Erkenntnis
https://doi.org/10.1007/s10670-023-00679-2

ORIGINAL RESEARCH

The Unmeasurability of Absolute Velocities from the Point of View of Epistemological Internalism

Joanna Luc[1]

Received: 17 August 2022 / Accepted: 13 March 2023

Abstract

Absolute velocities in Newtonian mechanics are commonly regarded as unmeasurable. Roberts (Br J Philos Sci 59(2):143–168, 2008) provides a justification for this thesis which appeals to the observational indistinguishability of boost-related models of Newtonian mechanics. Middleton and Murgueitio Ramírez (Australas J Philos, 2020) criticise his argumentation by pointing out that his analysis of the notion of measurement is too restrictive, and that, under a weaker analysis (based on counterfactuals), absolute velocities are measurable. Jacobs (Australas J Philos, 2020) opposes their view, arguing that, on a properly formulated counterfactual-based account of measurement, absolute velocities are unmeasurable. However, in his argument, observational indistinguishability plays no role, even though, intuitively, it is very relevant for the issue of (un)measurability. This paper's aim is to bring observational indistinguishability back to this discussion. I build upon the observation that there is an analogy between possible views on knowledge and justification on the one hand, and on measurement on the other. In particular, I explore the distinction between externalist and internalist approaches. Counterfactual-based accounts are all externalist, whereas the observational indistinguishability of boost-related models becomes relevant if we are interested in an internalist concept of measurement.

1 Introduction

Absolute velocities in Newtonian mechanics are commonly regarded as unmeasurable. This claim might be justified (roughly) as follows. Boost-related models of Newtonian mechanics are observationally indistinguishable, but differ with respect to absolute velocities. The latter premise is a mathematical fact, whereas the former requires some justification; here, I will assume that it is true. To finish this argument, one needs to show that the observational indistinguishability of boost-related models

✉ Joanna Luc
joanna.luc.poczta@gmail.com

1 Institute of Philosophy, Jagiellonian University, Grodzka 52, 31-044 Kraków, Poland

Published online: 03 April 2023

entails the unmeasurability of absolute velocities (see, e.g., Roberts 2008). However, Middleton and Murgueitio Ramírez (2020) have recently questioned this last step, pointing out that its validity delicately depends on the details of our view regarding what counts as a measurement. Additionally, they argued that Roberts's concept of measurement, which excludes the measurability of absolute velocities, is too strong, whereas another concept of measurement, which they find more plausible, does not exclude the measurability of absolute velocities. Jacobs (2020) criticises this argument, defending the traditional view that absolute velocities are unmeasurable, but without appealing to the observational indistinguishability of boost-related models. This might be surprising since observational indistinguishability seems to be, intuitively, very relevant for the discussed issue. This is the motivation for the current paper. I will suggest that observational indistinguishability disappeared from the debate about the (un)measurability of absolute velocities because Middleton and Murgueitio Ramírez, as well as Jacobs, use externalist notions of measurement (where "externalist" is understood in the same way as in the discussion about knowledge and justification in contemporary epistemology). In contrast to them, I will sketch an internalist account of measurement, and argue that, within this view, the connection between the unmeasurability of absolute velocities and the empirical indistinguishability of boost-related models can be properly established. Therefore, the upshot of this paper will be twofold: at a general level, I will advocate an internalist way of thinking about the notion of measurement (in opposition to purely externalist accounts), and at a more specific level, I will show that this approach gives us a better understanding of the reasons why absolute velocities should be regarded as unmeasurable.

The following is the plan for this paper. In Sects. 2–4, I will review the preceding stages of the debate on the (un)measurability of absolute velocities, that is, the papers by Roberts (2008), Middleton and Murgueitio Ramírez (2020) and Jacobs (2020). The latter two papers notice an analogy between the analyses of knowledge present in the epistemological literature and possible analyses of measurement. I will develop this analogy further by bringing into the debate the distinction between externalist and internalist approaches, known from the debates about justification and knowledge, but also applicable to the issue of measurement (Sect. 5). In Sect. 6, I will argue that, if we understand measurement internalistically, the observational indistinguishability of boost-related models is the reason why absolute velocities are unmeasurable. In Sect. 7, I will show why Middleton and Murgueitio Ramírez's response to what they call "Knowledge Objection" does not undermine my proposal. Finally, Sect. 8 will summarise the main points of this paper.

2 Roberts (2008): Absolute Velocities are Not Measurable

The topic of Roberts's (2008) paper is how to explain what he calls the "Measurability-Invariance Principle", which states that, for any physical theory, every quantity that is in principle empirically measurable according to that theory is invariant under all dynamical symmetries of that theory. However, we are only interested in a much narrower issue, namely, in his argumentation that absolute velocities are not measurable (Roberts 2008, pp. 159–161).

We will consider only possible worlds in which the laws of Newtonian mechanics are exactly true.[1] Roberts based his argument on three assumptions, which are as follows[2]

- Assumption 1: A quantity Q is measurable in a world w only if there is a set of conditions C and a physical quantity P such that the laws of w guarantee that whenever C is satisfied, the value of Q is equal to some function of the value of P, $f(P)$. Conditions C (the "set-up conditions") specify the construction of the measuring device, the physical circumstances in which it is used and the procedure of using it. Physical quantity P (the "pointer variable") registers the result of the measurement. (It might be, for example, the relative position of a needle with respect to a dial.)
- Assumption 2: Conditions C are invariant under the dynamical symmetries of the laws of w. (For example, the proper functioning of a device measuring absolute speed, if there is any such device, is preserved under Galilean boosts; that is, if it works properly in some physical situation, it also works properly in a boost-related physical situation.)
- Assumption 3: The pointer variable P is invariant under the dynamical symmetries of the laws of w. (For example, relative positions are invariant under Galilean boosts.)

Let me make some comments about these assumptions. First, notice that assumption 1 is modal: the expression "the laws of w guarantee..." makes it a statement about all possible worlds with the same laws of nature as w, and not only about w. This is reasonable because the mere satisfaction of $Q = f(P)$ in conditions C in w would be too weak a requirement. A measurement should have certain modal stability; it cannot rely on entirely accidental coincidences between quantities. However, the strength of this modal stability can be a matter of debate. In Roberts's proposal, this strength is rather high since it takes into account all nomologically possible worlds (from the point of view of w).

