

Critical Factors Governing the Residual Response of Reinforced Concrete Beams Exposed to Fire

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Abstract. This paper presents results from a set of numerical studies on critical factors influencing residual response of fire exposed reinforced concrete (RC) beams. The numerical model, developed using finite element computer program ABAQUS, accounts for distinct material properties of reinforcing steel and concrete during fire exposure (both heating and cooling phases) and residual (after cool down) phase. In addition, residual plastic deformations that develop in the beam during fire exposure are also taken into consideration in evaluating post-fire response of RC beams. The validity of the model is established by comparing predictions from the numerical analysis with response parameters measured during fire and residual capacity tests. The validated model is employed to conduct a parametric study for varying fire intensity, level of loading (load ratio), restraint conditions, and cross-sectional sizes of the beam. Results from the parametric study indicate that load level has significant influence on both post-fire residual capacity, as well as residual deformations in the beam. The cross-sectional size of the beam also influences residual capacity and increasing the depth of section leads to improved residual capacity in the RC beam. Presence of axial restraint however, has only moderate influence on the residual response of RC beams exposed to fire. Moreover, under most parametric fire scenarios, RC beams can retain up to 70% of their room temperature capacity provided tensile rebar temperature does not exceed 450°C.

Keywords: Reinforced concrete beams, Residual capacity, Finite element analysis, Fire resistance, Fire exposed beams

1. Introduction

Fire represents a severe hazard encountered by built infrastructure during its lifetime. Hence, structural members in buildings have to satisfy fire resistance requirements as fire safety is one of the key considerations in building design. However, historical survey data clearly suggests that while fires do occur in structures, complete collapse of structural system due to fire is a rare event [1]. The probability of such failure in reinforced concrete (RC) structural members is even lower due to low thermal conductivity, high thermal capacity, and slower degradation of mechanical properties of concrete with temperature. It is reasonable to assume,

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after most fire incidents, RC structures may be opened to re-occupancy with adequate repair and retrofitting, depending on severity of fire exposure [2–4].

In case of exposure to a severe fire, RC members might experience significant structural damage resulting from loss of concrete due to possible fire induced spalling, high rebar temperatures and relatively larger permanent deformations. Alternatively, exposure to moderate fire scenario may not result in noticeable deformations or loss of concrete section due to spalling, and thus loss of structural capacity may not be significant. Nonetheless, there is uncertainty regarding load bearing capacity of RC members owing to temperature induced degradation in material properties and extent of redistribution of stresses within the RC member after fire. Therefore, in such cases, it is imperative to assess if sufficient residual capacity exists in structural members prior to re-occupancy after the fire incident. Such an assessment forms also the basis for developing retrofitting measures in fire damaged RC structures.

In this paper, a finite element based numerical model is developed for evaluating residual response of fire exposed RC beams. The model developed in ABAQUS [5] is validated by comparing response predictions against experimental data generated in fire tests and residual capacity tests on RC beams [6]. The validated model is applied to quantify the influence of fire exposure scenario, load ratio, restraint conditions and size of beam on residual capacity of fire exposed RC beams.

2. Previous Studies

While a substantial number of studies investigated behavior of RC structures under fire [7–10], there are relatively limited studies in the literature that focus solely on residual response of RC beams following fire exposure [11–13]. Moreover, there are no clearly established principles or methods in current design standards or codes for evaluating residual capacity of RC beams after fire exposure. Some of the notable experimental and numerical studies pertaining to evaluation of residual capacity of RC beams after fire exposure are reviewed here.

Kumar and Kumar [14] tested five RC beams to generate data on residual capacity of fire exposed RC beams. All beams had a span of 3.96 m with 200 mm × 300 mm cross-section and fabricated with same grade of concrete (of compressive strength 17 MPa), reinforcement grade (of yield strength 415 MPa), reinforcement ratio, clear cover to reinforcement and stirrups. The maximum fire rating of these beams was 1.5 h under standard ISO 834 [15] fire as per IS: 3809-1979 [16]. One beam was tested to failure without being exposed to fire and served as a control specimen. Four beams were exposed to a standard ISO 834 [15] fire for 1 h, 1.5 h, 2 h and 2.5 h duration without any loads applied on the beam during fire exposure. The beam exposed to fire for 2.5 h duration failed due to excessive deflections during fire exposure itself and thus only other three beams were load tested to failure to ascertain residual capacity after cooling down in room temperature. Based on the results from residual tests, the authors concluded that the beam exposed to 1 h standard fire retains 83% of its room temperature

capacity, while similar beam after 2 h of fire exposure retains only 50% of its original capacity.

Kodur et al. [12] tested three RC beams for residual capacity after being exposed to design fire exposures with a decay phase. The beams were 3.952 m in span with 254 mm × 406 mm cross-section and identical reinforcement grade (of yield strength 420 MPa), reinforcement ratio and stirrups. One of these beams was made of normal strength concrete (NSC), having average compressive strength 55 MPa, while the other two beams were made of high strength concrete (HSC) having average compressive strength 105 MPa. The beams were exposed to short design fire or long design fire to simulate practical fire exposure scenarios. None of these beams failed under fire exposure and reached complete burn-out conditions. Based on the results of this study, the authors conclude that RC beams retain significant flexural capacity, after exposure to fire, especially when the temperature in reinforcing steel remains below 500°C. Moreover, they concluded that peak rebar temperature is the most important factor that governs residual flexural capacity after fire exposure.

Besides such tests, limited numerical studies have been reported on the residual capacity of fire exposed RC beams. Two types of approaches are utilized for evaluating residual capacity of RC beams. The first approach [11–13] is based on simplified sectional analysis, wherein strength equations similar to that for room-temperature capacity evaluation is utilized, but strength reduction factors (due to high temperature exposure) for concrete and reinforcing steel are applied in calculating residual capacity of fire exposed RC beams. The reduction factors are calculated based on maximum temperatures experienced within the cross section for a specified fire exposure in concrete and reinforcing steel.

The second approach utilized by researchers for evaluating post-fire response of RC beams is through detailed finite element analysis. A transient three dimensional thermo-mechanical numerical model was developed by Ozbolt et al. [17] to simulate the behavior of RC beams exposed to high temperatures. Predictions from the model were compared with measured data from tests reported in literature [14], to illustrate that such a model is an effective tool to numerically study behavior of RC beams, both during fire exposure and residual (after cool down) conditions.

The above literature review indicates that RC beams retain significant level of capacity after exposure to fire. However, the extent of residual capacity in a fire exposed RC beam is dependent on fire exposure scenario, peak rebar temperature, level of loading (load ratio), restraint conditions and sectional dimensions of beam. Many of these factors vary significantly in different scenarios and are inter-dependent. Therefore, the computed residual capacity of RC members after fire exposure can vary widely based on the assumptions used in the analysis [12]. Although earlier studies indicated these factors to be critical, the extent of influence of these parameters is not studied. This paper is aimed at conducting numerical studies in order to quantify the influence of critical factors governing residual response of fire exposed RC beams. Results from numerical studies are utilized to quantify the influence of various parameters on the response of fire exposed RC beams.

3. Numerical Model

In order to account for various critical factors that influence residual response of fire exposed RC beams, three stages of analysis are needed i.e. room temperature analysis (Stage 1), analysis during fire exposure (Stage 2), and residual (after cool down) analysis (Stage 3). This approach is similar to the one proposed by Aziz and Kodur [18] for evaluating residual capacity of steel bridge girders, and is implemented through a transient three dimensional finite element model.

3.1. General Procedure

The three stages of analysis for evaluating residual capacity of a fire exposed RC beams comprises of structural analysis at room temperature prior to fire exposure (Stage 1), fire resistance analysis during exposure to fire (Stage 2) and finally, post-fire residual analysis after cooling down of the beam (Stage 3). A flow chart shown in Figure 1 illustrates various steps required for evaluating residual capacity of fire exposed RC beams. This analysis procedure can be implemented using any finite element based package, such as ABAQUS [5].

In Stage 1, room temperature capacity of the RC beam is evaluated through a detailed finite element analysis by gradually incrementing the load on the structure till failure occurs. Alternatively, the ultimate capacity can also be estimated using specified strength equations in design standards [19]. The room temperature capacity determined in Stage 1 is utilized to assess relative level of loading (load ratio) on the beam during fire exposure (Stage 2 of the analysis). Also, the capacity calculated during Stage 1 analysis is used to estimate the extent of degradation in capacity once the residual capacity is ascertained in Stage 3 after cool down of the beam. For Stage 1 of analysis, room temperature mechanical properties of concrete and reinforcing steel are required.

In Stage 2 of the analysis, the response of RC beam is evaluated by exposing it to given fire exposure, load ratio, and boundary conditions. Realistic loading and fire exposure that are present throughout a typical fire event can be applied on the beam. This stage of the analysis is carried out at various time increments till the failure of the beam or till burn-out conditions. Response parameters from thermal and structural analysis are to be utilized at the end of each time increment to check the failure state of the RC beam under different failure limit states. For this stage of analysis, temperature dependent thermal and mechanical properties of concrete and reinforcing steel, that are distinct during heating and cooling phases of fire exposure, are to be incorporated in the analysis.

