Erkenntnis (2024) 89:1235–1255
https://doi.org/10.1007/s10670-022-00581-3

ORIGINAL RESEARCH

Credences are Beliefs about Probabilities: A Defense from Triviality

Benjamin Lennertz[1]

Received: 23 April 2021 / Accepted: 16 June 2022 / Published online: 6 March 2023

Abstract
It is often claimed that credences are not reducible to ordinary beliefs about probabilities. Such a reduction appears to be decisively ruled out by certain sorts of triviality results–analogous to those often discussed in the literature on conditionals. I show why these results do not, in fact, rule out the view. They merely give us a constraint on what such a reduction could look like. In particular they show that there is no single proposition belief in which suffices for having a particular credence, regardless of one's evidence. But if we allow such propositions to vary with evidence–as we should–then the results do not rule out a reduction. So, at least on this count, credences might very well just be beliefs about probabilities.

1 Introduction

A small group of theorists defend or are tempted by the thesis that credences are fundamentally ordinary beliefs about probabilities (Moon & Jackson, 2020 call the view "belief-first"; see also Lance, 1995; Hawthorne & Stanley, 2008; Weisberg, 2013; Dogramaci, 2018; Lennertz, 2021):

> **Credences are Beliefs about Probabilities (CBP)**: For every proposition, p, and real number, *n*, every attitude of confidence, or credence, of degree *n* in p is, fundamentally, a belief that the probability of p is *n*.

For example, my 50% credence that the cafeteria will serve Johnny Marzetti is fundamentally a belief that it is 50% probable that the cafeteria will serve Johnny Marzetti.

CBP has some advantages. It seamlessly explains ascriptions of what appear to be attitudes toward complex contents–like "Sally believes it is 50% likely that the cafeteria will serve Johnny Marzetti or they've changed the menu." This is because, according to CBP, such sentences *do* ascribe attitudes (beliefs) with complex

Benjamin Lennertz
belennertz@gmail.com

[1] Department of Philosophy, Colgate University, 13 Oak Dr, Hamilton, NY 13346, USA

contents, rather than requiring inventing new attitudes for each different sort of complexity (Lennertz, 2021; Moss, 2018). CBP also illuminates the role played by credences in reasoning about what to do, given a principle where knowledge is required to rely on something in reasoning (Hawthorne & Stanley, 2008; Weisberg, 2013). Finally, since CBP locates the probability in the content of the attitude, it supports a straightforward explanation of rational norms on credences as grounded in norms on belief (Moss, 2018).[1]

Despite these advantages, CBP is unpopular. It faces a number of objections.[2] I'll focus on one from formal triviality results (Russell & Hawthorne, 2016; Schroeder, 2018; Goldstein, 2019, though these authors don't take the results to directly show that CBP is false). These appear to show that if CBP and other natural assumptions are true, rational people are only ever 0% or 100% confident of each proposition; they never have intermediate levels of confidence. Since this conclusion appears unacceptable, we must drop one of the assumptions.

These triviality results take as an assumption a biconditional that appears to follow from CBP. Simplifying a bit, Moon and Jackson (2020, p. 654) suggest:

> **Biconditional**: S has a credence of degree *n* in p if and only if S believes that p is *n*-probable.

It is easy to interpret the triviality results as refuting Biconditional. Schroeder paraphrases his conclusion as follows: "there is no possible object of credence, q, such that believing it is necessary and sufficient for having a credence of *n* in p, where p is any object of credence and *n* is any value in (0, 1)" (2018, p. 302). A key assumption in Goldstein's proof is a transparency principle like Biconditional: "an agent believes [A is *n*-likely] just in case her credence in A is *n*" (2019, p. 192). Russell and Hawthorne state their assumption: "conditional on the claim that the probability of A is *x*, you should assign credence *x* to A" (2016, p. 313). Their interpretation of "conditional on the claim that" is to issue a conditional probability, but a nearby interpretation is "if you believe that", which yields the right-to-left direction of Biconditional.

In this paper, I uncover two relevant readings of Biconditional. I argue that the triviality results show the following: if the first reading of Biconditional is true, rational people must only have credences with extremal or trivial values (0 or 1); but the other reading appears consistent with people having the full range of strengths of credences. Given the plausible assumption that our credences aren't merely of trivial strengths, then the triviality results investigated in this paper show that the

[1] Moon and Jackson present other advantages of the view (2020, pp. 659–662).

[2] One claims that there is no interpretation (concept) of probability belief about which serves the same role as a credence (Maher, 1986, p. 367; Christensen, 2004, pp. 18–20; Ross, 2006, p. 189; Eriksson & Hájek, 2007, pp. 206–207; Staffel, 2013, p. 5357; Konek, 2016, p. 514; Moss, 2018, p. 2). Another claims that even if there is such a concept, young children and animals don't possess it, but still have credences (Price, 1986, p. 19; Frankish, 2009; see Moon & Jackson, 2020, pp. 662–665 for a reply). A third says that CBP can't account for credences in propositions which border on ungraspability (Jackson, forthcoming). A fourth suggests that CBP clashes with the linguistic data about related terms like "might" and "likely" (Yalcin, 2007, 2011). See Moon and Jackson (2020) for responses to objections 1, 2, and 4; they are silent on the topic of this paper.

first reading of Biconditional is false, but they leave open that the second could be true. One thing that follows from this is a constraint on the sort of proposition that is hypothesized to be believed when one has a credence. But this constraint is consistent with the most plausible way of cashing out CBP anyway.[3]

2 Triviality Results

I'll discuss Schroeder's (2018) result in detail and then relate it to Goldstein's (2019) and Russell and Hawthorne's (2016). None is inconsistent with CBP.

2.1 Schroeder's Result

After giving a pictorial version of his proof, Schroeder describes it:

> Let p and q be any objects of credence and n be any value in the interval (0, 1). If q is consistent with p, then there is a way of believing q—namely, by being certain of q—which entails having a credence of 1 in p, and hence which is insufficient to have a credence of n in p. And if q is consistent with ~p, then there is a way of believing q —namely, by being certain of q—which entails having a credence of 0 in p, and hence which is insufficient to have a credence of n in p. But if q is not consistent with p and q is not consistent with ~p, then q is itself inconsistent. So there is no consistent object of credence q such that belief in q is sufficient for a nonextreme credence of n in p, for any object of credence p. (Schroeder, 2018, p. 303)

Schroeder targets a claim that is stronger than Biconditional. He argues that there is no proposition of any sort, q (whether of the form the probability of p is *n* or not), such that a person has a credence of *n* in p just in case they believe q.

Schroeder's argument is sound, but it doesn't refute CBP. I'll formally reconstruct the beginning of his proof in a natural deduction system, which will help us see this clearly, as it highlights the key issue: quantifier scope.

