Erkenn (2008) 68:113–127
DOI 10.1007/s10670-007-9081-y

ORIGINAL ARTICLE

Quine, Putnam, and the 'Quine–Putnam' Indispensability Argument

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Received: 24 October 2006 / Accepted: 3 August 2007 / Published online: 28 September 2007

Abstract Much recent discussion in the philosophy of mathematics has concerned the indispensability argument—an argument which aims to establish the existence of abstract mathematical objects through appealing to the role that mathematics plays in empirical science. The indispensability argument is standardly attributed to W. V. Quine and Hilary Putnam. In this paper, I show that this attribution is mistaken. Quine's argument for the existence of abstract mathematical objects differs from the argument which many philosophers of mathematics ascribe to him. Contrary to appearances, Putnam did not argue for the existence of abstract mathematical objects at all. I close by suggesting that attention to Quine and Putnam's writings reveals some neglected arguments for platonism which may be superior to the indispensability argument.

Much recent debate in the philosophy of mathematics has revolved around the question of whether there are mathematical entities such as numbers, sets, and vector spaces. Platonists assert that there are such entities, and that they are abstract, whereas nominalists deny the existence of abstract mathematical entities.[1] Any argument for platonism is thus an argument against nominalism.

[1] What it is to be abstract has also been a topic of discussion. Perhaps the most popular approach has been to say that an entity is abstract iff it lacks spatio–temporal location and is causally inactive. Nothing I will say here hinges on what abstractness is taken to be; see Hale (1987, chapter 3) and Rosen (2001) for more on the debate.

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One argument for platonism which has recently received extensive discussion is known as the 'Quine–Putnam indispensability argument'. The argument claims that we should posit abstract mathematical entities because of the role mathematics plays in empirical science. The debate over this argument has been a lively one, with supporters of the argument—such as Michael Resnik (1997) and Mark Colyvan (2001)—attempting to repel the counter-arguments launched by Hartry Field (1980, 1989), Geoffrey Hellman (1999), Elliott Sober (1993), Penelope Maddy (1997), and Joseph Melia (2000), among others. The present paper makes no attempt to contribute to this debate. Instead, my perspective will be a scholarly one: I will argue that neither Quine nor Putnam ever used the argument that is usually called the 'Quine–Putnam indispensability argument'. Quine indeed argued for platonism, but his argument (I claim) relies on his distinctive views about how to answer ontological questions, whereas the so-called Quine–Putnam argument does not. Putnam didn't argue for platonism at all: close examination of the passages that are routinely read as arguments for this conclusion reveals that his interest was in the objectivity of mathematics rather than the existence of abstract mathematical objects.

The argument will proceed as follows. The first section examines some recent statements of the argument, to prove that it is commonly attributed to Quine and Putnam. Sections 2 and 3 outline Quine's method of arguing for existential claims and use this to show that Quine did not put forward the 'Quine–Putnam' argument. Finally, Sect. 4 turns to Putnam, arguing that his writings on the philosophy of mathematics have been misread. In the final section, I consider what implications all this holds for contemporary philosophy of mathematics.

1 What is the Indispensability Argument?

There are many different statements of the indispensability argument. To begin with, let us examine two recent formulations. Michael Resnik sees the argument as flowing from three (Quinean) premises:

> *Indispensability*: (a) Mathematical theories are indispensable components of our best scientific theories; (b) referring to mathematical objects and invoking mathematical principles is indispensable to the practice of science.
> *Confirmational Holism*: The evidence for a scientific theory bears directly upon its theoretical apparatus as a whole and not upon its individual hypotheses.
> *Naturalism*: Science is our ultimate arbiter of truth and existence (Resnik 1995: 166).

Resnik states the argument as follows:

> [I]f mathematics is an indispensable component of science, then, by holism, whatever evidence we have for science is also evidence for the mathematical objects and mathematical principles it presupposes. So, by naturalism, mathematics is true, and the existence of mathematical objects is as well grounded as that of the other entities posited by science (Resnik 1995: 166).

