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ORIGINAL ARTICLE

The Problem of Coherence and Truth Redux

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Abstract In "What price coherence?" (Analysis 54:129–132, 1994), Klein and Warfield put forward a simple argument that triggered an extensive debate on the epistemic virtues of coherence. As is well-known, this debate yielded far-reaching impossibility results to the effect that coherence is *not* conducive to truth, even if construed in a ceteris paribus sense. A large part of the present paper is devoted to a re-evaluation of these results. As is argued, all explications of truth-conduciveness leave out an important aspect: while it might not be the case that coherence is truth-conducive, it might be conducive to *verisimilitude* or *epistemic utility*. Unfortunately, it is shown that the answer for both these issues must be in the negative, again. Furthermore, we shift the focus from sets of beliefs to particular beliefs: as is shown, neither is any of the extant probabilistic measures of coherence truth-conducive on the level of particular beliefs, nor does weakening these measures to quasi-orderings establish the link between coherence and truth for an important amount of measures. All in all, the results in this paper cast a serious doubt on the approach of establishing a link between coherence and truth. Finally, recent arguments that shift the focus from the relationship between coherence and truth to the one between coherence and confirmation are assessed.

1 Introduction

The last two decades have seen considerable efforts in trying to answer two perennial challenges facing coherence theories of justification. On the one hand, still in 1999 Laurence BonJour renounced his coherence theory of justification recognizing that "the precise nature of coherence remains an unsolved problem"(BonJour 1999,

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p. 124). A paper by Shogenji (1999), published that very same year, is the starting point of a new branch of Bayesian epistemology engaged in the project of explicating the notion of coherence by means of probability theory. Among those who have contributed to this project are Douven and Meijs (2007), Fitelson (2003), Glass (2002), Meijs (2005, 2006), Olsson (2002), Roche (2013) and Schupbach (2011).

On the other hand, scholars addressed the problem of coherence and truth. In “What price coherence?” (1994), Klein and Warfield put forward an argument to the effect that “[c]oherence, per se, is not truth conducive” (p. 131), pointing out that “by increasing the coherence of a set of beliefs, the new, more coherent set of beliefs is often less likely to be true than the original, less coherent set” (p. 129). The responses to their paper are manifold. While Merricks (1995) suggests that “truth conduciveness should be evaluated – not on the level of systems or sets of beliefs – but on the level of particular beliefs” (p. 307), Shogenji (1999) and all other Bayesian solution proposals agree in this respect with Klein and Warfield. However, Shogenji (1999) and, more recently, Schupbach (2008) argued that evaluating truth-conduciveness along the lines of Klein and Warfield’s proposal is inadequate insofar as “[s]uch comparison may lump together the effects of two factors – coherence and total individual strength – on truth.” (Shogenji 1999, p. 342). Accordingly, they both present proposals to sidestep the skeptic conclusion demanding to hold constant the total individual strength when scrutinizing the truth-conduciveness of coherence.[1]

Another line of response has initially been brought forward by Bovens and Olsson (2002) and Olsson (2002, 2005a). Basically, Bovens and Olsson dispute Klein and Warfield’s explication of truth-conduciveness claiming that what we are to compare are not bare sets of propositions but *testimonial systems*, featuring for every proposition a report to the effect that the proposition is true. Associated with this transition to testimonial systems is an adaption of the explication of truth-conduciveness focusing not on whether a more coherent set is *initially* more likely to be true than a less coherent set, as in Klein and Warfield’s account, but whether it is more likely to be true, *given* the reports for each proposition. Formally, this modification amounts to replacing the joint prior probability by the joint probability *conditional on* the reports. A similar account is proposed by Bovens and Hartmann (2003). Notwithstanding their solution to Klein and Warfield’s challenge, Olsson (2005a) as well as Bovens and Hartmann (2003) consonantly draw similar negative conclusions with respect to the truth-conduciveness of coherence. More specifically, Olsson, Bovens and Hartmann prove far-reaching *impossibility results* concerning the very possibility of constructing truth-conducive, probabilistic measures of coherence. These arguments, if sound, would have devastating effects regarding the tenability of (Bayesian) coherence theories. Nonetheless, the constraints that are required for deriving the skeptical conclusion within each of these two models have not remained uncontested. Among those who contributed to this debate are Meijs and Douven (2007), Schupbach (2008), and Wheeler (2012). A reevaluation of these results is given in Sect. 3. However, before delving into the discussion of extant impossibility results, Sect. 2 briefly introduces the project of probabilistically measuring coherence.

[1] For a detailed analysis see Sect. 2.

Then, in Sect. 4 we want to pursue another strategy to try to counter the skeptic results on the truth-conduciveness of coherence based on yet another interpretation of the notion of truth-conduciveness. An indication for this interpretation is given in the following passage, to be found in the introduction to Olsson's *Against Coherence* (2005a). There Olsson asks (p. 1, italics added):

> Does coherence imply truth? This is our central problem, and one could in principle imagine many ways of attacking it. While it may be implausible to think that a system that is coherent is thereby guaranteed to contain only true propositions, it is conceivable that coherence could imply *verisimilitude*, so that a system, in virtue of being coherent, is at least *close to the truth*.

Yet even a glimpse into the comprehensive literature on verisimilitude reveals the inadequacy of equating verisimilitude and (a high) likelihood of truth, as it is done in the literature on the truth-conduciveness of coherence. Thus, the strategy that is pursued in order to try to recover from the impossibility results is to examine whether coherence is conducive to truth*likeness*, instead of being conducive to truth. Another departure from mainstream research on truth-conduciveness is that the focus is on (scientific) *theories*. Accordingly, the main question to be answered is the following: is coherence among theories a scientific virtue in the sense that coherence increases verisimilitude? As will be argued in Sect. 4, this question must be answered in the negative. In this context, the connection between coherence and *epistemic utility* is explored, where epistemic utility is construed as an average of posterior probability and informativity.

Section 5 is then devoted to an assessment of Merricks's (1995) suggestion on the truth-conduciveness of coherence on the level of particular beliefs. As is shown in this section, neither is any of the extant coherence measures truth-conducive in this sense nor an important amount of quasi-orderings that can be obtained from these measures, the only exception being the one based on the firmness measure of confirmation. All in all, the results of this paper cast a serious doubt on the approach of establishing a link between coherence and truth. At least, it seems hard, if not impossible, to establish a *deductive* link in the sense that more coherence *invariably* leads to truth. Accordingly, the paper substantiates the insight that the problem of coherence and truth poses a serious threat for coherentist approaches to epistemic justification (BonJour 1985; Harman 1986).

In the last section, we briefly consider Wheeler and Scheines' recent results on the relationship between coherence and *confirmation*. First, it is shown that their results can be generalized in important respects; second, however, we show that they do not hold for all extant coherence measures.

2 Probabilistic Measures of Coherence

This section is devoted to a brief summary of extant probabilistic measures of coherence. Without providing a detailed motivation for any of the accounts, the section introduces three different families of coherence measures. Formally, a probabilistic measure of coherence **Coh** is a (partial) function assigning each pair

$(S, \Pr)$, where S is a set of propositions and Pr a probability function over the algebra generated by the propositions in S, a real number representing S's degree of coherence under Pr. Usually, a coherence measure **Coh** features a threshold δ separating coherent from incoherent sets of propositions so that a set S is coherent according to measure **Coh** under probability distribution Pr if and only if $\mathbf{Coh}_{\Pr}(S) > \delta$.[2]

The first family of measures rests upon the idea of coherence as a deviation from probabilistic independence. It is well-known that propositions are called probabilistically independent if the probability that all of them are true equals the product of their marginal probabilities. Hence, in order to quantify the deviation from this point of neutrality, Shogenji (1999) endorses the following function as a measure of coherence:

$$\mathcal{D}(S) = \frac{\Pr\left(\bigwedge_{A \in S} A\right)}{\prod_{A \in S} \Pr(A)}$$

Obviously, if the propositions under consideration are probabilistically independent, then the assigned degree of coherence by $\mathcal{D}$ is equal to 1. In case of probabilistic dependence, the propositions are either assessed coherent (if it exceeds 1) or incoherent (otherwise).

Another family of measures quantifies the degree of coherence in terms of the propositions' relative set-theoretic overlap, i.e., by comparing the probability of the set-theoretic intersection of the models of the propositions under consideration with the probability of their set-theoretic union. The corresponding measure that has independently been proposed by Glass (2002) and Olsson (2002) reads as follows:

$$\mathcal{O}(S) = \frac{\Pr\left(\bigwedge_{A \in S} A\right)}{\Pr\left(\bigvee_{A \in S} A\right)}$$

The question how to determine a threshold separating coherence from incoherence for this measure is controversial. However, given that our consequent discussion will focus exclusively on comparative coherence judgments, we can dispense with a discussion of reasonable candidate thresholds.

Both the deviation measure and the overlap measure have been criticized for being unduly subset-insensitive in the sense that they only take into account the n-wise coherence of the set of propositions under consideration but neglect the coherence of proper subsets. To overcome this shortcoming, Meijs (2006) and Schupbach (2011) put forward refined versions of these measures. In what follows I introduce a general recipe for achieving subset-sensitivity that slightly differs from both these extant accounts: to have a unified framework, for each set m-membered set S let "$[S]^k$" denote the set of all subsets of S with cardinality $k \leq m$. Furthermore, a *weighting system* is a vector $(w_1, \ldots, w_{m-1})$ of positive weights $w_i \geq 0$ such that $\sum_{i=1}^{m-1} w_i = 1$.

[2] In what follows, reference to the probability function in $\mathbf{Coh}_{\Pr}$ is dropped whenever it is clear from the context.

Now let **Coh** be either the deviation measure or the overlap measure, then the following is a recipe for subset-sensitive versions of the above coherence measures:

$$\mathbf{Coh}^*(S) = \sum_{k=1}^{m-1} \sum_{S' \in [S]^{k+1}} w_k \times \mathbf{Coh}(S')$$

In what follows we will assume equal weights so that $w_i = 1/(m-1)$ for all $1 \leq i \leq m-1$.[3]

Yet another approach to measuring coherence is initially due to Fitelson (2003, 2004) and has been systematically developed by Douven and Meijs (2007). This family of measures is based on the suggestion that coherent propositions mutually confirm each other. Consequently, these accounts utilize probabilistic measures of confirmation (a.k.a. *support*) that have been widely discussed within philosophy of science (cf. Crupi et al. 2007; Festa 2012). Formally, a confirmation measure ξ is a (partial) function assigning triples $(A, B, \Pr)$ a real number that is supposed to represent the degree of confirmation that a piece of evidence B provides for a scientific hypothesis A under probability distribution Pr. Each confirmation measure ξ features a threshold θ separating confirmation from disconfirmation; usually, this point of neutrality is the case of probabilistic independence. Table 1 is a summary of prominent proposals.[4]

Given this list of confirmation measures, the following recipe allows for the construction of the confirmation-based family of coherence measures: for each set S let $[S]$ denote the set of all pairs of non-empty, non-overlapping subsets of S and denote the cardinality of $[S]$ by "κ". Given an ordering $(\hat{S}_1, \ldots, \hat{S}_\kappa)$ of the elements of $[S]$, the coherence measure $\mathcal{C}_\xi$ based on confirmation measure ξ is defined as follows[5]:

$$\mathcal{C}_\xi(S) = \sum_{i=1}^{\kappa} w_i \times \xi(\hat{S}_i)$$

Given that the recipe incorporates the *mutual* confirmation relations obtaining among the subsets of S, some of the above confirmation measures yield identical coherence measures. This is true for d and m on the one hand, and s and n on the other. Nonetheless, we are still left with a large number of rival accounts to measuring coherence. In recent papers it has been shown that these measures are not only non-equivalent, but also that they disagree with respect to the satisfaction and violation of various coherence-desiderata (cf. Schippers 2014a, b, 2015a). Some of the results contained in these and other studies might be considered a vindication of a particular measure establishing its superiority as compared to its competitors (see also Douven and Meijs 2007; Olsson and Schubert 2007; Roche 2013; Schippers

[3] For refined weighting systems see Schupbach (2011).