I think that the most helpful way to reconstruct Schroeder's reasoning is by considering three possibilities for the relationship between p and q (though this isn't precisely how he presents it in the above quotation): they are consistent, they are inconsistent though q is not a contradiction, and q is a contradiction. Formalizing the first possibility is sufficient for diagnosing why the proof doesn't refute CBP (Appendix A outlines the full proof). I underline names to distinguish them from variables. And for a person, S, and credence function, C, I use C^S to refer to C as

[3] The most commonly discussed triviality results concern principles involving conditionals. Lewis (1976) initiated a stream of such results. Edgington interprets Lewis's result to show that there are no conditional propositions that can serve as the objects of our attitudes (1995). Though I won't directly discuss conditionals, the specter of an analogous reaction to our results – that there are no probabilistic propositions of the sort required by CBP – motivates my response.

the credence function that represents S. To streamline, I appeal to a proposition (Consistency_a) and an inference rule (Certainty Entailment)[4] that are not axioms of probability theory but follow from them, as well as a substantive philosophical assumption (Certainty to Belief):[5]

1	For any person, S, proposition, p, and real number, n, 0 < n < 1, there is a proposition, q such that for any rational credence function, C, C^S(p) = n if and only if S believes q	Suppose for reductio
2	Let S be a person, p a proposition, and n a real number such that 0 < n < 1	Stipulation
3	There is a proposition, q such that for any rational credence function, C, C^S(p) = n if and only if S believes q	∀ Elimination 1[5]
4	For any rational credence function, C, C^S(p) = n if and only if S believes q	Suppose for ∃ Elimination
5	For any person, S, proposition, p, and rational credence function, C, if C^S(p) = 1, then S believes p	Certainty to Belief
6	For any propositions, p and q, if p and q are consistent, then there is some rational credence function, C, such that C(p&q) = 1	Consistency_a
7	If p and q are consistent, then there is some rational credence function, C, such that C(p&q) = 1	∀ Elimination 6
8	p and q are consistent	Suppose for reductio
9	There is some rational credence function, C, such that C(p&q) = 1	→ Elimination 7, 8
10	C^S(p&q) = 1	Suppose for ∃ Elimination
11	p&q entails p	Logical Truth
12	p&q entails q	Logical Truth
13	C^S(q) = 1	Certainty Entailment 10, 12
14	C^S(p) = 1	Certainty Entailment 10, 11
15	If C^S(q) = 1, then S believes q	∀ Elimination 5
16	S believes q	→ Elimination 13, 15
17	C^S(p) = n if and only if S believes q	∀ Elimination 4
18	C^S(p) = n	↔ Elimination 16, 17
19	1 = n < 1	2, 14, 18
20	1 = n < 1 (contradiction)	∃ Elimination 9, 10-19
21	It's not the case that p and q are consistent	~ Introduction, 8-20

We can see informally how the rest of the proof goes by noting why line 21 is counterintuitive. If supposition 1 were true, the proposition belief in which would be sufficient for a credence of n in p would, itself, be consistent with p. For if it were inconsistent, then we would be able to infer that p is false from our belief in that proposition. But then we shouldn't have a nonzero credence in p. In this manner, Schroeder's total proof refutes the initial reductio assumption (I here ignore the possibility of rejecting the other assumptions).

4

n	p entails q	
m	C(p) = 1	
	C(q) = 1	Certainty Entailment n, m

5 Strictly speaking, the ∀E rule only allows instantiating one variable at a time, but I streamline here.

Again, Schroeder's proof is sound. But note how it proceeds, fixing q before the relevant credence function. This shows that we can't take a proposition, p, and degree, n, and find another proposition, q, such that belief in q is necessary and sufficient for having credence of degree n in p, no matter what our rational credence function is. But it does not show that the following is impossible: fix p, n, and a credence function and *then* find a proposition q such that believing q is necessary and sufficient for having credence of n in p, given that credence function. That is, it doesn't show that we can't, if we fix the credence function first, find a proposition, belief in which is sufficient for credence n in p.

Here's another way of putting my point: this proof shows that there is no proposition, *the probability of p is n*, full stop. For any such candidate proposition, belief in it doesn't suffice for having a credence of *n* in p, for every credence function.[6] But this doesn't show that there aren't many propositions, *the probability$_C$ of p is n*, one of which is believed each time a person has a credence of *n* in p.[7] Indeed, it seems eminently plausible that one can utter "the probability of p is n" in one context and express something true while uttering it in another and express something false. One who, at t_1, knows nothing other than that a two-sided coin will be tossed and says, "the probability of heads is 50%" says something true. If they learn at t_2 that the coin is biased toward heads and, again, say, "the probability of heads is 50%", they say something false. A plausible explanation is that sentences of the form, "the probability of p is n", are context-sensitive. The speaker at t_1 uses this sentence to express the proposition *that the probability, given their evidence at t_1, of heads is 50%*. At t_2 they use it to express the proposition *that the probability, given their evidence at t_2, of heads is 50%*. These are different propositions, since one is true and one is false. And it is possible that for each time, believing the proposition *that the probability, given one's evidence at that time, of heads is 50%*, is, at that time, having a credence of 50% that the coin lands heads. Nothing about the result above rules this out.

A third way of viewing my point appreciates that Biconditional is not explicit about the binding of variables:

> **Biconditional**: S has a credence of degree *n* in p if and only if S believes that p is *n*-probable. (Moon & Jackson, 2020)

We can distinguish (at least) two readings[8]:

[6] Similarly, Rothschild says the corresponding triviality results about conditionals show that "our semantic theory will not be able to assign a general meaning to $a \rightarrow c$ [in our case, "the probability of p is n"] which... applies across different credal states" (2013, p. 53).

[7] Charlow notes that it is "well-known that appeals to context-sensitivity offer a way around purported Triviality results" (2016, p. 550). Many responses to triviality results for conditionals exploit this observation (Bacon, 2015; Gillies, 2009; Khoo & Mandelkern, 2019; Kratzer, 2012; van Fraassen, 1976).

[8] Hájek and Hall (1994, pp. 75–76) distinguish *four* readings of what they call *The Hypothesis*, an underspecified statement about the relationship between probabilities of conditionals and conditional probabilities. One of the readings is the target of classic triviality results:

Stalnaker's Thesis: For a rational credence function, C, and propositions, p and q: if $C(p) > 0$, then $C(p \rightarrow q) = C(q \mid p)$.

Biconditional$_{\mathbf{f}}$: For any person, S, proposition, p, and real number, *n*, there is a proposition, that p is *n*-probable, such that for any credence function, C, $C^S(p) = n$ if and only if S believes that p is *n*-probable.

Biconditional$_{\mathbf{v}}$: For any person, S, proposition, p, real number, *n*, and credence function, C, $C^S(p) = n$ if and only if there is a proposition, that p is *n*-probable, such that S believes that p is *n*-probable.

The f subscript of Biconditional$_f$ indicates that the proposition targeted for belief is *fixed* prior to the agent's credence function; the v subscript of Biconditional$_v$ indicates that that proposition can *vary* with the credence function. Biconditional$_f$ implies the reductio assumption in Schroeder's proof, if we treat p is *n*-probable as a name for a proposition (it is stronger than Schroeder's reductio assumption which considers only $0 < n < 1$). However, supposing that "the probability of p is *n*" expresses different propositions in different contexts, it is the only weaker Biconditional$_v$ that follows from CBP:

CBP: For every proposition, p, and real number, *n*, every attitude of confidence, or credence, of degree *n* in p is, fundamentally, a belief that the probability of p is *n*.

So, the proof, though sound, doesn't show that our target thesis is false (or trivialized).

