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METHODS FOR CONTROLLING ACCURACY OF PREDICTION OF POLYMER TEXTILE MATERIALS RELAXATION PROCESSES

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Issues of methodology and adequacy of mathematical modeling of polymer textile materials relaxation processes are examined. The developed integral criteria of confident prediction of polymer textile materials relaxation processes are based on minimization of integral functional convolution corresponding to a specific equation of state. The developed criteria of confident prediction of polymer textile materials relaxation processes can be introduced by their computerization.

Prediction of polymer textile materials relaxation process depends on several factors, such as accuracy of experimental data, adequacy of physical model, and optimum construction of the mathematical model. Higher experiment accuracy can be achieved by improving measuring devices and increasing the volume of tests to increase the representativeness of the experimental data [1].

For improving the physical model of relaxation processes it is necessary to combine study of samples of textile materials not only at the macrolevel, but also rheologically at the microlevel, which is associated with objective difficulties. An alternative of increase in accuracy of prediction of polymer textile materials relaxation processes is selection of an optimum mathematical model as a part of the referred physical model [2].

The physical approach based on the Aleksandrov–Gurevich's idea of taking account of the activating influence of deformation on acceleration of relaxation processes is universally accepted in the theory of nonlinear hereditary relaxation of polymers [3].

The foundation of construction of mathematical models of polymer textile materials relaxation processes, however, is the Boltzmann principle of superposition of responses onto the prehistory of deformation, which is expressed analytically in the Boltzmann–Volterra integral equation where the following determining integral equation corresponds to nonlinear-hereditary relaxation [4]:

$$\sigma_t = E_0\varepsilon_t - \left(E_0 - E_\infty\right)\cdot\int_0^t \varepsilon_\theta \varphi'_{\varepsilon;t-\theta} d\theta \tag{1}$$

where t – time; ε_t – deformation; σ_t – stress; E_0 – modulus of elasticity; E – modulus of viscoelasticity; $\varphi'_{\varepsilon t}$ – relaxation kernel.

As a rule, one of the following functions is chosen as the relaxation function $\varphi_{\varepsilon t}$: probability integral (PI) characterizing normal distribution [5], hyperbolic tangent (HT) [6], Kohlrausch function (KF) [7], the graph of which does not have the property of symmetry in contrast to PI and HT, but has a simpler analytical form, and normalized arctan of logarithm of reduced time (NAL) [8].

The NAL function corresponds to Caucy probability distribution, which has an important property: the arithmetic mean of random quantities distributed according to the Caucy normalized law is also distributed according to the Caucy normalized law.

This fact justifies selection of NAL function as the foundation of the mathematical model of polymer textile materials relaxation processes because any textile material is a complex macrosturcture consisting of simpler components (filaments, fibers, macromolecules, etc.). Consequently, the relaxation spectrum is a sum of elementary spectra that correspond to simpler elements distributed also in accordance with the Caucy law [9].

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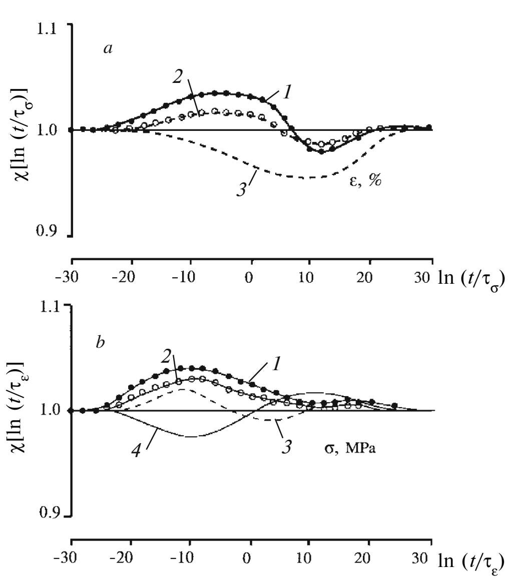

Fig. 1.

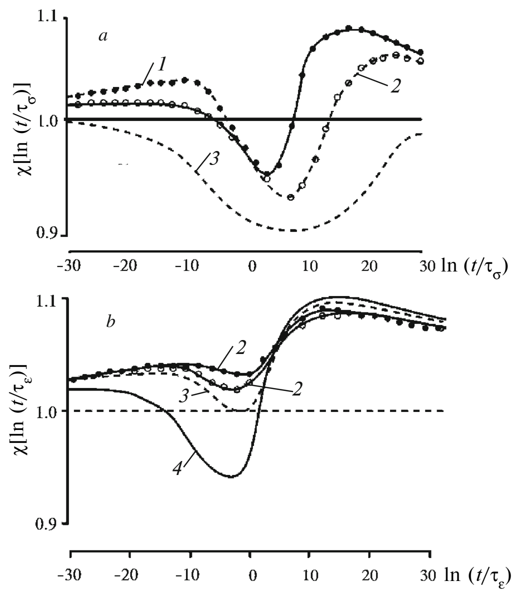

Fig. 2.

