



# Assessment of healthcare waste treatment methods using an interval-valued intuitionistic fuzzy double normalization-based multiple aggregation approach

Abdullah Al-Barakati<sup>1</sup> · Pratibha Rani<sup>2</sup> 

Received: 5 November 2021 / Accepted: 11 March 2023 / Published online: 24 April 2023  
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## Abstract

Healthcare waste management has been an extensively attractive topic recently since it is one of the key concerns regarding both environment and public health, predominantly in developing nations. The optimization of the treatment procedure for healthcare waste is indeed a complex “multi-criteria decision-making (MCDM)” problem that involves contradictory and interweaved critical criteria. To successfully handle this issue, this study extends the original method, named the “double normalization-based multi-aggregation (DNMA)” approach, with “interval-valued intuitionistic fuzzy sets (IVIFSs)” for decision-making problems taking criteria in terms of benefit or cost types. This method involves two target-based normalizations and three subordinate utility models. To estimate the criteria weights, we propose a new parametric divergence measure and discuss the feasibility of the developed divergence measure based on existing divergence measures for IVIFSs. Further, the developed framework is implemented to elucidate the “healthcare waste treatment (HCWT)” problem. The comparative and sensitivity analyses of the outcomes indicate that the proposed approach efficiently tackles the problem of HCWT selection. The outcomes show that steam sterilization (0.462) is the optimal one for HCWT. The prioritization options, obtained by presented approach, are dependable and suitable, which are steam sterilization > microwave > incineration > landfilling.

**Keywords** Healthcare waste treatment · Waste residuals · Double normalization-based multiple aggregation method · Interval-valued intuitionistic fuzzy sets · Divergence measure

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✉ Abdullah Al-Barakati  
aalbarakati@kau.edu.sa

Pratibha Rani  
pratibha138@gmail.com

<sup>1</sup> Faculty of Computing and Information Technology, King Abdulaziz University, Jeddah 21589, Saudi Arabia

<sup>2</sup> Department of Mathematics, Rajiv Gandhi National Institute of Youth Development, Sriperumbudur, Tamil Nadu 602105, India

## 1 Introduction

More healthcare facilities, then more “Healthcare Waste (HCW)” (Aung et al., 2019; Wu et al., 2019). Literature and reports show a substantial surge in HCW (Adamović et al., 2018; Rafiee et al., 2018). As a result, HCW management has become one of the most demanding challenges for public sectors all over the world, especially in emerging nations in which the HCW is usually combined with the solid wastes that come out of cities (Debere et al., 2013). As described by “World Health Organization (WHO),” the HCW is “waste generated by healthcare activities including a broad range of materials, from used needles and syringes to soiled dressings, body parts, diagnostic samples, blood, chemicals, pharmaceuticals, medical devices, and radioactive materials” (WHO, 2018). The WHO reported that roughly 85% of the HCW is not dangerous, but the remaining 15% can be a highly dangerous material that might be toxic, infectious, or even radioactive. To proficiently segregate the dangerous and non-dangerous wastes, numerous regulations are adopted by most of the developed and developing countries. It is possible to treat the non-dangerous wastes simply with the municipal solid wastes, but in case of disposing of the dangerous HCW, we are in need of some strict, demanding treatments. If the HCW is managed poorly, it might result in infectious effects on waste handlers, healthcare personnel, patients, and the whole society. This can make the environment polluted, which can negatively impact all people living in that society. Consequently, the HCW must be strictly segregated right where they are generated, be treated properly, and then disposed of prudently. It should be noted that this procedure is done improperly, the living environment will be seriously polluted, diseases such as typhoid and cholera are transmitted, unpleasant odors will be discharged, and different species of insects and worms will grow in the place where people live (Du Mortier et al., 2016). These threatening issues have resulted in the attractiveness of the topic of HCW management to academic scholars, governments, and practitioners (Awodele et al., 2016; Wu et al., 2019).

Many scholars (Liu et al., 2019; Wang et al., 2018) have pointed out the high importance of choosing the appropriate technology for waste treatment. This is definitely a complex MCDM concern. To tackle such type of problem efficiently, there is a requirement to propose MCDM models. On the other hand, it is not easy for “decision experts (DEs)” to offer crisp assessments of options on various attributes (Arcos-Aviles et al., 2018; Ekmekçioğlu et al., 2010; Mardani et al., 2015, 2016, 2017). Sometimes, crisp values may be unable to observe the imprecision. Thus, to treat the vagueness and uncertainty, “fuzzy sets (FSs)” (Zadeh, 1965) have been broadly used to various technologies. The theory of “interval-valued intuitionistic fuzzy sets (IVIFSs)” (Atanassov & Gargov, 1989), as an extension of “intuitionistic fuzzy sets (IFSs)” (Atanassov, 1986), was proposed to illustrate information precisely, where both “belongingness degree (BD)” and “non-belongingness degree (ND)” are defined in interval values.