Could Schroeder's proof be salvaged with a reductio assumption implied by the weaker Biconditional$_v$? No. It is instructive to see the difference between Biconditional$_v$ and Biconditional$_f$ to see what would happen if we tried:

1	For any person, S, proposition, p, real number, n, $0 < n < 1$, and rational credence function, C, $C^S(p) = n$ if and only if there is a proposition, q such that S believes q	For reductio
2	Let $\underline{S}$ be a person, $\underline{p}$ a proposition, $\underline{n}$ a real number such that $0 < \underline{n} < 1$, and $\underline{C}$ a rational credence function	Stipulation
3	$\underline{C}^{\underline{S}}(\underline{p}) = \underline{n}$ if and only if there is a proposition, q, such that $\underline{S}$ believes q	$\forall$ Elimination 1

The original proof made a supposition for existential elimination before instantiating the credence function with $\underline{C}$. But this isn't possible here. There are some false starts we could take if we were to, at the next step make an assumption of $\underline{C}^{\underline{S}}(\underline{p}) = \underline{n}$ so that we could use biconditional elimination, but none leads to the result we want. So, the original proof shows that there is no proposition belief in which suffices for a credence of *n* in p (for $0 < n < 1$), regardless of the agent's credence function. But we cannot show that for each particular credence function, there is not a proposition belief in which suffices for having that credence function assign *n* to p (for $0 \leq n \leq 1$).

A toy model illustrates how Biconditional$_v$ is consistent with assigning non-trivial credence values to simple propositions. Take a toy domain of 3 worlds, w_1, w_2, w_3. Suppose an initial rational credence function, Cr, makes every world equiprobable: $Cr(w_1) = Cr(w_2) = Cr(w_3) = 1/3$. Suppose that p is true in w_1 and w_2, and false elsewhere. Now suppose that S learns some piece of evidence. Biconditional$_v$ implies that S's credence in p is one-half if and only if there is a proposition, that p is ½-probable, such that S believes that p is ½-probable. In this case there are two things that the evidence learned might have been that would yield that S has a credence of one-half in p. So there are two propositions which, if S came to believe them, would suffice for $Cr_{S\text{'s evidence}}(p)=½$. The evidence could be the information that w_2 isn't the actual world (E_1), in which case we have:

p is ½-probable$_1 = \{w_1, w_3\}$.

Or the evidence could be the information that w_1 isn't the actual world (E_2), in which case we have:

p is ½-probable$_2 = \{w_2, w_3\}$.

Suppose S is rational and has gained some evidence. If S either comes to believe p is ½-probable$_1$ or comes to believe p is ½-probable$_2$, then $Cr_{S\text{'s evidence}}(p)=½$. And if $Cr_{S\text{'s evidence}}(p)=½$, then S will either believe p is ½-probable$_1$ or believe p is ½-probable$_2$. Thus we have a case where Biconditional$_v$[9] is true, at least for simple propositions, but S has a credence in p that is neither 0 nor 1 – so, is not trivialized.

More programmatically, credences, or probabilities, evolve with evidence. We need to track this evidence in advocating for or arguing against theses about the connection between belief and credence. As evidence evolves we are either dealing with new probability functions or previous ones conditionalized on more evidence.[10] Even before seeing Schroeder's result, we should have thought it surprising to find a proposition belief in which is necessary and sufficient for a credence of *n* in p *regardless* of one's credence function and evidence.

2.2 Other results

Goldstein proves multiple triviality results. The one that is similar to Schroeder's relies on the following thesis (2019, p. 192):

> **Belief Transparency**: For any proposition, p, and real number, n, where C represents an agent S's rational credence function: S believes p is n-likely if and only if C(p) = n.

I detail Goldstein's proof in Appendix B. One way of viewing why his result holds but doesn't trivialize CBP is that in his quasi-logical language, "p is n-likely" expresses the same proposition in every context. But the result doesn't hold if we drop that assumption. Here's another way of viewing it: Belief Transparency is an alternative formulation of Biconditional, and as with Biconditional, there are two possible readings, one where *p is n-likely* picks out a proposition that is fixed just by p and *n*, Biconditional$_f$, and another where it varies based on the credence function of the relevant agent, Biconditional$_v$:

[9] This case makes it particularly obvious that Biconditional$_f$ is not true, since there is no single proposition that p is ½-probable.

[10] Goldstein (2021) focuses on making sense of the resulting credence after update, but he does so in a different way. Instead of relying on shiftiness of propositional contents, he enriches the arguments of probability functions to world-information state pairs. He also enriches his update rule so that changes in the world parameter get reflected in the information state, yielding the desired shiftiness.

Belief Transparency$_f$: For any proposition, p, and real number, n, there is a proposition, *p is n-likely*, such that where C represents an agent S's rational credence function: S believes p is n-likely if and only if C(p) = n.
Belief Transparency$_v$: For any proposition, p, and real number, n, where C represents an agent S's rational credence function, there is a proposition, *p is n-likely*, such that S believes *p is n-likely* if and only if C(p) = n.

Like Schroeder's, Goldstein's result trivializes Belief Transparency$_f$, but not Belief Transparency$_v$–or the corresponding Biconditional$_v$. And, again, Biconditional$_v$ is consistent with CBP.

Russell and Hawthorne prove a plethora of triviality results. The closest to our focus trivializes the following principle (2016, p. 313):

Exact Probability: For any proposition, p, real number, n, and rational credence function, C, if C(the probability of p is n) > 0, then C(p | the probability of p is n) = n.

This principle is not as obviously related to CBP, but it still causes trouble for it as follows: Suppose credence 1 is sufficient for belief. Then the consequent of Exact Probability implies that a rational person who has credence 1–and, so, believes–that the probability of p is n will have credence of degree n in p. So, if Exact Probability were trivialized, then CBP would be too for any beliefs of which a person is certain.

Exact Probability also requires the contentious assumption that there is a single object of credence expressed by the quasi-logical sentence "the probability of p is n". As with our other results, we can distinguish different readings:

Exact Probability$_f$: For any proposition, p, and real number, n, where $0 < n < 1$, there is a proposition, *the probability of p is n*, such that for any rational credence function, C, if C(the probability of p is n) > 0, then C(p | the probability of p is n) = n.
Exact Probability$_v$: For any proposition, p, real number, n, where $0 < n < 1$, and rational credence function, C, there is a proposition, *the probability of p is n*, such that if C(the probability of p is n) > 0, then C(p | the probability of p is n) = n.

As discussed in Appendix C, Russell and Hawthorne's result follows from Exact Probability$_f$, whereas Exact Probability$_v$, in which what is meant by "the probability of p is n" is not fixed, is consistent with CBP.[11]

2.3 Taking Stock

I'll draw three main lessons from these formal results. (1) There is no single proposition that we might call *the probability of p is n*, which a person believes if and only

[11] Russell and Hawthorne don't, themselves, conclude that Exact Probability$_f$ is false. They explore a dynamic picture of contents according to which the proof fails at a different step. For discussion, see Appendix C.

if they have credence (based on any rational credence function with any evidence) of n in p. Nonetheless, (2) CBP may still be true (non-trivially). Finally, (3) scenarios that witness the consistency of CBP and the triviality results have a particular character. For each credence of n in a proposition, p, there is some proposition, q, such that a person's credence of n in p is their belief in q–in a way that q varies based on the person's credence function and evidence.