Following the cooling down of the beam, and if there is no failure of the beam in Stage 2, Stage 3 of the analysis is to be carried out. In this stage of the analysis, the cooled down RC beam is loaded incrementally till failure and the structural response of the beam is traced. The residual capacity corresponds to maximum load that the beam can sustain before attaining failure. For this analysis, residual properties of concrete and steel reinforcement are required.

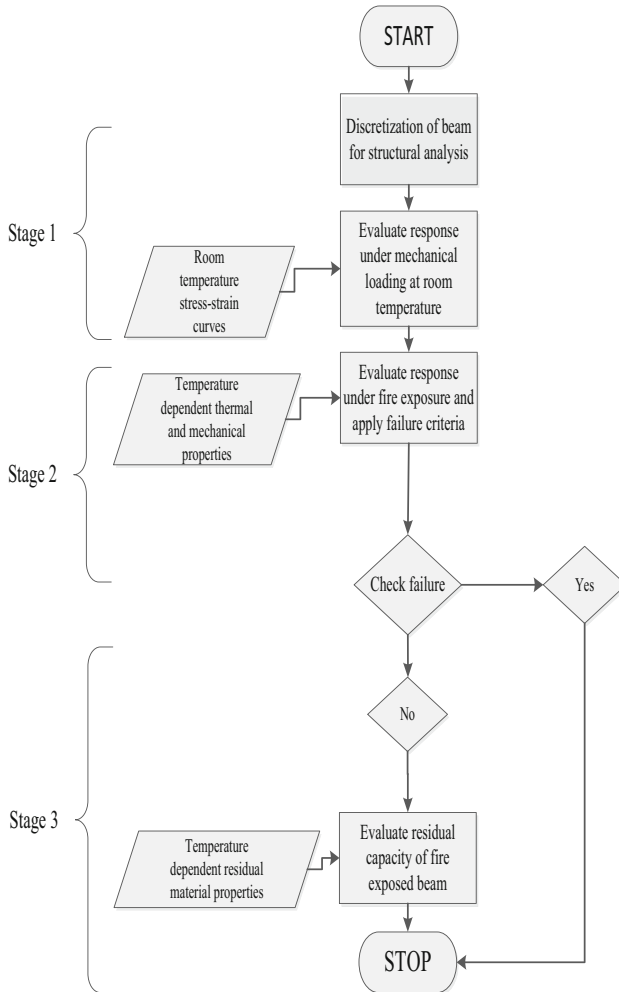


Figure 1. Flow chart describing the three stages involved in residual capacity analysis.

3.2. Modeling Assumptions

The following assumptions are made in the development of the numerical model:

- A perfect bond is assumed at the interface between reinforcing steel and concrete. While cracks that develop in the tension zone of concrete may result in the weakening of bond and subsequent slippage of reinforcement, over certain length of the beam (beam segment) that includes several cracks, the average strain in both the reinforcement and the concrete remains approximately equal [20].

- Fire induced spalling has not been explicitly modeled in analysis implying the model is applicable only for normal strength concrete (NSC) with concrete compressive strengths 70 MPa or lower [21].
- Transient creep strain is considered implicitly in the model. This implies that this strain is conservatively neglected upon cooling down of the beam. This simplifying assumption has been shown to have very limited influence on analysis results [22].
- The boundary conditions are assumed to be unheated since this is the case in most practical situations in buildings wherein the supports are embedded in the walls (and slab) and the end supports remain unexposed to fire.

3.3. Details of the Numerical Model

The numerical model is developed using the finite element computer program ABAQUS [5]. The modeling of RC beams exposed to fire is undertaken using the sequentially coupled thermo-mechanical procedure. In this procedure, the mechanical analysis relies on the heat transfer analysis at each time step, but no reverse dependency exists [5]. In Stage 1 of the analysis, the room temperature load bearing capacity is evaluated. The response of RC beam during fire exposure (Stage 2 analysis) is traced through two sets of discretization models, one for undertaking thermal analysis and the other for undertaking mechanical (strength) analysis. Results from thermal analysis are applied as thermal-body-loads on the structural model, uniformly along the RC beam. Following the cooling of the beam, its residual load bearing capacity is evaluated by undertaking Stage 3 analysis. Temperature dependent thermal and mechanical properties of concrete and reinforcing steel that are different at various stages are input to the analysis.

3.4. Discretization of the Beam

Two sub-models are needed to carry out the three stages of analysis, namely, structural and thermal models. A structural model is needed to carry out strength analysis in Stages 1, 2, and 3 while Stage 2 of the analysis requires heat transfer calculations to compute temperatures in the fire exposed RC beam.

For the structural analysis, the concrete in the beam is discretized with eight-noded continuum elements with reduced integration (C3D8R) and reinforcing steel is discretized with two-noded link elements (T3D2). C3D8R element has eight nodes with three degrees of freedom; namely three translations in x, y, and z directions. This element can be used for 3D modeling of solids with or without reinforcement and it is capable of accounting for cracking of concrete in tension, crushing of concrete in compression, creep and large strains [5]. T3D2 elements are used to model one-dimensional reinforcing bars that are assumed to deform by axial stretching only. They are pin jointed at their nodes; only translational displacements and the initial position vector at each node are used in the discretization. When the strains are large, the formulation is simplified by assuming that the trusses are made of incompressible material. This approach has been used effectively to model reinforcement explicitly wherein nodes of reinforcement are

coincident with corresponding nodes of concrete [5]. The interaction between concrete and reinforcement is achieved by using the embedded region constraint, i.e. no slip is allowed between reinforcing steel and concrete. Restraint at the supports is provided using axial spring elements available in the ABAQUS library.

For the thermal analysis, in 3D space, concrete and reinforcement are discretized using eight noded linear brick element (DC3D8) and 2 noded link element or truss element (DC1D2) available in ABAQUS library. These elements have nodal temperature as the only active degree of freedom. A tie constraint is used to apply temperatures from concrete to steel at the shared nodes. The values of the convective heat transfer coefficient in the analysis is taken to be $25 \text{ W/m}^2 \text{ }^\circ\text{C}$ for fire exposed concrete surfaces and $9 \text{ W/m}^2 \text{ }^\circ\text{C}$ for unexposed concrete surfaces while the emissivity factor is assumed to be 0.7 for all concrete surfaces, as per Eurocode 2 [23] recommendations. Such a modeling strategy has been found to yield satisfactory results in other studies for reinforced concrete structures exposed to high temperatures [17, 24].

3.5. Input Parameters for Analysis

Various parameters such as level of loading, boundary conditions, fire scenario and material properties are to be input to carry out different stages of analysis. In Stage 1 of the analysis, the ultimate capacity of the RC beam prior to fire exposure is evaluated. Based on ultimate capacity evaluated in Stage 1 a percentage of the ultimate load is applied during fire exposure in Stage 2 of analysis. If the beam survives fire exposure during Stage 2, the analysis proceeds to Stage 3 wherein residual response of the RC beam is evaluated.

Room temperature stress–strain relationships for concrete and steel are input for Stage 1 of analysis. A damaged plasticity constitutive model proposed by Lubliner et al. [25] and later modified by Lee and Fenves [26] is used to model the complex behavior of concrete, involving strong nonlinearity and different failure mechanisms under compression and tension (crushing or cracking). The stress–strain relation for concrete in tension is represented through a bilinear relation which is elastic up to the peak stress. A metal plasticity model that utilizes Mises yield surface with associated plastic flow and isotropic hardening is adopted for the reinforcing steel constitutive model [5].

In Stage 2 of analysis, during heating phase, temperature dependent thermal and mechanical properties of reinforcing steel and concrete are input into analysis and these property relations are assumed as per Eurocode 2 and 3 [27, 28] recommendations. The variation of mechanical and thermal properties with respect to temperature, is different in heating phase and cooling phase, and is dependent on the maximum temperature reached during heating phase. During the cooling phase, a linear interpolation between the elevated and residual mechanical properties after cool down is adopted [29]. Thermal properties of reinforcing steel and concrete, namely; thermal expansion, thermal conductivity, and specific heat are assumed to be fully reversible during the decay (cooling) phase.

In Stage 3 of analysis, after cooling of the fire exposed RC beam, the residual uniaxial compressive and tensile strength of concrete (after cooling down to room

temperature) is assumed to be 10% less than the strength attained at the maximum temperature. This assumption is based on Eurocode 4 [29] provisions. The residual stress–strain relationship for reinforcing steel is calculated using degradation trends reported by Neves et al. [30].

3.6. Failure Criteria

In undertaking residual strength analysis of an RC beam, different failure criteria are to be applied at each stage of the analysis depending on the relevant failure limit states. In Stage 1 of analysis (at ambient conditions), for evaluating capacity of the beam, strength limit state generally governs the failure and the ultimate capacity of an RC beam and corresponds to the point at which failure (due to flexure or shear) occurs. In Stage 2 of analysis (under fire exposure), the RC beam under fire exposure, experiences high temperatures and undergoes large deflections increases due to degradation in strength and modulus properties of concrete and reinforcing steel. In addition to the strength limit state, deflection limit state is to be applied as a reliable performance index to evaluate the failure in the second stage of the analysis. In the third stage, to evaluate residual capacity of the fire exposed RC beam, strength and deflection limit states generally govern.