Second, Roberts does not impose any further constraints on the function f. I think that at this stage this is appropriate: the value of Q should be uniquely determined by the value of P, but for this, the fact that f is a function suffices (notice that we require

[1] We know that they are not exactly true in the actual world, but still these considerations could tell us something about the actual world: first, because the laws of Newtonian mechanics are approximately true in the actual world, and second, because a similar argument can be run for many other theories and quantities that vary under their dynamical symmetries, including those theories that are more accurate than Newtonian mechanics.

[2] My formulation of the assumptions and argument is not exactly the same as in the original text. Assumption 1 should be regarded as a definition (or an analysis) of the notion of measurement rather than a physical assumption; it has this status in Middleton and Murgueitio Ramírez's reconstruction of this argument.

that Q is a function of P, not the other way around). However, further constraints on f can be justified within the internalist view on measurement—see footnote 13.

Third, P and Q might be regarded as functions whose arguments are time and a physical system and whose values are real numbers. Therefore, the equality $Q = f(P)$ should be understood as holding for any time t and any pair of physical systems, s_1 and s_2, provided that conditions C are satisfied (formally: $Q(t, s_1) = f(P(t, s_2))$ in conditions C). Here, s_1 is the system that is measured and s_2 is a measurement device. Conditions C need to include (among other things) some information about the relationship between s_1 and s_2 at a given time.

Fourth, Roberts ignores the dynamical and temporal aspects of measurement (stressed, e.g., by Wallace 2022); that is, the fact that a measurement is a process that starts with a device being in a certain "ready" state and, only after some time (during which the device interacts with the measured system), the value of pointer variable P is such that $Q = f(P)$. However, it seems that this complication would not substantially change the discussion reviewed here. Wallace himself seems to presuppose a strong notion of measurement that is similar to Roberts's (but adjusted by taking into account the dynamical and temporal aspects), since he concludes from the lack of universal functional dependence between the pointer variable (in his notation: some function of O) and symmetry-variant quantity (in his notation: any quantity dependent on g) that the latter is unmeasurable (2022, pp. 329–330).

The argument itself runs as follows. Consider a world w_1 such that all its dynamical laws are invariant under the Galilean group of transformations. Assume for *reductio* that in w_1 there exist absolute speedometers—that is, measuring devices that measure their absolute speeds S and register them by means of some pointer variable P. By assumption 1, this implies that there are conditions C such that the laws of w_1 guarantee that whenever C holds, $S = f(P)$ for some function f. Let us assume that in some region of w_1 conditions C are satisfied, so that $S = f(P)$ there (if this is not the case in the particular world we have started with, surely there will be some world with the same laws where this is true). Now, consider another possible world w_2 with the same laws as w_1, which can be "obtained" from w_1 by applying a (non-trivial) Galilean boost to w_1. Since all dynamical laws of w_1 are invariant under Galilean boosts, it follows from assumption 2 that conditions C also hold in w_2; and in the region where this is the case, the value of P is the same as in the corresponding region of w_1 (by assumption 3). Therefore, the value of $f(P)$ is also the same, since this is a function of P. Assumption 1 then entails that $S = f(P)$ in the region of w_2 under consideration (because conditions C are satisfied there). However, the absolute speed S *does* change under (non-trivial) Galilean boosts, so it must be different in w_2 than it is in w_1. This means that $S \neq f(P)$ in w_2, which is a contradiction.

One controversial feature of this argument is that it is formulated in terms of symmetry-related possible worlds. As such, it presupposes that there are different possible worlds such that one of them can be "obtained" by applying a boost transformation (or, more generally, any dynamical symmetry) to the other. However, this excludes a plausible view that there are no symmetry-related possible worlds: there are only symmetry-related models, but each of them represents the same possible world.[3] A way out of this difficulty is to observe that in this argument we consider possible worlds *in which there are absolute velocities*, so that two boost-related models represent two different possible worlds with different values of absolute velocities (provided that these boosts are interpreted actively). Even if one is sceptical about the existence of absolute velocities in the actual world, their existence does not involve a logical contradiction, so one can reasonably ask questions such as: in (logically) possible worlds where there are absolute velocities, are they measurable? Moreover, if such a scepticism is warranted, its main justification is based precisely on the fact that absolute velocities would be unmeasurable if they existed, so our considerations are relevant for both advocates and deniers of the existence of absolute velocities.

For our purposes, the crucial insight of this argument is that the unmeasurability of absolute velocities is closely related to the fact that two possible worlds differing only by a boost (which entails their difference with respect to absolute velocities) would be observationally indistinguishable for epistemic subjects inhabiting them. That is, at least if we grant assumption 3, there would be no way for such subjects to convince themselves that they live in a world w_1 rather than in a boost-related world w_2; and for this reason, any differences between w_1 and w_2 would be beyond their epistemic reach. This is because the only physical quantities to which they have perceptual epistemic access are the pointer variables, and those are assumed to be invariant under the dynamical symmetries; the epistemic access to all other physical quantities is mediated by the pointer variables.

3 Middleton and Murgueitio Ramírez (2020): Absolute Velocities are Measurable

Middleton and Murgueitio Ramírez (2020) object to Roberts's argument for the unmeasurability of absolute velocities by arguing that his construal of the notion of measurement is too demanding. They also claim that what they regard as a more adequate analysis of this notion allows absolute velocities to be measurable.[4] Importantly (for our further analysis), they stress that they do endorse the view that

[3] At least if we consider these models as representing entire possible worlds. If these models represent subsystems, then the differences between them might become physically meaningful (cf. Luc (2022: 5–6)).

[4] Wallace (2022, p. 333) also claims that, *in some sense* symmetry-variant quantities can sometimes be measured. However, this is possible only if such quantities can be interpreted as relational quantities that capture a relation between the original system and some other system. For example, it might seem that the speedometer measures the velocity of the car *simpliciter*, which is a symmetry-variant feature of the car (call this "the first interpretation"); however, what it in fact measures is the car's velocity relative to the road, which is a symmetry-invariant feature (at least in classical mechanics) of the larger system consisting of the car and the road ("the second interpretation"). In the terminology of Luc (2022), in the first interpretation, the car is represented explicitly, whereas the road is represented implicitly—that is, the reference frame is here regarded as corresponding to a physical object (in our example—the road), and not merely as a formal device for describing the physical situation. The two interpretations are connected in the following way: the second represents explicitly the physical object that in the first was represented only implicitly (which is why these are two different ways of modelling *the same* physical situation). Clearly, neither Wallace's view nor mine can be used to argue that absolute velocities are measurable because they are not interpretable as relational quantities.

boost-related worlds are perceptually indistinguishable; they only reject the claim that “the perceptual indistinguishability of worlds related by boosts entails the non-measurability of absolute velocity” (2020, p. 2).