Susan Vineberg explains the indispensability argument similarly:

> Both Quine and Putnam have argued that acceptance of our scientific theories as true forces us to accept abstract mathematical entities as well. The reason stems from the fact that mathematics forms an essential part of many of our scientific theories. … For both philosophers, since mathematical theory forms an essential part of our well confirmed scientific theories, we have reason to regard the mathematical theory used in these theories as true as well. Furthermore, they claim that the variables in our mathematical theories range over abstract entities and hence their existence is presupposed by the truth of our scientific theories of which mathematics is a part … Thus, to accept the truth of our scientific theories while denying the existence of abstract mathematical objects is simply to fail to accept the consequences of affirming the truth of our scientific theories (Vineberg 1996: S257).

These quotations suggest the following argument:

(1) We should believe the claims made by well confirmed scientific theories.
(2) Some pure mathematical theories (e.g. arithmetic, analysis) are essential parts of well confirmed scientific theories.
(3) Scientific experiments do not confirm single sentences (‘The Earth’s orbit takes 365 days’), or even single theories (Copernican astronomy, for instance); rather, they confirm theories along with the auxiliary hypotheses that are used to derive the predictions to be tested.
(4) We should believe these pure mathematical theories. (From (2) and (3)).
(5) If these pure mathematical theories are true, then there are abstract mathematical entities.
(6) So we should believe that there are abstract mathematical entities. (From (4) and (5)).

Here, premises (2), (3) and (1) correspond to Resnik’s Indispensability, Confirmational Holism, and Naturalism respectively. Premiss (5) has no counterpart in Resnik’s argument, but needs to be present in order for the argument to show that there are abstract mathematical entities. The only book-length study of the argument, Colyvan 2001, presents it in essentially the same way.

The indispensability argument is very commonly attributed to W. V. Quine and Hilary Putnam. As we saw above, Vineberg credits Quine and Putnam with the argument. Other citations are legion. Putnam 1971 is the *locus classicus* for the argument: citations include Field (1980: 107 n. 4), Horgan (1984: 529), Maddy (1990: 29–30), Shapiro (1997: 46); Melia (2000: 455), Yablo (2000: 197), and Colyvan (2001: 10). Along with many other authors, Horgan (1984: 529), Maddy (1990: 29), Papineau (1993: 191–192), and Colyvan (2001: 10) attribute the argument to Quine.[2] In short, it is very common for philosophers of mathematics to attribute the indispensability argument to Quine or Putnam or both. In the following

[2] A few authors differ: Resnik (1997: 45 n. 3) is unsure whether Quine and Putnam intended the argument they are usually credited with; he produces a quotation from Putnam 1971 which he interprets as expressing a different, ‘pragmatic’ argument for platonism (Resnik 1997: 47 n. 7). And see footnote 5 below.

sections, I will show that this attribution is mistaken, starting with Quine. I begin by outlining Quine's preferred way of arguing for existential claims; then I show that the 'Quine–Putnam' indispensability argument was not put forward by Quine.

2 Quine's Ontological Method

Quine held that it is philosophically fruitful to paraphrase our best scientific theories into a 'canonical notation' (first-order predicate calculus, to be precise). Here the word 'paraphrase' does not mean that the original sentence and the paraphrase have the same meaning, or even the same truth-conditions (as Quine 1960: 159, 242, and 258–259 make clear): Quine avoided this idea, since he was suspicious of the notions of analyticity and synonymy. Neither does Quine propose that scientists should use the formal paraphrases instead of ordinary scientific language: he emphasises that there is no reason for scientists to use formal languages in their day-to-day work (Quine 1960: 161–162). Instead, Quine regards the paraphrases as philosophically valuable because they wear their ontological commitments on their faces. Once scientific theories have been paraphrased into canonical notation, we can use them to discover what exists.

To understand this idea, it will be helpful to say a little more about the notion of ontological commitment. The usual way of explaining it is this:

> A sentence is ontologically committed to an entity iff it entails that the entity exists, that is, if it could not possibly be true unless the entity existed.

So if a theory is true—that is, if all the sentences included in it are true—then its ontological commitments must exist. A theory's ontological commitments are its implicit existential claims.

As Quine pointed out, this way of understanding ontological commitment will not do if we want to find out what *sorts* of things our best theories say exist. For a theory might have no ontological commitments in the sense just introduced, but 'still not tolerate an empty universe either' (1969a: 96). Take the sentence:

> (A) There is a book.