Fig. 1. Application of criteria (3) – (*a*) and (4) – (*b*) with NAL function to multicomponent yarn at 20 °C: *a* (at ε, %): *1* – 1, *2* – 2, *3* – 3; *b* (at σ, MPa): *1* – 19.8, *2* – 26.4, *3* – 33.0, *4* – 34.0.

Fig. 2. Application of criteria (3) –(*a*) and (4) – (*b*) with PI to multicomponent yarn at 20 °C: *a* (at ε, %): *1* – 1, *2* – 2, *3* – 3; *b* (at σ, MPa): *1* – 19.8, *2* – 26.4, *3* – 33.0, *4* – 37.0.

Availability of several mathematical models using different relaxation functions is reasonable because it makes it possible to get results of relaxation process prediction independent of each other. Relaxation characteristics obtained by averaging characteristics determined by different methods have a high degree of reliability than the characteristics obtained by a single method [10].

When approximating relaxation process by normalized function question arises about the legitimacy of such approach. The probability criteria, use of which enables determination of the relative error of a similar approximation, offer the answer to this question. Computerization of these criteria helps get a visual means of confirming the reliability of the calculated relaxation characteristics.

A reciprocal process for relaxation is creep process, the determining equation for which has the form:

$$\varepsilon_t = D_0\sigma_t + \left(D_\infty - D_0\right)\cdot\int_0^t \sigma_\theta \varphi'_{\sigma;t-\theta}\, d\theta, \tag{2}$$

where D_0 – initial elastic compliance; D_∞ – limit-equilibrium compliance; $\varphi'_{\sigma,t}$ – delay kernel.

We get the analytical correlation between relaxation and compliance from the nonlinear hereditary relaxation equation (1) at σ = const:

$$E_0 D_{\sigma,t} + \int_0^t D_{\sigma\theta} E'_{\varepsilon;t-\theta}\, d\theta = 1 \tag{3}$$

at $E'_{\varepsilon t} = \partial E_{\varepsilon t} = \partial(\ln(t/t_1))$.

Likewise, from nonlinear hereditary creep equation (2) we get at ε = const:

$$D_0 E_{\varepsilon t} + \int_0^t E_{\varepsilon\theta} D'_{\sigma;t-\theta}\, d\theta = 1 \tag{4}$$

at $D'_{\sigma,t} = \partial E_{\sigma t} = \partial(\ln(t/t_1))$.

The more successful the selection of integral kernels, the less deviation of the left parts of equations (3) and (4) from unity.

These equations may be regarded as integral criteria of correspondence of modulus of relaxation and compliance and, consequently, the other viscoelastic characteristics as well [11].

A multicomponent yarn obtained by circular spinning (cotton – 30 %, flax – 20 %, and lavsan – 50 %, linear density 29 tex, breaking stress σ_b = 225 MPa, breading deformation ε_b = 14 %) is proposed as the sample on which the criteria (3) and (4) were tested.

Graphical interpretation of the criteria (3) and (4) is a visual means of best selection of integral relaxation and delay kernels from a number of proposed ones. Designating

$$\chi\left(\ln\frac{t}{\tau_\sigma}\right)=E_0D_{\sigma t}+\int_0^t D_{\sigma\theta}E'_{\varepsilon;t-\theta}d\theta, \quad (5)$$

$$\chi\left(\ln\frac{t}{\tau_\varepsilon}\right)=E_0D_{\varepsilon t}+\int_0^t E_{\varepsilon\theta}D'_{\sigma;t-\theta}d\theta, \quad (6)$$

let us present the results of application of the criteria (3) and (4) to a multicomponent yarn (Fig. 1), which were obtained for integral kernels corresponding to NAL function and similar results for kernels corresponding to probability integral (Fig. 2). These results visually show the advantage of the first of these for modeling relaxation properties of the studied yarn.

The integral criteria (3) and (4) may also be regarded as the optimum selection of the mathematical model of relaxation of textile materials [12]:

$$\left|E_0D_{\sigma t}+\int_0^t D_{\sigma\theta}E'_{\varepsilon;t-\theta}d\theta-1\right|\to\min, \quad (7)$$

$$\left|D_0E_{\varepsilon t}+\int_0^t E_{\varepsilon\theta}D'_{\sigma;t-\theta}d\theta-1\right|\to\min. \quad (8)$$

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