Middleton and Murgueitio Ramírez consider (what they call) the Basic World, which is Newtonian-like (i.e., it has absolute space and its dynamical laws have boosts as symmetries) and consists of a single car moving along an infinitely long, straight road in a fixed direction. Additionally, they assume that this car is equipped with an ordinary speedometer, which measures its velocity[5] with respect to the road, and that the road is always at absolute rest. From these assumptions, it follows that the number on the speedometer is equal to the car's absolute velocity, because this has the same value as the velocity of the car with respect to the road (which is measured by the speedometer). Does this mean that this speedometer in the Basic World *measures* the car's absolute velocity? Roberts's argument reviewed in the previous section can be applied to this case, so if this argument is correct, then the speedometer in the Basic World does not measure absolute velocity.

The crucial ingredient in Middleton and Murgueitio Ramírez's reconstruction of Roberts's argument is the analysis of the notion of measurement that they take to be presupposed by Roberts (2020, p. 4):

> *The Perception-Grounded Modal Analysis*: A device d with pointer variable P measures a quantity Q at time t in world w iff there exists a collection of conditions C such that (i) C is perceivable, (ii) C is satisfied at t in w, and (iii) for every world w' with the same laws of nature as w, if d exists at time t' in w' and C is satisfied at t' in w', then $P(t', w') = Q(t', w')$.

Another assumption that they attribute to Roberts is “Perceptual Invariance”, which states that if C is perceivable, then it is preserved by boosts (and the same holds for P, which is perceivable by definition). The Perception-Grounded Modal Analysis corresponds to Roberts's assumption 1, and the Perceptual Invariance corresponds to assumptions 2 and 3. The only important difference (as far as I can see) is that Middleton and Murgueitio Ramírez require $P(t', w') = Q(t', w')$, whereas Roberts imposes a weaker requirement—namely, $Q = f(P)$.[6] I think that the latter is more reasonable because the functional relationship between Q and P might not be linear, and if it is, whether the exact equality holds will depend on the choice of units,[7] whereas whether the functional relationship between Q and P holds is independent of the choice of units (although the exact mathematical formula for this relationship could be different for different choices). This will be important for me later

[5] Strictly speaking, it measures only the value of its velocity; that is, its speed. However, I will follow the authors in this loose way of speaking.

[6] As noted earlier (third comment on p. 5), I think that the first argument of Q and P should be more fine-grained—not an entire possible world, but a system in such a world. This is because we attribute physical quantities to various systems, most of which are smaller than the entire world.

[7] Recall that our pointer variable is typically the relative position of a needle with respect to a dial, whereas the measured quantities are very diverse (velocity, temperature, voltage, etc.). The authors presumably postulate $P = Q$ because they think about the car's absolute velocity in the Basic World as being measured by its relative velocity with respect to the road. However, this is a simplification (the pointer variable of the speedometer is the relative position, not the relative speed), and even granting this, typically P and Q would be quantities of different kinds.

(see footnote 17), but I do not regard this as an objection to the authors because all their considerations can be easily amended to take this into account. Middleton and Murgueitio Ramírez's (2020, p. 4) reconstruction of the reasoning based on these assumptions is similar to what was presented in Sect. 3, so it will be omitted here.

One can retain the thesis that the speedometer measures the absolute velocity of the car in the Basic World only if one rejects one of the above assumptions. Middleton and Murgueitio Ramírez (2020, p. 5) propose to reject the Perception-Grounded Modal Analysis because they find it too restrictive. According to them, requiring the pointer variable to correctly capture the measured quantity in all nomologically possible worlds in which d exists and C holds is too strong. They give the following counterexample: even though it is nomologically possible that the volume of mercury in the thermometer undergoes a large fluctuation so that it shows the wrong value of the temperature at that moment, we should not conclude from this possibility that the thermometer in the actual world does not measure temperature (assuming that the value of the pointer variable in the actual world correctly captures the value of the measured quantity). More generally, the authors claim that "for any collection of perceivable set-up conditions, we can find unlikely yet nomologically possible scenarios in which the measurement device malfunctions even though the set-up conditions are satisfied" (2020, p. 5).

To avoid such counterexamples, the authors propose that we should require the measurement devices to be reliable only in the nearby possible worlds, not in all nomologically possible worlds. One such analysis, inspired by Nozick's (1981) analysis of knowledge, is as follows (Middleton and Murgueitio Ramírez 2020, p. 7):

> *The Counterfactual Analysis*: A device d with pointer variable P measures quantity Q at time t iff (1) $P(t) = Q(t)$ and (2) for any $x \neq Q(t)$ in the range of d, if it had been the case that $Q(t) = x$, then it would have been the case that $P(t) = x$.

It seems that a more transparent formulation of condition (2) would be as follows: if x is the actual value of $Q(t)$, then for any $x' \neq x$ in the range of d, if it had been the case that $Q(t) = x'$, then it would have been the case that $P(t) = x'$. As before, I think that instead of requiring the values of the measured quantity Q and the pointer variable P to be equal, we should only postulate the former to be a function of the latter. (The above Counterfactual Analysis can easily be amended in this way.) The restriction to nearby possible worlds is implicit in the counterfactual in condition (2).[8] Middleton and Murgueitio Ramírez (2020, p. 7) claim that this analysis avoids the counterexample with the thermometer because in the nearby possible worlds the fluctuation cannot be much larger than in the actual world. In general, they suggest that this analysis is extensionally adequate, although they argue for this by considering only a very limited range of examples.

Moreover, according to Middleton and Murgueitio Ramírez (2020, p. 7), in light of this analysis, the speedometer in the Basic World measures the car's absolute

[8] The following truth-conditions for counterfactuals are presupposed here: "if it had been the case that A, then it would have been the case that B" is true iff in all possible worlds sufficiently close to the actual world in which A holds, B also holds (or: in the closest possible world in which A holds, B also holds).

velocity. Condition (1) is satisfied by construction. To evaluate condition (2), we need to ask ourselves what would be the case if the car's absolute velocity at t had some value $v' \neq v$, where v is its value in the Basic World. The authors claim that condition (2) is satisfied because "the closest world to the Basic World in which the car is moving with an absolute velocity of v' at t is a world in which the road remains at absolute rest and only the absolute velocity of the car is altered" (2020, p. 7). In such a world, the speedometer shows v', which coincides with the value of the car's absolute velocity in that world. The authors conclude that "there exists at least one reasonable analysis of measurement according to which the speedometer in the Basic World measures the absolute velocity of the car" (2020, p. 6).