In this study, for the first time, we introduce a framework based on a divergence measure to develop the extended DNMA framework under the IVIFSs environment to elucidate the MCDM problems, which include criteria in terms of benefit and cost types. The utility degree-based approaches are valuable in managing the MCDM problems with massive experts and criteria. Liao and Wu (2020) suggested the “double normalization-based multiple aggregation (DNMA)” model, which considers the advantages of diverse normalization process and “aggregation operators (AOs)” and integrates them in a suitable mode. The “overall utility degree (OUD)” of IVIF-DNMA

method extensively utilizes the “subordinate utility degrees (SUDs)” and the priorities of options. The contribution of the paper is discussed as follows:

- First, we develop an extended utility value-based framework, namely, the DNMA method with IVIFSs. This method is composed of two normalization functions and three subordinate utility models.
- Secondly, to evaluate the criteria weights, this paper introduces an innovative parametric divergence measure of IVIFSs and makes a comparison with the existing IVIF-divergence measures.
- Thirdly, we discuss a case study on the selection of “Healthcare Waste Treatment (HCWT)” methods to exemplify the procedure of the presented framework in solving MCDM problems.
- Finally, to validate of the results, a sensitivity investigation is discussed through varying over parameter values of the presented IVIF-DNMA framework. We discuss a comparison of the IVIF-DNMA framework with extant models to express the reliability of the outcomes.

The rest of the work is prepared as follows: Sect. 2 confers the review of decision-making in the discipline of HCWT, how to choose the related criteria, and the hierarchical configuration of the study. Section 3 presents the elementary ideas of IVIFSs. Section 4 discusses the parametric divergence measure for IVIFSs. Section 5 proposes a novel IVIFS-DNMA method. Section 6 demonstrates the developed framework by an empirical case study of HCWT assessment. It also illustrates the comparative discussion with extant models and sensitivity investigation with different utility parameter values. Section 7 shows the conclusions and further recommendations of the study.

## 2 Literature review

### 2.1 Interval-valued intuitionistic fuzzy sets (IVIFSs)

The principle of IVIFSs has been proposed to elucidate information, where both the BD and ND are defined in interval values. The IVIFSs have attained a number of theoretical achievements in terms of measure theory (Zhang et al., 2011), comparison methods (Xu, 2007; Bai, 2013; Zhang et al., 2018), and MCDM methods (Mishra & Rani, 2018a; Oztaysi et al., 2017). Khoshnava et al. (2020) presented an “interval-valued intuitionistic fuzzy-improved score function and weighted divergence based approximation (IVIF-ISF-WDBA)” model for prioritize the stakeholders. Mishra et al. (2020c) discussed the “complex proportional assessment (COPRAS)” on IVIFSs to assess suitable safety and health facility in “hazardous waste recycling systems (HWRSS).” Mishra et al. (2020d) reviewed the IVIF-information measures and illustrated some counter-intuitive concerns. Alrasheedi et al. (2021) applied “IVIF-combined compromise solution (CoCoSo)” model to prioritize the indicators of “green growth to sustainable manufacturing (GGSM).” Mishra et al. (2022a) developed the “additive ratio assessment (ARAS)” model on IVIFSs to assess and select the suitable “low-carbon tourism strategy (LCTS).” Chen et al. (2021) proposed the IVIF-projection model to assess “third party reverse logistic providers (3PRLPs)” with sustainability to promote the “circular economy (CE).”

Under the IVIFSs setting, there are many promising ranking techniques developed by researchers for MCDM problems, such as the IVIF-TOPSIS method (Bai, 2013), IVIF-VIKOR method (Mishra & Rani, 2019a; Rani et al., 2018), IVIF-TODIM method (Mishra & Rani, 2018a), IVIF-WASPAS (Mishra & Rani, 2018b), IVIF-COPRAS (Wang et al., 2016; Hezam et al. 2022b), and IVIF-MABAC (Mishra et al., 2020b). There are three representative approaches of the utility degree-based MCDM approaches, i.e., the TOPSIS, VIKOR, and MULTIMOORA (He et al. 2021). They are different from the normalization and aggregation techniques. It is known that various normalization procedures have different advantages and disadvantages (Jahan & Edwards, 2015), and several “Aggregation Operators (AOs)” have different functions (Llopis-Albert et al., 2017; Liao & Wu, 2020; Mishra et al., 2019b). However, the classical utility value-based methods apply only one normalization technique, which would limit their applications. In this regard, Liao and Wu (Liao & Wu, 2020) proposed a new utility degree-based DNMA model to treat the MCDM concerns. Wang and Rani (2021) gave the IF-DNMA model for prioritizing and evaluating the “sustainability risk factors (SRFs)” in “supply chain management (SCM).” Recently, Saha et al. (2022) presented the q-ROF- “full consistency method (FUCOM)”-DNMA” approach to deal with the HCWTT assessment. Hezam et al. (2022a) introduced a hybrid MCDM methodology by combining “method based on the removal effects of criteria (MEREC)-ranking sum (RS)-DNMA” approach with IFSs and applied to evaluate the “alternative fuel vehicles (AFVs)” problem.