This sentence is not ontologically committed to any entity: for no entity *e* does (A) entail that *e* exists. The existence of any book suffices to make the (A) true; no *specific* book is required. However, (A) does entail that there is *some* book: the sentence will be false if there are no books at all. We ought to be able to say that the sentence is ontologically committed to the existence of books (that is, to the existence of at least one book). The following formulation lets us do this:

> A sentence *s* is ontologically committed to *F*s iff *s* entails that there exists an *F* (cf. Quine 1969a: 93).

Just what existential claims are entailed by sentences of canonical notation? To answer that question, we have to understand how the formal language is interpreted. The predicates are interpreted by using English expressions. For instance, we may say that the predicate 'D' applies to all and only those things that are dogs. For convenience, Quine often adopts the device of using English expressions in place of predicates when writing down paraphrases, and I will follow him in this, writing '∃x x is-a-dog' instead of '∃x Dx'. How about the quantifiers? Stipulating how the formal language is to be interpreted, Quine writes: 'Existence is what existential quantification expresses. There are things of kind F if and only if ∃x Fx' (Quine 1969a: 97, notation slightly altered; cf. Quine 1981c). It follows that:

(OC1) If a sentence *s* of canonical notation entails a sentence of the form '$\exists x Fx$', *s* entails that there are *F*s.

More generally, a sentence of the form '∃x ϕ(x)' (where ϕ(x) is an open sentence) is true just in case there is some entity satisfying ϕ(x). The sentence is said to *quantify over* such entities, and the bound variable x is said to *range over* them. Let us consider the special case where ϕ(x) is of the form 'x = *a*', for some constant *a*. Since '=' signifies identity, '∃x ϕ(x)' will be true just if there is some entity that is identical to *a*—that is, just if *a* exists. So:

(OC2) If a sentence *s* of canonical notation entails a sentence of the form '$\exists x\, x = a$', *s* entails that *a* exists.

By the definition of 'ontological commitment', it follows that any sentence entailing '∃x *F*x' is ontologically committed to *F*s, and any sentence entailing '∃x x = *a*' is committed to *a*. For instance, consider:

(B) ∃x (x is-a-ruler & x = k); and
(C) ∃x x is-a-pencil.

(B) entails '∃x x = k'; (OC2) says that (B) entails that k exists. So (B) is ontologically committed to k. (B) is also ontologically committed to at least one ruler, since it entails '∃x x is-a-ruler'. In the same way, (C) entails the existence of at least one pencil. We can therefore say that it is ontologically committed to pencils, though not to any particular one.

When we know how to read off the existential implications of sentences in canonical notation, we have a naturalistic way of answering ontological questions, giving entities a place in our ontology for 'essentially scientific reasons' (Quine 1969a: 97). We paraphrase our scientific theories into canonical notation; then we look to see what the paraphrases quantify over. Quine assumes that we should believe the paraphrases of well confirmed empirical theories, and everything that they entail. So if the paraphrases are ontologically committed to *F*s, we should include *F*s in our ontology, and if the paraphrases entail that *a* exists, we should include *a* in our ontology. We can argue for ontological conclusions in this way:

Our best theories, once paraphrased, are ontologically committed to *F*s (or: are ontologically committed to *a*).
We should believe the paraphrases of our best theories.
Therefore:
We should believe that there are *F*s (or: believe that *a* exists).[3]

[3] In this exposition of Quine's ontological method, I have deliberately ignored his theses of ontological relativity and inscrutability of reference. For an explanation of why these doctrines do not undermine the project of ontology, see Hylton (2004), §V.

3 Quine's Argument and the Indispensability Argument

With this background in place, it is easy to state Quine's argument for platonism. In many of our best scientific theories, quantitative language abounds. For instance, scientists say things like

(S) The surface area of Saturn is 1.08×10^{12} km^2.

The only way to paraphrase such sentences into canonical notation involves quantifying over numbers. Therefore we should accept the existence of numbers.