[4] The confirmation measures in Table 1 are based on two different qualitative notions of confirmation, sometimes called *incremental confirmation* and *absolute confirmation*. The details of this distinction are not important in the present context.

[5] Here and in what follows we assume the following notational convention: if S and S' are sets of propositions, then $\xi(S, S')$ denotes $\xi(\bigwedge S, \bigwedge S')$, where $\bigwedge S = \bigwedge_{A \in S} A$.

Table 1 A survey of prominent probabilistic measures of confirmation

Confirmation measure	Definition	Advocate
$d(A, B)$	$\Pr(A\|B) - \Pr(A)$	Carnap (1962)
$r(A, B)$	$\Pr(A\|B)/\Pr(A)$	Keynes (1921)
$n(A, B)$	$\Pr(B\|A) - \Pr(B\|\neg A)$	Nozick (1981)
$s(A, B)$	$\Pr(A\|B) - \Pr(A\|\neg B)$	Christensen (1999)
$m(A, B)$	$\Pr(B\|A) - \Pr(B)$	Mortimer (1988)
$l(A, B)$	$\Pr(B\|A)/\Pr(B\|\neg A)$	Good (1984)
$k(A, B)$	$n(A,B)/[\Pr(B\|A) - \Pr(B\|\neg A)]$	Kemeny and Oppenheim (1952)
$z(A, B)$	$\begin{cases} d(A,B)/[1-\Pr(A)], & \text{if } \Pr(A\|B) \geq \Pr(A) \\ d(A,B)/[\Pr(A)], & \text{otherwise} \end{cases}$	Crupi et al. (2007)
$f(A, B)$	$\Pr(A\|B)$	Carnap (1962)

2014c; Schubert 2012a, b). However, if it could be shown that one of the measures is truth-conducive in a well-specific sense, then this should be considered an important contribution to the discussion on the pros and cons of the various measures. Accordingly, the next section starts with a reevaluation of extant impossibility results taking into account all measures introduced in this section.

3 Reconsidering Extant Impossibility Results

3.1 Propositional Truth-Conduciveness

According to a coherentist position in epistemology, coherence is a means for justifying beliefs: "beliefs are justified by being inferentially related to other beliefs in the overall context of a coherent system" (BonJour 1985, p. 90). However, justification does not seem to have any intrinsic value, but is to be seen as a means for achieving *truth*. Thus, like any other epistemological account to justification, the coherentist has to clarify the connection between coherence and truth. In BonJour's own words:

> [...] one crucial part of the task of an adequate epistemological theory is to show that there is an appropriate connection between its proposed approach to epistemic justification and the cognitive goal of *truth*. That is, it must be somehow shown that justification as conceived by the theory is *truth-conducive*, that one who seeks justified beliefs is at least likely to find true ones. (1985, pp. 108–109, emphasis BonJour's)

Consequently, the question to be addressed in this section is the following: "why, if a system of empirical beliefs is coherent (and more coherent than any rival system), is it thereby justified in the epistemic sense, that is, why is it thereby likely to be true?" (1985, p. 93). Put another way, this section examines whether coherence is a guide to truth in the sense that a more coherent set of beliefs is more likely to be true than a less coherent one. According to Klein and Warfield (1994) the answer must be in the negative. To show why, they ask us to consider the following example:

> A detective has gathered a large body of evidence that provides a good basis for pinning a murder on Mr. Dunnit. In particular, the detective beliefs that Dunnit had a motive for the murder and that several credible witnesses claim to have seen Dunnit do it. However, because the detective also believes that a credible witness claims that she saw Dunnit two hundred miles away from the crime scene at the time the murder was committed, her belief set is incoherent (or at least somewhat incoherent). Upon further checking, the detective discovers some good evidence that Dunnit has an identical twin whom the witness providing the alibi mistook for Dunnit. (1994, pp. 130–131)

Obviously, adding beliefs about the evidence regarding Dunnit's twin brother renders the belief set more coherent by relieving the tension between the evidence for and against Dunnit. However, "since the new belief set contains more beliefs than the old set and the added beliefs neither have an objective probability of 1 nor are they entailed by the old set, the *more* coherent set is less likely to be true than is the original, *less* coherent set." Thus, Klein and Warfield conclude, "coherence, *per se*, is not truth-conducive" (1994, p. 131, emphasis Klein and Warfield's). The characterization of truth-conduciveness underlying Klein and Warfield's argument is the following:

Definition 3.1 Coherence is *propositionally truth-conducive* iff for all sets S, S', if S is more coherent than S', then $\Pr(\bigwedge_{A\in S} A) > \Pr(\bigwedge_{B\in S'} B)$ for all probability distributions Pr.

In the Dunnit-example, S' is obtained by adding a proposition to S that is neither entailed by the propositions in S nor has a probability of 1; in this case, it is a simple probabilistic fact that $\Pr(\bigwedge_{A\in S} A) > \Pr(\bigwedge_{B\in S'} B)$, although S' is intuitively more coherent. Thus, coherence can not be propositionally truth-conducive in the sense of Definition 3.1. In particular, there can be no probabilistic coherence measure that is propositionally truth-conducive in this sense. Klein and Warfield's argument, however, has not remained uncontested. According to Shogenji (1999), "we cannot evaluate truth-conduciveness of coherence simply by checking whether more coherent beliefs are more likely to be true together than less coherent beliefs. Such comparison may lump together the effects of two factors – coherence and total individual strength – on truth." Hence, according to Shogenji, "we need to check whether more coherent beliefs are more likely to be true together than less coherent *but individually just as strong* beliefs" (1999, p. 342, emphasis Shogenji's).

Thus, according to Shogenji coherence is only truth-conducive ceteris paribus. What needs to be fixed when assessing the truth-conduciveness of coherence is the sets' total individual strength, which Shogenji identifies with the product of the marginal probabilities of all propositions within each set. Thus, we get the following alternative characterization of propositional truth-conduciveness:

Definition 3.2 Coherence is *propositionally truth-conducive*$_{cp}$ iff for all sets S, S' that share the same total individual strength, if S is more coherent than S', then $\Pr(\bigwedge_{A \in S} A) > \Pr(\bigwedge_{B \in S'} B)$ for all probability distributions Pr.

Observation 3.1 Except Shogenji's measure, none of the coherence measures on the market are truth-conducive in the sense of Definition 3.2.[6]

That is, the deviation measure $\mathcal{D}$ is the only coherence measure that is truth-conducive in the sense of Definition 3.2. However, this should come as no surprise: given Shogenji's characterization of the total individual strength and his definition of coherence, it is just a trivial consequence that in this sense "coherence per se, when it is isolated from the total individual strength of beliefs, is truth conducive" (1999, p. 343). Because, assuming equal total individual strength entails that the degree of coherence as measured by $\mathcal{D}$ is proportional to the likelihood of the propositions' being true together.

Should this result be interpreted as a vindication of the Shogenji measure as the only viable option for measuring coherence if one strives for truth-conduciveness? This conclusion seems premature. Evidence abounds that Shogenji's coherence measure is not an adequate explication of the pretheoretic notion of coherence.[7] Its subset-sensitive counterpart $\mathcal{D}^*$, although able to cushion some of the criticisms against Shogenji's measure, is *not* truth-conducive in the sense of Definition 3.2. What is more, Olsson (2002, 2005a, b) discusses a simple measure that is even truth-conducive without assuming equal total individual strength:

$$\mathcal{C}_0(S) = \Pr\left(\bigwedge_{A \in S} A\right)$$

Olsson draws the following conclusion:

> According to $\mathcal{C}_0$, coherence *is* the joint probability, and so this measure, unsurprisingly, comes out as truth-conducive in the propositional sense. This fact actually tells against, rather than in favor of, the reasonableness of our explication [of truth-conduciveness]. $\mathcal{C}_0$ is not a plausible measure of coherence, and it would be highly surprising, therefore, if it turned out to be, in any interesting sense, truth-conducive. (2002, p. 251)

Similarly, based on the negative evidence regarding $\mathcal{D}$'s adequacy as a measure of coherence, we conclude that the fact that only Shogenji's measure is truth-conducive in the sense of Definition 3.2 tells against this very definition of truth-conduciveness.[8]

[6] For a proof of this and other observations see the Appendix. For a small subset of measures, this observation has already been proved by Meijs and Douven (2007).

[7] Cf. Bovens and Hartmann (2003), Fitelson (2003), Glass (2005), Koscholke (2015), Olsson (2002), Roche (2013), Schippers (2014c), Schupbach (2011), Siebel (2005), Siebel and Wolff (2008), and Wheeler (2009).

[8] Olsson (2001) also points out that Shogenji (1999) argument for why the total individual strength ought to be kept fixed is far from conclusive.

3.2 Doxastic and Testimonial Truth-Conduciveness

An altogether different strategy to counter Klein and Warfield's skeptic result is pursued by Olsson (2002, 2005a, b).[9] To motivate his alternative account of truth-conduciveness, Olsson draws our attention to a hitherto neglected aspect of Klein and Warfield's Dunnit-example: "It is part of the example that the detective *believes these propositions to be true*." Thus, he concludes that "what we are to compare are, in fact, not two sets of bare propositions, but two doxastic systems" (2005, p. 107), where a doxastic system $\mathbf{S}$ features for each proposition A_i a belief $Bel\,A_i$ to the effect that A_i is true. For each doxastic system $\mathbf{S} = \{\langle A_1, Bel\,A_1\rangle, \ldots, \langle A_n, Bel\,A_n\rangle\}$ let "$\mathbf{Coh}_{\mathrm{Pr}}(\mathbf{S})$" denote the degree of coherence of the ordered pair of propositional contents $\langle A_1, \ldots, A_n\rangle$ of $\mathbf{S}$ depending on probability distribution Pr, and denote the joint probability of the contents of the doxastic system conditional on the beliefs by "$\Pr(\mathbf{S})$". Now, according to Olsson, the proper concept of truth-conduciveness is captured by the following definition[10]:

Definition 3.3 Coherence is *doxastically truth-conducive* iff for all doxastic systems $\mathbf{S}$ and $\mathbf{S}'$, if $\mathbf{S}$ is more coherent than $\mathbf{S}'$, then $\Pr(\mathbf{S}) > \Pr'(\mathbf{S}')$ for all probability distributions $\Pr, \Pr'$.

Call an extension of a set S of propositions *non-trivial* if the added proposition is neither entailed by the propositions in S nor has a probability of one. Klein and Warfield's result utilizes the fact that any non-trivial extension of a set of propositions renders the resulting set less probable than the original set. However, there is no corresponding theorem for non-trivial extensions of doxastic systems and $\Pr(\mathbf{S})$. In fact, a counter-example to this condition is stated by Bovens and Olsson (2002). Accordingly, doxastic truth-conduciveness might indeed be considered a more promising route to truth-conducive measures of coherence. However, the question remains whether this is in fact the case.