“Divergence measure (DiM) has been widely used for the measurement of discrimination among objects. In FSs theory, Pal (1993) was the first scholar who gave the idea of DiM. Later, Vlachos et al. (2007) initiated the philosophy of DiM in the case of IFSs and applied it to the study areas such as pattern recognition and medical diagnosis. Ansari et al. (2018) defined innovative IF-DiM and implemented to develop a technique for edge detection. Montes, et al. (2015) discussed the IF-DiM with a new definition and axioms. Mishra et al. (2017a) introduced new IF-DiMs with their application in the correlative MCDM approach. Mishra et al. (2017b) proposed some novel Jensen-exponential DiMs to develop a new MCDM procedure with partial information on the weights of criteria for IFSs. The DiM has also been extended to IVIFSs. For instance, Zhang et al. (2011) studied a DiM for IVIFSs to evaluate the pattern recognition problems. Ye (2011) introduced an optimized MCDM model using novel IVIF-DiMs. Meng et al. (2013) presented a new IVIF-DiM, which was utilized to compute the attribute weights for the proposed decision-making method. Meng et al. (2015) argued a framework using a DiM and a Shapley function. Mishra et al. (2020b) integrated the method on the basis of “multi-attributive border approximation area comparison (MABAC)” model using “IVIF-entropy and DiM” to evaluate “programming language assessment (PLA)” problems. Mishra and Rani (2018b) defined IVIF-entropy-DiM using the exponential function and utilized it to discuss the WASPAS method under the IVIFSs environment.

## 2.2 Application of decision-making and criteria selection in healthcare waste treatment

A great part of research recently carried out into the ways of choosing the appropriate HCWT technologies has applied the fuzzy set-based “linguistic values (LVs)” to signify DEs’ views. For example, a fuzzy MCDM approach was introduced by Dursun et al., (2011a, 2011b), aiming at assessing four prominent technologies related to waste treatment. The basis of their approach was the concepts of fuzzy measure and fuzzy integral (Morrissey & Browne, 2004). In another

study, an MCDM procedure was developed by Dursun et al., (2011a, 2011b) based on fuzzy logic and to evaluate the methods of HCW disposal. Furthermore, Liu et al. (2013) discussed a new VIKOR model for the selection of HCWT methods. They validated their method by handling an HCWT assessment problem in China. Also, Liu, et al. (2014) used MULTIMOORA (Brauers & Zavadskas, 2010) method with “interval 2-tuple linguistic numbers (I2TLNs)” to choose the best waste treatment process for a case in Shanghai, China. Next, Voudrias (2016) discussed the detailed explanations related to the HCWT procedure and then evaluated them by means of “analytic hierarchy process (AHP).” Lee et al. (2016) pointed out a study in England in which the AHP was applied as a mechanism for the assessment of the “optimal” method that can be implemented for HCWT. Shi, et al. (2017) gave an MCDM model with MABAC and I2TLNs for evaluating the “best” procedure for HCWT. Xiao (2018) proposed a D-number doctrine-based method, conducted an investigation on the HCWT procedures. More recently, Liu et al. (2019) recommended a technique using the Hamy-mean operator and “intuitionistic uncertain linguistic terms sets (IULTSs)” for the assessment of the optimal HCWT method. Hinduja and Pandey (2018) established an integrated procedure with “decision-making trial and evaluation laboratory (DEMATEL)-AHP” on IFSs for estimating and choosing the optimal HCWT. Mishra et al., (2020a) discussed the “evaluation based on distance from average solution (EDAS)” model on IFSs for estimating the suitable disposal method for HCWT. Rani et al. (2020) presented “Pythagorean fuzzy-“stepwise weight assessment ratio analysis (SWARA)-ARAS” model for choosing the HCWT options. Liu et al. (2021) recommended the “Pythagorean fuzzy-CoCoSo” model for assessing and ranking “medical waste treatment methods (MWTMs). Puška et al. (2021) combined the “full consistency method (FUCOM)” and “compromise ranking of alternatives from distance to ideal solution (CRADIS)” models for selecting the type of incinerators to treat the problem of HCW in healthcare organizations in Bosnia and Herzegovina. Rani et al. (2021) presented the CRITICMULTIMOORA model on SVNSSs for choosing the food waste treatment method. Chauhan and Singh (2021) gave a hybrid model with a “linear programming (LP)” model to treat the concern of safe disposal of hazardous and infectious HCWs. Torkayesh et al. (2021) proposed a model with the “geographic information system (GIS),” the BWM, and the “measurement of alternatives and ranking based on compromise solution (MARCOS)” tools on grey interval set for landfill location selection for the HCW system during COVID-19 era. Mishra et al. (2022b) discussed the IVPF- “complex proportional assessment (COPRAS)” model for treating the “waste-to-energy (WTE)” methods for “municipal solid waste (MSW)” treatment. Torkayesh and Simic (2022) discussed a novel hybrid model with the “hierarchical stratified best–worst method (H-SBWM),” the CoCoSo and the WASPAS methods for recycling facility location selection in urban healthcare plastic waste.