Why should we think that the paraphrases must quantify over numbers? Early on, Quine experimented with alternative paraphrases (see Goodman and Quine 1947). But these did not deal with much of mathematics, and Quine later came to believe that quantifying over numbers is required. To paraphrase (S) into canonical notation, we must say something like:

(S*) 1.08×10^{12} is-the-surface-area-in-km^2-of Saturn

(see Quine 1960: 245). And (S*) entails '$\exists x\ x = 1.08 \times 10^{12}$'. By (OC2), it is ontologically committed to 1.08×10^{12}. Assuming that numbers are abstract objects, it follows that if (S*) is true, there are abstract objects. According to Quine, we have 'essentially scientific reasons' to be platonists.[4]

As I have mentioned, there is an alternative use of the word 'paraphrase', avoided by Quine, according to which a sentence shares truth-conditions with its paraphrase. Many philosophers fail to share the distaste for analyticity that led Quine to avoid this notion. Suppose that such a philosopher wishes to give an account of the truth-conditions of a group of sentences. One possibility is to try to go directly from the original sentences to some specification of their truth-conditions. Another is to make use of an intermediate language (L, say). On this approach, we translate each sentence of the original group into L, as well as providing an account of the truth-conditions of sentences of L. Put together, these furnish truth-conditions for the original sentences. I'll call this way of giving truth-conditions *proxy semantics*, following Sainsbury (1991: 326). Within philosophy, Donald Davidson (1980, 1984) has been the most prominent advocate of proxy semantics for natural languages.

There is an obvious way of mimicking the Quinean pattern of argument within this alternative framework. Let us suppose that the theorist writes down their proxy translations in the same language as Quine uses, with the same interpretation, so that the ontological commitments of the proxies can be found by applying (OC1) and (OC2). We can take a sentence from a well confirmed scientific theory, translate it into the proxy language, then find out what ontological commitments the translation has by using (OC1) and (OC2). Since the original sentence and the translation have the same truth-conditions, the ontological commitments of the proxy belong to the original sentence too. If the original sentence is true, then these entities must exist. But since we should believe our best scientific theories, we should believe the

[4] For versions of this argument, see Quine (1969a: 97–97) and Quine (1981b).

original sentence, so we should believe that these entities do exist. Once we believe a theory, its commitments become ours. The pattern of argument is as follows:

> Our best scientific theories are ontologically committed to *F*s (or: *a*).
> We should believe our best scientific theories.
> Therefore:
> We should believe that there are *F*s (or: believe that *a* exists).

This is what Davidson 1977 calls ‘the method of truth in metaphysics’.

> We can use this to construct an argument for platonism:
> Some of our scientific theories are ontologically committed to abstract mathematical objects.
> We should believe our well confirmed scientific theories.
> Therefore:
> We should believe that there are abstract mathematical objects.

This is a telescoped version of the argument (1)–(6) from Sect. 1, which is customarily attributed to Quine. (The first premiss here corresponds to (4), the second to (5).) We can now see that this is not Quine’s argument, since it does not follow his favoured method of arguing for existential claims. There is no mention of paraphrase into canonical notation; the first premiss of the argument makes a claim about the ontological commitments of our *current* theories, rather than the ontological commitments of regimented versions. Quine, on the other hand, believes that there is no fact of the matter about the commitments of ordinary talk before we paraphrase it: ‘a fenced ontology is just not implicit in ordinary language’ (Quine 1981b: 9). Those who attribute the indispensability argument to Quine are actually crediting him with a Davidsonian argument that he would not have endorsed.

4 Putnam and the Indispensability Argument

The following passage from Putnam’s *Philosophy of Logic* (1971) sounds very much like the indispensability argument:

> So far I have been developing an argument for realism along roughly the following lines: quantification over mathematical entities is indispensable for science, both formal and physical; therefore we should accept such quantification; but this commits us to accepting the existence of the mathematical entities in question. This type of argument stems, of course, from Quine, who has for years stressed both the indispensability of quantification over mathematical entities and the intellectual dishonesty of denying the existence of what one daily presupposes (Putnam 1971: 347).