4 Jacobs (2020): Absolute Velocities are Not Measurable

Jacobs (2020) challenges Middleton and Murgueitio Ramírez by claiming that their proposed analysis of measurement is neither reasonable nor entails the measurability of absolute velocities.

Concerning the first thesis (i.e., the unreasonability of the Counterfactual Analysis), Jacobs begins by recalling Nozick's (1981) full analysis of knowledge, which is as follows: S knows that p iff (1) p is true; (2) S believes that p; (3) if p were false, S would not believe that p; and (4) if p were true, S would believe that p. He then constructs an analogue for the notion of measurement (Jacobs 2020, p. 3):

> *Truth-Tracking*: P accurately measures Q iff (i) $P(t) = Q(t)$; (ii) if it were the case that $Q(t) = x'$ for $x' \neq Q(t)$, then it would be the case that $P(t) = x'$; and (iii) if it were the case that $Q(t) = x$ for $x = Q(t)$, then it would be the case that $P(t) = x$.

Condition (ii) might look self-contradictory as it seems to require that $Q(t) = x' \neq Q(t)$; however, the first occurrence of $Q(t)$ refers to the world that is a subject of the counterfactual supposition, whereas the second occurrence of $Q(t)$ refers to the actual world (this difference is not visible in the notation). Similarly, condition (iii) might seem trivial, but it is not, for exactly the same reason. I think that a more transparent formulation of these conditions is as follows: if x is the actual value of $Q(t)$, then: (ii) for any $x' \neq x$, if it were the case that $Q(t) = x'$, then it would be the case that $P(t) = x'$, and (iii) if it were the case that $Q(t) = x$, then it would be the case that $P(t) = x$.

The problem with the Counterfactual Analysis[9] by Middleton and Murgueitio Ramírez is, according to Jacobs, that it does not include condition (iii), which is the analogue of Nozick's condition (4). Jacobs concludes from this that the Counterfactual Analysis provides only necessary conditions for the measurement, which are not jointly sufficient. To test whether the speedometer in the Basic World measures the car's absolute velocity in light of the Truth-Tracking analysis, we need to ask what

[9] Jacobs's analysis is also based on counterfactuals; however, I will use the expression "Counterfactual Analysis" (with capital letters) to denote Middleton and Murgueitio Ramírez's specific proposal.

happens in the nearby worlds in which the car has the same absolute velocity as in the Basic World. These are worlds in which the road's absolute velocity is slightly different than in the Basic World. However, in such worlds, the car's absolute velocity is not the same as its relative velocity with respect to the road, so (iii) is not satisfied.[10] Therefore, the speedometer does not measure the car's absolute velocity in the Basic World.

Concerning the second thesis (i.e., that Counterfactual Analysis does not classify the speedometer in the Basic World as measuring the car's absolute velocity), its truth depends crucially on what possible world in which the car has a different absolute velocity than in the Basic World counts as nearby with respect to the Basic World. Is this the world in which the road remains at absolute rest and the car has a different relative velocity with respect to it ("the Relative World"), or the world in which all velocities are boosted by a constant factor, so the car's relative velocity with respect to the road is the same as in the Basic World ("the Boosted World")? Middleton and Murgueitio Ramírez (2020, p. 7) opt for the former, whereas Jacobs (2020, pp. 4–5) argues for the latter. However, in the Boosted World, the car's absolute and relative velocities are not the same, so if the Boosted World is indeed more similar to the Basic World than the Relative World, then even the Counterfactual Analysis gives the verdict that the speedometer does not measure the car's absolute velocity.

As should be visible from the above summary of Jacobs's paper, even though his general conclusion concerning absolute velocities is the same as that of Roberts, their argumentative strategies significantly differ. In particular, the fact that boost-related worlds are observationally indistinguishable, which was crucial for Roberts, plays no role in Jacobs's account. Jacobs appeals to the Boosted World, but he is interested in its similarity to the Basic World (in order to assess its relevance for the evaluation of counterfactuals), not in its observational indistinguishability from the Basic World.

This observation is my point of departure because it seems highly intuitive that the observational indistinguishability of boost-related worlds should be the main motivation for regarding absolute velocities as unmeasurable. If we cannot tell

[10] It seems to me that this conclusion is not unavoidable. If instead of changing the road's absolute velocity, we changed some other detail of the Basic World (e.g., the size of the car), while leaving that absolute velocity the same, then condition (iii) would be satisfied. The crucial question is: which possible worlds should be regarded as sufficiently similar to the Basic World to be taken into account in the evaluation of counterfactuals such as (ii) and (iii)? Changing the road's absolute velocity might seem to be more relevant in the context of (iii) than changing the size of the car, but why is this so? Is this because the former change results in the mismatch between the car's absolute velocity and its relative velocity with respect to the road, which is our pointer variable, and our only source of knowledge about the values of any physical quantities are the values of pointer variables? Or should we vary any physical quantity that can be changed without changing the absolute velocity of the car? These considerations might point to a more general problem for counterfactual analyses of epistemological concepts: if the decision concerning which features of the actual world should be varied and which should be fixed in possible worlds that are taken into account in the evaluation of such countefactuals depends on whether these features are epistemically relevant, then an important part of the conceptual job is done by the assessment of the epistemic relevance of these features rather than by the enagement of counterfactuals.

apart two worlds differing by absolute velocities, how can we empirically establish the absolute velocities of the objects in any of these worlds? Whatever hypothesis concerning absolute velocities we accept in one of them, this hypothesis would be equally reasonable to accept in the other—but it would be true in, at most, one of them. Why, then, has the observational indistinguishability disappeared from the picture? And on what grounds could it be reintroduced, if at all?

5 Externalism and Internalism in Epistemology

The discussion about measurement reviewed above is closely related to one debate in epistemology concerning the nature of epistemological concepts such as knowledge and justification, namely to the debate between externalism and internalism. Let us first see what these two positions amount to, and later we will relate them to the views on measurement previously presented.