This third lesson can look unsatisfying. If one has a credence of n in p, then we can find some q belief in which serves as a proxy for that credence.[12] But, as intended, q is not just a random proxy. Rather it is a proposition about the probability of p given the relevant probability function and evidence in that context. This seems exactly the sort of thing that a credence might be. More specifically, we can say that a credence of n in p is a belief that the probability of p is n, given your evidence.[13] This allows two people to have a credence of the same strength in a given proposition, where, for each person, that credence is a belief in a different proposition. For example, consider a case where Fred knows that a fair coin was flipped and has landed, but knows nothing else. Fred's credence of 0.5 that the coin landed heads is a belief that the probability that the coin landed heads given his current evidence is 50%. Contrast this with a case where Sally knows that a coin was flipped and there is a testifier who saw the toss and tells Sally it landed heads, but who Sally knows tells the truth exactly 50% of the time. Sally's credence of 0.5 that the coin landed heads is a belief that the probability that the coin landed heads given her current evidence is 50%. Since Fred's and Sally's bodies of evidence are different, this view makes the plausible prediction that they have different beliefs that are sufficient for their credences (in contrast with Biconditional$_f$).[14] This way of viewing the issue also makes sense of why, in *intra*personal cases, the proposition that serves as a proxy for a credence of n in p might change as a person's evidence changes. Either the probability function that proposition is about or the evidence that the probability function is conditioned on changes. This is consistent with our ordinary conception of credence functions: when a rational person gains evidence, either the function

[12] Earman mentions "find[ing] a proxy event that will do duty for the state of affairs of the objective probability's lying between specified limits" (1992, p. 9). The so-called basic tenability result in (Hájek & Hall, 1994, p. 93) about conditionals is analogous to the claim that there is some q that serves the needed role for each choice of p and n. The question then is whether this q has the desired features. For them, these are features of the conditional (e.g. licensing modus ponens for conditionals with ordinary antecedents and consequents). For helpful discussions, see (Edgington, 1995; Hájek & Hall, 1994). We, however, want to know whether q will act like a proposition of the form the probability of p is n; we'd also like this picture to apply generally and, in particular, in cases where p itself is a conditional. I'm sorry to say that I don't have a proof of tenability to offer at this point; exploring this will have to wait until future work. Note that the informal toy model produced above is largely illustrative; it doesn't show tenability generally. For instance, it is too simple to model cases where the proposition at issue, p, is a conditional or is itself a proposition about the probability of a proposition; nor does it address conditional credences/probabilities–and, in particular an agent's conditional credence in a proposition given some probability assignment to that proposition.

[13] Moon and Jackson suggest that the relevant proposition is one that is relative to the believer's evidence (2020, p. 658 at footnote 23). This suggests that they too have in mind a reading of Biconditional as Biconditional$_v$ rather than Biconditional$_f$.

[14] Thanks to an anonymous reviewer for suggesting this example.

that represents their credences changes or, if it stays the same, it is understood as updated by that evidence. It would be counterintuitive to think that belief in the same proposition would occur whenever one had a particular credence in a proposition despite those evidential changes, as in Biconditional$_f$.[15] Updating the proposition to be about current evidence facilitates beliefs about such propositions playing the role that credences do in reasoning, decision-making, and action. In such cases, it is almost always the probability of propositions given one's current evidence that grounds the practical relevance of our attitudes about them.

3 A Single Proposition?

In this section, I'll entertain objections to this CBP-consistent picture of credences. These objections are to this *particular* picture; they are independent of the triviality results, which, as we've seen, don't by themselves show that CBP is false.

My defense of CBP relies on there being many propositions about the probability of p being *n*, which may involve different probability functions or probability functions conditionalized on different evidence. This might appear to treat our belief states as excessively and objectionably shifty. The core of this objection can be seen from a philosophy of mind perspective and a philosophy of language perspective. I'll discuss each in turn.

First, the philosophy of mind: when a person, say Jorge, gains some evidence, he is required, not just to change the degree to which he believes a proposition is probable, but to change the notion of probability that his beliefs are about. Earlier his belief was about what was probable given his evidence *then*. Now Jorge's belief is about what is probable given his evidence *now*. There are two ways to conceptualize this: either his earlier and later beliefs are about different probability functions or they are about the same probability function, conditioned on different evidence. If his earlier belief is about $P_E(\bullet)$, his later belief is either about $P_{E\&E'}(\bullet)$ or $P_E(\bullet \mid E')$.[16] Either way, something beyond the strength of his thought's degree has changed. We might think that this is an objectionably shifty way to conceive of changes in confidence.

I'll begin my response by conceding that it would be implausible if a change in confidence required Jorge to explicitly reflect on the probability function his thought is about or to list the evidence that that function is conditionalized upon. But that is not required. Instead, Jorge might have that belief in a way that he would describe to himself as "p is n-likely given my current evidence." Suppose that he gets some evidence but doesn't change his confidence. He might again say to himself "p is n-likely given my current evidence." The second time he says this to himself, the proposition

[15] van Fraassen (1976, pp. 273–276) argues similarly against a "fixed interpretation" for conditionals. He blames the desire for a fixed interpretation on Lewis's metaphysical realism, though this underplays independent considerations in favor of a fixed interpretation, which are related to my discussion in Sect. 3.

[16] The former conception is more general, since it allows both that (i) rational updating may not always happen by conditionalization and (ii) Jorge may simply not update by conditionalization, even if that is uniquely rational.

expressed is a bit different. But there is no great mental energy that needs to be expended here, nor is it implausible that our thoughts evolve like this. Consider an analogy to thoughts about time. Suppose Francesca believes what she could express by saying to herself "I am sitting on the couch now" and then, ten minutes later, has a belief that she would express with the same phrase. Her beliefs at those two times seem to have different contents, but this doesn't mean that beliefs about time are too objectionably shifty to be captured by our usual model of belief. It is not hard for us to have beliefs like this, nor is it hard for theorists to understand them. The same is true in our case, where a person's degree of confidence changes.

Furthermore, I contend that there is nothing gained by thinking of these evolutions of mind fundamentally in terms of credences rather than beliefs, as we might if we rejected CBP. Consider Tomiko, who has some degree of credence in a proposition. The popular way of modeling her state, provided that she is rational, is as a probability function yielding that degree as its value for that proposition, as argument. But once Tomiko receives more evidence, what counts as her credence function is either a different probability function or the same function conditionalized on her updated evidence. That is, if her earlier credence function was represented by $P_E(\bullet)$, her later credence function will be represented by either $P_{E\&E'}(\bullet)$ or $P_E(\bullet \mid E')$. These are just the probability functions that featured in Jorge's belief on the CBP picture. Tomiko's state appears to have the same sort of shiftiness as Jorge's. So, rejecting CBP gets us no further in addressing any excessive shiftiness than accepting it does. There simply is shiftiness in representing these states of mind no matter how we divide up the attitude and content. But that should convince us that the shiftiness is not objectionable.[17]

A different take on the shiftiness objection is associated more closely with concerns in the philosophy of language. Lewis mentions it in his original results about conditionals (1976, p. 133). It seems like two people agree if they both say, "It is 87% likely that it will rain on July 30, 2049 in central Lagos" and two people disagree if one says "It is 87% likely that it will rain on July 30, 2049 in central Lagos" and the other says "It is *not* 87% likely that it will rain on July 30, 2049 in central Lagos." It is common to think that these sentences are used to express credences of 87% and something other than 87%, respectively, that it will rain on July 30, 2049 in central Lagos. It would be easy to explain this agreement and disagreement if there were a single proposition belief in which sufficed for credence of 0.87 that it will rain on July 30, 2049 in central Lagos (and one which sufficed for a credence of some other degree in that proposition). But we've seen that there can be no such proposition. And the agreement and disagreement don't look easily explicable on a picture where the two people who appear to agree utter the sentence based on different evidence and, so, express different propositions (similarly for disagreement).