3.7. Model Validation

The validity of the above developed finite element model is established by comparing the predicted response parameters against measured data on two concrete beams (designated as beam B1 and B2) tested under fire exposure by Dwaikat and Kodur [6]. The test parameters and results of the tested beams and are summarized in Table 1. The beams had identical reinforcement with yield strength of 420 MPa and were made of normal strength concrete (NSC), having average compressive strength 55 MPa. These beams are selected for validation since the authors reported comprehensive results on temperatures and deflections and this helps to undertake detailed comparisons. Also, since the heating rate in these cases is between 2 K/min and 50 K/min, Eurocode material models are applicable even when the tests are carried out as per ASTM E119 [31]. The tested beams are made of normal strength concrete (designated as B1 and B2 as per Table 1). The dimensions and reinforcement details of both these beams were identical and are shown in Figure 2. Both beams were tested under two point loads of 50 kN (as

Table 1
Summary of Test Parameters and Results Used for Model Validation

| Beam designation | Fire exposure | Support conditions | Maximum rebar temperature (°C) | | Residual deformation (mm) | | Residual load bearing capacity (kN) | |
|------------------|---------------|--------------------|--------------------------------|-------|---------------------------|-------|-------------------------------------|-------|
| | | | Measured | Model | Measured | Model | Measured | Model |
| B1 | ASTM E119 | Simply supported | 577 | 579 | – | – | – | – |
| B2 | SF | Axially restrained | 493 | 496 | 13.7 | 20.9 | 119.5 | 120.8 |

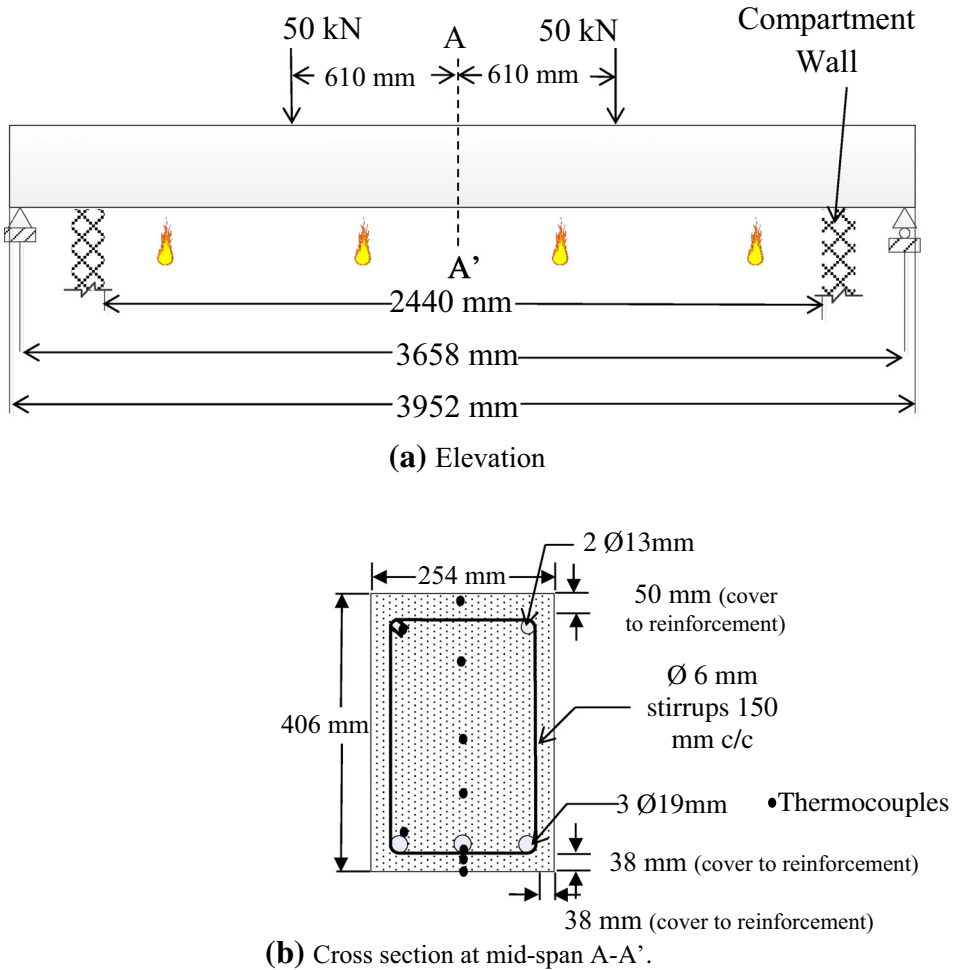


Figure 2. Dimensions, loading and reinforcement details of RC beams selected for validation [6].

depicted in Figure 2) which produced a bending moment equal to 55% of the flexural capacity of the beam, according to ACI 318 [19] capacity equation. In the tests, one of the beams is subjected to ASTM E119 [31] standard fire and the other beam is subjected to a short design fire (SF). Beam B1 was simply supported while beam B2 was axially restrained during fire exposure.

In the experiments, load is applied 30 min before the start of the fire and is maintained till no further increase in deformation could be measured. This is selected as the initial condition for measuring the deflections in the beam during subsequent fire exposure. The load is then maintained constant throughout the duration of fire exposure. There was no spalling in these beams during fire tests. Beam B1 failed during fire exposure after 180 min. In the case of beam B2, no

failure occurs during fire exposure and hence this beam was tested for evaluating residual capacity after the beam cooled down to ambient conditions [12]. This beam was then subjected to incremental loading at the rate of 3 kN/min till failure occurred.

These two beams (B1 and B2) were analyzed in ABAQUS by applying the above discussed procedure for residual capacity evaluation. The axial restraint in beam B2 was simulated as in tests using a nonlinear axial spring of stiffness 13 kN/mm with a maximum cut-off force of 120 kN [6]. The validation process involved comparison of thermal and structural response predictions from the analysis with that reported in the two fire tests on beams B1 and B2.

A comparison of measured and predicted temperatures at various locations in beams B1 and B2 is plotted in Figure 3. Results in Figure 3a indicate that temperatures in concrete (at quarter-depth and mid-depth of section) as well as reinforcing steel on the tension side of the beam B1, monotonically increase throughout the duration of ASTM E119 [31] fire exposure. Failure occurs in the beam at around 180 min into fire exposure when rebar temperature reaches 577°C which is marginally lower than the failure rebar temperature of 593°C prescribed as failure temperature as per prescriptive limit state [32]. In beam B2, exposed to a short design fire (SF), the rate of rise of temperatures in concrete and rebars are slightly higher than that in the case of beam B1 during the heating phase of the fire (see Figure 3b). This can be attributed to steeper rise in fire temperatures in the case of “SF” than “ASTM E119” fire during early stages of fire exposure. After attaining a maximum value, temperatures in both concrete and reinforcing steel begin to decrease in the beam due to presence of a decay phase in this design fire exposure in beam B2. It should be noted that peak concrete and rebar temperatures in beam B2 occur during the decay phase of the fire exposure. This is due to relatively higher specific heat and lower conductivity of concrete, which results in a lag between rise in fire temperatures and the increase in temperatures within beam cross section. This temperature lag increases with depth from the fire exposed surface. A closer examination of plotted temperatures in Figure 3 indicate that there is good agreement between predicted and measured values during both fire tests, except during initial phase of fire exposure (until first 30 min), wherein predicted values are slightly lower.

The measured and predicted mid-span deflection response in two beams B1 and B2, during fire exposure is shown in Figure 4. It can be seen that mid-span deflection increase during early stages of fire exposure due to temperature induced degradation of strength and stiffness properties in concrete and reinforcing steel. Since temperatures in beam B1 continue to increase monotonically during the entire duration of fire exposure, failure occurs in the beam at 180 min due to significant degradation in strength properties of concrete and reinforcing steel. For beam B2, however, due to presence of cooling phase, the peak temperatures experienced in concrete and reinforcing steel of the beam are relatively lower and due to presence of a decay (cooling) phase in fire, a recovery in mid-span deflection occur during the decay phase of fire. Overall, predicted mid-span deflection from the developed model agrees well with measured deflection response during the fire exposure. The beam failed in flexural mode in both fire tests and numerical