I will rely on the presentation in BonJour's (2009) book entitled "Epistemology: Classic Problems and Contemporary Responses" (cf. also, e.g., Pappas 2017 and references therein). Externalism can be characterised (BonJour 2009, pp. 203–204) as a view

> that epistemic justification can depend in part or perhaps even entirely on matters to which the believer in question need have no cognitive access at all, matters that are entirely external to his or her cognitive viewpoint. Thus, to take the most widely-held recent externalist view, a belief might allegedly be justified for a particular believer simply because the causal process that led to its adoption is cognitively reliable, that is, is a process of a general kind that in fact produces true beliefs in a high proportion of the cases in which it occurs—even if both the nature of the process and its reliability are entirely unknown and cognitively inaccessible to the believer in question.

In contrast, internalism is associated with the first-person perspective (BonJour 2009, p. 204):

> The fundamental claim of internalism (...) is that epistemological issues arise and must be dealt with from within the individual person's first-person cognitive perspective, appealing only to things that are accessible from that standpoint. The basic rationale is that what justifies a person's beliefs must be something that is available or accessible to him or her, that something to which he has no access cannot give him a reason for thinking that one of his beliefs is true (...)

It should now be clear that both the concept of measurement used by Middleton and Murgueitio Ramírez, as well as the concept of measurement used by Jacobs, are externalist ones. In both papers, an explicit reference is made to Nozick (1981), who is one of the main advocates of externalism with respect to the concept of knowledge.[11] Both the Counterfactual Analysis and the Truth-Tracking analysis of measurement are developed in close analogy to Nozick's analysis of knowledge.

This observation suggests that there might be an unexplored territory of analyses of the concept of measurement that rely on the analogy to internalist rather than externalist stances in epistemology.[12] This leads us to the question of whether we lose anything by analysing the concept of measurement in a purely externalist way. Does an internalist have anything interesting to add to the analysis of the concept of measurement? This is a subtle issue. First, it is disputable whether the correct analysis of epistemological concepts such as knowledge and justification should be purely externalist, purely internalist, or perhaps some mixture of both. Second, even if we managed to get an opinion on this matter, it does not straightforwardly carry over to the issue of measurement. It is conceivable, in principle, that justification and knowledge should be viewed in an internalist way, while measurement should rather be conceived externalistically (or the other way around); some more complex positions are also possible. Concerning the concepts of knowledge and justification, I believe that, even though the externalist perspective might provide some interesting insights about them, the internalist perspective is indispensable and more fundamental. Let me invoke BonJour (2009, pp. 216–219, all emphases are in the original text), who argues for this thesis:

> (...) the internalist approach pertains to epistemological issues that are raised from what is essentially a *first-person* rather than a third-person perspective, that is, to the situation where I ask what reasons *I* have for thinking that my *own* beliefs, rather than someone else's, are true. (...)
> No matter how much work may be done in delineating externalist conceptions of knowledge or justification or reliability and in investigating how those apply

[11] Another leading externalist is Goldman (2012), the author of the reliabilist conception mentioned by BonJour in the above quote.

[12] On the externalist side, there is also room for further developing the analogy between views on knowledge and justification on the one hand and measurement on the other. Let me make two remarks. First, Armstrong (1973) develops his externalist view on knowledge in analogy to his view on measurement, which is similar to Roberts's in that he requires the correctness of the readings of a measurement device to be guaranteed by the laws of nature. This shows that inspiration can go in both directions. Second, such a strong conception can be weakened in not only a Nozick-like way (by using counterfactuals), but also in a Goldman-like way—that is, by replacing in condition (iii) of the Perception-Grounded Modal Analysis "for every world" with "for most worlds" or "for a high proportion of worlds". The Goldman-like approach deals with the large fluctuation issue (see p. 10) because such fluctuations occur in a small proportion of nomologically possible worlds, but classifies absolute velocities as unmeasurable (because each possible world in which the relative velocity measured by a speedometer coincides with the absolute velocity corresponds to infinitely many worlds in which they are not equal, "obtained" by boosting the initial world). Another variant of Goldman-like analysis could be formulated in terms of objective probabilities: in conditions C, the functional relationship $f(P) = Q$ is required to hold only with a sufficiently high probability, possibly lower than 1. This would likely also render absolute velocities unmeasurable (although the detailed answer would require specifying the precise meaning of objective probability in this case, which is a non-trivial task). However, in both versions of the Goldman-like approach, the key reason for the unmeasurability of absolute velocities is the fact that in most possible cases absolute velocity and relative velocity (or any other pointer variable) do not coincide, and not the fact that symmetry-related possible worlds are empirically indistinguishable. In contrast, the internalist view on measurement proposed in this paper restores the importance of the latter.

> to various kinds of beliefs or areas of investigation, there is a way in which all such results are merely hypothetical and insecure as long as they cannot be arrived at from the resources available within a first-person epistemic perspective. If, for example, an epistemologist claims that a certain belief or set of beliefs, whether his or her own or someone else's, has been arrived at in a reliable way, but says this on the basis of cognitive processes of his or her own whose reliability is at best an external fact to which he or she has no first-person access, then the proper conclusion is merely that the belief or beliefs originally in question are reliably arrived at (and perhaps thereby are justified or constitute knowledge in externalist senses) *if* the epistemologist's own cognitive processes are in fact reliable in the way that he or she no doubt believes them to be. But the only apparent way to arrive at a result that is not ultimately hypothetical in this way is for the reliability of at least some cognitive processes to be establishable on the basis of what the epistemologist can know directly or immediately from his or her first-person epistemic perspective.

It seems plausible that a similar argument can also be made in the case of the concept of measurement. The mere fact that our measurement devices are reliable indicators of the values of physical quantities (considered in abstraction from whether we can have any reasons for believing that this is so) does not by itself help us in deciding whether we should believe that the value of a given quantity in the case under consideration is (approximately) equal to the one displayed by our device. For the measurement device to be useful for our cognitive enterprises, what is needed is not only the reliability of this device, but also some reasons to think that this device is indeed reliable. In the next section, I will argue that this internalist criterion cannot be met in the case of absolute velocities, which means that absolute velocities are unmeasurable in the internalist sense.

6 The Unmeasurability of Absolute Velocities: An Internalist Perspective

Roberts, whose argument was our starting point, seems to appreciate the internalist way of thinking about cognition and measurement, as he writes (2008, p. 163):

> In order to count as an empirical measurement procedure, a procedure must be such that we can use it to acquire empirical knowledge. We can hardly do that if we cannot determine, ultimately on the basis of perception, that the procedure is being used, and what the value of its pointer variable is.