[17] This point extends to conceptions of credences that take them to be not genuine attitudes but theoretical constructs which model choice behavior. According to this picture, credences don't describe some feature of the agent's mind, but are a useful way of representing one feature of what goes into predicting her behaviors from a third person perspective. Suppose that this picture is right. Then, since we're not getting at some underlying reality, it seems equally reasonable to represent these entities as beliefs about probabilities as it does to represent them as credences.

Thus, the way we've developed CBP appears unable to explain agreement and disagreement.[18]

The debate over this issue has raged in the literature on epistemic and deontic modals in the last couple decades. I can't fully engage the range of possible responses here. Some theorists question whether there truly is disagreement in cases like the one I've described,[19] but I'm going to suppose there is some genuine disagreement and lay out two popular and attractive paths for answering the objection.

The first suggests that even though statements about what is likely or probable are shifty, in these cases of agreement and disagreement, the two interlocutors are focusing on a common parameter–as would be so if we both use "that" while pointing at the same object. We might, when speaking with another person about what is likely or not, presuppose that our evidence and probability functions are relevantly similar (López de Sa, 2008). My assertion that it is 87% likely, given *my* evidence, that it will rain on July 30, 2049 in central Lagos is inconsistent with your assertion that it is not 87% likely, given *your* evidence, that it will rain on July 30, 2049 in central Lagos, given the presupposition that you and I share the relevant evidence and probability function. A different proposal of this general kind holds that speakers are talking about some evidence and probability function that will be obtained before it needs to be used to solve the relevant question or problem (Yanovich, 2014). Thus, what I really assert is that it is 87% likely, given the evidence that will be obtained before we need to act on these topics, that it will rain on July 30, 2049 in central Lagos, and what you really assert is it is not 87% likely, given the evidence that will be obtained before we need to act on these topics, that it will rain on July 30, 2049 in central Lagos. These assertions are inconsistent. A very similar proposal, which yields the same kind of solution, is that these assertions are about what the shared evidence will be after the participants take on board the very utterances being made (Mandelkern, 2020). All of these

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18 It is sometimes said that rational inconsistency is the intrapersonal analog of interpersonal disagreement–and so we may want to explain it in terms of inconsistency of contents about probabilities (Lennertz, 2021; Moss, 2018). Nonetheless, the challenge of explaining interpersonal disagreement does not extend to explaining rational inconsistency. Consider a person who thinks that it is 87% likely that it will rain on July 30, 2049 in central Lagos and thinks that it is *not* 87% likely that it will rain on July 30, 2049 in central Lagos. Both thoughts feature the same probability function, since their evidence is the same. So we can straightforwardly explain why they are irrational.

19 It is popular to try to diffuse the objection by questioning the data. For instance, Dowell (2011, 2013) suspects that these sorts of situations are often underdescribed. When the details are properly filled in – when we realize the first speaker has different evidence or is talking about a different probability function than the second–we won't judge them to be agreeing (or disagreeing) in uttering sentences that look on the surface to be expressing the same propositions (or negations of each other). Dorr and Hawthorne (2013) also find more nuance in intuitions about a related problem of giving a response to Yalcin's puzzle (2007). Another way to diffuse the objection claims that even if we do have the simple intuitions that I described above, it could be that these intuitions are in error (Björnsson & Almér, 2010; Björnsson & Finlay, 2010; Finlay, 2014). We might be systematically confused in thinking that there is no shiftiness in terms like "likely" by such cases when, in fact, looking at the entire semantic and pragmatic theory shows that such an account is most plausible. In the main text, I survey two ways to respond to this data that do not import error to either theorists or speakers.

suggestions explain agreement and disagreement in virtue of what is asserted (perhaps together with what is presupposed).

The other popular and attractive path for dealing with the objection concedes there is no single proposition that is asserted by the first speaker and denied by the second, as the simple version of the shifty account suggests. Nonetheless, the interlocutors might agree or disagree based on something else pragmatically conveyed by uttering those sentences in the context. For instance, a speaker might 'put in play', without committing to, a proposition that the hearer responds to–perhaps it is 87% likely, given *the conversational group's* evidence, that it will rain on July 30, 2049 in central Lagos (von Fintel & Gillies, 2011). Or a speaker might suggest to degree 0.87 that it will rain on July 30, 2049 in central Lagos and the hearer may disagree with this act of suggestion (Montminy, 2012). Finally, it might be that a speaker expresses a stance of taking it as 87% likely in their reasoning and deliberation that it will rain on July 30, 2049 in central Lagos and the hearer refuses to take up that stance (Lennertz, 2014).

4 Conclusion

I want to close by refocusing on my goal. I aimed to show that the triviality results we've explored don't refute the claim that credences are fundamentally beliefs about probabilities (CBP). In Sect. 2, I showed that these results don't by themselves rule out CBP. Nonetheless, they do show that there is no single proposition belief in which is necessary and sufficient for having a particular credence. That leaves us with a view that appears consistent with CBP which says that as a person gains evidence, the proposition belief in which constitutes their credence changes. In Sect. 3 I explored a couple of ways of objecting to the shiftiness of this view, but I suggested that none of these objections are decisive, drawing on the large literature on these questions to chart possible ways forward. Importantly, the main goal of this paper–to show that the considered triviality results are consistent with CBP–holds independent of any particular defense from Sect. 3 of the shifty view.

Appendix A

Here is a reconstruction of Schroeder's entire proof. In addition to what we saw in the text, I assume an instance of a probability axiom, Additivity, and an instance of a truth derived from the axioms, which we can call Maximality:[20]

[20] Strictly speaking, the ∀E rule only allows instantiating one variable at a time, but I streamline here.

