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MATHEMATICAL MODELING AND METHODS OF DETERMINATION OF FUNCTIONAL-USE RELAXATION-RECOVERY PROPERTIES OF POLYMER TEXTILE MATERIALS

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Mathematical models and methods of determination of functional-use relaxation-recovery properties of textile industry materials having decisive importance for comparative analysis and qualitative sampling of materials having specific properties are reviewed.

Functional-use and performance properties of textile industry materials are based primarily on determination of the physicomechanical properties of these materials, to the study of which paramount attention must be paid.

For a comprehensive study and prediction of the functional-use and performance properties of textile industry materials to improve the quality of goods therefrom, it is proposed to conduct investigations of the basic relaxation-recovery and deformation-performance processes, i.e., relaxation and creep, which characterize the key physicomechanical properties of the materials [1-3].

It is expedient to conduct such study making use of mathematical modeling, followed by computer-aided prediction of relaxation and creep.

Although relaxation and creep processes are different in physical nature, they are, in fact, reciprocal processes harmoniously complementing each other. Because of this, the study of relaxation and deformation properties of textile industry materials relating primarily to the class of viscoelastic solids is an essential and, in some cases, an urgent task [4-6].

Relaxation process implies a change in the stress σ_t (or force F_t) applied to a material over a time t under the action of deformative ε:

$$\sigma = \sigma(t) = \frac{F(t)}{s}. \tag{1}$$

A key property of relaxation process is relaxation modulus $E_{\varepsilon t} = \sigma_t/\varepsilon$, which has two asymptotic values:modulus of viscoelasticity

$$E_\infty = \lim_{t\to\infty} E_{\varepsilon t} \tag{2}$$

and modulus of elasticity [7-9]

$$E_0 = \lim_{t\to 0} E_{\varepsilon t}. \tag{3}$$

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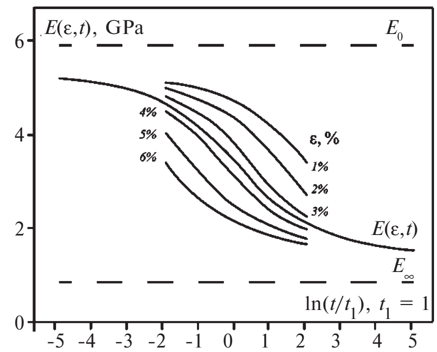

Fig. 1. Graph depicting experimental "family" of relaxation curves of 33.3 tex Nitron yarn at constant deformation values.

The relaxation modulus $E_{\varepsilon t}$ can be modeled mathematically using increasing normalized relaxation function $\varphi_{\varepsilon t}$ that acquires significance in the segment [0.1]:

$$E_{\varepsilon t} = E_0 - \left(E_0 - E_\infty\right)\varphi_{\varepsilon t}. \tag{4}$$

Let us take the normalized arc tangent of logarithm (NAL) that characterizes the Cauchy integral distribution as the relaxation function $\varphi_{\varepsilon t}$ [10, 14]:

$$\varphi_{\varepsilon t} = \frac{1}{2} + \frac{1}{\pi}\operatorname{arctg}\left(\frac{1}{b_{n\varepsilon}}\ln\frac{t}{\tau_\varepsilon}\right), \tag{5}$$

where τ_ε – a characteristic of mean relaxation time and $b_{n\varepsilon}$ – a characteristic of relaxation intensity.

For ease of modeling, logarithmic scale of reduced dimensionless time is used.

Instead of the mean relaxation time parameter τ_ε, which is defined by the deformation-time function of shears in logarithmic time scale, it is proposed to consider the mean relaxation time $\bar{\tau}_\varepsilon$ determined by the equation

$$\bar{\tau}_\varepsilon = \frac{1}{\varepsilon_2 - \varepsilon_1}\int_{\varepsilon_1}^{\varepsilon_1} \tau_\varepsilon d\varepsilon\ , \tag{6}$$

where ε_1, ε_2 – minimum and maximum values from the studied deformation range [15-18].