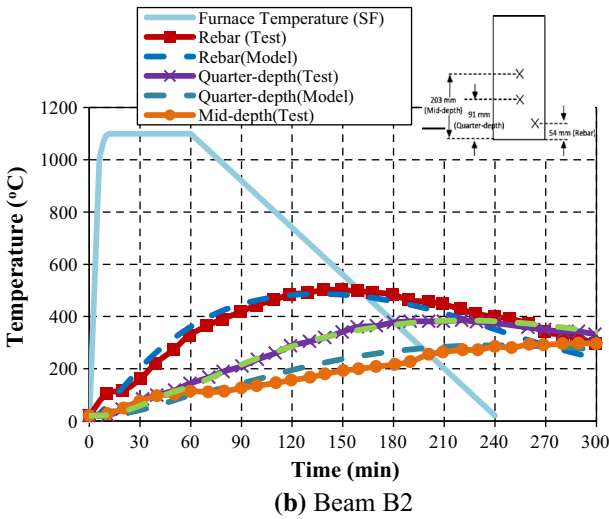
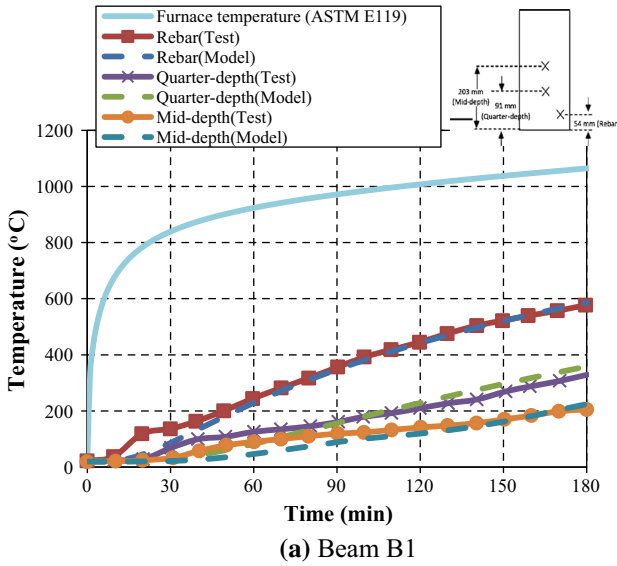


Figure 3. Comparison of predicted and measured cross sectional temperatures for beams B1 and B2 during fire exposure.

analysis. A longitudinal crack occurred at the level of tension bar in the beam during fire test, which resulted in significant spalling at the bottom surface of the beam. This type of fire induced spalling in concrete does not significantly influence the fire response of an RC beam as it occurs just prior to failure [6]. Since spalling has not been explicitly accounted for in the current model, the numerical analysis predicts a more ductile failure than that observed experimentally.

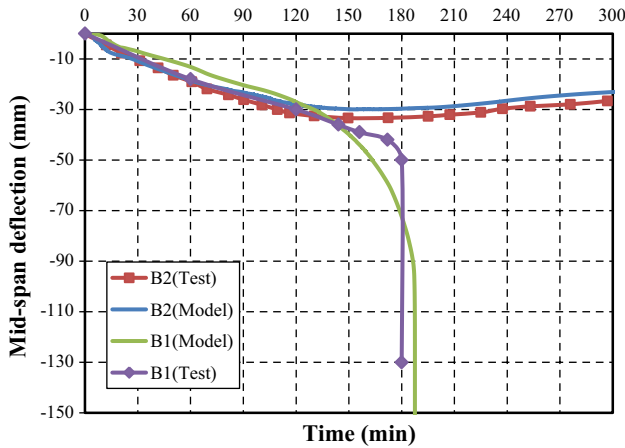


Figure 4. Comparison of the predicted and measured mid-span deflections for beams B1 and B2 during fire exposure.

Finally, the predicted residual load–deflection response in beam B2, which did not fail during fire exposure, is compared to measured response in Figure 5. There is a residual deflection of 13.7 mm, left over from fire exposure, which was assumed to be the initial state for the residual strength analysis. The predicted load–deflection trend by the finite element model is similar to the measured response during the residual capacity test. However, the residual deformation predicted by the model is relatively higher than the measured value during the test. This difference can be attributed to the fact that cooling phase properties adopted in the study are based on Eurocode 4 [5] provisions which are conservative and lead to higher prediction of post-fire residual deformations. Besides, there is some

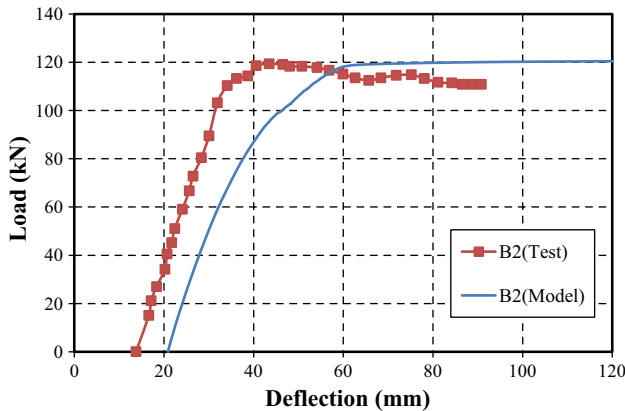


Figure 5. Comparison of the predicted and measured residual load–deflection response for beam B2.

uncertainty regarding the level of loading that were maintained on the beam in the test as it cooled down to room temperature [6]. Also, the rate of unloading adopted before carrying out the residual capacity test is not specified. It should be noted that the predicted, as well as measured residual capacity in the test is higher than room temperature capacity computed as per ACI 318 [19] strength equations. This is mainly due to the effect of strain hardening of steel reinforcement which is not taken into account in ACI 318 [19] strength equations, but is accounted for in the analysis.

Overall, predictions from the proposed model are in good agreement with the reported test data in Stages 1, 2 and 3. The slight differences in deflection predictions can be attributed to minor variations in idealization adopted in the analysis, such as stress-strain relationship of steel and concrete.

4. Parametric Studies

The validated finite element model is applied to study influence of critical factors on residual response of fire exposed RC beams. The factors considered in this parametric study include fire exposure scenario, load ratio, axial restraint and size of beam.

4.1. Characteristics of Selected Beams

As part of this parametric study, three beams (designated beam BX1, BX2 and BX3) of varying cross sectional sizes are analyzed. A summary of the design parameters of these beams is given in Table 2. These simply supported beams are 6 m in span and have different cross-sectional sizes of 125 mm × 250 mm (BX1), 180 mm × 300 mm (BX2) and 300 mm × 480 mm (BX3) respectively. The beams are subjected to a uniformly distributed load along its span only 5 m of the central span is exposed to fire. A schematic representation of the beams, together with loading arrangement, is presented in Figure 6.

Material property relations for concrete and reinforcing steel at various stages are adopted based on discussion provided in Sect. 3.4. All three beams are

Table 2
Summary of Test Parameters and Results Used for Parametric Study

| Beam designation | Beam dimensions (mm) | Flexural reinforcement | | Room temperature capacity (kN) | | Fire resistance (ACI 216) (min) |
|------------------|----------------------|------------------------|--------------|--------------------------------|-------|---------------------------------|
| | | Top bars | Bottom bars | ACI 318 | Model | |
| BX1 | 125 × 250 | 2 mm Ø 12 mm | 3 mm Ø 16 mm | 74.6 | 89.7 | 60 |
| BX2 | 180 × 300 | 2 mm Ø 12 mm | 3 mm Ø 20 mm | 143 | 168.8 | 120 |
| BX3 | 300 × 480 | 2 mm Ø 12 mm | 3 mm Ø 25 mm | 351 | 403.5 | 120 |

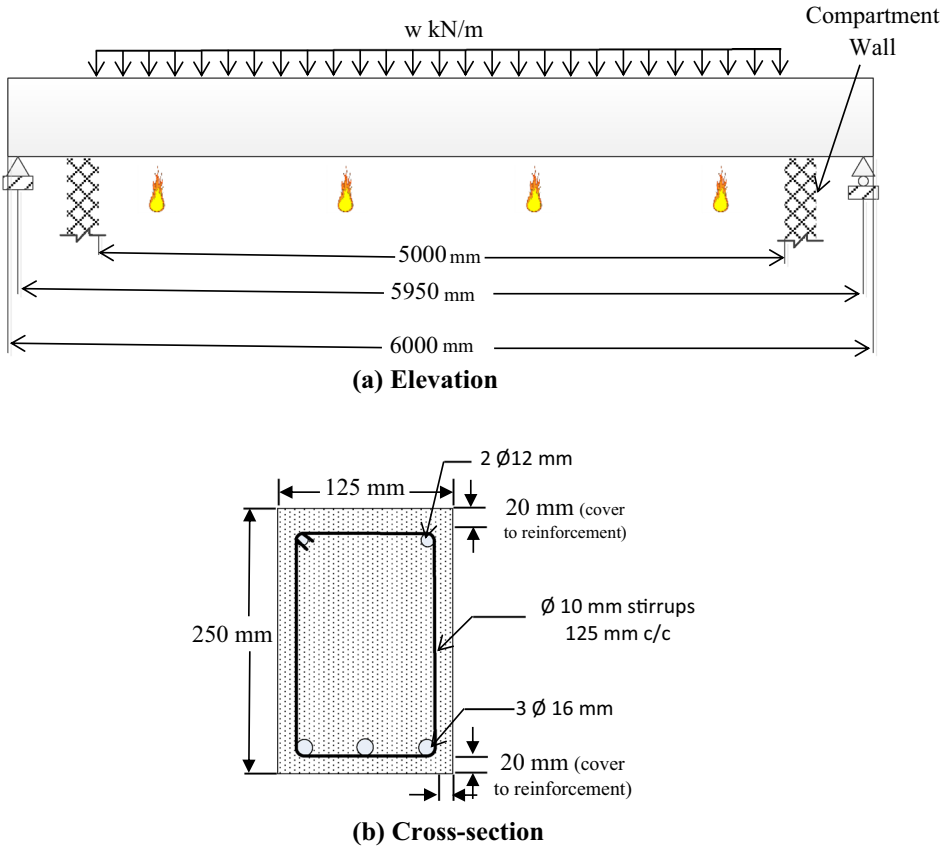


Figure 6. Dimensions, loading and reinforcement details of beam BX1 selected for parametric study.

assumed to be made of concrete of compressive strength of 40 MPa with reinforcing bars and stirrups having yield strength of 420 MPa and 280 MPa respectively. The flexural reinforcement in each of the three beams is provided such that it satisfies strain conditions for tension-controlled sections as per ACI 318 [19] guidelines. The shear reinforcement comprises of stirrups of $\phi 10$ mm spaced at 125 mm center to center. Reinforcement details for beam BX1 are illustrated in Figure 6b.