What he emphasises here is that a measurement procedure not only must satisfy certain subject-independent conditions to deserve this name, but it must also be possible for epistemic subjects using it to determine that these conditions are indeed satisfied. For example, it is not sufficient that we use the correct measuring procedure—we must also be able to find out that we are doing so, as otherwise the results of this procedure would not be of any value for our epistemic enterprises.

The above list of conditions can be extended as follows: we can hardly acquire any empirical knowledge by means of a measurement procedure if we cannot determine, ultimately on the basis of perception, whether the relation between the quantity of our interest Q and the pointer variable P is indeed $Q = f(P)$ (for some explicitly given function), so that we can reconstruct the value of Q from the value of P. For our measurement of Q the fact that the functional relationship $Q = f(P)$ is satisfied does not suffice; we also need some reasons to think that it is indeed satisfied. However, this is exactly what is impossible in the case of absolute velocities if we grant (as all parties in the debate do) that boost-related worlds are observationally indistinguishable. Epistemic subjects living in possible worlds with absolute velocities could perhaps have at their disposal reliable devices that track absolute velocities (if our understanding of reliability is sufficiently weak), but they would not have any reasons to think that their devices indeed track absolute velocities. In other words, even if they would be able to measure absolute velocities in the externalist sense of measurement, they would not be able to measure velocities in the internalist sense of measurement because for the latter the mere satisfaction of the functional relationship $Q = f(P)$ (even appropriately modally stable!), is not sufficient.

The internalist view on measurement can be formulated in various ways. I propose the following:

> *The Justifiability of the Functional Relationship*: For a device d with a pointer variable P to measure a quantity Q, it must be the case that for some conditions C, (i) $Q = f(P)$ holds always (or almost always)[13] in conditions C, and (ii) we can provide some reasons, ultimately based on observation, that (i) obtains.

This is clearly not a full internalist analysis of the notion of measurement, but only a necessary condition.[14] A measurement here is conceived as performed by an epistemic subject, not by a measuring device considered in abstraction from its being used by an epistemic subject (otherwise the talk about providing reasons would make no sense). Moreover, as advertised, it is conceived in an internalist way: for a physical process to count as a measurement, there must not only be a relationship between the quantity we want to measure Q and the pointer variable P, but it must also be possible for epistemic subjects to argue (on the basis of observation, which is something that is available from their first-person epistemic perspective) that this functional relationship indeed holds.

It can be objected that we often use measurement devices (e.g., speedometers or thermometers) without knowing how exactly they work, and there does not seem to be anything epistemically inappropriate about this. We just buy a car that includes a

[13] Should it hold in the actual world (or in the chosen world of reference w, if we are sceptical about the actual existence of absolute velocities), in the nearby possible worlds, in most nomologically possible worlds (from the point of view of w), or in all such worlds? Since I am only formulating a necessary condition here, it suffices to restrict (i) to a single possible world. In the case of absolute velocities, the Justifiability of the Functional Relationship is violated because condition (ii) is not satisfied.

[14] On the basis of this condition, one can argue that f should satisfy some further constraints besides being a function, such as continuity and monotonicity, since without them it would be difficult to justify that $Q = f(P)$ holds.

speedometer and read out from it the values of the car's speed. For this reason, the Justifiability of the Functional Relationship might seem too demanding. However, I think that this impression is wrong. Even non-specialists who use measurement devices need some reasons for thinking that the readings of these devices correctly capture the values of the quantity they intend to measure. However, in this case, the reasons do not concern the physical layout of the devices, but the structure of the society they live in—namely, that it contains experts who know how to construct reliable measurement devices and that devices bought in such-and-such places are likely to be constructed on the basis of such expert knowledge. The experts themselves also need reasons for thinking that their devices are reliable (i.e., that Q and P are related by the function f), but in this case, these reasons directly concern the physical details of these devices.

However, there is also a more serious objection to the Justifiability of the Functional Relationship. One can wonder whether establishing the relation $Q = f(P)$ is possible in any case at all. This is because it seems that to establish such a relation, we would need to independently know the value of Q and the value of P in order to check whether they are indeed related by a function f. However, by assumption, Q itself is not perceptually accessible to us; we can only measure it by means of some pointer variables. One idea could be to measure Q by means of another pointer variable P' such that $Q = f'(P')$, and then compare the values of Q obtained in this way with the values of P; however, for this proposal to work, we would need to independently know that $Q = f'(P')$ holds, which leads to exactly the same problem as we have with Q and P.

This is a complicated problem, but not unsolvable. Chang (2004) has investigated it for the case of temperature measurement. His proposed answer is that scientific knowledge has a coherentist and iterative nature. According to Chang, there are two rules that govern the development of knowledge: the principle of respect and the imperative of progress. The principle of respect is a tentative affirmation of the existing system of beliefs. This system is not regarded as true in all details, but forms a starting point for further investigations. Each new stage of the system's development builds on the previous one, but its results might lead to the correction of certain assumptions made in the previous stage. In the case of measuring temperature, one can distinguish three stages (Chang 2004, pp. 47–48). The first stage is the bodily sensation of hot and cold. This allows us to observe certain correlations between the changes of temperature and the behaviour of some substances; for example, we can see that the volume of fluids is different depending on whether the environment is hot or cold. This observation forms the basis for the second stage, which is the construction of thermoscopes—that is, devices that enable us to detect whether the temperature of one object is higher or lower than the temperature of the other. Thermoscopes only use an ordinal scale, which means that, even though their readings can be assigned numbers, it does not make sense to perform arithmetical operations on them. The principle of respect is used here to legitimise the accuracy of thermoscopes by their agreement with our senses, whereas the imperative of progress is satisfied here because thermoscopes provide us with more fine-grained verdicts than our senses. The third stage is the construction of thermometers using a numerical scale. Again, their validation appeals to the previous stage—that is, to thermoscopes. In particular, thermoscopes were used to establish fixed points (i.e., phenomena that always occur at the same temperature), which were crucial for constructing the numerical scale of thermometers. The verdicts concerning temperature at each stage should be consistent among themselves and in broad agreement with the verdicts reached at earlier stages (this is the coherentist aspect of Chang's view). However, certain corrections are allowed, provided that they concern a sufficiently narrow class of cases. If the agreement between thermoscopes and our senses is broad enough, sometimes we can trust thermoscopes rather than our senses if they happen to disagree (and the same, *mutatis mutandis*, holds for thermometers).