It is no surprise, then, that Putnam 1971 is the *locus classicus* for the indispensability argument. In this section, I will first explain why this attribution seems compelling: then I will show that, far from mounting the indispensability argument, Putnam did not argue for platonism at all.[5]

In 1967, Putnam published an essay which suggested interpreting mathematical sentences as modal logical claims, a view not dissimilar to Geoffrey Hellman's (1989) modal structuralism (Putnam 1967b). These interpretations avoid quantifying over numbers by using a modal primitive: claims about numbers are revealed to be claims about possible structures. Putnam's view is that this understanding of mathematics and the traditional platonist understanding in terms of abstract objects are 'equivalent descriptions' (46) and can be used to clarify each other. Underlying this attitude is a deflationary approach to ontology:

> [W]e do not have to choose between Platonism… and Nominalism… The old question 'Would a list of all the things in the world include chairs *and* numbers, or only such things as chairs?' is not a good question (Putnam 1979: xi, xii).

In 'What is mathematical truth?' (1975), Putnam argues that the 'consistency and [mathematical] fertility' (73) of classical mathematics is evidence for its truth. He then asserts that the role mathematics plays in physical theory means that 'it is not possible to be a realist with respect to physical theory and a nominalist with respect to mathematical theory' (73). Putnam sketches an argument, referring the reader to his 1971 for a more detailed treatment.

It might appear, then, that Putnam repudiated his doctrine of 'equivalent descriptions' between 1967 and 1975. Further evidence is seemingly provided by this passage, which occurs just after his claim that scientific realists cannot be nominalists:

> In a sense, this means that our intuitions are inconsistent. For I believe that the position most people find intuitive—the one that I certainly found intuitive—is realism with respect to the physical world and some kind of nominalism or if-thenism with respect to mathematics (1975: 74).

Putnam's 1975 position appears to be that the modal logical understanding of mathematics is inadequate for the needs of science; he seems to claim that 'mathematics as modal logic' is not equivalent to platonism but decidedly inferior. The modal translations concern what follows from what, making no reference to numbers; surely this is the 'nominalism' and 'if-thenism' attacked by Putnam's arguments, both here and in his 1971?

That this interpretation is widely shared is indicated by the many citations of Putnam 1971 as suggesting an indispensability argument for platonism, in the sense of that term I introduced above. To see what is wrong with this interpretation of Putnam 1971, let us start with his 1975 paper. He certainly says there that he is arguing against nominalism and for realism (75, 73). But by 'realism' he means the

[5] Not every author has agreed with this attribution. During a discussion of Colyvan, Pincock (2004: 67) asserts that *Philosophy of Logic* is an argument for Harvard realism, not platonism; and whilst surveying the varieties of nominalism found in print, Burgess and Rosen (1997: 201) spend a paragraph arguing for the same interpretation. Since these passages have met with a resounding silence, I judge that a more detailed discussion is required.

view that mathematical sentences are either true or false (bivalence) and are made true by 'something *external*… not (in general) our sense data, actual or potential, or the structure of our minds, or our language, etc.' (1975: 70). Call this *Harvard realism*. Clearly, lots of nominalist positions would count as Harvard realist, because they respect the mind-independence of mathematical truth. Harvard realism is more to do with objectivity than the existence of mathematical objects. Putnam explicitly says that realism—that is, Harvard realism—is not committed to the existence of abstract mathematical objects, because the modal logical picture is realist too: there is no need to '"buy" the Platonist ontology' to be a realist in his sense (1975: 70, 72).

Following Quine (e.g. Quine 1960: 233), Putnam uses the word 'nominalist' in an unusual way as well. According to Putnam, 'mathematics as modal logic' is 'not intended to satisfy the nominalist. The nominalist, good man that he is, cannot accept modal notions any more that he can accept the existence of sets. We leave the nominalist to satisfy himself' (1975: 70). By 'nominalism' Putnam means not the rejection of abstract entities but the rejection of abstract entities *and modality*. Call this *Harvard nominalism*.

With its terminology understood as intended, Putnam 1975 begins to look very different. Putnam's argument for 'realism' aims to establish not platonism but instead the mind-independence of mathematics. Rather than being aimed at those who deny the existence of mathematical objects, the argument's target is those who deny the objectivity of mathematics—in Putnam's opinion, intuitionists and fictionalists.[6] How is the 1975 argument anti-nominalist? Putnam seems to assume that the only Harvard realist positions are platonism and 'mathematics as modal logic' (see for example 1975: 70; 1979: xi.) Since neither of these counts as nominalist in Putnam's framework, his argument for Harvard realism is automatically an argument against Harvard nominalism.