1	For any person, S, proposition, p, and real number, n, $0 < n < 1$, there is a proposition, q such that for any rational credence function, C, $C^{S}(p) = n$ if and only if S believes q	Suppose for reductio
2	Let $\underline{S}$ be a person, $\underline{p}$ a proposition, and $\underline{n}$ a real number such that $0 < \underline{n} < 1$	Stipulation
3	There is a proposition, q such that for any rational credence function, C, $C^{\underline{S}}(\underline{p}) = \underline{n}$ if and only if $\underline{S}$ believes q	∀ Elimination 1[20]
4	For any rational credence function, C, $C^{\underline{S}}(\underline{p}) = \underline{n}$ if and only if $\underline{S}$ believes $\underline{q}$	Suppose for ∃ Elimination
5	For any person, S, proposition, p, and rational credence function, C, if $C^{S}(p) = 1$, then S believes p	Certainty to Belief
6	For any propositions, p and q, if p and q are consistent, then there is some rational credence function, C, such that $C(p\&q) = 1$	Consistency_a
7	If $\underline{p}$ and $\underline{q}$ are consistent, then there is some rational credence function, C, such that $C(\underline{p}\&\underline{q}) = 1$	∀ Elimination 6
8	$\underline{p}$ and $\underline{q}$ are consistent	Suppose for reductio
9	There is some rational credence function, C, such that $C(\underline{p}\&\underline{q}) = 1$	→ Elimination 7, 8
10	$\underline{C}^{\underline{S}}(\underline{p}\&\underline{q}) = 1$	Suppose for ∃ Elimination
11	$\underline{p}\&\underline{q}$ entails $\underline{p}$	Logical Truth
12	$\underline{p}\&\underline{q}$ entails $\underline{q}$	Logical Truth
13	$\underline{C}^{\underline{S}}(\underline{q}) = 1$	Certainty Entailment 10, 12
14	$\underline{C}^{\underline{S}}(\underline{p}) = 1$	Certainty Entailment 10, 11
15	If $\underline{C}^{\underline{S}}(\underline{q}) = 1$, then $\underline{S}$ believes $\underline{q}$	∀ Elimination 5
16	$\underline{S}$ believes $\underline{q}$	→ Elimination 13, 15
17	$\underline{C}^{\underline{S}}(\underline{p}) = \underline{n}$ if and only if $\underline{S}$ believes $\underline{q}$	∀ Elimination 4
18	$\underline{C}^{\underline{S}}(\underline{p}) = \underline{n}$	↔ Elimination 16, 17
19	$1 = \underline{n} < 1$	2, 14, 18
20	$1 = \underline{n} < 1$ (contradiction)	∃ Elimination 9, 10-19
21	It's not the case that $\underline{p}$ and $\underline{q}$ are consistent	~ Introduction, 8-20
22	$\underline{q}$ is consistent	For reductio
23	For any proposition p, if p is consistent, then there is some rational credence function, C, such that $C(p) = 1$	Consistency_a
24	If $\underline{q}$ is consistent, then there is some rational credence function, C, such that $C(\underline{q}) = 1$	∀ Elimination 23
25	There is some rational credence function, C, such that $C(\underline{q}) = 1$	→ Elimination 22, 24
26	$\underline{C'}^{\underline{S}}(\underline{q}) = 1$	Suppose for ∃ Elimination
27	If $\underline{C'}^{\underline{S}}(\underline{q}) = 1$, then $\underline{S}$ believes $\underline{q}$	∀ Elimination 5
28	$\underline{S}$ believes $\underline{q}$	→ Elimination 26, 27
29	$\underline{C'}^{\underline{S}}(\underline{p}) = \underline{n}$ if and only if $\underline{S}$ believes $\underline{q}$	∀ Instantiation, 4
30	$\underline{C'}^{\underline{S}}(\underline{p}) = \underline{n}$	↔ Elimination 28, 29
31	$\underline{C'}^{\underline{S}}(\underline{q}\text{ or }\underline{p}) = \underline{C'}^{\underline{S}}(\underline{q}) + \underline{C'}^{\underline{S}}(\underline{p})$	Additivity 21
32	$\underline{C'}^{\underline{S}}(\underline{q}\text{ or }\underline{p}) \leq 1$	Maximality
33	$\underline{C'}^{\underline{S}}(\underline{q}) + \underline{C'}^{\underline{S}}(\underline{p}) \leq 1$	31, 32
34	$1 + \underline{C'}^{\underline{S}}(\underline{p}) \leq 1$	26, 33
35	$\underline{C'}^{\underline{S}}(\underline{p}) \leq 0$	34
36	$0 < \underline{n} \leq 0$	2, 30, 35
37	$0 < \underline{n} \leq 0$ (contradiction)	∃ Elimination 25, 26-36
38	$\underline{q}$ is inconsistent	~ Introduction 22-37
39	Any proposition that plays the role of q in premise 1 is consistent	Assumption (discussed below)
40	$\underline{q}$ plays the role of q in 1	4
41	$\underline{q}$ is consistent	∀ Elimination 39
42	$\underline{q}$ is consistent or ($\underline{r}$&~$\underline{r}$)	∨ Introduction 41
43	$\underline{r}$&~$\underline{r}$	∨ Elimination 38, 42
44	$\underline{r}$&~$\underline{r}$ (contradiction)	∃ Elimination 3, 4-43
45	1 (our supposition for reductio) is false	~ Introduction 1-44

The assumption in 39 seems plausible. Any candidate for the proposition, belief in which is necessary and sufficient for credence *n* in p cannot be a contradiction. If it were a contradiction, one could not rationally have credence n in p (for doing so would require believing a contradiction). And this would be so for every p and every n, where $0 < n < 1$.

Appendix B

Here is a formalization of Goldstein's result (2019) as a standard triviality proof, not a reductio. I'll let $C_p(\bullet) = C(\bullet \mid p)$. $C_p(\bullet)$ is a rational credence function if $C(\bullet)$ is. Again I use some standard truths of probability, like the Law of Total Probability and a couple of rules that I call CP and Negation CP:[21,22,23]

[21] Schroeder's proof did not employ Lockeanism, which some may reject, though see (Goldstein, 2019, p. 193 note 10) for a partial defense.

[22] I don't know a good name for this derived rule, but I avoid the tedium of proving it from the logical axioms. 9, 12, and 15 are related to the principle that Goldstein calls Credal Transparency.

[23] The same proposition, p is n-likely, is the object of these different credence functions. One way to try to reconstruct the derivation while accepting Belief Transparency$_v$ rather than Belief Transparency$_f$ allows these propositions to vary. But then a step like 17 would not follow from the law of total probability. This is analogous to how the law of total probability doesn't apply in triviality proofs for conditionals if one accepts that the conditional is shifty (Khoo & Mandelkern, 2019, p. 517, note 32).