Even though Chang (2004) is not interested in the debate between internalist and externalist views on epistemological concepts, his account clearly can be given an internalist reading. What we are interested in at each stage of our iterative process is the reasons that the results of this stage (such as our readings of temperature) are correct; and these should be reasons that are accessible to us and which we can (at least in principle) explicitly formulate. The mere objective reliability of an instrument, which cannot be argued for, is of no use for our measuring practices, and as such, is not sufficient for calling a procedure a measurement.

For our purposes, it is crucial to notice that the first stage of an iterative process is an observational difference between (at least) two values (or two classes of values) of the quantity under investigation. It might be either actually observed, as in the case of temperature ("hot" vs. "cold"), or based on an empirical prediction of some physical theory.[15] However, in the case of absolute velocities, this is in principle impossible; so *a fortiori*, all the subsequent stages of an iterative process, leading to the construction of a full-blown measuring device, are impossible in this case, as they would need to be based on this first stage.[16]

Why is the first stage of an iterative process impossible in the case of absolute velocities? Choose some possible world with absolute velocities and boost-invariant laws of nature, including at least one object in motion (call one of these objects *o*).

---

[15] Although we cannot directly measure temperature (in temperature measurements, the pointer variable is relative position, not temperature itself), we can at least access it perceptually in a coarse-grained way. In contrast, some other quantities (e.g., spin) are purely theoretical—that is, we do not have any perceptual access to them. However, they are still postulated to explain something observable. In such cases, the iterative procedure must also have some observational differences as an input, although their relation to the quantity under investigation is more hypothetical.

[16] The relationship between observation and measurement is also investigated, for example, by Ismael and van Frassen (2003, p. 376), who define observable quantities as those that are "distinguishable by even a gross discrimination of colour, texture, smell, and so on", and measurable quantities as those "whose values make some discernible impact on gross discrimination of colour, texture, smell, and so on, but it doesn't matter how attenuated the connection is, how esoteric the impact, or how special the conditions under which it can be discerned". According to them, any measurement relies on the observable quantities on the one hand (i.e., the difference in measurable quantities needs to lead to a difference in observable ones, even if only in very special conditions and because of subtle connections), and on the theory on the other (because to establish these connections, we might need to make certain theoretical assumptions). They seem to believe that these theoretical assumptions, in some cases, cannot be given any further epistemic support ("A theory is tested by means of measurements of quantities on the assumption that the theory is satisfied; for certain quantities nothing more basic is possible", Ismael and van Frassen (2003, p. 377)), which seems to me a less adequate view than the coherentist approach pro-

Now, consider the set of all possible worlds that are boost-related to it. All worlds in this set are observationally indistinguishable (in particular, all relative velocities and relative positions are the same in these worlds), but for any possible value of absolute velocity $\vec{v}_a$, there is some possible world in this set such that o has absolute velocity $\vec{v}_a$ in that world. Therefore, there is no pair of values (or classes of values) of absolute velocities such that it makes an observational difference that one of them is instantiated rather than the other. However, this is precisely what is needed for the iterative process to take off.

Notice that what the above reasoning (if correct) establishes is *not* that it is *impossible to know or to justify* that one has measured an absolute velocity, but that it is *impossible to measure* an absolute velocity. This is because the view under consideration here is internalism with respect to measurement itself, not with respect to the knowledge or justification that measurement has taken place (although I endorse both).

7 The Unmeasurability of Absolute Velocities and the "Knowledge Objection"

So far, I have not mentioned that Middleton and Murgueitio Ramírez (2020, pp. 9–10) consider something in line with my internalist account of measurement under the name of "the Knowledge Objection" (which is an objection to their thesis that the speedometer in the Basic World measures the absolute velocity of the car). They formulate it as follows: a car driver in the Basic World is not able to gain knowledge about the absolute velocity of the car by looking at the speedometer because, to gain such knowledge, he would need to first know that the road is in absolute rest, which is something he cannot do. However, since the speedometer does not provide knowledge about the absolute velocity of the car, it cannot be said to measure it. According to the authors, this objection is not valid because it relies on the following principle, which they claim is false (Middleton and Murgueitio Ramírez 2020, p. 10)[17]:

Footnote 16 (continued)

posed by Chang (2004), which allows the mutual support of the measurement procedures by the theory and the other way around. I also agree with the criticism of Ismael and van Fraassen by Read and Møller-Nielsen (2020), who claim that theoretical assumptions are needed for *every* observation and that "theories *themselves* are ultimately the best guides we have as to what we take ourselves to observe, and what we take ourselves to observe in turn provides epistemic support (or refutation) of those same theories" (2020, p. 95). When talking about observation and observational (in)distinguishability in this paper, I subscribe to the thesis that it is always theory-laden to some degree. However, I think that the distinction between observable and measurable quantities, which Read and Møller–Nielsen rejected, is legitimate: not because the former are entirely free of theoretical assumptions (as Ismael and van Fraassen claimed), but because the latter (which I take to include e.g. spin) are not perceivable for humans at all, but can be measured thanks to their relationship to perceivable quantities. Observational (in)distinguishability is about observable quantities, not measurable ones; however, since differences in measurable quantities at least sometimes lead to differences in observable ones (by the very definition of measurability), this does not make measurable quantities irrelevant for observational (in)distinguishability.

[17] It is used in the Knowledge Objection with the following substitutions: d is the speedometer, F is the car's absolute velocity, and p is the proposition that the road is at absolute rest.

The Foundation Principle: If d gives true information about F only when p is true, then an agent a can use d to get knowledge about F only when a first knows p.

They reject this principle because they think that it has a counterexample (2020, p. 10); namely, "I can know on the basis of my visual experiences that I have hands without first having to know that my visual system is correctly calibrated to produce accurate visual experiences".

The relationship between the Knowledge Objection and the internalist view on measurement is not straightforward because the notion of knowledge can be understood in various ways, and the Justifiability of the Functional Relationship does not use the concept of knowledge. However, the Knowledge Objection relies on the fact that we cannot establish the functional relationship between the car's absolute velocity and its relative velocity with respect to the road, and this fact is crucial for the speedometer not being able to measure absolute velocities in light of the internalist condition proposed in this paper. Therefore, we should consider whether Middleton and Murgueitio Ramírez's response to the Knowledge Objection undermines the internalist view on measurement.