The heart of Olsson's *Against Coherence* (2005a) is an impossibility result demonstrating that at least in certain respects the answer must be in the negative. More precisely, Olsson proves that in basic Lewis scenarios all truth-conducive coherence measures are non-informative in a way to be defined below.[11] For present purposes it suffices to characterize a *basic Lewis scenario* as one in which two independent witnesses report equivalently on the same proposition.[12] Like doxastic systems, these scenarios are an instance of the general class of *testimonial systems* $\mathbf{T} = \{\langle A_1, E_1, R_1\rangle, \ldots, \langle A_n, E_n, R_n\rangle\}$ featuring for each proposition A_i a report E_i to the effect that A_i is true with a certain degree of reliability R_i. Again "$\mathbf{Coh}_{\mathrm{Pr}}(\mathbf{T})$" denotes the degree of coherence of $\langle A_1, \ldots, A_n\rangle$ under probability distribution Pr and

[9] See also Bovens and Olsson (2002) and Cross (1999).

[10] Cf. Olsson (2005a). Olsson's earlier definition of truth-conduciveness (2002) differs insofar as also the probability distribution is not allowed to vary between both sets.

[11] See also Olsson (2005b).

[12] Furthermore, it is assumed that each witness i is either completely reliable (R_i) or completely unreliable (U_i), and that their *reliability profile* is incompletely known, i.e. $\Pr(R_1) = \Pr(R_2) > 0$ and $\Pr(R_i) + \Pr(U_i) = 1$.

"$\Pr(\mathbf{T})$" equals the joint probability of the testimonies' contents conditional on the testimonies. Given these characterizations, Olsson is able to prove the following impossibility theorem (cf. Olsson 2005a, b):

Theorem 3.1 *There are no informative coherence measures that are truth-conducive in a basic Lewis scenario, where* $\Pr(R_i) = \Pr'(R_i)$.

Now, a coherence measure **Coh** is called *informative* in a basic Lewis scenario **S** if there are probability distributions $\Pr, \Pr'$ such that $\mathbf{Coh}_{\Pr}(\mathbf{S}) \neq \mathbf{Coh}_{\Pr'}(\mathbf{S}')$. Therefore, what Olsson has shown is only that *if* there is coherence measure that is truth-conducive in a basic Lewis scenario, *then* it must be one that is not informative in such a scenario. This, however, might not be that problematic after all: many scholars agree in that to assign equal degrees of coherence to sets of equivalent testimonies even constitutes an adequacy constraint for the viability of *any* probabilistic measure of coherence. In this regard, Meijs and Douven (2007) rightly point to the fact that Bovens and Hartmann (2003, p. 52) and Fitelson (2003, p. 194) "have argued that making sets consisting of equivalent propositions maximally coherent (and, consequently, equally coherent to all other such sets) is a *sine qua non* for any adequate measure of coherence" (2007, p. 350). Similarly, for Siebel and Wolff (2008) a case of equivalent testimonies constitutes even a "touchstone for coherence measures".

Thus, it seems problematic to argue against the truth-conduciveness of coherence by focusing on basic Lewis scenarios. The reason for Olsson to focus on this class of scenarios is that "even in the simplest of cases there can be no coherence measure that is truth-conducive in this weak sense [where independence and equal individual credibility are assumed]" (2005b, p. 395). So the argument seems to run as follows: given that there is no probabilistic measure of coherence that is truth-conducive in these simple cases constituted by basic Lewis scenarios, there is *a fortiori* no coherence measure that is truth-conducive *tout court*. But, since intuitions regarding the degree of coherence of equivalent propositions seem heterogeneous, Olsson's generalization is blocked: even in light of Olsson's findings, it might be the case that there are truth-conducive coherence measures that are *not* informative in basic Lewis scenarios, where the latter fact counts not as a vice, but as a virtue of a coherence measure. Put another way, Olsson's alleged impossibility result may even be read as a vindication of the view of equivalent testimonies being maximally coherent.

A similar approach to tackle the problem of truth-conduciveness for testimonial systems is considered by Bovens and Hartmann (2003, 2005, 2006). The basic intuition underlying their account is that coherence is to be modeled indirectly as a confidence-boosting property. Thus, given that we can identify a certain set of factors that have an impact on an agent's degree of confidence regarding a certain amount of information, and assuming that coherence is among these factors, then we can assess the impact of coherence as follows: given that we hold constant all but one of the confidence-boosting factors, and given that for two information sets S_1 and S_2 the confidence-boost for S_1 exceeds the corresponding boost for S_2, then this boost must be due to a difference in coherence among the individual pieces of information in both sets. Therefore, if coherence makes a difference regarding confidence, then this difference can indirectly be utilized in order to compare the coherence ordering of information sets. The main part of Bovens and Hartmann's impossibility result focuses on the following set of conditions:

Separability For all information sets S, S', if S is no less coherent than S', then our degree of confidence that the content of S is true is no less than the corresponding degree of confidence regarding S', ceteris paribus.

Probabilism The binary relation of "... being no less coherent than ..." over the set $\mathbb{S}$ of all information sets is fully determined by the probabilistic features of the information sets contained in $\mathbb{S}$.

Ordering The binary relation of "... being no less coherent than ..." is an ordering, i.e., the relation is transitive and complete.

These three conditions constitute the core of a position that is labeled "Bayesian Coherentism" by Bovens and Hartmann. Now, in order to assess the compatibility of these conditions, Bovens and Hartmann consider a testimonial system, where for each information item A_i ($1 \leq i \leq n$) there is a report E_i from a less than fully reliable source to the effect that A_i is true. According to Bovens and Hartmann, what affects our degree of confidence $\Pr(\mathbf{S})$[13] regarding the veridicality of the information conditional on reports are the following three factors:

(i) *How expected is the information?* Bovens and Hartmann suggest to explicate the expectedness by means of the joint prior probability of all pieces of information within **S**.

(ii) *How reliable are the sources?* Reliability is modeled by means of a reliability parameter $r = 1 - q/p$, where $q := \Pr(E_i|\neg A_i)$ is the probability of receiving a report to the effect that A_i is true when in fact it is not and $p := \Pr(E_i|A_i)$ is the true positive rate.

(iii) *How coherent is the information?*

Given this taxonomy of confidence-boosting factors, we can explicate the above ceteris paribus conditions contained in the separability-condition in more detail: given two information sets S and S' with an equal degree of expectedness of the information *and* equally reliable sources, coherence and confidence should covary, i.e. if S is more coherent than S', then our degree of confidence that the information in S is true should exceed our corresponding degree of confidence regarding the content of S'.

Surprisingly, Bovens and Hartmann are able to show that *separability, probabilism* and *ordering* are jointly inconsistent. That is, given the ceteris paribus conditions and given that the taxonomy of confidence-boosting factors is exhaustive, there cannot be a coherence ordering that satisfies both other conditions. More precisely, they show that there are information sets S and S' such that S's posterior probability exceeds the one of S' for some values of the reliability

[13] Like in our discussion of Olsson's account of testimonial systems, $\Pr(\mathbf{S})$ denotes the joint *posterior* probability of the information set conditional on the reports.

parameter, while for other values it is the other way round. Since coherence should be immune to differences in the sources' degree of reliability, separability fails to hold. Thus, according to Bovens and Hartmann, the conclusion to be drawn is that there cannot be a probabilistic coherence measure that induces an ordering on information sets and is truth-conducive in the sense that more coherence entails a higher degree of confidence ceteris paribus. The moral Bovens and Hartmann draw is that we have to dispense with the *ordering* condition: there are pairs of information sets such that we lack intuitions regarding which is the more coherent one. Consequently, only a quasi-ordering, (i.e. a binary relation that is reflexive and transitive but not complete) is tenable. However, as Meijs and Douven (2007) rightly remark, this is not the only conclusion that might be drawn. Instead of abandoning *ordering*, they point out that,

> the coherentist is free to declare all combinations of two sets for which it is the case that one of them has a higher posterior probability than the other, given some values of the reliability parameter, and a lower posterior probability than the other, given other values of the reliability parameter, as being equally coherent.[14] (2007, p. 350)

Furthermore, there might be other ways to determine the expectedness of the information. For example, Meijs (2007) considers a revised model that explicates the expectedness by means of the *marginal* probabilities.[15] Eventually, it is far from clear that Bovens and Hartmann's taxonomy of confidence-boosting factors is exhaustive. Just to mention one aspect, Bovens and Hartmann stipulate that equivalent reports are maximally coherent. But, given their explication of the expectedness of the information in terms of the joint prior probability, there is no difference between a set of two witnesses reporting equivalently that A is the case on the one hand, and ten witness doing so on the other. However, regarding our degree of confidence matters might be considerably different. If A is highly unlikely, we will presumably be much more inclined to assign a high degree of confidence to A given ten equivalent testimonies, whereas two equivalent testimonies might not have such a substantial impact.[16] Thus, we conclude that Bovens and Hartmann's impossibility result is far from being conclusive as an argument against the tenability of Bayesian coherentism.

4 Coherence, Verisimilitude and Epistemic Utility

The former sections reported on extant impossibility theorems that purport to show that coherence is not truth-conducive, even in fortunate settings. In this section, now, we draw attention to two other possible interpretations of the merits of coherence in scientific contexts. The first of these is motivated by a passage in the introduction to Olsson's landmark *Against coherence*, in which he suggests that "coherence could

[14] For a similar remark see Olsson (2005b, p. 403).

[15] Note that this approach is more akin to Shogenji's requirement of equal total individual strength.

[16] Cf. Meijs (2007).

imply verisimilitude, so that a system, in virtue of being coherent, is at least close to the truth" (2005a, p. 1). In a nutshell, the idea of *verisimilitude* is the following: "A theory is highly verisimilar if it says many things about the target domain, and if many of these things are (almost exactly) true" (Cevolani and Tambolo 2013, p. 922). According to a verisimilitudinarian position, scientific progress is to be thought of as succession of theories increasing in verisimilitude or approximating truth. In this sense, "such theory-changes as the as the transition from Newton's to Einstein's theory are progressive because, although the new theory is, strictly speaking, presumably false, it is estimated to be closer to the truth than the superseded one" (Cevolani and Tambolo 2013, p. 922). So the question is whether this verisimilitudinarian account also makes for a better understanding of the merits of coherence.