4.2. Range of Parameters

In order to study the influence of critical factors on residual response of RC beams, four sets of analysis are carried out using the previously described numerical model. One parameter is varied within a practical range, whereas all the other properties are kept constant within each analysis set. In the first set of analysis, the residual response of RC beams is studied after exposure to four fire exposure scenarios. The time-temperature curves for these four fire exposure scenarios, which represent a wide range of compartment characteristics including fuel load

and ventilation conditions, are calculated based on Eurocode 1 [23] provisions. In the second set of analysis, the influence of load level present on the beam during fire exposure is studied. The load ratio during fire exposure is varied from 30% to 60% of the ultimate capacity of the beam which to account for different level of loading that can be present during a fire exposure. In the third set of analysis, the level of axial restraint at the beam supports is varied using four different values of axial stiffness, i.e., 10 kN/mm, 25 kN/mm and 50 kN/mm. No explicit rotational restraint is provided during analysis. The results from this set of analysis are used to illustrate effect of fire induced axial restraint on residual response of RC beams. Finally, in the fourth set of analysis, three beams with different cross sectional dimensions are considered for evaluating the effect of beam size on residual response after fire exposure. The beam dimensions and reinforcement details are chosen based on the three different range of widths as in tabulated fire resistance values in ACI 216.1 [32]. The test parameters and the results from the tested beam have been summarized in Table 3. Analysis results for each of these parameters are discussed in detail in the subsequent sections.

4.3. Analysis Details

The various parameters including geometry of beam, level of loading, boundary conditions, fire exposure scenario and material properties were input to the model based on the discussion in Sects. 4.1 and 4.2. The analysis is carried out in small time steps, with the size of each time step being automatically chosen by the computer program with a specified minimum time step of 0.2 min to ensure numerical convergence. In order to investigate the convergence of the finite element mesh, beam B1 tested by Dwaikat and Kodur [6], was modeled using different meshes. Displacement response of the beam converged within a tolerance of 1% when an element size of 25 mm × 25 mm × 25 mm was used. Therefore, to strike a good balance between accuracy and efficiency, this element size was adopted in all the subsequent numerical simulations for the parametric study. Also, the effect of geometric non-linearity is incorporated in analysis using the updated Lagrangian method [5], to account for large deflections that occur in RC beams during fire exposure. [33, 34] Newton–Raphson method with a tolerance limit of 0.02 on the displacement norm is employed as the solution technique. The line search function is activated for rapid convergence [35, 36].

5. Results from Parametric Studies

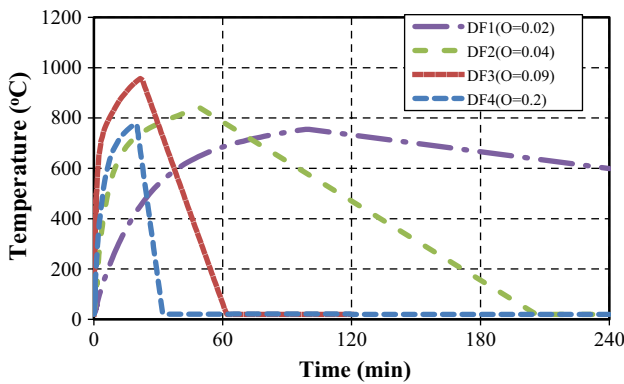
Data generated from four sets of parametric studies is utilized to trace the residual response of under different conditions. The comparative response is further utilized to quantify the effect of critical factors on residual capacity of fire exposed RC beams.

5.1. Effect of Fire Exposure

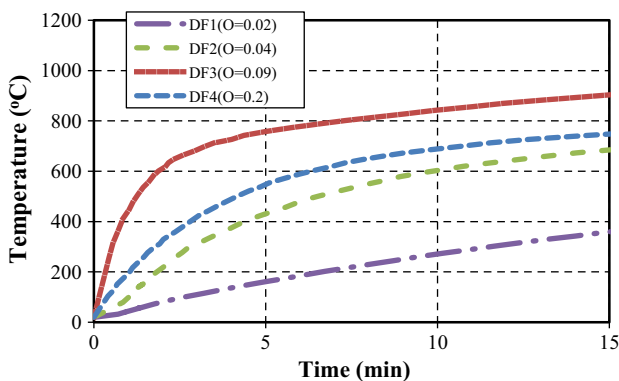
To study the effect of fire scenario on residual capacity, an RC beam, BX1, is analyzed under four different fire exposures scenarios. The flexural capacity of this beam was evaluated to be 67.3 kN-m or a total load capacity of 89.7 kN from Stage 1 of analysis. A uniformly distributed load corresponding to 50% load ratio is applied on the beam during fire exposure.

Four parametric fire exposures are considered to cover a wide range of ventilation and fuel characteristics encountered in buildings. The respective time-temperature curves for the four parametric fire exposure scenarios (DF1, DF2, DF3 and DF4) are calculated according to Eurocode 1 [23] recommendations and shown in Figure 7.

Response parameters from Stage 2 of analysis is plotted in Figure 8a to show comparative thermal response of RC beam under different fire scenarios. The tem-



(a) Different fire scenarios



(b) Zoomed in fire scenarios (First 15 minutes)

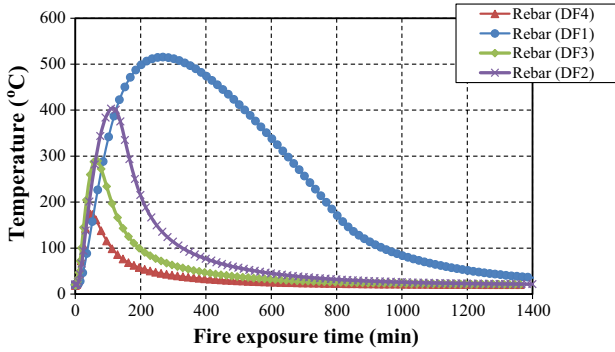
Figure 7. Parametric time-temperature curves for a total compartment area of 360 m², fire load density 600 MJ/m² and four opening factors ($O = A_v \sqrt{h_{eq}} / A_t$) accounting for different area of openings (A_v), equivalent height of openings (h_{eq}), and total area (A_t).

perature in corner tension rebar is plotted as a function of fire exposure time. The rebar does not attain its failure temperature (593°C) as per prescriptive requirements [32] in any of the considered parametric fire exposures. The rate of temperature rise, in the rebar in the case of fire scenario DF1, is lower than the rate of temperature rise in fire scenarios DF2, DF3 and DF4. These temperature trends in rebars follow temperature rise in respective fire time–temperature curves. However, results plotted in Figure 8a indicate that while the peak fire temperature attained in fire scenario DF 1 (about 750°C) is significantly lower than in fire scenario DF3 (about 950°C), peak rebar temperature in fire scenario DF1 is greater by about 200°C . This can be attributed to the longer heating phase in fire scenario DF1 of about 100 min followed by a slower cooling rate, as compared to 20 min heating phase of about and followed by a faster cooling rate in fire scenario DF3. Thus, it can be inferred that peak fire temperature alone does not give a clear indication of the maximum temperature attained in the rebar during fire exposure. Moreover, fires with lower peak temperatures may result in higher peak rebar temperatures, depending on the duration of heating phase and subsequent rate of cooling.

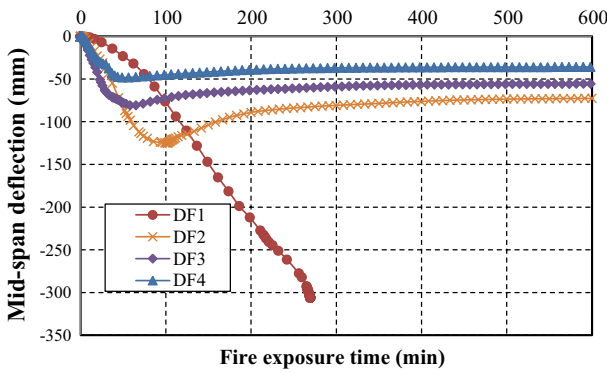
The progression of mid-span deflection with fire exposure time is plotted in Figure 8b to illustrate comparative structural response of RC beams under four fire scenarios. The deflections at early stages of fire exposure increase at a much slower rate during fire scenario DF1 than that under fire scenarios DF2, DF3 and DF4. The rate of rise in mid-span deflection correlates with the rate of rise in rebar temperature. However, due to the prolonged heating phase present in fire scenario DF1, the beam fails during fire exposure since maximum deflection limit state is exceeded. Therefore the analysis for this case does not proceed to residual capacity evaluation (Stage 3). In the remaining three fire scenarios (DF2, DF3 and DF4); mid-span deflections that occur during fire exposure begin to recover just as the rebar temperature starts decreasing after attaining peak value. However, part of the deflection remains as residual deflection even after cool down.