One puzzling detail remains: Putnam's rejection of 'if-thenism'. To dispel the impression that this term refers to the modal logical picture of mathematics, let us consider Putnam's 1967a, 'The thesis that mathematics is logic'. This paper, which was written before Putnam's espousal of modal logic and the thesis of 'equivalent descriptions' (1979: xiii), argues for a view of mathematics which is explicitly called 'if-thenism' (1967a: 20). The footnote added to this paper when it was printed in Putnam 1979 makes it crystal clear that this, not the modal view, was Putnam's target.[7]

We have said enough to show that Putnam 1975 is not an argument for the existence of abstract mathematical objects. What about Putnam 1971, which is more commonly cited as a source of the indispensability argument? On the surface, Putnam seems to give a perfectly explicit statement of the indispensability argument (see the quotation above), concluding that we should accept the existence of the mathematical entities used in science. If this is right, then Putnam's progress must

[6] The references to Errett Bishop (a prominent intuitionist) and the scorn heaped on the view that numbers and functions are 'mere fictions' (74) make this clear.

[7] 'This [if-thenist view of applied mathematics] is wrong. Cf. chapter 4 [i.e. Putnam 1975] in which it is argued that one cannot consistently be a realist in physics and an if-thenist in mathematics' (33).

have been along these lines. Between 1967 and 1971, Putnam convinced himself that the applicability of mathematics was good evidence for platonism, and abandoned the ‘equivalent descriptions’ idea. Soon afterwards he wrote his 1971, which argues that indispensability supports platonism. But before 1975 he regained his belief in ‘equivalent descriptions’, and restated the indispensability argument as an argument for Harvard realism in his 1975 paper.

Given that few of Putnam’s philosophical opinions have proved immune from revision, this might seem a compelling view. But there are two pieces of evidence that tell against it. Firstly, Putnam 1975 summarises Putnam 1971 in these words: ‘a reasonable interpretation of the *application* of mathematics to the physical world *requires* a [Harvard] realistic interpretation of mathematics’. No change in view is signalled; indeed, the reader is referred to Putnam 1971 for a fuller exposition of the argument. The other piece of evidence is the final section of Putnam 1971, ‘Unconsidered complications’. In this passage, Putnam tells the reader that ‘the realm of mathematical fact admits of many “equivalent descriptions”‘, and regrets that he lacks the space to deal with this issue (1971: 356–357). Although he does not mention the modal logical picture, I think we can be confident that he means the same by ‘equivalent descriptions’ as he does in his 1967b and 1975. Given that his 1971 was intended to be an introductory work, it seems right to say that Putnam chose to give a simplified version of his argument for Harvard realism, conflating it with platonism for simplicity’s sake.[8]

This account of the development of Putnam’s views dovetails neatly with his more recent writings on the philosophy of mathematics. He has repeatedly attacked Quine’s mathematical realism: ‘[W]ouldn’t mathematics have worked exactly as well even if the “intangible objects” didn’t exist? … But doesn’t this already show that positing immaterial objects to account for the success of mathematics is a useless shuffle?’ (1996: 247, italics removed; see also 2004: 65–67, which is part of a lecture entitled ‘Objectivity without objects’). Nowhere does he signal that he used to endorse the doctrine he attacks. Elsewhere he is explicit in noting changes of view; for instance, Putnam (1990: 259 n. 29) highlights a disagreement with Putnam 1975.[9]

In summary, I agree with Burgess and Rosen when they write:

> It may very well be, then, that if [Putnam] had had the space to discuss ‘equivalent descriptions’ [in his 1971] he would instead have said that

[8] One ‘unconsidered complication’ of my own is that ‘mathematics as modal logic’ changed between 1967b and 1975. In the earlier paper, Putnam uses logical necessity in his modal translations; in the later one, he invokes ‘a strong and uniquely mathematical sense of “possible” and “impossible”’ (1975: 70). Field (1988: 270 n. 36) is baffled by the change, but I think that he himself provides an explanation (at 1984: 85 n. 7): if we use purely logical modality, every consistent theory will come out as equally ‘good’, and, since this is incompatible with bivalence (which the ‘objects picture’ is intended to support), the ‘equivalent descriptions’ thesis will be hard to sustain. Shifting to a less permissive sort of modality resolves the problem. I do not think that admitting Putnam’s change of heart threatens the argument in the text.