When determination only of qualitative properties of the materials is needed, switch from functional relationship to a constant is justified for mathematical modeling of relaxation-recovery properties. Such switch greatly simplifies the mathematical model, which is of considerable importance for investigating qualitative viscoelastic properties. Note that in more detailed studies of viscoelastic-relaxation processes, for example, from the spectral analysis premises, such switch to a simplified mathematical model is unjustified [19-21].

Thus, equation (4) is a mathematical model of polymer material relaxation process.

The choice of the function NAL as the foundation of the mathematical model of relaxation is not accidental because the Cauchy probability distribution, the integral function of distribution of which it is, possesses a unique property, i.e., the sum of the characteristics distributed in accordance with the Cauchy probability law also has by its distribution the Cauchy probability distribution. For textile industry materials obeyance to this law is extremely important because any complex textile object is an aggregate of simpler textile objects (yarns consist of fibers, fabrics consist of yarns, etc.). So, if the parameters of simpler textile materials obey the Cauchy probability distribution law, the parameters of more complex textile materials will also obey this distribution law [22-25].

The method of determination of functional-use relaxation-recovery properties of textile and light industry materials is based on numerical treatment of experimental "family" of relaxation curves for values of deformation constant $\varepsilon = \text{const}$ obtained on a stress relaxometer.

Table 1. Technical Characteristics of Studied Textile Yarns.

Name	Linear density, tex	Breaking stress, GPa	Breaking deformation, %
Capron-91	91	1.08	13.2
Capron-149	149	1.10	17.0
Lavsan-15.6	15.6	0.52	24.3
Lavsan-114	114	0.83	11.5
Capron-187	187	0.78	15.5
Capron-189	189	0.91	16.8
Nitron -33.3	33.3	0.92	12.8
Capron-410	410	0.68	23.3

An example of the graph of "family" of relaxation curves for Nitron polymer yarn with a linear density of 33.3 tex at constant values of deformation ε is given in Fig. 1.

Introducing the formula for the studied normalized relaxation function (5)

$$\varphi_{\varepsilon t} = \frac{1}{2} + \frac{1}{\pi}\operatorname{arctg}(W_{\varepsilon t}) = \varphi(W_{\varepsilon t}) \tag{7}$$

with the argument

$$W_{\varepsilon t} = \frac{1}{b_{n\varepsilon}}\ln\frac{t}{\tau_\varepsilon} = \frac{1}{b_{n\varepsilon}}\left[\ln\left(\frac{t}{t_1}\right) + \ln\left(\frac{t_1}{\tau_\varepsilon}\right)\right] \tag{8}$$

and differentiating (4) further we get the expression for the derivative from the relaxation modulus:

$$E'_{\varepsilon t} = \frac{\partial E_{\varepsilon t}}{\partial \ln \frac{t}{t_1}} = -(E_0 - E_\infty)\bar{r}_{\varepsilon t} = -\frac{1}{b_{n\varepsilon}}\cdot\frac{1}{\pi}\cdot(E_0 - E_\infty)\frac{1}{1+W_{\varepsilon t}}, \tag{9}$$

which contains the relaxation core $\bar{r}_{\varepsilon t}$:

$$\bar{r}_{\varepsilon t} = \frac{\partial \varphi_{\varepsilon t}}{\partial \ln t} = \frac{1}{b_{n\varepsilon}}\cdot\frac{1}{\pi}\cdot\frac{1}{1+W_{\varepsilon t}^2}. \tag{10}$$

The quantity t_1 means the value of the "base" time, which we will assume for convenience as $t_1 = 60$ sec. The base time value is needed so that a dimensionless quantity lay under the sign of the logarithm.