Figure 8c depicts load–deflection response of beam BX1 from Stage 3 of analysis (post-fire residual response), along with room temperature response prior to fire exposure. There is residual deformation in the beam, even after the beam cools down to room temperature, resulting from plastic deformations that occurred in the loaded beam during fire exposure. The extent of these residual deformations varies with fire severity, i.e. peak fire temperatures, duration of heating phase and rate of cooling experienced during fire exposure. Residual deformations observed in fire scenario DF2 are about 50% more than those observed in fire scenario DF4, which is much less severe. Also, more severe fire exposure leads to lower leads to higher temperature induced degradation in mechanical properties resulting in lower residual capacity. The reduction in load bearing capacity of beam BX1 is 30%, 25% and 22% after exposure to fire scenarios DF2, DF3 and DF4 respectively.

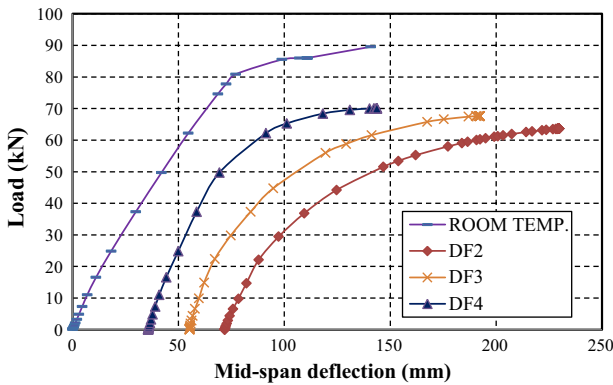
The maximum rebar temperature, peak deformation during fire exposure, residual deformation and residual capacity for beam BX1 exposed to four different fire exposure scenarios are summarized in Table 3. Under fire scenario DF1, beam attains the highest rebar temperatures and hence there is no residual capacity left in the beam after fire exposure. Amongst the other three fire exposures, beam BX1 experiences maximum degradation in residual capacity, about 34% of room



(a) Maximum corner tension rebar temperatures for beam for different scenarios



(b) Mid-span deflections for different fire scenarios for first 120 minutes



(c) Residual post-fire load-deflection response for different fire scenarios

Figure 8. Effect of fire scenario on residual response of fire exposed RC beam BX1.

temperature capacity, in fire scenario DF3, where peak rebar temperature experienced during fire exposure is the highest. The residual deformation in this case (of magnitude 72 mm) is greater than the other two cases (less than 55 mm). Therefore, it can be inferred that higher rebar temperatures as well as higher residual deformations lead to a lower residual capacity in fire exposed RC beams. Moreover, the magnitude of peak rebar temperatures experienced during fire exposure and residual deformations that occur after cool down vary greatly depending on the fire exposure scenario.

5.2. Effect of Load Ratio

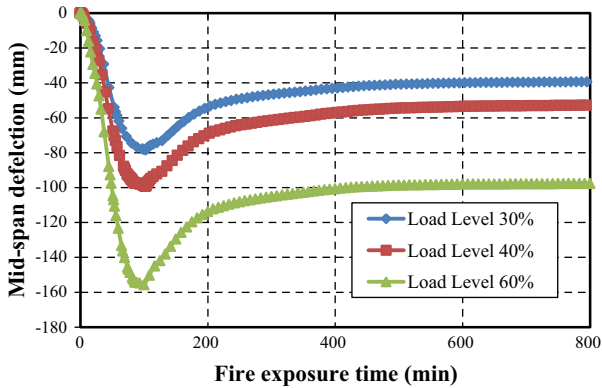
The magnitude of loading during fire exposure can have significant influence on residual capacity of fire exposed RC beams. To study the effect of level of loading on residual capacity, beam BX1 is subjected to varying load ratio of 30%, 40% and 60%, and exposed to fire scenario DF2. Results from the analysis are presented in Figure 9a, b and Table 3 to illustrate the effect of varying load ratio on residual capacity of fire exposed RC beams.

Results plotted in Figure 9a show that both the rate of rise in deflection, as well as maximum deflection, occurring during fire exposure increases with increasing load ratio. Also, the ratio of peak deflection during fire exposure to residual deflection, when the beam cools down to room temperature, reduces from about 2 to 1.5 indicating higher residual deflection (refer Table 3). This is primarily because a larger load ratio causes early yielding of steel reinforcement and hence higher plastic strains during fire exposure.

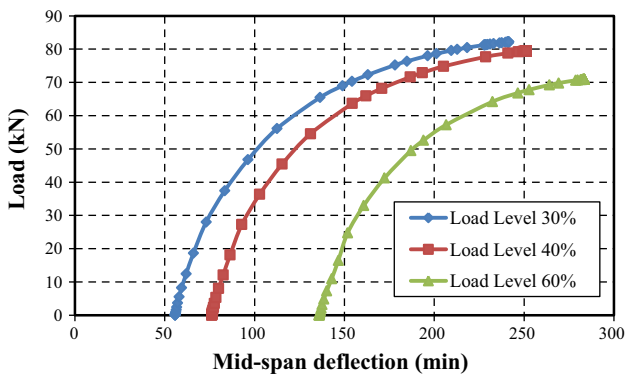
Figure 9b shows residual load–deflection response in the beam after exposure to fire under different load ratios. It can be seen that heavily loaded beams, i.e. with load ratio greater than 50%, have a significantly lower residual capacity. Results summarized in Table 3 show that for a load ratio of 30%, degradation in load bearing capacity of beam BX1 after fire exposure is about 15%. However, when the load ratio is increased to 60%, the corresponding reduction in capacity is about 26%. Also, while the peak rebar temperature remains unchanged under different load ratios, higher residual deformations occur at higher load ratios which result in lower residual capacity after fire exposure. These trends in results suggest that residual deformation after fire exposure is a better indicator of residual capacity than peak rebar temperature alone.

5.3. Effect of Axial Restraint

RC beams can develop significant restraint forces under fire exposure and the extent of these forces depends on support conditions. These restraint forces influence behavior of RC beams both during and after fire exposure. For studying the effect of axial restraint, residual response of beam BX2 is studied under parametric fire scenario DF4 for three different levels of axial stiffness at the end supports. Axial restraint is introduced in the beam by means of springs at the end supports corresponding to three different axial stiffness values, i.e., 10 kN/mm, 25 kN/mm and 50 kN/mm. The results from this set of analysis are plotted in Figure 10a, b and summarized in Table 3.



(a) Mid-span deflections during fire exposure

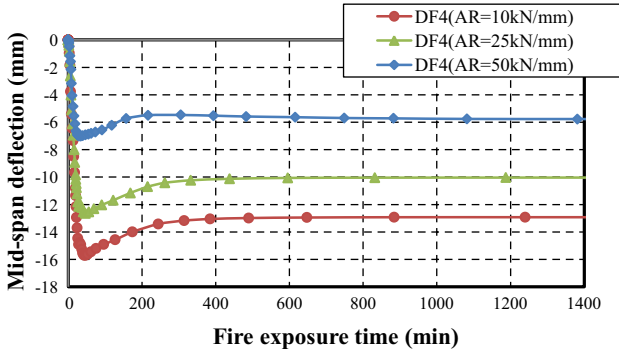


(b) Residual load-deflection response

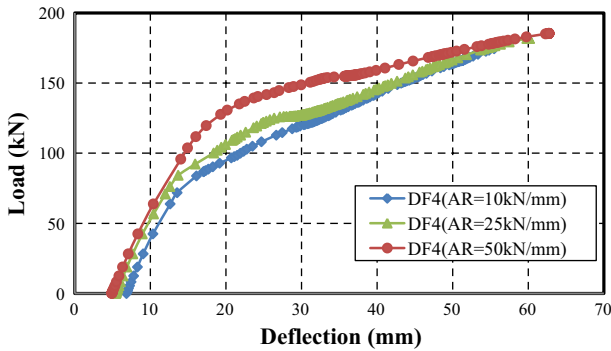
Figure 9. Effect of load ratio for beam BX1 under fire exposure scenario DF2.

To compare the mid-span deflection in beam BX2 during fire exposure DF4 for different support conditions is plotted in Figure 10a. Results indicate that the mid-span deflections are significantly reduced with increasing axial stiffness (refer Table 3). The peak deformation during fire exposure is 16 mm for an axial stiffness of 10 kN/mm which reduces to 7 mm for the case with a much higher axial stiffness of 50 kN/mm. This trend can be attributed to the arch action that gets generated in restrained RC beam due to the development of fire induced axial restraint force and this force (moment) counteracts the applied bending moment [37]. Moreover, a higher stiffness results in greater restraint forces and thereby lower deflections during fire.