Before we start out for an answer to this question, we have to tackle an issue that arises from the fact that theories are usually modeled as conjunctions in these approaches, so that it is not clear from the outset how to apply the concept of coherence. This is because coherence is usually assumed to be a relation that requires at least two propositions in order to be meaningfully applied.[17] One such possibility is within the relevant elements account to verisimilitude that has been developed by Schurz and Weingartner (1987, 2010). Their main concern is to formulate a consequence-based account to verisimilitude that is immune to the objections that Popper's definition faced (cf. Miller 1974; Tichý 1974). The main intuition underlying this approach is that a theory A is more verisimilar than another theory A' if A has more true consequences than A' and does *not* have more false consequences than t', or vice versa. However, as Miller and Tichý pointed out, the set of consequences needs to be restricted in order to prevent the ordering of false theories with respect to verisimilitude from collapsing. Schurz and Weingartner's characterizations of relevant elements can be spelled out by means of the notion of a prime implicate as follows: let $\mathcal{L}_n$ be a propositional language with n atomic propositions n atomic propositions $p_1, \ldots, p_n$, then a *clause* is a disjunction of literals $\bigvee_{i \leq k} \pm p_i$ in distinct and alphabetically ordered propositional variables; a literal $\pm p_i$ is either an atomic proposition p_i or the negation of an atomic proposition $\neg p_i$. Given these conventions, the set of implicates of a formula A can be characterized as the set of clauses entailed by A. Similarly, a formula B is a *prime implicate* of a formula A if it is an implicate of A and there is no logically stronger implicate B' of A. The set of prime implicates of a formula A will be denoted by A_Π. So, for $(p_1 \wedge p_2)_\Pi = \{p_1, p_2\}$ and $((p_1 \rightarrow p_2) \wedge p_3)_\Pi = \{\neg p_1 \vee p_2, p_3\}$. Accordingly, we can partition each theory A into its set of true prime implicates A_{Π_1} and its set of false prime implicates A_{Π_0}. To illustrate, let $A = p_1 \wedge \neg p_2$ and assume that in the domain under consideration both p_1 and p_2 are true, then $A_{\Pi_1} = \{p_1\}$ and $A_{\Pi_2} = \{\neg p_2\}$. Similarly, if $A = (p_1 \rightarrow p_2) \wedge (p_2 \rightarrow \neg p_3)$, then $A_\Pi = \{\neg p_1 \vee p_2, \neg p_2 \vee \neg p_3, \neg p_1 \vee \neg p_3\}$ and therefore $A_{\Pi_1} = \{\neg p_1 \vee p_2\}$ and $A_{\Pi_0} = \{\neg p_2 \vee \neg p_3, \neg p_1 \vee \neg p_3\}$. If we partition theories into their prime implicates, then we can capture the main intuition underlying consequence-based definitions of

[17] For exceptions see Akiba (2000) and Fitelson (2003). Fitelson even maintains that "intuitively, all propositions 'cohere with themselves' (maximally), except for necessary falsehoods" (2003, p. 198). This makes it even harder to image a possible application of the concept of coherence within the common accounts to verisimilitude.

verisimilitude as follows: a theory A is *at least as verisimilar as* another theory A' ($A \geq_{\Pi} A'$) if $A_{\Pi_1} \vDash A'_{\Pi_1}$ and $A'_{\Pi_0} \vDash A_{\Pi_0}$.[18] Moreover, $A >_{\Pi} A'$ if $A \geq_{\Pi} A'$ and not $A' \geq_{\Pi} A$. In this sense, $p_1 \wedge p_2$ is more verisimilar than both $p_1 \wedge \neg p_2$ and $p_1 \wedge (p_2 \vee p_3)$.

Given that we partition each theory into its prime implicates, it is at least possible to compare a subset of rival theories with respect to coherence, viz., each theory A such that A_{Π} contains at least two elements. In what follows we limits consideration to *conjunctive theories*, i.e., theories that are stated as conjunctions of literals. For this kind of theories, Cevolani, Crupi and Festa developed a "basic feature approach" (cf. Cevolani et al. 2010, 2011) to versimilitude that is a special case of a number of existing of existing proposals.[19] The basic idea of this approach can be restated in our present context as follows: given that the degree of verisimilitude of a theory A is an average of A's truth-content and A's falsity-content, the following is a straightforward verisimilitude measure for conjunctive theories, where c denote the true constituent, i.e., the maximal conjunction of true literals in $\mathcal{L}_n$:

$$\mathcal{V}(A, c) = (|A_{\Pi_1}| - |A_{\Pi_0}|)/n$$

According to this measure, the degree of verisimilitude increases with the number of true prime implicates and decreases with the number of false prime implicates, ceteris paribus. Given this measure, the question regarding the relationship between coherence and verisimilitude can now be stated precisely as follows: is it for all theories A, A' the case that if $\mathbf{Coh}(A_{\Pi}) > \mathbf{Coh}(A'_{\Pi})$, then also $\mathcal{V}(A, c) > \mathcal{V}(A', c)$?

There is still an important void in this explication which is due to the fact that the coherence orderings induced by **Coh** are relative to probability distributions. Therefore, it might be the case that for some probability distribution Pr it is the case that $\mathbf{Coh}_{\Pr}(A_{\Pi}) > \mathbf{Coh}_{\Pr}(A'_{\Pi})$ while for another distribution Pr$'$ it is the other way round. This, however, cannot be the case for the verisimilitude orderings. In order to close this gap, one can either resort to *all* possible probability distributions or a well-specified subset thereof. The first option drastically narrows down the range of possible pairs of theories, viz., to those theories such that the coherence ordering is insensitive to the chosen probability distribution. But, in combination with the fact that all elements of A_{Π} are logically independent (cf. Schippers 2014d), it seems hard, if not impossible to find such pairs. To illustrate, if $c = p_1 \wedge p_2 \wedge p_3$, then $A_{\Pi} = \{p_1, p_2\}$ might be more coherent than $A'_{\Pi} = \{p_1, \neg p_2\}$ for some probability distribution, while being less coherent than A'_{Π} for other distributions. On the other hand, $\mathcal{V}(A, c)$ will exceed always $\mathcal{V}(A', c)$ irrespective of the chosen distribution. Hence, the only available option seems to be the one based on a well-specified subset of distributions. A promising candidate seems to be a distribution that incorporates the total amount of available evidence regarding some domain under consideration. If we now additionally assume that A is adjusted in light of conflicting evidence, then it seems indeed reasonable to conclude that a higher degree of coherence (in the long run) will

[18] It is assumed that $\Delta \models \Gamma$ iff for all $A \in \Gamma$: $\Delta \models A$.

[19] Among these are the ones proposed by Kuipers (1982), Oddie (1986), Schurz and Weingartner (1987, 2010), Brink and Heidema (1987) and Gemes (2007); cf. Cevolani et al. (2011).

eventually lead to a higher degree of truthlikeness (cf. BonJour 1976, p. 300ff.).[20] We leave this question for further research.

Instead, we now turn to yet another interpretation of the merits of coherence in scientific theory choice. It might be asked what the negative results in the former sections on the different explications of truth-conduciveness of coherence tell us about the well-known thesis in philosophy of science that coherence is a virtue in theory choice. At first glance, the results might seem to provide equally negative evidence regarding the utility of a coherence-based decision making in scientific contexts (independent of issues of verisimilitude): given that coherence does not lead to truth, it seems that there is no reason to prefer a coherent theory over a less coherent rival theory, if both theories are on a par with respect to all other aspects that we can base our decision on (like simplicity, explanatory power, etc.). But, recall the common feature of all definitions of truth-conduciveness: to be truth-conducive meant to be conducive to an increase in posterior probabilities. However, in his *Logic of Scientific Discovery* (1968) Popper insisted that high posteriors are *not* what scientists are aiming at when scrutinizing their theories. Popper claimed:

> Science does not aim, primarily, at high probabilities. It aims at a high informative content, well backed by experience. But a hypothesis may be very probable simply because it tells us nothing, or very little. A high degree of probability is therefore not an indication of "goodness" – it may be merely a symptom of low informative content. (Popper 1968, p. 399)

Thus, according to Popper, good scientific theories should combine the ideas of truth and content (cf. Popper 1963, p. 237). A similar approach originates from cognitive decision theory (Hempel 1960; Levi 1967). The basic idea is that the acceptance of a scientific theory depends on a rule of maximizing 'epistemic utility', where epistemic utility is a function of the theory's truth-value and its information value.

In a nutshell, the keynote of epistemic utility theory can be illustrated as follows: suppose that there is a finite set of possible actions $\mathcal{A}$ and a finite set of possible worlds $\mathcal{W}$, then an agent's utility function u takes an action $\alpha \in \mathcal{A}$ together with a possible world $w \in \mathcal{W}$ and returns a real number that is supposed to represent the degree to which the agent values the outcome of act α in w. Now given a situation with a number of different possible acts, the *Principle of Expected Utility* requires to opt for the act α that maximizes the value of the following formula:

$$EU(\alpha, \mathcal{W}) = \sum_{w \in \mathcal{W}} \Pr(w) \cdot u(\alpha, w)$$

In our present context of scientific theory choice the possible worlds to take into account are just the worlds where t is true and the ones where t is false. Furthermore, we assume to have a certain amount of evidence e out our disposal. Then, the utility

[20] Another option is to investigate the relationship between coherence and *estimated verisimilitude*, where the latter basically is an expectation value for verisimilitude in the light of a certain set of relevant pieces of evidence (cf. Niiniluoto 1987, ch. 7). Schippers (2015b) investigates the relationship between coherence and estimated verisimilitude based on the idea that what we are to compare are not the theories' degree of coherence and its degree of verisimilitude but whether a higher degree of coherence between a theory and the available evidence leads to a higher degree of estimated verisimilitude.

of endorsing a true theory t can be identified with t's informational value, which is inversely proportional to t's prior probability and measured by $\text{cont}(t) = 1 - \Pr(t)$; on the other hand, the utility of endorsing a false theory t is measured by $-\text{cont}(\neg t)$, which is inversely proportional to the potential gain of endorsing the true theory $\neg t$. In order to incorporate the evidence, we replace the 2-place function $u(\alpha, w)$ by a 3-place function $u(t, x, e)$ where x is either 1, if t is true and 0 otherwise (cf. Niiniluoto 1987, ch. 12).

(i) $u(t, 1, e) = \text{cont}(t)$
(ii) $u(t, 0, e) = -\text{cont}(\neg t)$

Given these characterizations, the expected epistemic utility of theory t in the light of evidence e is given by the following formula:

$$EU(t, e) = \Pr(t|e) \cdot u(t, 1, e) + \Pr(\neg t|e) \cdot u(t, 0, e)$$

Given the particular choice for the utilities involved in this equation, it is a straightforward to show that (4) can be rewritten as the difference between the posterior probability of t given e and the t's prior probability. Hence, to maximize (4), t should be a theory with both a high posterior probability given e and a high information content.

In order to evaluate this idea of epistemic utility theory as it relates to vindicating coherentism, we assume that each theory t is the union of a set of non-empty but possibly overlapping models $m_1, \ldots, m_n$ such that each model m_i assembles the relevant propositions in t that are necessary in order to account for the corresponding set of relevant pieces of evidence e_i.[21] The probability of t is then simply identified with the probability of the conjunction of all of t's models m_i.

What does it mean to say that e_i is evidence for m_i? According to the standard Bayesian concept of confirmation, e_i is evidence for m_i if and only if the posterior probability of m_i given e_i exceeds the prior probability of m_i. As is well known, this is equivalent to stating that the probability of e_i given m_i exceeds the probability of e_i given $\neg m_i$. Furthermore, assuming both probabilities to be non-extreme so that our evidence is neither completely reliable nor completely unreliable, we can equivalently say that e_i is evidence for m_i if and only if the inverse likelihood ratio $x(e_i, m_i) = 1/l(e_i, m_i)$ lies strictly between 0 and 1.[22]

Given that we want to scrutinize the impact of the coherence of a scientific theory on its degree of epistemic utility, we have to shield off the possible impact of all other factors. Accordingly, we assume this inverse likelihood ratio to be identical for all models and all corresponding pieces of evidence, i.e. $x(e_i, m_i) = x(e_j, m_j) =: x$ for all

[21] This model-theoretic explication of the concept of a scientific theory relies heavily on Bovens and Hartmann (2003, pp. 53–55). Cf. Hartmann (2008). This representation of a theory accounts for the fact that propositions are usually not tested in isolation, but is on the other hand fine grained enough to allow for testing of proper parts of a given theory. Furthermore, by making allowance for overlapping sets it takes into consideration that some propositions in t (for example scientific laws) might play a prominent role in more than one model.

[22] What is here called the 'inverse likelihood ratio' is sometimes also simply called the likelihood ratio (cf. Howson and Urbach 2006, p. 21).

$1 \leq i, j \leq n$. Furthermore, let $\bar{x} = 1 - x$.[23] Finally, we assume that each model m_i *screens off* e_i from all other model variables m_j and all other evidence variables e_j. Utilizing standard notation (Pearl 2000) we thus assume that for all $1 \leq i \leq n$

$$e_i \perp\!\!\!\perp m_1, e_1, \ldots, m_{i-1}, e_{i-1}, m_{i+1}, e_{i+1}, \ldots, m_n, e_n | m_i$$

Now it is the time to render precise the idea of coherence being conducive to epistemic utility:

Definition 4.1 Coherence is *conducive to epistemic utility* iff for all theories t and t' with corresponding evidence sets e and e' the following claim is true: if t is more coherent than t', then $EU(t, e) > EU(t', e')$ for all probability distributions Pr *ceteris paribus*.