1	For any person, S, proposition, p, and real number, n, where $0 < n < 1$, there is a proposition, *p is n-likely*, such that for any rational credence function, C, S believes p is n-likely if and only if $C^S(p) = n$	Belief Transparency_f
2	There is some threshold, t, such that, for any person, S, rational credence function, C, proposition, p, S believes p if and only if $C^S(p) \geq t$	Lockeanism[21]
3	Let $\underline{S}$ be a person, $\underline{C}$ a rational credence function, $\underline{p}$ a proposition, and $\underline{n}$ a real number, $0 < \underline{n} < 1$	Stipulation
4	There is a proposition, *p is n-likely*, such that for any rational credence function, C, $\underline{S}$ believes p is n-likely if and only if $C^{\underline{S}}(\underline{p}) = \underline{n}$	∀ Elimination 1
5	For any person, S, rational credence function, C, proposition, p, S believes p if and only if $C^S(p) \geq \underline{t}$	Suppose for ∃ Elimination
6	For any rational credence function, C, $\underline{S}$ believes $\underline{\text{p is n-likely}}$ if and only if $C^{\underline{S}}(\underline{p}) = \underline{n}$	Suppose for ∃ Elimination
7	$\underline{S}$ believes $\underline{\text{p is n-likely}}$ if and only if $\underline{C_p}^{\underline{S}}(\underline{p}) = \underline{n}$.	∀ Elimination 6
8	$\underline{S}$ believes $\underline{\text{p is n-likely}}$ if and only if $\underline{C_p}^{\underline{S}}(\underline{\text{p is n-likely}}) \geq \underline{t}$	∀ Elimination 5
9	$\underline{C_p}^{\underline{S}}(\underline{p}) = \underline{n}$ if and only if $\underline{C_p}^{\underline{S}}(\underline{\text{p is n-likely}}) \geq \underline{t}$	Hypothetical Syllogism (Biconditional) 7, 8[22]
10	$\underline{S}$ believes $\underline{\text{p is n-likely}}$ if and only if $\underline{C_{\sim p}}^{\underline{S}}(\underline{p}) = \underline{n}$	∀ Elimination 6
11	$\underline{S}$ believes $\underline{\text{p is n-likely}}$ if and only if $\underline{C_{\sim p}}^{\underline{S}}(\underline{\text{p is n-likely}}) \geq \underline{t}$	∀ Elimination 5
12	$\underline{C_{\sim p}}^{\underline{S}}(\underline{p}) = \underline{n}$ if and only if $\underline{C_{\sim p}}^{\underline{S}}(\underline{\text{p is n-likely}}) \geq \underline{t}$	Hypothetical Syllogism (Biconditional) 10, 11
13	$\underline{S}$ believes $\underline{\text{p is n-likely}}$ if and only if $\underline{C}^{\underline{S}}(\underline{p}) = \underline{n}$	∀ Elimination 6
14	$\underline{S}$ believes $\underline{\text{p is n-likely}}$ if and only if $\underline{C}^{\underline{S}}(\underline{\text{p is n-likely}}) \geq \underline{t}$	∀ Elimination 5
15	$\underline{C}^{\underline{S}}(\underline{p}) = \underline{n}$ if and only if $\underline{C}^{\underline{S}}(\underline{\text{p is n-likely}}) \geq \underline{t}$	Hypothetical Syllogism (Biconditional) 13, 14
16	For all rational credence functions, C, and propositions, p and q, $C(q) = C_p(q) \times C(p) + C_{\sim p}(q) \times C(\sim p)$	Law of Total Probability
17	$\underline{C}^{\underline{S}}(\underline{\text{p is n-likely}}) = \underline{C_p}^{\underline{S}}(\underline{\text{p is n-likely}}) \times \underline{C}^{\underline{S}}(\underline{p}) + \underline{C_{\sim p}}^{\underline{S}}(\underline{\text{p is n-likely}}) \times \underline{C}^{\underline{S}}(\sim\underline{p})$	∀ Elimination 16[23]
18	For any rational credence function, C, and proposition, p, $C_p(p) = C(p \mid p) = 1$	CP
19	$\underline{C_p}^{\underline{S}}(\underline{p}) = 1$	∀ Elimination 18
20	$\underline{C_p}^{\underline{S}}(\underline{p}) \neq \underline{n}$	3, 19
21	$\underline{C_p}^{\underline{S}}(\underline{\text{p is n-likely}}) < \underline{t}$	Modus Tollens for Biconditionals 9, 20
22	For any rational credence function, C, and proposition, p, $C_{\sim p}(p) = C(p \mid \sim p) = 0$	Negation CP
23	$\underline{C_{\sim p}}^{\underline{S}}(\underline{p}) = 0$	∀ Elimination 22
24	$\underline{C_{\sim p}}^{\underline{S}}(\underline{p}) \neq \underline{n}$	3, 23
25	$\underline{C_{\sim p}}^{\underline{S}}(\underline{\text{p is n-likely}}) < \underline{t}$	Modus Tollens for Biconditionals 12, 24
26	$\underline{C}^{\underline{S}}(\underline{\text{p is n-likely}}) < \underline{t} \times \underline{C}^{\underline{S}}(\underline{p}) + \underline{t} \times \underline{C}^{\underline{S}}(\sim\underline{p})$	17, 21, 25
27	$\underline{C}^{\underline{S}}(\underline{p}) + \underline{C}^{\underline{S}}(\sim\underline{p}) = 1$	Negation
28	$\underline{C}^{\underline{S}}(\underline{\text{p is n-likely}}) < \underline{t}$	26, 27
29	$\underline{C}^{\underline{S}}(\underline{p}) \neq \underline{n}$	Modus Tollens for Biconditionals 15, 28
30	$\underline{C}^{\underline{S}}(\underline{p}) \neq \underline{n}$	∃ Elimination 4, 6-29
31	$\underline{C}^{\underline{S}}(\underline{p}) \neq \underline{n}$	∃ Elimination 2, 5-30
32	For any person, S, rational credence function, C, proposition, p, and real number, n, where $0 < n < 1$, $C^S(p) \neq n$	∀ Introduction 31

32 trivializes Belief Transparency_f. If it is true, then rational people only have extreme credences. As in Schroeder's proof, we can reject Belief Transparency_f but retain Belief Transparency_v. However, if we do so, we cannot derive the triviality result in this way.

Appendix C

Here is a formalization of Russell and Hawthorne's result (2016). Again, I'm taking truths and inference rules based on probability and logic as basic, rather than proving them from the axioms[24]:

1	For any proposition, p, and real number, n, where 0 < n < 1, there is a proposition, *the probability of p is n*, such that for any rational credence function, C, if C(the probability of p is n) > 0, then C(p | the probability of p is n) = n	Suppose for reductio (Exact Probability$_f$)
2	Let p be a proposition and n a real number such that 0 < n < 1	Stipulation
3	There is a proposition, the probability of p is n, such that for any rational credence function, C, if C(the probability of p is n) > 0, then C(p | the probability of p is n) = n	∀ Elimination 1
4	For any rational credence function, C, if C(the probability of p is n) > 0, then C(p | the probability of p is n) = n	Suppose for ∃ Elimination
5	~p and the probability of p is n are consistent	For reductio
6	For any propositions, p and q, if p and q are consistent, then there is some rational credence function, C, such that C_p(q) > 0	Consistency$_b$
7	If ~p and the probability of p is n are consistent, then there is some rational credence function, C, such that $C_{\sim p}$(the probability of p is n) > 0	∀ Elimination 6
8	There is some rational credence function, C, such that $C_{\sim p}$(the probability of p is n) > 0	→ Elimination 5, 7
9	$C_{\sim p}$(the probability of p is n) > 0	Suppose for ∃ Elimination
10	If $C_{\sim p}$(the probability of p is n) > 0, then $C_{\sim p}$(p | the probability of p is n) = n	∀ Elimination 4
11	$C_{\sim p}$(p | the probability of p is n) = n	→ Elimination 9, 10
12	For any rational credence function, C, and proposition, p, $C_{\sim p}$(p) = C(p | ~p) = 0	Negation CP
13	$C_{\sim p}$(p) = 0	∀ Elimination 12
14	$C_{\sim p}$(p | the probability of p is n) = 0	Probabilistic Persistence 9, 13
15	0 < n = 0	2, 11, 14
16	0 < n = 0 (contradiction)	∃ Elimination 8, 9-15
17	~p and the probability of p is n are inconsistent	~ Introduction 5-16
18	For any propositions ~p and q, if ~p and q are inconsistent, then q entails p	Inconsistency to Entailment
19	If ~p and the probability of p is n are inconsistent, then the probability of p is n entails p	∀ Elimination 18
20	the probability of p is n entails p	→ Elimination 17, 19
21	For any proposition, p, and real number, n, if the probability of p is n entails p, then the probability of p is n is a contradiction or n = 1	Probability Entailing Content
22	If the probability of p is n entails p, then the probability of p is n is a contradiction or n = 1	∀ Elimination 21
23	the probability of p is n is a contradiction or n = 1	→ Elimination 20, 22
24	the probability of p is n is a not contradiction	Logical Truth
25	n = 1	∨ Elimination 23, 24
26	1 = n < 1	2, 25
27	1 = n < 1 (contradiction)	∃ Elimination 3, 4-26
28	1 (our supposition for reductio) is false	~ Introduction 1-27

24 Consistency$_b$, Inconsistency to Entailment, and Probability Entailing Content are stated in the proof. Probabilistic Persistence is an inference schema:

m	C(p) = 0	
n	C(q) > 0	
	C(p | q) = 0	Probabilistic Persistence m, n

Russell and Hawthorne don't, themselves, draw the conclusion in 28. Instead they think that the problem is in the derivation to step 19. They explore a different way of thinking of contents in a dynamic framework. According it, either Inconsistency to Entailment (18) is not true–if statements like <u>the probability of p is n</u> count as propositions–or 19 does not express an instantiation of Inconsistency to Entailment – if they do not (see Russell & Hawthorne, 2016 for details).