Results from residual load–deflection response of the beam for different levels of axial restraint are plotted in Figure 10b. Axial restraint has a relatively moderate influence on residual capacity than deformations during fire. Moreover, while



(a) Mid-span deflections for beam BX2



(b) Residual load-deflection response of beam BX2

Figure 10. Response of beam BX2 under design fire exposure DF4 for different levels of axial restraint.

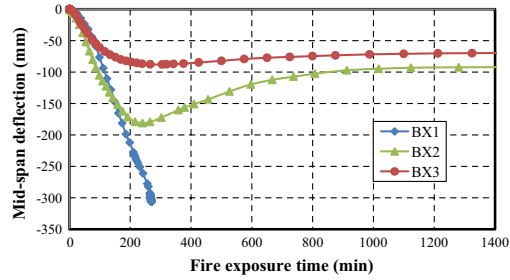
peak deformations during fire exposure vary significantly for different levels of axial restraint, the residual deformations after cool down of the beam are very similar. However, a greater axial restraint results in lower post fire deformations and hence a higher residual capacity for the same fire exposure (refer Table 3).

It should be noted that the load carrying capacity of the beam with axial restraint is higher than the simply supported case due to restraint forces experienced at the supports.

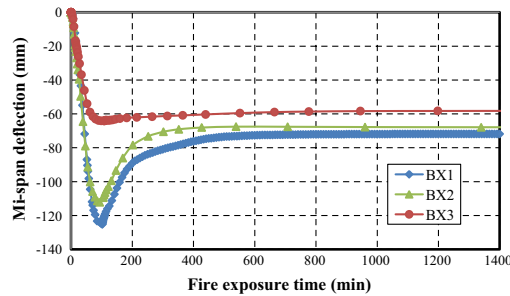
5.4. Effect of Varying Size of Beam

The influence of cross-sectional size of the RC beam on post-fire residual capacity is studied by exposing beams of three different cross-sections, i.e. beam BX1, BX2 and BX3, to four parametric fire scenarios (DF1, DF2, DF3 and DF 4). The response of these three beams for each of the fire scenario is summarized in Figures 11a–d, 12a–c and Table 3.

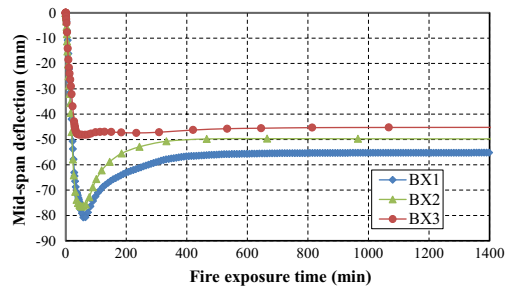
Results plotted in Figure 11 a-d show that larger depth in the case of beam BX1 leads to relatively lower mid-span deflections during fire exposure. This is



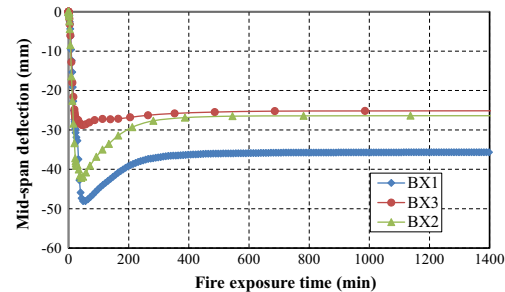
(a) Design fire exposure DF1



(b) Design fire exposure DF2

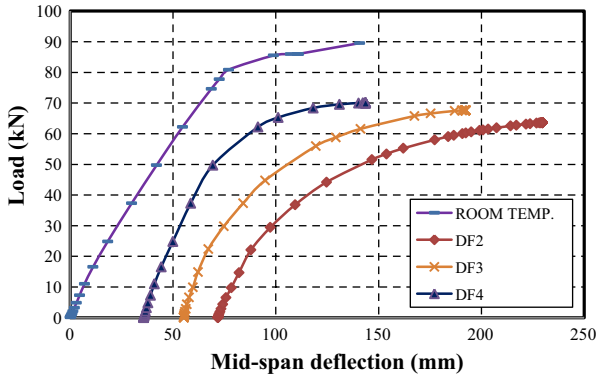


(c) Design fire exposure DF3

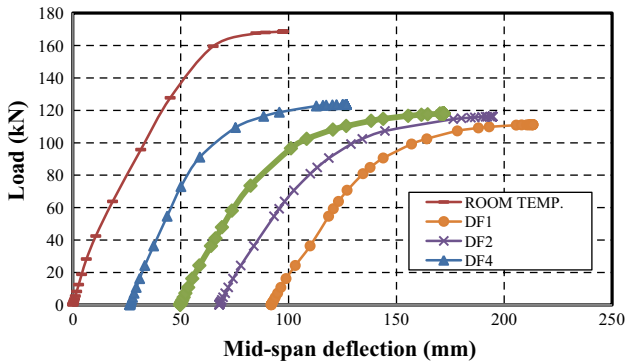


(d) Design fire exposure DF4

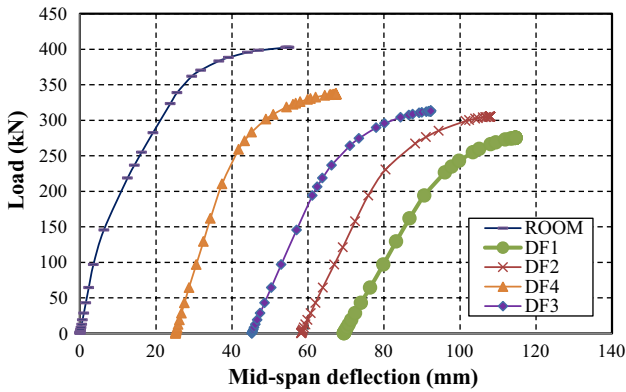
Figure 11. Effect of cross section on fire response of RC beams under different fire exposures.



(a) Beam BX1



(b) Beam BX2



(c) Beam BX3

Figure 12. Effect of fire scenario on residual load-deflection response in RC beams with varying cross section.

Table 3
Summary of Varied Parameters and Results from Parametric Study

| Parameter | Beam | Fire exposure scenario | Peak rebar temperature (°C) | Load ratio (%) | Support conditions | Peak deformation (mm) | Residual deformation (mm) | Predicted residual load capacity (kN) | | |
|-----------------|------|------------------------|-----------------------------|----------------|--|-----------------------|---------------------------|---------------------------------------|-----|-----|
| Fire scenario | BX1 | DF1 | 515 | 50 | SS ^a | 204 | — | — | | |
| | | DF2 | 403 | | | 125 | 72 | 64 | | |
| | | DF3 | 290 | | | 81 | 55 | 67 | | |
| | | DF4 | 174 | | | 48 | 35 | 70 | | |
| Load ratio | BX1 | DF2 | 403 | 30 | SS | 77 | 39 | 82 | | |
| | | | | 40 | | 52 | 79 | | | |
| | | | | 60 | | 97 | 71 | | | |
| Axial restraint | BX2 | DF4 | 161 | 50 | AR ^b = 10 kN/mm AR = 25 kN/mm AR = 50 kN/mm | 16 | 7 | 174 | | |
| | | | | 12 | | 5 | 181 | | | |
| Size of beam | BX1 | DF1 | 515 | 50 | SS | 204 | — | — | | |
| | | | | | | DF2 | 403 | 125 | 72 | 64 |
| | | | | | | DF3 | 290 | 81 | 55 | 67 |
| | | | | | | DF4 | 174 | 48 | 35 | 70 |
| | BX2 | DF1 | 498 | 274 | 379 | 111 | 181 | 92 | 111 | |
| | | | | | | | DF2 | 112 | 68 | 116 |
| | | | | | | | DF3 | 77 | 50 | 118 |
| | | | | | | | DF4 | 42 | 26 | 122 |
| | BX3 | DF1 | 470 | 350 | 252 | 142 | 88 | 69 | 275 | |
| | | | | | | | DF2 | 64 | 58 | 305 |
| | | | | | | | DF3 | 48 | 45 | 312 |
| | | | | | | | DF4 | 29 | 25 | 337 |

^a Simply supported

^b Axially restrained

due to slower temperature rise in concrete and rebar due to the larger mass of concrete. Mid-span deflections for beams BX1 and BX2, on the other hand are relatively large due to higher temperatures attained in the cross section. Moreover, mid-span deflection in the beam recovers with decrease in rebar temperatures as the fire temperature starts to decay. The ratio between peak deformation during fire exposure and residual deformation after the beam cools down to room temperature varies between 1.6 and 1.4 for beams BX1 and BX2. However, for beam BX3, this ratio varies from 1.3 to 1.1. This implies that deformations in beam BX3, although relatively lower in magnitude, are irrecoverable in nature. On the other hand, deformations in beam BX1 and BX2 are larger, with a greater percentage of the deformations being recovered after fire exposure. Also, while peak rebar temperature in beams of different cross section is similar since the cover to reinforcement is maintained constant, the variation in residual deformations is different and depends on cross-sectional size of the beam.

