatching rules based on a queueing approximation, which is then used to determine the proper fleet size. Simulation experiments under various conditions positively confirmed the accuracy of the new model. The new model retains its estimation accuracy for different numbers of AGVs in the system and over a wide range of the coefficient of variance of inter-service request time. However, when the travel time between pick-up and drop-off locations is probabilistic, the estimation accuracy decreases.

The proposed model may be also used for such systems as the transportation systems of people in low-density areas, transportation systems of handicapped or elderly people, parcel pick-up and delivery systems, and container transportation service systems as well as the AGV systems considered in this paper.

Some modeling assumptions are made in this paper to make the problem manageable, including deterministic travel time between two locations, PASTA property in delivery requests, and stochastic independency of vehicle idleness. Although the new model significantly increases the accuracy of the estimation for vehicle usage and part waiting time compared with existing models, more research is needed on these assumptions to further increase the estimation accuracy. When more time and input information is available, the fleet size obtained from this analytical model could be used as a starting value for more detailed analysis such as simulation.

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