The following notational conventions are adapted from Bovens and Hartmann (2003) to yield a concise representation of the t's epistemic utility given e as a function of the likelihood-ratio x. Let a_i denote the sum of the joint probabilities of all combinations of i negative values and $n - i$ positive values of the model variables $m_1, \ldots, m_n$. Accordingly, for a theory t with models $m_1, \ldots, m_n\}$ call $\langle a_0, \ldots, a_n \rangle$ the *weight vector* of the models in t, then we get the following representation for the epistemic utility function, where $\gamma = \sum_{i=0}^{n} a_i \bar{x}^i$.

$$EU(t, e) = a_0 \cdot \left(\frac{1 - \gamma}{\gamma} \right)$$

Now let **Coh** be a probabilistic measure of coherence, then a theories degree of coherence as measured by **Coh** must be a function of the weight vector $\langle M_1, \ldots, M_n \rangle$. Thus, to evaluate the EU-conduciveness of **Coh**, all we have to do is to fix the weight vectors for two theories t, t'.[24] Let $\langle .05, .30, .10, .55 \rangle$ and $\langle .05, .20, .70, .05 \rangle$ be the weight vectors for the models m_i in t and m'_i in m'. The corresponding epistemic utilities of t and t' are plotted in Fig. 1. This figure shows that there cannot be a probabilistic measure of coherence that is EU-conducive in the sense outlined above. Assume that according to Coh theory t is more coherent than t', then for every $x \in (0, 0.2)$ the condition in Definition 4.1 is violated. Alternatively, stipulating t to be less coherent than t', every value $x \in (0.2, 1]$ again falsifies the conditions of Definition 4.1. Thus, Coh is not EU-conducive. Consequently, since no constraints were required for Coh, the result generalizes: there cannot be a probabilistic coherence measure that is conducive to epistemic utility in the sense of Definition 4.1.

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[23] Note that a constant inverse likelihood ratio is also stipulated in Bovens and Hartmann (2003) model. Furthermore, the likelihood-ratio is provably equivalent to the Bayes factor which is a popular measure of evidence in Bayesian statistics (cf. Kass and Raftery 1995). Furthermore, the results are independent of the choice of $\bar{x}$. The only condition is that $\bar{x}$ is a continuous and strictly decreasing function of x.

[24] The following example is due to Bovens and Hartmann (2003, p. 20). However, there are many more weight vectors featuring differences in a_0 that nonetheless lead to similar negative results.

5 Truth-Conduciveness on the Level of Particular Beliefs

A common feature of all approaches that have been considered so far is that truth-conduciveness is explicated on the level of sets (of beliefs, propositions, etc.). However, Merricks (1995) suggests that "truth-conduciveness should be evaluated – not on the level of systems or sets of beliefs – but on the level of particular beliefs" (1995, p. 307). To motivate his idea, Merricks re-examines Klein and Warfield's Dunnit-example. Let "S" denote the initial set of testimonies and "S^*" the extended set containing additionally the information that Dunnit has an identical twin, then Merricks claims that

> [t]he important question is whether any *particular belief* is less likely to be true when part of the more coherent S^* than when part of the less coherent S. (1995, p. 309, emphasis Merricks', notation adapted)

As Merricks notes, it seems that this question must be answered in the negative with respect to the Dunnit-example. Just to mention an example, consider the probability of the proposition that Dunnit committed the murder: while this proposition is highly unlikely to be true given the other pieces of evidence contained in the initial set S, its posterior probability is raised considerably by adding the testimony that Dunnit had an identical twin whom the witness providing the alibi mistook for Dunnit (cf. Merricks 1995, p. 309). In order to examine the general validity of Merricks' suggestion concerning the truth-conduciveness of particular beliefs, we have to formalize what it means that a particular belief is less likely to be true when

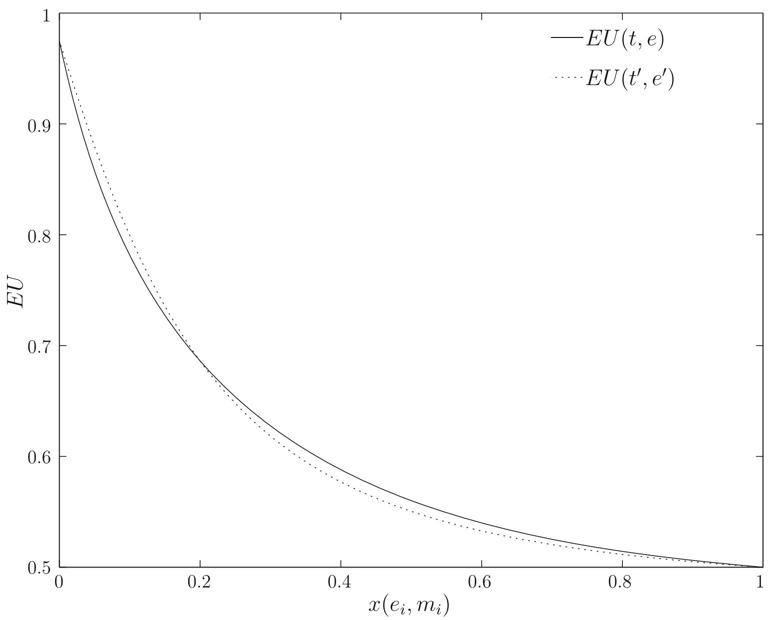

Fig. 1 Epistemic utilities of two theories t and t'

part of a more coherent set than when part of a less coherent set. A first attempt at a formal rendition is the following:

Definition 5.1 Coherence is *truth-conducive on the level of particular beliefs* iff for all non-disjoint sets S, S': if S is more coherent than S', then $\Pr(A| \bigwedge_{B \in S \setminus \{A\}} B) > \Pr(A| \bigwedge_{B' \in S' \setminus \{A\}} B')$ for all $A \in S \cap S'$.

That is, coherence is truth-conducive in the sense of Definition 5.1 if an increase in coherence from S to S' is accompanied by an increase in the probability of all propositions in the sets' intersection conditional on the remainder of the set. Accordingly, a coherence measure **Coh** is truth-conducive on the level of particular beliefs if it satisfies Definition 5.1. Unfortunately, it turns out that none of the extant coherence measures is truth-conducive in this sense.

Observation 5.1 None of the extant probabilistic measures of coherence is truth-conducive in the sense of Definition 5.1.

So it seems, again, that this is bad news for probabilistic approaches to coherence: even though we moved from truth-conduciveness on the level of sets of beliefs to truth-conduciveness on the level of particular beliefs, there is still no sign of recovery. However, there are two issues to be considered: for one, regarding probabilistic coherence measures there is disagreement as regards the right weighting schema for how to aggregate the multiplicity of confirmation relations into a unique value that represents the coherence of the whole set. We will dwell upon this point in more detail below. For another, Definition 5.1 is not the only possible explication of Merricks' suggestion on truth-conduciveness: as was outlined above, Merricks focuses on pairs of sets such that one is a superset of the other. This is indeed more akin to Klein and Warfield's initial proposal. Instead of focusing on an arbitrary pair of sets such that one is more coherent than the other, their discussion, too, takes into account sets of propositions and their supersets. More precisely, Klein and Warfield (1994, p. 130) consider the following two strategies to render a set of beliefs more coherent:

The Subtraction Strategy	In order to render a set of beliefs more coherent, one can subtract one or more "troublesome" beliefs from it.
The Addition Strategy	In order to render a set of beliefs more coherent, one can add one or more beliefs to it.

Thus, by subtracting or adding beliefs one can for example relieve the tension between elements of a set and render it thereby more coherent. In what follows, we limit considerations to adding a single belief.[25] Another question that needs to be addressed is how to couch formally the phrase of "rendering a set of propositions more coherent". One possibility, the one that is in the spirit of the preceding sections, is the following: a belief A renders S more coherent if and only if the superset $S \cup \{A\}$ is more coherent than S itself. This leads to the following definition of truth-conduciveness:

[25] However, *mutatis mutandis*, these considerations can analogously be extended to the case of subtracting/adding any finite number of beliefs.

Definition 5.2 Coherence is *truth-conducive on the level of particular beliefs* iff for all sets S: if $S \cup \{A\}$ is more coherent than S, then $\Pr(x|\bigwedge_{B\in S} B \wedge A) > \Pr(x|\bigwedge_{B\in S} B)$ for all $x \in S$.

Observation 5.2 None of the extant probabilistic measures of coherence is truth-conducive in the sense of Definition 5.2.[26]

However, what is meant by the Addition Strategy seems to be something different. Given that Klein and Warfield consider the Dunnit example "a typical example in which the Addition Strategy would be employed" (p. 131), it seems that what is meant is more like the following: if adding a belief renders the set of beliefs S more coherent, than this comparison involves only *one* belief set, viz. S, whose coherence is assessed with respect to different backgrounds, where the second results from the first by adding the information about A. Thus, the comparison involves something like conditioning on new information rather than set-theoretically adding new information. Applied to the Dunnit-example, we might say that S is incoherent insofar as it is highly unlikely that Dunnit committed the crime given the alibi. On the other hand, adding the information about his twin-brother relieves this tension and renders *the very same set S* more coherent. Or, to put it another way, in the light of the information about his twin-brother, S is not incoherent anymore. Accordingly, another formal rendition of Merricks' suggestion is the following:

Definition 5.3 Coherence is *truth-conducive on the level of particular beliefs* iff for all sets S: if S's degree of coherence is increased in the light of A, then $\Pr(x|\bigwedge_{B\in S\setminus\{x\}} B \wedge A) > \Pr(x|\bigwedge_{B\in S\setminus\{x\}} B)$ for all $x \in S$.

Now we turn to the issue of how to aggregate the determinants of each coherence measure into a unique coherence value. In the literature, the following suggestions can be found: while Shogenji as well as the Glass and Olsson only consider n-wise coherence, Fitelson and Douven and Meijs consider straight averages of confirmation measures, and Schupbach (2011) provides two further alternative weighting systems. Whereas one of them assigns higher weights to subsets of *lower* cardinality, the other one assigns the more weight the *higher* the cardinality of the subset under consideration. So far there is no principled reason for preferring any particular weighting system whatsoever. It might even be preferable to have a contextual factor built into the measures so that depending on the context different weighting strategies come into play. However, here we dispense with a detailed investigation of these issues. Instead we focus on an approach that is immune to the preferred weighting schema. Thus, we look for a robust account of the relation of "... being more coherent than ...". Note that this account can only meaningfully be applied when comparing sets of equal cardinality. This demand is satisfied in the present context where one and the same set is assessed with respect to different backgrounds. It is easy to see that the most robust account requires Definition 5.3 to be satisfied for *all* weighting systems assigning non-negative weights to each

[26] There is a small caveat: $\mathcal{O}$ trivially satisfies Definition 5.2, but this is only due to the fact that according to $\mathcal{O}$ it is impossible to increase coherence by adding any proposition whatsoever (proof omitted).

determinant such that the weights sum to 1. As can easily be shown, since all but one weight is allowed to equal zero, there is an equivalent requirement based on the following quasi-orderings:

Definition 5.4 Let S be a set of propositions, then S is more coherent given A according to the refined deviation measure of coherence iff $\mathcal{D}^*(S^*|A) > \mathcal{D}^*(S^*)$ for all $S^* \subseteq S$.