Considering that the extreme value of the derivative from the relaxation modulus $E'_{\varepsilon t}$ is attained at $W_{\varepsilon t} = W_\tau = 0$, let us determine its corresponding characteristic value of relaxation modulus E_τ:

$$E_\tau = E_0 - (E_0 - E_\infty)\cdot 0.5 = \frac{(E_0 + E_\infty)}{2}, \tag{11}$$

which enables determination of the width of the band of relaxation modulus values:

$$2\Delta E_\tau = 2\cdot(E_0 - E_\infty) = E_0 - E_\infty = 2\cdot(E_\tau - E_\infty), \tag{12}$$

whence we get the elasticity modulus value

$$E_0 = E_\tau + \Delta E_\tau \tag{13}$$

and the viscoelasticity modulus value

$$E_\infty = E_\tau - \Delta E_\tau. \tag{14}$$

Table 2. Technical Characteristics of Studied Capron Tapes.

Name	Linear density, tex (warp/weft)	Width, mm	Breaking stress, GPa	Breaking deformation, %
CT-2	16/16	2	1.53	18
CT-13	16/16	13	1.69	20
CT-15	29/29	15	1.82	25
CT-16	29/29	16	1.91	26
CT-25	29/29	25	4.41	25
CT-26	29/29	26	5.-89	20
ShK	303/14	5	1.96	30

Table 3. Technical Characteristics of Studied Technical Fabrics.

Name	Width, mm	Composition, %	Linear density, tex (warp/weft)	Breaking stress, GPa	Breaking deformation
TF-11	140	Lavsan-60, Capron-40	6.7/18.8	2.9	18
TF-13	100	Capron-100	3.3/5.0	3.7	22
TF-14	140	Lavsan-100	7.1/16.3	3.4	19
TF-16	150	Lavsan-80, Capron-20	6.9/14.3	3.6	21
TF-17	100	Capron-100	5.0/5.0	3.1	18

Hence, under the condition of attainability of the extremum at the midpoint of the band at $W_\tau = 0$ we can determine using expression (9) the characteristic value for the derivative from the relaxation modulus:

$$E'_\tau = -\frac{1}{b_{n\varepsilon}} \cdot \frac{1}{\pi} \cdot (E_0 - E_\infty) = -\frac{1}{b_{n\varepsilon}} \cdot \frac{2}{\pi} \cdot \Delta E_\tau . \tag{15}$$

Further, from expressions (9) and (15) we get

$$E'_{\varepsilon\tau} = E'_\tau \cdot \frac{1}{1 + W_{\varepsilon t}^2}, \tag{16}$$

or

$$W_{\varepsilon\tau} = \pm\sqrt{\frac{E'_\tau}{E'_{\varepsilon t}} - 1} . \tag{17}$$

Subtracting equation (4) from equation (11) and taking account of (12) we get:

$$E_{\varepsilon t} - E_\tau = (E_0 - E_\infty) \cdot (0.5 - \varphi_{\varepsilon t}) = 2\Delta E_\tau (0.5 - \varphi_{\varepsilon t}), \tag{18}$$

or

$$\Delta E_\tau = -0.5 \frac{(E_{\varepsilon t} - E_\tau)}{\varphi(W_{\varepsilon t}) - 0.5}, \tag{19}$$

whence we get the characteristic value of the relaxation intensity $1/b_{n\varepsilon}$:

$$\frac{1}{b_{\varepsilon t}} = -\frac{\pi}{2} \cdot \frac{E'_\tau}{\Delta E_\tau} . \tag{20}$$

The characteristic of the mean relaxation time τ_ε is obtained as a parameter of time shift of the relaxation curve obtained for the deformation ε value until coincidence with the generalized relaxation curve obtained by equation (4) [26-28].

Table 4. Calculated Relaxation-Recovery Properties of Textile Yarns.