Let me begin with one disanalogy between the case of observing one's hands and the case of measuring some physical quantity Q. For the former, there is an obvious candidate for an initially distinguished hypothesis (namely, the hypothesis that if I see my hands, then I have hands), whereas for the latter, there is no such distinguished hypothesis: if we construct a certain device with pointer variable P, then we need to check what is the functional relationship (if any) between P and the physical quantity Q that we want to measure.[18] (If we have previously constructed a device of this type and have checked that the relation $Q = f(P)$ holds, then we have good reasons to think that it holds for the new one; however, we are interested in establishing this relation for the first time.) For example, if we construct for the first time a speedometer (considered here as a device measuring the car's relative velocity with respect to a road), then we need to check that the position of the needle is correlated with the velocity of the car, and how exactly it is correlated (does the needle's being in the middle of the dial indicate the velocity of 50 km/h or 100 km/h or some other value?). Before checking, there is no distinguished hypothesis concerning this issue. In other words, we need to check whether the relation of the form $Q = f(P)$ holds, and if so, what exactly is f.[19] There are, in general, many options for f, which is another contrast with the case of knowing that one has hands (the latter essentially involves two hypotheses: either one correctly sees that one has hands or

[18] Here, the difference between requiring $Q = P$ and requiring $Q = f(P)$ becomes important—if we ask about the former, there are only two possible answers ("equal" or "unequal"), whereas if we ask about the latter, there are many options because there are many functions.

[19] In practice, the phases of checking how exactly a given device works and the construction or adjustment of this device are often not separated. For example, we might want the needle of the speedometer to be in the middle of the dial when the relative velocity is 80 km/h, so if it does not behave in this way, we change the details of this device until it does.

not). Of course, since we usually construct a given device on the basis of some idea about what it should do, we usually have some initially distinguished *rough* hypothesis concerning its functioning. However, learning the exact shape of the function f would likely always require calibration. Moreover, this distinguished hypothesis comes from our previous experience with (at least partially) similar devices or natural structures, which moves the problem one step back; but, as we observed in the previous section, at the beginning of this process, there must have been some experiences that establish the relation between P and Q (although they might be highly theory-laden). This suggests that, *even if* we agree that in the case of hands we do not need to check whether our visual system is able to detect their presence, in the case of measurement devices, such checking is indispensable.

More generally, for Middleton and Murgueitio Ramírez's argument against the Knowledge Objection to be valid, it needs to be the case that the example with hands undermines the Foundation Principle, and that the Foundation Principle is the only motivation for the Knowledge Objection. However, our concern here is the internalist view on measurement, not the Knowledge Objection itself. Is the former motivated by something similar to the Foundation Principle? I think that it does not need to be motivated in this way because the Justifiability of the Functional Relationship does not need to be related to a foundationalist epistemology. In the previous section, we saw an account of measurement that is coherentist but endorses this condition. This is in contrast to the Foundation Principle, which grows out of the foundationalist epistemology since, according to it, to know some things, we need to *first* know some other things. In the coherentist approach, the relation of dependence between various pieces of knowledge does not induce such a hierarchy of primacy.[20]

8 Summary

In this paper, I have suggested that measurement can be understood in an externalist or internalist way, analogously to knowledge and justification. The approaches by Middleton and Murgueitio Ramírez, as well as by Jacobs, belong to the former category. In contrast, I have sketched an internalist account based on the Justifiability of the Functional Relationship, which postulates that a necessary condition for measurement is that we can provide reasons for there being a functional relationship between the physical quantity Q that we want to measure and the pointer variable P. In the case of absolute velocities, this condition cannot be satisfied because boost-related models are observationally indistinguishable.

[20] In this way, we found a link connecting the issue of measurement with another big debate in epistemology: namely, between foundationalism and coherentism (in addition to the previously noticed link with the debate between externalism and internalism). According to foundationalism, some truths are more basic than others, and the justification of the latter is built upon the former. In contrast, the coherentist epistemology does not postulate such a hierarchy: here, the justification comes from the multidirectional relations of mutual support within the web of propositions.

Acknowledgements I would like to thank Tomasz Placek and two anonymous reviewers for helpful comments on an earlier version of this paper. This publication has been supported by a grant from the Faculty of Philosophy under the Strategic Programme Excellence Initiative at Jagiellonian University.

References

Armstrong, D. M. (1973). *Belief, truth and knowledge*. Cambridge University Press.

BonJour, L. (2009). *Epistemology: Classic problems and contemporary responses*. Rowman & Littlefield Publishers Inc.

Chang, H. (2004). *Inventing temperature: Measurement and scientific progress*. Oxford University Press.

Goldman, A. I. (2012). *Reliabilism and contemporary epistemology: essays*. Oxford University Press.

Ismael, J., & van Fraassen, B. (2003). Symmetry as a guide to superfluous theoretical structure. In K. Brading & E. Castellani (Eds.), *Symmetries in physics: Philosophical reflections* (pp. 371–392). Cambridge University Press.

Jacobs, C. (2020). Absolute velocities are unmeasurable: Response to Middleton and Murgueitio Ramírez. *Australasian Journal of Philosophy*. https://doi.org/10.1080/00048402.2020.1849327

Luc, J. (2022). Arguments from scientific practice in the debate about the physical equivalence of symmetry-related models. *Synthese, 200*(72).

Middleton, B., & Murgueitio Ramírez, S. (2020). Measuring absolute velocity. *Australasian Journal of Philosophy*. https://doi.org/10.1080/00048402.2020.1803938

Nozick, R. (1981). *Philosophical explanations*. Harvard University Press.

Pappas, G. (2017). Internalist vs. externalist conceptions of epistemic justification. In: Zalta, E. N. (Ed.), *The Stanford encyclopedia of philosophy*. https://plato.stanford.edu/archives/fall2017/entries/justep-intext.

Read, J., & Møller-Nielsen, T. (2020). Redundant epistemic symmetries. *Studies in History and Philosophy of Modern Physics, 70*, 88–97.

Roberts, J. T. (2008). A puzzle about laws, symmetries and measurability. *British Journal for the Philosophy of Science, 59*(2), 143–168.

Wallace, D. (2022). Observability, redundancy, and modality for dynamical symmetry transformations. In J. Read & N. J. Teh (Eds.), *The philosophy and physics of Noether's theorems* (pp. 322–353). Cambridge University Press.