Definition 5.5 Let S be a set of propositions, then S is more coherent given A according to the refined overlap measure of coherence iff $\mathcal{O}^*(S^*|A) > \mathcal{O}^*(S^*)$ for all $S^* \subseteq S$.

Definition 5.6 Let S be a set of propositions, then S is more coherent according to $\mathcal{C}_\xi$ given A iff $\xi(S', S''|A) > \xi(S', S'')$ for all $(S', S'') \in [S]$.

On the one hand, it is straightforward to show that if S_1 is more coherent than S_2 in the sense of these definitions, then for any coherence measure, S_1 is assigned a higher degree of coherence than S_2 irrespective of the chosen weighting system. On the other hand, these definitions only induce quasi-orderings on propositional sets: any pair of sets S_1, S_2 such that some of the relevant determinants of coherence mentioned in the definitions are higher for S_1, while others are higher for S_2, is not ordered. Hence, the relation of "... being more coherent than ..." induced by these definitions is *not* complete.

Definition 5.3 requires furthermore to spell out the degree of coherence of some set S "in the light of" some other proposition A. We do this by means of the well-known idea of (strict) conditionalization. Thus, the degree of coherence of S in the light of A, $\mathbf{Coh}(S|A)$ is quantified by replacing all unconditional probabilities $\Pr(x)$ involved in assessing S's coherence by conditional probabilities $\Pr(x|A)$, and by replacing all conditional probabilities $\Pr(x|y)$ involved in assessing S's by $\Pr(x|y \wedge A)$. Confirmation measures are adapted likewise.

As the following observation reveals, even weakening coherence measures to quasi-orderings does not allow to establish the connection between coherence and truth for the vast majority of measures:

Observation 5.3 The partial orderings induced by Definitions 5.4 and 5.5 are not truth-conducive in the sense of Definition 5.3. The partial orderings induced by Definition 5.6 are only truth-conducive for $\xi = f$.

Thus, only the coherence quasi-ordering induced by f is truth-conducive in this sense. This, however, should come as no surprise given that f is based solely on the relevant conditional probabilities.[27]

[27] Recently, Shogenji (2013) proposed to change the focus from questions of truth-conduciveness of coherence to the question whether coherence boosts the transmission of support from a set of pieces of evidence to a hypothesis. Basically, he argues that it can be shown that the *less* coherent a set of pieces of evidence is, the *higher* the support it transmits to the hypothesis under consideration, given a suitable amount of ceteris paribus conditions. However, his result suffers from some limitations: (i) all that the proof shows is that we can easily change (conditional and unconditional) probabilities of conjunctions to terms involving Shogenji's coherence measure $\mathcal{D}$, viz., by replacing $\Pr(\bigwedge_{i \leq n} A_i)$ by $\prod_{i \leq n} \Pr(A_i) \cdot \mathcal{D}(A_1, \ldots, A_n)$. The same applies to conditional probabilities like $\Pr(\bigwedge_{i \leq n} A_i|B)$; this,

6 A Possibility Result?

Another shift in the debate on the merits of a coherentist position in epistemology is due to Wheeler and Scheines (2013). In their eyes, the main ingredients of a coherentist position are that "coherent beliefs are more likely to be true than incoherent beliefs, and that coherent evidence *provides more confirmation* of a hypothesis when the evidence is made coherent by the explanation provided by that hypothesis (p. 135, my italics). It is this second part in particular that their analysis focuses on. More precisely, they show that "ceteris paribus, it is not the coherence of the evidence that boosts confirmation, but rather the ratio of the coherence of the evidence to the coherence of the evidence conditional on a hypothesis" (p. 135).

The concept of *focused correlation* (Wheeler 2009) that Wheeler and Scheines utilize in order to model the difference between the degree of coherence of the evidence as such and the corresponding degree of coherence conditional on the hypothesis is identical to the ratio between $\mathcal{D}(S)$ and $\mathcal{D}(S|A)$ as introduced above. In the light of our critical remarks on the adequacy of Shogenji's measure as a measure of coherence, we think that it is worth exploring the robustness of Wheeler and Scheines' results as regards other extant coherence measures. This will be the topic of the current section. Accordingly, our generalized concept of *focused coherence* reads as follows: let $S \cup \{A\}$ be a set of propositions, then S's degree of coherence focused on A as measured by **Coh**, i.e. $\mathbf{Coh}(S, A)$, is given by the following equation[28]:

Footnote 27 continued
however, does not show that the degree of coherence as measured by $\mathcal{D}$ should be considered to have some impact on whatever quantity we measure, because the adequacy of $\mathcal{D}$ as a measure of coherence is not beyond reasonable doubt (cf. Schippers 2014c; Siebel 2005; Siebel and Wolff 2008). Wheeler (2009), instead, proposes to interpret $\mathcal{D}$ as a measure of correlation, and recently Brössel (2015) highlights the fact that $\mathcal{D}$ has been proposed by Keynes (1921) as a coefficient of dependence. Accordingly, some more argumentation seems wanting to conclude from the simple replacement of probabilistic terms to the impact of coherence. (ii) One of Shogenji's arguments for why coherence has a negative impact on the transmission of support is based on the following equation:

$$r(H, E_1 \wedge \ldots \wedge E_n) = \prod_{i \leq n} r(E_i, H) \cdot \frac{\mathcal{D}(E_1, \ldots, E_n | H)}{\mathcal{D}(E_1, \ldots, E_n)}$$

This equation is supposed to show that "other things being equal, the more coherent the pieces of evidence $E_1, \ldots, E_n$ are, the *less* probabilistic support H receives from $E_1, \ldots, E_n$" (Shogenji 2013, p. 2532, emphasis Shogenji's). However, Shogenji's result is limited to confirmation measures satisfying a number of "minimum requirements" that are not shared by all extant confirmation measures. Furthermore, in anticipation of the Sect. 6 the above formula can also be interpreted as saying that the higher the degree of focused coherence, the higher the transmission of support (see Sect. 6). All in all, I think that Shogenji's argument deserves a detailed investigation that is, unfortunately, beyond the scope of the present paper.

[28] Note that for some of the above coherence measures a *difference* between $\mathbf{Coh}(S|A)$ and $\mathbf{Coh}(S)$ might seem more appropriate. More generally, every function that is strictly monotonically increasing in $\mathbf{Coh}(S|A)$ and decreasing in $\mathbf{Coh}(S)$ could be chosen. We leave this issues for future research.

$$\mathbf{Coh}(S, A) = \frac{\mathbf{Coh}(S|A)}{\mathbf{Coh}(S)}$$

Given this formal rendition of focused coherence, the question that Wheeler and Scheines try to answer can now be stated as follows: let $\mathbf{E}$ and $\mathbf{E}'$ be two evidence sets for a hypothesis H such that $\mathbf{Coh}(\mathbf{E}, H) > \mathbf{Coh}(\mathbf{E}', H)$, then what (if any) are the conditions such that this already entails $\xi(H, \mathbf{E}) > \xi(H, \mathbf{E}')$ for some incremental confirmation measure ξ?

As regards ceteris paribus conditions, Wheeler and Scheines distinguish the following types of evidence sets:

- $\mathbf{E}$ is a *positive evidence set* for H if $\Pr(H|E_i) > \Pr(H) > \Pr(H|\neg E_i)$ for all $E_i \in \mathbf{E}$.
- $\mathbf{E}$ is an *equal evidence set* for H if $\Pr(H|E_i) = \Pr(H|E_j)$ and $\Pr(H|\neg E_i) = \Pr(H|\neg E_j)$ for all $E_i, E_j \in \mathbf{E}$.

That is, positive evidence sets for a hypothesis H are sets whose members are all probabilistically relevant to H in the sense that H's unconditional probability lies strictly between its conditional probabilities given any of the set's members on the one hand and its negation on the other. On the other hand, if all conditional probabilities of H given any of $\mathbf{E}$'s members (or their negations) are identical, then $\mathbf{E}$ is an equal evidence set. Furthermore, $\mathbf{E}$ is called an *equal positive evidence set* for $\mathbf{E}$ if $\mathbf{E}$ is *both* a positive evidence set *and* and an equal evidence set. Given these characterizations, Wheeler and Scheines prove, among others, the following observation.

Observation 6.1 If $\mathbf{E}$ is a positive evidence set for H and $\mathcal{D}(\mathbf{E}, H) > 1$, then $\xi(H, \mathbf{E}) > \theta$ for $\xi \in \{r, l, k\}$.[29]

It is straightforward to show that Wheeler and Scheines' result can be generalized to other incremental confirmation measures[30]:

Observation 6.2 If $\mathbf{E}$ is a positive evidence set for H and $\mathcal{D}(\mathbf{E}, H) > 1$, then $\xi(H, \mathbf{E}) > \theta$ for all incremental confirmation measures ξ.

This does, indeed, look like good evidence for the connection between coherence and confirmation. Wheeler and Scheines go on to prove another observation for a slightly more restricted class of evidence sets. This observation reads as follows:

Observation 6.3 If $\mathbf{E} = \{E_1, E_2\}$ and $\mathbf{E}' = \{E_1, E_3\}$ are sets such that $\mathbf{E} \cup \mathbf{E}'$ is an equal positive evidence set, then $\mathcal{D}(\mathbf{E}, H) > \mathcal{D}(\mathbf{E}', H)$ if and only if $\xi(H, \mathbf{E}) > \xi(H, \mathbf{E}')$ for $\xi \in \{r, l, k\}$.

Again, it is easy to show that this observation can be generalized; however, this time the mentioned property does not hold for *all* incremental confirmation

[29] More precisely, Wheeler and Scheines consider six confirmation measures among which are r, l and k.

[30] See footnote 4.

measures but only for those satisfying the following property of *final probability incrementality* (cf. Crupi et al. 2013):

(FPI) If $\Pr(H|E) > \Pr(H|E')$, then $\xi(H,E) > \xi(H,E')$.

(FPI) is widely recognized as a highly plausible condition for confirmation measures; Eells and Fitelson (2000) even observe that "it is not an exaggeration to say that most Bayesian confirmation theorists would accept (**FPI**) as a desideratum for Bayesian measures of confirmation" (p. 670). Accordingly, it is a virtue of Wheeler and Scheines' model that we can prove the following generalization of the latter observation:

Observation 6.4 If $\mathbf{E} = \{E_1, E_2\}$ and $\mathbf{E}' = \{E_1, E_3\}$ are sets such that $\mathbf{E} \cup \mathbf{E}'$ is an equal positive evidence set, then $\mathcal{D}(\mathbf{E}, H) > \mathcal{D}(\mathbf{E}', H)$ if and only if $\xi(H, \mathbf{E}) > \xi(H, \mathbf{E}')$ for all measures ξ satisfying (FPI).

On the other hand, this observation indicates where the limits of Wheeler and Scheines' observation might be: what about measures that violate (FPI)? In this regard, we get the following result:

Observation 6.5 Even if $\mathbf{E} = \{E_1, E_2\}$ and $\mathbf{E}' = \{E_1, E_3\}$ are sets such that $\mathbf{E} \cup \mathbf{E}'$ is an equal positive evidence set, then there are cases such that $\mathcal{D}(\mathbf{E}, H) > \mathcal{D}(\mathbf{E}', H)$ and $\xi(H, \mathbf{E}) < \xi(H, \mathbf{E}')$ for $\xi \in \{n, s, m\}$.