[9] In conversation with the author in Sheffield, 2005, Putnam confirmed that his 1971 and 1975 were intended to establish Harvard realism rather than platonism.

> classical mathematics, either in its usual version with abstracta, or in an 'equivalent description' with modality, is indispensable (1997: 201).

The 'Quine–Putnam indispensability argument' is misnamed: Putnam never held that we should believe in mathematical entities because of the role of mathematics in science.[10]

5 Summary and Implications

As we have seen, both Quine and Putnam thought that the scientific utility of mathematics provides evidence for the truth of some mathematical claims, but the argument from mathematical truth to platonism stems from Quine alone. The current debate over the metaphysics of mathematics focuses on an argument that is not Quine's (though it is similar to Quine's). In other words, neither Quine nor Putnam put forward the argument that is generally called the 'Quine–Putnam indispensability argument'. Alex Oliver (1999: 269) writes: 'Neither logicians nor grammarians can be trusted to tell the history of either grammar or logic'. My main conclusion is that, likewise, philosophers of mathematics cannot be trusted to tell the recent history of their own discipline.

To finish, let me consider what implications the misattribution I have exposed has for contemporary debate in the philosophy of mathematics. Does reflecting on Quine and Putnam's actual arguments show that there is anything wrong with the argument that is standardly attributed to them? And does it help to point philosophers of mathematics in any new directions? I will take these questions in turn.

To answer the first question, let me begin with Quine. As we have seen, the 'Quine–Putnam' argument involves claims about the ontological commitments of natural language sentences, whereas Quine's argument does not. Quine claims that there is no fact of the matter about the commitments of ordinary talk. It is plausible to suppose that Quine is led to this view by his belief in the indeterminacy of translation: if, as Quine believes, the criterion of ontological commitment applies to sentence of canonical notation,[11] then natural language sentences have determinate ontological commitments only if how they should be translated into canonical notation is a determinate matter. Since translation is indeterminate, natural language sentences lack determinate ontological commitments.

[10] Indeed, there are passages where he seems to suggest that 'mathematics as modal logic' is superior to the standard picture: 'The theory of mathematics as the study of special *objects* has a certain implausibility which, in my view, the theory of mathematics as the study of ordinary objects with the aid of a special concept does not…[P]uzzles… as to how one can refer to mathematical objects… can be clarified with the aid of modal notions' (1975: 72).

[11] Quine (1960: 242) argues that there is no syntactic criterion of ontological commitment that applies directly to natural language sentences: '[A]t best there is no simple correlation between the outward forms of ordinary affirmations and the existences implied'. To show this, Quine points out that the sentence 'Agnes has fleas' can be interpreted as '∃x (Fx & Gx)' whereas other sentences of the same form, such as 'Tabby eats mice' and 'Ernest hunts lions', cannot. (see also Quine 1969a: 106.)

But it is far from clear that we should be persuaded by Quine's arguments for the intedeterminacy of translation. This is not the place to report in detail the debate over the doctrine, but one source of weakness in Quine's arguments is their claim that the only evidence relevant to translation is behavioural facts about assent and dissent (see Miller 1998: 131–132). To say the very least, it is far from clear that Quine's arguments establish their conclusion (see Miller 1998: 128–150 and Kirk 2004). Quine has not established that there is anything worrying about arguments that involve claims about the ontological commitments of natural language sentences. So reflection on Quine's actual argument provides no reason to abandon discussion of the 'Quine–Putnam' argument.