Name	E_0, GPa	E_∞, GPa	$1/b_{n\varepsilon}$	$\bar{\tau}_\varepsilon$, sec
Capron-91	3.9	1.2	0.43	193
Capron-149	2.8	1.5	0.41	98
Lavsan-15,6	13.9	6.5	0.13	138
Lavsan-114	14.0	4.0	0.09	132
Capron-187	2.4	1.5	0.16	142
Capron-189	3.1	1.2	0.42	139
Nitron -33,3	5.9	0.8	0.30	104
Capron-410	3.9	1.8	0.32	207

Table 5. Calculated Relaxation-Recovery Properties of Studied Capron Tapes.

Name	E_0, GPa	E_∞, GPa	$1/b_{n\varepsilon}$	$\bar{\tau}_\varepsilon$, sec
CT-2	65.0	28.5	0.213	942
CT-13	45.7	14.8	0.313	832
CT-15	43.8	15.7	0.334	984
CT-16	42.2	16.2	0.249	791
CT-25	40.4	18.8	0.356	718
CT-26	38.2	19.2	0.241	637
ShK	49.8	21.5	0.312	587

Table 6. Calculated relaxation-recovery properties of studied technical fabrics.

Name	E_0, GPa	E_∞, GPa	$1/b_{n\varepsilon}$	$\bar{\tau}_\varepsilon$, sec
TF-11	53.5	22.6	0.709	394
TF-13	51.7	19.1	0.696	372
TF-14	57.4	18.3	0.647	389
TF-16	48.3	21.6	0.682	351
TF-17	59.8	16.9	0.584	384

The relaxation characteristics E_0, E_∞, τ_ε, and $b_{n\varepsilon}$ of textile industry materials obtained by the method of modeling of the functional-use relaxation-recovery properties of these materials are proposed to be used further for evaluation of their quality. Note as well that the relaxation properties obtained by mathematical modeling using NAL function obey Cauchy probability distribution, which is particularly important for materials that have a complex (composite) macroscopic structure.

The method of determination of functional-use relaxation-recovery properties of textile industry materials is applicable for a variety of textile materials, such as textile yarns (technical characteristics listed in Table 1), Capron tapes (Table 2), and technical fabrics (Table 3).

The calculated relaxation-recovery properties of textile yarns are listed in Table 4, of Capron tapes, in Table 5, and of technical fabrics, in Table 6.

A preliminary evaluation of the functional-use relaxation-recovery properties of the studied materials can be made from the calculation parameters of the functional-use relaxation-recovery properties of textile industry materials, namely, elasticity modulus E_0, viscoelasticity modulus E_∞, relaxation process intensity $1/b_{n\varepsilon}$, and mean relaxation time $\bar{\tau}_\varepsilon$ [29-32].

For example, among the textile yarns, Nitron yarn having a linear density of 33.3 tex is the one whose elastic properties are restored most fully after deformation because its viscoelasticity modulus is the lowest (E = 0.8 GPa). The elastic properties of Lavsan yarn having a linear density of 15.6 tex are least restored because its viscoelasticity modulus is the highest (E_∞ = 6.5 GPa).

At the same time, the restoration is most intense for Capron yarn having a linear density of 91 tex because its intensity parameter value is the highest ($1/b_{n\varepsilon} = 0.43$). The restoration process is least intense for Lavsan yarn because its intensity parameter has the lowest value ($1/b_{n\varepsilon} = 0.09$).

If the textile yarns are compared in terms of mean relaxation time, it can be stated here that among the textile yarns the elastic properties of Capron yarn having a linear density of 149 tex are restored fastest because its mean relaxation time is the least ($\bar{\tau}_\varepsilon$ = 98 sec) and the elastic properties of Capron yarn with a linear density of 410 tex are restored the slowest because its mean relaxation time is the longest ($\bar{\tau}_\varepsilon$ = 207 sec).

Thus, application of the mathematical model and the method of determination of functional-use relaxation-recovery properties of textile industry materials is tested on a representative group of textile materials, for which predictable relaxation-recovery parameters and properties, which have a decisive importance for comparative analysis and qualitative sapling of materials possessing specific properties, were obtained.

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