Based on results summarized in Table 3, it can be seen that reduction in capacity is lower in beams with larger cross-sectional size (such as beam BX1) or breadth to depth ratio. Results show that beam BX1 retains approximately 78% of its room temperature capacity after fire exposure scenario DF4, while beam BX3 having almost twice the breadth and depth of beam BX1, has a residual capacity of 84% after the same fire exposure scenario. Alternatively, the residual capacity of beam BX3 after fire exposure is 10% more than the post-fire residual capacity in beam BX1. This is due to lower overall temperatures within the cross section resulting from high specific heat and low conductivity of concrete. While rebar temperatures are similar in all three beams due to same amount of cover, reinforcing steel recovers most of its strength after cool down. Concrete on the other hand does not depict such recovery in strength when exposed to high temperatures immediately after cool down [29]. Thus, lower temperatures across the cross section, due to larger size of beam, result in improved residual response of RC beams.

6. Summary

A nonlinear finite element analysis is applied to quantify the influence of critical factors on residual response of fire exposed RC beam. Results from the studies clearly infer that residual capacity of fire exposed RC beams is a function of type of fire exposure, load level, cross sectional dimensions and level of axial restraint in RC beams. Specifically, RC beams exposed to most parametric fires can retain up to 70% of room temperature capacity, provided tensile rebar temperature does not exceed 450°C. Similarly, the reduction in residual capacity of fire exposed RC beams almost doubles when the load level increases from 30% to 60% of room temperature capacity of the beams. In addition, cross sectional dimensions of a fire exposed has an influence on residual capacity of fire exposed beams; with larger beam cross-section leading to higher residual capacity. Finally, the extent of axial restraint has a moderate influence on residual capacity of fire exposed beams.

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References

1. Beitel JJ, Iwankiw NR (2005) Historical survey of multi-story building collapses due to fire. *Fire Prot Eng* 3rd Quart 27:42
2. Gosain N, Drexler F, Choudhuri D (2008) Evaluation and repair of fire-damaged buildings. *Struct Mag* 9:18–22
3. Alonso C (2009) Assessment of post-fire reinforced concrete structures: determination of depth of temperature penetration and associated damage. *Concr Repair Rehabil Retrofit II*:471–478
4. Awoyera P, Akinwumi I (2014) Forensic investigation of fire-affected reinforced concrete buildings. *IOSR J* 11:17–23
5. ABAQUS (2012) Version 6.12 Documentation. Dassault Systemes Simulia Corp., Providence, RI
6. Dwaikat M, Kodur V (2009) Response of restrained concrete beams under design fire exposure. *J Struct Eng* 135:1408–1417
7. Gustafarro AH (1966) Factors influencing the fire resistance of concrete. *Fire Technol* 2:187–195. doi:[10.1007/BF02588551](https://doi.org/10.1007/BF02588551)
8. Lie TT (1978) Calculation of the fire resistance of composite concrete floor and roof slabs. *Fire Technol* 14:28–45. doi:[10.1007/BF01997259](https://doi.org/10.1007/BF01997259)
9. Chang JJ, Moss PJ, Dhakal RP, Buchanan AH (2009) Effect of aspect ratio on fire resistance of hollow core concrete floors. *Fire Technol* 46:201–216. doi:[10.1007/s10694-009-0087-7](https://doi.org/10.1007/s10694-009-0087-7)
10. Kodur V, Mcgrath R (2003) Fire endurance of high strength concrete columns. *Fire Technol* 39:73–87. doi:[10.1023/A:1021731327822](https://doi.org/10.1023/A:1021731327822)
11. Hsu JH, Lin CS (2008) Effect of fire on the residual mechanical properties and structural performance of reinforced concrete beams. *J Fire Prot Eng* 18:245–274. doi:[10.1177/1042391507077171](https://doi.org/10.1177/1042391507077171)
12. Kodur VKR, Dwaikat MB, Fike RS (2010) An approach for evaluating the residual strength of fire-exposed RC beams. *Mag Concr Res* 62:479–488. doi:[10.1680/mac.2010.62.7.479](https://doi.org/10.1680/mac.2010.62.7.479)
13. Bai LL, Wang ZQ (2011) Residual bearing capacity of reinforced concrete member after exposure to high temperature. *Adv Mater Res* 368:577–581. doi:[10.4028/www.scientific.net/AMR.368-373.577](https://doi.org/10.4028/www.scientific.net/AMR.368-373.577)
14. Kumar A, Kumar V (2003) Behaviour of RCC beams after exposure to elevated temperatures. *Inst Eng India Civ Eng Div* 84:165–170
15. ISO 834-1 (1999) Fire resistance tests-elements of building construction. Part 1: general requirement. ISO, Geneva
16. IS:3809-1979 (1979) Fire resistance test of structures (first revision). Bureau of Indian Standards, New Delhi

17. Ožbolt J, Bošnjak J, Periškić G, Sharma A (2014) 3D numerical analysis of reinforced concrete beams exposed to elevated temperature. *Eng Struct* 58:166–174. doi:[10.1016/j.engstruct.2012.11.030](https://doi.org/10.1016/j.engstruct.2012.11.030)
18. Aziz E, Kodur V (2013) An approach for evaluating the residual strength of fire exposed bridge girders. *J Constr Steel Res* 88:34–42. doi:[10.1016/j.jcsr.2013.04.007](https://doi.org/10.1016/j.jcsr.2013.04.007)
19. ACI (2008) ACI 318-08: Building code requirements for reinforced concrete, vol 552. ACI, Detroit
20. Kodur VKR, Dwaikat M (2008) A numerical model for predicting the fire resistance of reinforced concrete beams. *Cem Concr Compos* 30:431–443. doi:[10.1016/j.cemconcomp.2007.08.012](https://doi.org/10.1016/j.cemconcomp.2007.08.012)
21. Dwaikat MB, Kodur VKR (2009) Fire induced spalling in high strength concrete beams. *Fire Technol* 46:251–274. doi:[10.1007/s10694-009-0088-6](https://doi.org/10.1007/s10694-009-0088-6)
22. Lu L, Yuan Y, Caspeele R, Taerwe L (2015) Influencing factors for fire performance of simply supported RC beams with implicit and explicit transient creep strain material models. *Fire Saf J* 73:29–36. doi:[10.1016/j.firesaf.2015.02.009](https://doi.org/10.1016/j.firesaf.2015.02.009)
23. ECS (2002) EN 1991-1-2: actions on structures. Part 1-2: general actions—actions on structures exposed to fire. ECS, Brussels
24. Gao WY, Dai J-G, Teng JG, Chen GM (2013) Finite element modeling of reinforced concrete beams exposed to fire. *Eng Struct* 52:488–501. doi:[10.1016/j.engstruct.2013.03.017](https://doi.org/10.1016/j.engstruct.2013.03.017)
25. Lubliner J, Oliver J, Oller S, Onate E (1989) A plastic-damage model for concrete. *Int J Solids* 25:299–326
26. Lee J, Fenves G (1998) Plastic-damage model for cyclic loading of concrete structures. *J Eng Mech* 124:892–900
27. ECS (2004) EN 1992-1-1: Design of concrete structures. Part 1-1: general rules and rules for buildings. ECS, Brussels
28. ECS (2005) EN 1993-1-2: Design of steel structures. Part 1-2: general rules-structural fire design, vol 2. ECS, Brussels
29. ECS (2005) EN 1994-1-2: Design of composite steel and concrete structures. Part 1-2: general rules-structural fire design, vol 2. ECS, Brussels
30. Neves I, Rodrigues JPC, Loureiro ADP (1996) Mechanical properties of reinforcing and prestressing steels after heating. *J Mater Civ Eng* 8:189–194
31. ASTM E119-07 (2007) Standard methods of fire test of building construction and materials. ASTM International, West Conshohocken
32. ACI 216.1-97 (1997) Standard method for determining fire resistance of concrete and masonry construction assemblies. ACI, Farmington Hills
33. Wu B, Lu JZ (2009) A numerical study of the behaviour of restrained RC beams at elevated temperatures. *Fire Saf J* 44:522–531. doi:[10.1016/j.firesaf.2008.10.006](https://doi.org/10.1016/j.firesaf.2008.10.006)
34. Rafi M, Nadjai A, Ali F (2008) Finite element modeling of carbon fiber-reinforced polymer reinforced concrete beams under elevated temperatures. *ACI Struct J* 105:701–710
35. Crisfield M (1982) Accelerated solution techniques and concrete cracking. *Comput Methods Appl Mech* 33:585–607
36. Schweizerhof K (1993) Consistent concept for line search algorithms in combination with arc-length constraints. *Commun Numer Methods Eng* 9:773–784
37. Kodur VKR, Dwaikat M (2007) Performance-based fire safety design of reinforced concrete beams. *J Fire Prot Eng* 17:293–320. doi:[10.1177/1042391507077198](https://doi.org/10.1177/1042391507077198)