Let me turn now to Putnam. As we saw in Sect. 4, Putnam does not aim to establish platonism. His deflationary attitude to ontological debates means that he doubts whether there is any intelligible question concerning the existence of abstract mathematical objects. But does Putnam offer any convincing justification for this attitude? Putnam's writings on this remain controversial, with several authors arguing that he does not (see Gross 2004; van Inwagen 2002; Horgan and Timmons 2002; Sider 2001: introduction; Sider forthcoming; see also the other essays in Chalmers et al., forthcoming). Stated very roughly, Putnam's idea is that ontologists with rival theories are talking past each other—they mean different things by the words and phrases like 'exists' and 'there are'—and that this shows that ontological debates are unworthy of our attention. Putnam's critics argue that it is not clear how his claim that ontologists are talking past each other can be developed into a tenable semantic account of the operation of these words and phrases; and they suggest that Putnam's arguments do little to undermine the project of ontology. It is then, not clear that Putnam provides us with any reason to stop investigating ontological theses, such as platonism, or to stop investigating arguments in their support, such as the 'Quine–Putnam' argument.

Luckily, then, the misattribution of the argument to Quine and Putnam is no threat to its continued discussion—though if the controversial doctrines these philosophers espoused *do* turn out to be defensible, then the 'Quine–Putnam' argument will be in trouble.

To close, I will suggest that consideration of Quine and Putnam's original arguments points out some neglected ways to argue for platonism. Consider this passage from Quine:

> Measures have sometimes been viewed as impure numbers: nine miles, nine gallons. We do better to follow Carnap in construing each scale of measurement as a polyadic general term relating physical objects to pure numbers. Thus 'gallon *xy*' means that the presumably fluid and perhaps scattered physical object *x* amounts to *y* gallons, and 'mile *xyz*' means that the physical objects *x* and *y* are *z* miles apart. Pure numbers, then, apparently belong in our ontology. (Quine 1981b: 14, footnote omitted; cf. Quine 1960: 245)

This passage argues for the existence of numbers on the grounds that measurement sentences such as length- and volume-ascriptions are best regimented using quantification over numbers. There is no mention of pure mathematical sentences here. Similarly, one of Putnam's (1971: Sect. 5) main examples is from applied mathematics, not pure mathematics: he argues that we have to quantify over numbers in order to state Newton's law of gravitation. Quine (1981b: 14) also mentions laws. These passages suggest two arguments for platonism that do not go via pure mathematics:

(1a) We should believe the measurement claims made by well confirmed scientific theories—for instance, astronomy's claim: 'Saturn has surface area 1.08×10^{12} km^2'.
(2a) If these measurement claims are true, then there are abstract mathematical entities.
(3a) So we should believe that there are abstract mathematical entities.

(1b) We should believe the law-statements that figure in well confirmed scientific theories.
(2b) If these law-statements are true, then there are abstract mathematical entities.
(3b) So we should believe that there are abstract mathematical entities.

Unlike the standard 'Quine–Putnam' argument given in Sect. 1, these arguments do not invoke confirmational holism. This is an advantage, because some of the most important attacks on the indispensability argument target this premiss (Sober 1993, Maddy 2005).[12] These attacks are no threat to the two arguments just given. That said, these arguments do involve claims about the entailments of measurement sentences and law-statements ((2a), (2b))—claims which are not *obviously* true (nor *obviously* false). But the advocate of platonism may find these easier to defend than the confirmational holism the standard argument invokes. So attention to Quine and Putnam's writings is important, not just to set the record straight, but because it reveals some new argumentative strategies for platonists to exploit.

Acknowledgments This paper draws on chapter 3 of my Ph.D. thesis (*Nominalist accounts of Mathematics*, University of Sheffield, UK). I would like to give the warmest of thanks to Rosanna Keefe and Chris Hoookway for their painstaking supervision. I would also like to thank Hilary Putnam for discussing his work with me. For comments and advice, thanks to Chris Daly, Michael Scott, and an anonymous referee.

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[12] Maddy (1997: 133–143) attacks the conjunction of confirmational holism and Quine's criterion of ontological commitment. Only recently has she chosen to lay the blame specifically with confirmational holism. For commentary on Sober, see Resnik (1997, chapter 7), Colyvan (2001: 126–134), and Leng (2002). For commentary on Maddy, see Colyvan (2001, chapter 5) and Leng 2002).

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