This proof, like Schroeder's and Goldstein's, is sound. But, we cannot replicate this sort of derivation while relying on Exact $\text{Probability}_{\text{v}}$ rather than Exact $\text{Probability}_{\text{f}}$, and CBP is consistent with Exact $\text{Probability}_{\text{v}}$:

> **Exact $\textbf{Probability}_{\textbf{v}}$**: For any proposition, p, real number, n, where $0 < n < 1$, and rational credence function, C, there is a proposition, *the probability of p is n*, such that if C(the probability of p is n) > 0, then C(p ∣ the probability of p is n) = n.

We might think, however, that there should be more to say about which proposition of the form, *the probability of p is n*, is the one that makes C(p ∣ the probability of p is n) = n true for a given C. A natural suggestion is that it is the proposition where probability just is the credence function that the person has at that time. So, credences about *the probability of p* are credences about that very credence function. I'll write the principle as follows:

> **Exact $\textbf{Probability}_{\textbf{r}}$**: For any rational credence function, C, proposition, p, and real number, n, where $0 < n < 1$, if $C(C(p) = n) > 0$, then $C(p \mid C(p) = n) = n$.

Is this principle subject to a triviality result like the one for Exact $\text{Probability}_{\text{f}}$? This might seem particularly pressing since the way I've wiggled out of earlier triviality proofs involved disambiguating between the scopes of quantifiers that bind credence functions and propositions about probabilities. But there could be no such ambiguity in this principle. Unfortunately, I can't guarantee that there is no result trivializing it. But I will give some license for optimism by showing that proceeding in similar ways to the triviality result for Exact $\text{Probability}_{\text{f}}$ won't yield such a result:

1	For any rational credence function, C, proposition, p, and real number, n, where 0 < n < 1, if C(C(p) = n) > 0, then C(p ∣ C(p) = n) = n	For reductio
2	Let $\underline{C}$ be a rational credence function, $\underline{p}$ a proposition, and $\underline{n}$ a real number such that $0 < \underline{n} < 1$	Stipulation
3	$\sim\underline{p}$ and $\underline{C}(\underline{p}) = \underline{n}$ are consistent	For reductio
4	For any propositions, p and q, if p and q are consistent, then there is some rational credence function, C, such that $C_p(q) > 0$	$\text{Consistency}_{\text{b}}$
5	If $\sim\underline{p}$ and $\underline{C}(\underline{p}) = \underline{n}$ are consistent, then there is some rational credence function, C, such that $C_{\sim\underline{p}}(\underline{C}(\underline{p}) = \underline{n}) > 0$	∀ Elimination 4
6	There is some rational credence function, C, such that $C_{\sim\underline{p}}(\underline{C}(\underline{p}) = \underline{n}) > 0$	→ Elimination 3, 5
7	$\underline{C}_{\sim\underline{p}}(\underline{C}(\underline{p}) = \underline{n}) > 0$, where $\underline{C}$ is a rational credence function	Suppose for ∃ Elimination?

But we can't use $\underline{C}$ as our proxy in 7, since we've already made assumptions about $\underline{C}$ elsewhere (namely that its assigning $\underline{n}$ to $\underline{p}$ is consistent with $\sim\underline{p}$). We could choose a different proxy:

7	$\underline{C^*}_{\sim\underline{p}}(\underline{C}(\underline{p}) = \underline{n}) > 0$, where $\underline{C^*}$ is a rational credence function	Suppose for ∃ Elimination
8	If $\underline{C^*}_{\sim\underline{p}}(\underline{C}(\underline{p}) = \underline{n}) > 0$, then $\underline{C^*}_{\sim\underline{p}}(\underline{p} \mid \underline{C}(\underline{p}) = \underline{n}) = \underline{n}$	∀ Elimination 1?

But now 8 doesn't follow from 1 by universal instantiation, since in 1, the same variable over credence functions appears with wider and narrower scope. 1 makes no claims about the relationship between different credence functions. Perhaps we should reexamine our reductio assumption at step 3, focusing not on $\underline{C}$ but $\underline{C}_{\sim\underline{p}}$. By doing so, we don't need to proceed by reductio; we can reason directly as follows:[25]

1	For any rational credence function, C, proposition, p, and real number, n, where $0 < n < 1$, if $C(C(p) = n) > 0$, then $C(p \mid C(p) = n) = n$	For reductio
2	Let $\underline{p}$ be a proposition, $\underline{n}$ a real number such that $0 < \underline{n} < 1$, and $\underline{C}_{\sim\underline{p}}$ a rational credence function	Stipulation
3	For any rational credence function, C, and proposition, p, $C_{\sim p}(p) = C(p \mid {\sim}p) = 0$	Negation CP
4	$\underline{C}_{\sim\underline{p}}(\underline{p}) = 0$	∀ Elimination 3
5	$\underline{C}_{\sim\underline{p}}(\underline{p}) \neq \underline{n}$	2, 4
6	$\sim\underline{p}$ and $\underline{C}_{\sim\underline{p}}(\underline{p}) = \underline{n}$ are inconsistent	Provable Negation 5[25]
7	For any propositions p and q, if p and q are inconsistent, then q entails $\sim p$	Inconsistency to Entailment
8	If $\sim\underline{p}$ and $\underline{C}_{\sim\underline{p}}(\underline{p}) = \underline{n}$ are inconsistent, then $\underline{C}_{\sim\underline{p}}(\underline{p}) = \underline{n}$ entails $\underline{p}$	∀ Elimination 7
9	$\underline{C}_{\sim\underline{p}}(\underline{p}) = \underline{n}$ entails $\underline{p}$	→ Elimination 6, 8
10	For any proposition, p, real number, n, and rational credence function, C, if $C(p) = n$ entails p, then $C(p) = n$ is a contradiction or $n = 1$.	Probability Entailing Content*
11	If $\underline{C}_{\sim\underline{p}}(\underline{p}) = \underline{n}$ entails $\underline{p}$, then $\underline{C}_{\sim\underline{p}}(\underline{p}) = \underline{n}$ is a contradiction or $\underline{n} = 1$	∀ Elimination 10
12	$\underline{C}_{\sim\underline{p}}(\underline{p}) = \underline{n}$ is a contradiction or $\underline{n} = 1$	→ Elimination 9, 11
13	$\underline{C}_{\sim\underline{p}}(\underline{p}) = \underline{n}$ is a not contradiction	?

It's not hard to show that $\sim\underline{p}$ and $\underline{C}_{\sim\underline{p}}(\underline{p}) = \underline{n}$ are inconsistent. But the remainder of the proof stalls when we must say that the probability of p is n is not a contradiction. When we cash this out in the way we decided in the proof, we see that it *is* a contradiction. It can't be true that $\underline{C}_{\sim\underline{p}}(\underline{p}) = \underline{n}$. Again, this doesn't mean that the English sentence, "The probability of p is n" is a contradiction on every interpretation, since this can be used to pick out any number of credence functions. But on the interpretation used in the proof, it is a contradiction. We should also recognize that this latest attempt to carry out the proof couldn't be on the right track, since we didn't use our reductio assumption in the proof. Even if we had been able to derive a contradiction, some other principle, not the reductio assumption, would be the culprit.

[25] Here's the rule I've called Provable Negation:

For all p, if $\vDash \sim p$, then for all q, q and p are inconsistent.

This is a metalogical rule. It isn't simply line 5 that justifies this step, but the fact that we reached that line without any substantive assumptions. I use this rule because it gets us more quickly to the important part of the proof.

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