Hence, Observation 6.4 is robust in the following sense: it holds no matter which incremental confirmation measure is chosen as long as the chosen measure satisfies (FPI) and these measures are by far the most prominent measures. Another question that naturally arises in this context is whether it also holds for all other measures of focused coherence. That this is not the case is shown by the following observation:

Observation 6.6 If $\mathbf{E} = \{E_1, E_2\}$ and $\mathbf{E}' = \{E_1, E_3\}$ are sets such that $\mathbf{E} \cup \mathbf{E}'$ is an equal positive evidence set, then there are cases such that $\mathcal{O}^*(\mathbf{E}, H) > \mathcal{O}^*(\mathbf{E}', H)$ and $\xi(H, \mathbf{E}) < \xi(H, \mathbf{E}')$ (even for measures satisfying (FPI)).

On the other hand, there are coherence measures other than the one chosen by Wheeler and Scheines for which Observation 6.4 holds true.

Observation 6.7 If $\mathbf{E} = \{E_1, E_2\}$ and $\mathbf{E}' = \{E_1, E_3\}$ are sets such that $\mathbf{E} \cup \mathbf{E}'$ is an equal positive evidence set, then $\mathcal{C}_f(\mathbf{E}, H) > \mathcal{C}_f(\mathbf{E}', H)$ if and only if $\xi(H, \mathbf{E}) > \xi(H, \mathbf{E}')$ for all measures ξ satisfying (FPI).

The proof of this observation utilizes the following lemma:

Lemma 6.1 *If* $\mathbf{E} = \{E_1, E_2\}$ *and* $\mathbf{E}' = \{E_1, E_3\}$ *are sets such that* $\mathbf{E} \cup \mathbf{E}'$ *is an equal positive evidence set, then* $Pr(E_i) = \Pr(E_j)$ *and* $\Pr(E_i \wedge \pm H) = \Pr(E_j \wedge \pm H)$ *for all* $1 \leq i, j \leq 3$.

Thus, although Observation 6.4 does not hold for *all* extant coherence measures when replacing $\mathcal{D}$ by any of these, it holds for at least one other measure (and might also turn out to be valid for other measures). But note that Observation 6.4 only applies to a very restricted class of evidence sets. Wheeler and Scheines also discuss how to relax these conditions; however, an in-depth investigation into various different classes of evidential sets is beyond the scope of the present paper. Nevertheless, in the light of these aforementioned observations we conclude that (i) Wheeler and Scheines' proposal for switching the focus from truth-conduciveness to confirmation-conduciveness seems very promising; on the other hand, (ii) more needs to be done in order to establish a relationship between *coherence* and confirmation along these lines in full detail. We leave these investigations for future research.[31]

7 Conclusion and Outlook

This paper contributes to the debate on the truth-conduciveness of coherence in various respects. Different arguments taken from the literature were re-evaluated in the light of recent developments; nonetheless, in the vast majority of considered cases, the conclusions remained largely skeptical as regards the prospects of a Bayesian coherentist position that is conducive to truth. However, the focus within this paper was the search for a deductive link between coherence and (a high likelihood of) truth. Even in the light of our negative findings, there my nonetheless be an *inductive* link in the sense that in the majority of probability distributions an increase in coherence is accompanied by an increase in likelihood of truth (either on the level of systems of beliefs or on the level of particular beliefs). We leave these questions for future research.

On the other hand, the negative findings may be due to a misconception of the idea of truth-conduciveness of coherence. The common assumption of all approaches to truth-conduciveness that have been considered above is that coherence is only taken into account as a static feature of the belief set. In contrast, what BonJour (1985) emphasized is that "the force of a coherentist justification depends ultimately on the fact that the systems of beliefs in question is not only coherent at a moment (a result which could be achieved by arbitrary fiat), but remains coherent in the long run. It is only such long-run coherence which provides compelling reason for thinking that the beliefs of the system are likely to be true" (p. 153). In this respect, Cross (1999) stresses that "long-run coherence is *not* a matter of how well a single set of propositions hang together: it is a matter of whether a sufficiently high degree of hanging-together is preserved across times in the belief history of an actual agent" (p. 187, italics in the original). Reexamining Klein and Warfield's argument against the background of this long-run perspective on coherentism, Cross concludes that "the truth

[31] We dispense with a discussion of Wheeler and Scheines' interesting ideas on coherence and causal structure. Although they provide us with very stimulating observations, all of them rest on interpreting $\mathcal{D}$ as a measure of coherence. Nonetheless, we grant that Wheeler and Scheines highlight a number of interesting connections between confirmation, causal structure and *correlation*, which is what $\mathcal{D}$ seems to measure. An in-depth analysis of these further results, however, must be postponed to another paper and can not be the focus of the present paper, which is solely concerned with probabilistic measures of coherence.

conduciveness of [coherentist] justification on BonJour's theory is not refuted by the Dunnit example or, in general, by the fact that a coherent set of beliefs will often turn out to be more likely to contain a falsehood than some of its less coherent subsets. Since this latter fact does not constitute a reason to reject BonJour's theory, it does not constitute a reason to reject the very idea of a coherence theory of justification" (p. 193). Likewise, it seems that we must conclude that a long-run perspective on coherentism might not be affected by any of the above negative results. Given that so far no one seems to have seriously engaged in this project, there is still some work to be done.

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Appendix 1: Proof of Observation 3.1

Consider the following probability distributions, where $x = 1 - \Pr(A_1 \vee A_2 \vee A_3)$:

A_1	A_2	A_3	Probability	A_1	A_2	A_3	Probability
T	T	T	1/5	F	T	T	11/65
T	T	F	2/41	F	T	F	7/44
T	F	T	3/37	F	F	T	1/18
T	F	F	$\frac{122786261713}{1248935336885}$	F	F	F	x

A'_1	A'_2	A'_3	Probability	A'_1	A'_2	A'_3	Probability
T	T	T	34/171	F	T	T	13/71
T	T	F	2/13	F	T	F	1/52
T	F	T	1/25	F	F	T	3/20
T	F	F	$\frac{15124126187369}{13521705438129525}$	F	F	F	x'

The table only gives a small sample of the complete distribution over six variables. However, note that $\Pr(\bigwedge_{i\leq 3} A_i) = 0.200 > 0.199 \approx \Pr(\bigwedge_{i\leq 3} A'_i)$ and $\prod_{i\leq 3}\Pr(A_i) = \prod_{i\leq 3}\Pr(A'_i) = 0.125$. Straightforward calculations yield the following results:

	$\mathcal{O}$	$\mathcal{D}^*$	$\mathcal{O}^*$	$\mathcal{C}_d$	$\mathcal{C}_r$	$\mathcal{C}_s$	$\mathcal{C}_l$	$\mathcal{C}_k$	$\mathcal{C}_z$	$\mathcal{C}_f$
$\{A_1, A_2, A_3\}$	0.246	1.292	0.381	0.119	1.276	0.212	1.792	0.246	0.228	0.572
$\{A'_1, A'_2, A'_3\}$	0.266	1.367	0.425	0.125	1.281	0.230	2.176	0.263	0.247	0.587

All mentioned measures assign a *lower* degree of coherence to the set $\{A_1, A_2, A_3\}$ which is in conflict with the requirements of Definition 3.2.

Appendix 2: Proof of Observation 5.1

For the vast majority of measures it is possible to show that Definition 5.1 is already violated for pairs of propositions. For these measures consider two sets of propositions $\{A_1, A_2\}$ and $\{A_1, A_3\}$ and the following probability distribution, where $x = 1 - \Pr(A_1 \vee A_2 \vee A_3)$:

A_1	A_2	A_3	Probability	A_1	A_2	A_3	Probability
T	T	T	7/62	F	T	T	5/51
T	T	F	14/65	F	T	F	5/52
T	F	T	6/73	F	F	T	1/182
T	F	F	1/220	F	F	F	x

The coherence values for the relevant measures are given in the following table.

	$\mathcal{O}$	$\mathcal{D}^*$	$\mathcal{O}^*$	$\mathcal{C}_d$	$\mathcal{C}_s$	$\mathcal{C}_l$	$\mathcal{C}_k$	$\mathcal{C}_z$	$\mathcal{C}_f$
$\{A_1, A_2\}$	0.539	1.574	0.539	0.241	0.453	2.921	0.480	0.463	0.710
$\{A_1, A_3\}$	0.376	1.514	0.376	0.205	0.316	2.370	0.402	0.326	0.562

Given that $\Pr(A_1|A_2) \approx 0.628 < 0.653 \approx \Pr(A_1|A_3)$, all these measures are *not* truth-conducive in the sense of Definition 5.1. Note that in order to prove the analogous result for the two missing coherence measures $\mathcal{D}$ and $\mathcal{C}_r$, we have to consider at least one set of propositions with more than two elements. This is because for the considered pairs of sets, $\Pr(A_1|A_2) < \Pr(A_1|A_3)$ already entails $\mathcal{D}(A_1, A_2) < \mathcal{D}(A_1, A_3)$ and the same holds for $\mathcal{C}_r$.

Therefore, let $S = \{A_1, A_2\}$ and $S' = \{A_1, A_2, A_3\}$ and consider the following probability distribution, where $x = 1 - \Pr(A_1 \vee A_2 \vee A_3)$:

A_1	A_2	A_3	Probability	A_1	A_2	A_3	Probability
T	T	T	12/49	F	T	T	12/37
T	T	F	2/25	F	T	F	3/44
T	F	T	9/73	F	F	T	1/72
T	F	F	6/53	F	F	F	x

Against the background of this distribution we get the desired result that even though A_1's posterior probability given A_2 (approx. 0.453) exceeds its posterior probability given both A_2 and A_3 (approx. 0.430), we have

$$\mathcal{D}(S) \approx 0.807 < 0.861 \approx \mathcal{D}(S'')$$
$$\mathcal{C}_r(S) \approx 0.807 < 0.936 \approx \mathcal{C}_r(S'')$$

Hence, these measures are *not* truth-conducive in the sense of Definition 5.1, too.

Appendix 3: Proof of Observation 5.2

Consider again the former probability distribution:

A_1	A_2	A_3	Probability	A_1	A_2	A_3	Probability
T	T	T	12/49	F	T	T	12/37
T	T	F	2/25	F	T	F	3/44
T	F	T	9/73	F	F	T	1/72
T	F	F	6/53	F	F	F	x

We can easily extend the calculated coherence values to all considered measures. The following table contains additional values for all measures but $\mathcal{O}$.

	$\mathcal{D}^*$	$\mathcal{O}^*$	$\mathcal{C}_d$	$\mathcal{C}_s$	$\mathcal{C}_l$	$\mathcal{C}_k$	$\mathcal{C}_z$	$\mathcal{C}_f$
$\{A_1, A_2\}$	0.807	0.341	−0.124	−0.350	0.594	−0.256	−0.193	0.516
$\{A_1, A_2, A_3\}$	0.930	0.417	−0.036	−0.082	0.940	−0.064	−0.028	0.565

As the table shows, these measures agree with $\mathcal{D}$ and $\mathcal{C}_r$ in that the extended set $\{A_1, A_2, A_3\}$ is more coherent than its subset $\{A_1, A_2\}$. Taking into account that nonetheless A_1' posterior probability is *lower* for this extended set, this result shows that all considered measures (except $\mathcal{O}$) are not truth-conducive in the sense of Definition 5.2.

Appendix 4: Proof of Observation 5.3

First of all, we can utilize the following probability distribution in order to show that the orderings induced by Definition 5.6 are not truth-conducive for all confirmation measures but *f*:

A_1	A_2	A_3	Probability	A_1	A_2	A_3	Probability
T	T	T	1/30	F	T	T	5/44
T	T	F	4/59	F	T	F	10/61
T	F	T	1/153	F	F	T	53/105
T	F	F	6/55	F	F	F	x

The confirmation values for the relevant measures that are currently of interest are given in the following table:

Confirmation	d	r	s	l	k	z
$\xi(A_1, A_2)$	0.050	1.232	0.081	1.317	0.137	0.064
$\xi(A_1, A_2 \vert A_3)$	0.166	3.745	0.214	4.550	0.640	0.177
$\xi(A_2, A_1)$	0.088	1.232	0.112	1.435	0.179	0.141
$\xi(A_2, A_1 \vert A_3)$	0.613	3.745	0.652	17.743	0.893	0.789

As the table indicates, all considered confirmation measures agree in that the there is a larger degree of confirmation between A_1 and A_2 when A_3 is taken for granted. This, however, is in sharp contrast with the relevant conditional probabilities: as was mentioned before, A_1's conditional probability given A_2 *exceeds* its conditional probability given A_2 *and* A_3. Accordingly, these measures are not truth-conducive in the sense of Definition 5.3.

The latter distribution also suffices to show that the refined deviation measure $\mathcal{D}^*$ is not truth-conducive in this sense. This is due to the fact that

$$\mathcal{D}^*(A_1, A_2) \approx 1.232 < 3.745 \approx \mathcal{D}^*(A_1, A_2 | A_3)$$

To show that the refined overlap measure $\mathcal{O}^*$ is not truth-conducive in the sense of Definition 5.3 we utilize the following distribution involving four propositions with $x = 1 - \Pr(A_1 \vee A_2 \vee A_3 \vee A_4)$:

A_1	A_2	A_3	A_4	Probability	A_1	A_2	A_3	A_4	Probability
T	T	T	T	1/25	F	T	T	T	1/43
T	T	T	F	1/23	F	T	T	F	1/36
T	T	F	T	3/56	F	T	F	T	1/44
T	T	F	F	5/69	F	T	F	F	2/39
T	F	T	T	4/39	F	F	T	T	1/59
T	F	T	F	1/57	F	F	T	F	6/71
T	F	F	T	1/56	F	F	F	T	2/53
T	F	F	F	27/94	F	F	F	F	x

According to the refined overlap measure all non-singleton subsets of $\{A_2, A_3, A_4\}$ are assigned a higher degree of coherence conditional on A_1. However, A_2's conditional probability given A_1, A_3 and A_4 is *lower* than its conditional probability given only A_3 and A_4. Hence, $\mathcal{O}^*$ also violates Definition 5.3.

Now we turn to the remaining confirmation measure f. In order to show that this measure is truth-conducive in the sense of Definition 5.6, note that for each pair $(S', S'') \in [S]$ the following claim holds by definition:

($\dagger_f$) If $f(S', S''|A) > f(S', S'')$, then $\Pr(S'|S'', A) > \Pr(S'|S'')$.

Hence, let $S' = \{x\}$ for some $x \in S$ and $S'' = S \setminus \{x\}$, then the fact that $f(S', S''|A) > f(S', S'')$ by definition together with ($\dagger_f$) entails the desired claim.

Appendix 5: Proof of Observation 6.2

If $\mathcal{D}(\mathbf{E}, H) > 1$ for some set $\mathbf{E} = \{E_1, \ldots, E_n\}$, then we get the following derivation:

$$\begin{aligned}\mathcal{D}(\mathbf{E}, H) > 1 &\Rightarrow \frac{\Pr(H|E_1, \ldots, E_n)\Pr(H)^{n-1}}{\Pr(H|E_1)\cdot \ldots \cdot \Pr(H|E_n)} > 1 \\ &\Rightarrow \Pr(H|E_1, \ldots, E_n) > \underbrace{\frac{\Pr(H|E_1)}{\Pr(H)}}_{>1} \cdot \ldots \cdot \underbrace{\frac{\Pr(H|E_{n-1})}{\Pr(H)}}_{>1} \cdot \Pr(H|E_n) \\ &\Rightarrow \Pr(H|E_1, \ldots, E_n) > \Pr(H|E_n) \\ &\Rightarrow \Pr(H|E_1, \ldots, E_n) > \Pr(H)\end{aligned}$$

This latter fact means that $\xi(H, \mathbf{E}) > \theta$ for all relevance-sensitive ξ. □

Appendix 6: Proof of Observation 6.4

Keeping in mind that by assumption $\Pr(H|E_2) = \Pr(H|E_3)$, we get

$$\begin{aligned}\mathcal{D}(\mathbf{E}, H) > \mathcal{D}(\mathbf{E}', H) &\Leftrightarrow \frac{\Pr(H|E_1, E_2)\cdot \Pr(H)}{\Pr(H|E_1)\Pr(H|E_2)} > \frac{\Pr(H|E_1, E_3)\cdot \Pr(H)}{\Pr(H|E_1)\Pr(H|E_3)} \\ &\Leftrightarrow \Pr(H|E_1, E_2) > \Pr(H|E_1, E_3)\end{aligned}$$

Thus, if ξ satisfies (FPI), then $\Pr(H|E_1, E_2) > \Pr(H|E_1, E_3)$ entails that $\xi(H, \mathbf{E}) > \xi(H, \mathbf{E}')$. □

Appendix 7: Proof of Observation 6.5

H	E_1	E_2	E_2	Probability	H	E_1	E_2	E_2	Probability
T	T	T	T	1/495	F	T	T	T	1/33
T	T	T	F	3/46	F	T	T	F	1/47
T	T	F	T	6/41	F	T	F	T	1/11
T	T	F	F	1/41	F	T	F	F	401/182172
T	F	T	T	1663/39606	F	F	T	T	1/68
T	F	T	F	5099/39606	F	F	T	F	4621/58938
T	F	F	T	1/21	F	F	F	T	1/114
T	F	F	F	1/177	F	F	F	F	x

Straightforward calculations yield the following results: $\Pr(H|E_1) \approx .622 > .462 \approx \Pr(H) > .363 \approx \Pr(H|\neg E_1)$ and $\Pr(H|E_i) = \Pr(H|E_j)$ as well as $\Pr(H|\neg E_i) = \Pr(H|\neg E_j)$ for all $1 \leq i, j \leq 3$. Hence, $\mathbf{E} \cup \mathbf{E}'$ is an equal positive evidence set for H. Furthermore, $\mathcal{D}(\mathbf{E}, H) \approx .676 > .657 \approx \mathcal{D}(\mathbf{E}', H)$; however, $n(H, \mathbf{E}) \approx .050 < .096 \approx n(H, \mathbf{E}')$, $m(H, \mathbf{E}) \approx .027 < .052 \approx m(H, \mathbf{E}')$ and $s(H, \mathbf{E}) \approx .118 < .121 \approx s(H, \mathbf{E}')$.

Appendix 8: Proof of Observation 6.6

H	E_1	E_2	E_2	Probability	H	E_1	E_2	E_2	Probability
T	T	T	T	1/12	F	T	T	T	1/31
T	T	T	F	1/205	F	T	T	F	1/306
T	T	F	T	1/1406	F	T	F	T	1/651
T	T	F	F	1/150	F	T	F	F	431/12852
T	F	T	T	3293/817950	F	F	T	T	1/918
T	F	T	F	6763/2017610	F	F	T	F	4513/132804
T	F	F	T	1/133	F	F	F	T	1/28
T	F	F	F	20/67	F	F	F	F	x

Given this probability distribution we calculate: $\Pr(H|E_1) \approx .575 > .409 \approx \Pr(H) > .376 \approx \Pr(H|\neg E_1)$ and $\Pr(H|E_i) = \Pr(H|E_j)$ as well as $\Pr(H|\neg E_i) = \Pr(H|\neg E_j)$ for all $1 \leq i, j \leq 3$. Hence, $\mathbf{E} \cup \mathbf{E}'$ is an equal positive evidence set for H. Furthermore, $\mathcal{O}(\mathbf{E}, H) \approx 1.444 > 1.428 \approx \mathcal{O}(\mathbf{E}', H)$; however, $\Pr(H|\mathbf{E}) \approx .7129 < .7132 \approx \Pr(H|\mathbf{E}')$ and therefore all (FPI)-measures will agree in that $\xi(H, \mathbf{E}) < \xi(H, \mathbf{E}')$.

Appendix 9: Proof of Observation 6.7

The proof of Observation 6.7 utilizes Lemma 6.1:

$$
\begin{aligned}
&\mathcal{C}_f(\mathbf{E},H) > \mathcal{C}_f(\mathbf{E}',H)\\
&\qquad \Leftrightarrow \frac{\Pr(E_1|E_2,H)+\Pr(E_2|E_1,H)}{\Pr(E_1|E_2)+\Pr(E_2|E_1)} > \frac{\Pr(E_1|E_3,H)+\Pr(E_3|E_1,H)}{\Pr(E_1|E_3)+\Pr(E_3|E_1)}\\
&\qquad \Leftrightarrow \Pr(H|E_1,E_2)\cdot\frac{\sum_{i=1,2}\Pr(H\wedge E_i)^{-1}}{\sum_{i=1,2}\Pr(E_i)^{-1}} > \Pr(H|E_1,E_3)\cdot\frac{\sum_{j=1,3}\Pr(H\wedge E_j)^{-1}}{\sum_{j=1,3}\Pr(E_j)^{-1}}\\
&\qquad \Leftrightarrow \frac{\Pr(H|E_1,E_2)}{\Pr(H|E_1,E_3)} > \frac{\sum_{i=1,2}\Pr(H\wedge E_i)^{-1}}{\sum_{i=1,2}\Pr(E_i)^{-1}}\cdot\frac{\sum_{j=1,3}\Pr(E_j)^{-1}}{\sum_{j=1,3}\Pr(H\wedge E_j)^{-1}}\\
&\qquad \Leftrightarrow \frac{\Pr(H|E_1,E_2)}{\Pr(H|E_1,E_3)} > 1 \qquad (\text{Lemma } 6.1)
\end{aligned}
$$

This completes the proof of Observation 6.7.

Appendix 10: Proof of Lemma 6.1

If $\mathbf{E}\cup\mathbf{E}'$ is an equal positive evidence set for H, then (i) $\Pr(H|E_i) > \Pr(H) > \Pr(H|\neg E_i)$ and (ii) $\Pr(H|\pm E_i) = \Pr(H|\pm E_j)$ for all $1 \leq i, j \leq 3$. Now we get:

$$
\begin{aligned}
\Pr(H) &= \Pr(H\wedge E_i) + \Pr(H\wedge\neg E_i)\\
&= \Pr(H|E_i)\cdot\Pr(E_i) + \Pr(H|\neg E_i)\cdot\Pr(\neg E_i)\\
&\overset{(ii)}{=} \Pr(H|E_j)\cdot\Pr(E_i) + \Pr(H|\neg E_j)\cdot\Pr(\neg E_i)
\end{aligned}
$$

and also

$$\Pr(H) = \Pr(H|E_j)\cdot\Pr(E_j) + \Pr(H|\neg E_j)\cdot\Pr(\neg E_j)$$

Hence we get

$$\Pr(H|E_j)\cdot\Pr(E_i) + \Pr(H|\neg E_j)\cdot\Pr(\neg E_i) = \Pr(H|E_j)\cdot\Pr(E_j) + \Pr(H|\neg E_j)\cdot\Pr(\neg E_j)$$

and thus

$$\Pr(H|E_j)\cdot(\Pr(E_i) - \Pr(E_j)) = \Pr(H|\neg E_j)\cdot(\Pr(\neg E_j) - \Pr(\neg E_i))$$

from which we conclude that either $\Pr(H|E_i) = \Pr(H|\neg E_i)$ in contradiction to (i) or $\Pr(E_i) = \Pr(E_j)$. With this latter identity and $\Pr(H|E_i) = \Pr(H|E_j)$ we conclude that also $\Pr(H\wedge\pm E_i) = \Pr(H\wedge\pm E_j)$.

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