To make a sustainable HCW management system, the system should be socially fit, economically practical, and environmentally efficient (Xiao, 2018). Consequently, the evaluation of HCWT options, which considers the need for trade-off various contradictory attributes with intrinsic imprecision and vagueness, is actually a MCDM problem of high significance. The MCDM methods that conventionally take into account the deterministic or random processes are not capable of efficiently handling those decision-making problems that include inaccurate linguistic information. In addition, in the case that many performance indicators are needed for discussion during the process of assessment, researchers often have a tendency to organize the attributes in a multi-level hierarchy to take out an assessment of high effectiveness. Thus, the present study is centered upon the complete multi-criteria assessment of HCWT options, aiming at identifying the best one that can be well applied to the case of Istanbul, which is the largest Turkish city. This work is mainly designed to offer an MCGDM model on the basis of the fuzzy measure principles

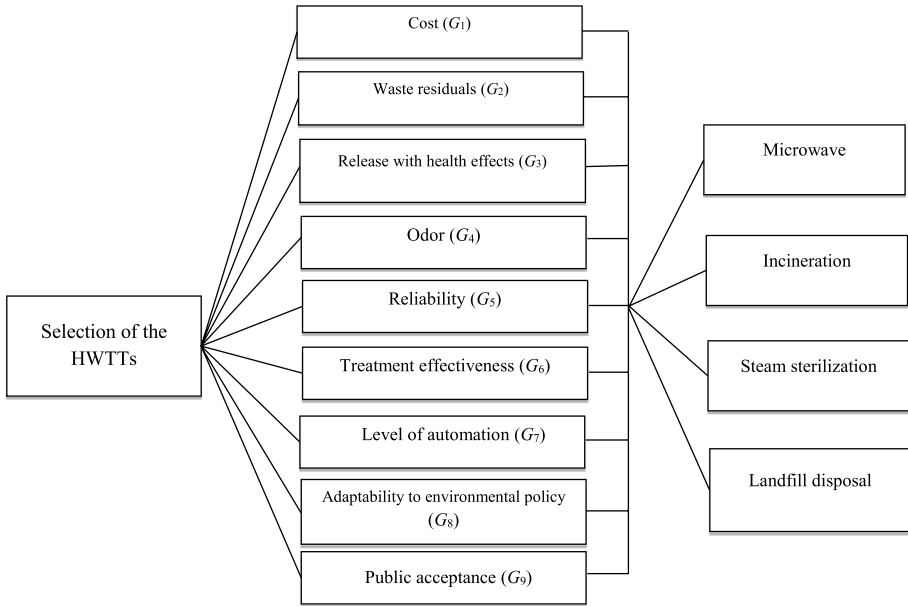


Fig. 1 A framework of selecting HCWT technologies

to evaluate the HCWT options. It helps to integrate into the analysis of those imprecise data that are represented as linguistic variables.

For evaluating and taking the proper criteria to assess the optimal best HCWT methods, there is a requirement to take a decision related to the framework to monitor the development of assessing these sources and then certify that the considered attributes are proficient of covering all facets of the considered approach. Thus, this paper offered a structure (Fig. 1) to discuss the key criteria that can be discussed in selecting the optimum sources (see Table 1).

The alternative HCWT technologies are  $B_1$ : “Incineration,”  $B_2$ : “Steam sterilization,”  $B_3$ : “Microwave” and  $B_4$ : “Landfill disposal.” These HCWTs are assessed over the four prime aspects, “economic, environmental, technical and social,” which have nine criteria, and are depicted in Fig. 1.

### 3 Preliminaries

Here, some essential concepts of IVIFSs and divergence measure are discussed.

Atanassov and Gargov (1989) extended IVIFSs based on IFSs to handle the uncertainty, which is exemplified by the BD and the ND in interval form.

**Definition 1** Atanassov and Gargov (1989). Let  $C = \{c_1, c_2, \dots, c_n\}$  be a fixed set. An IVIFS  $P$  on  $C$  is given by  $P = \{ \langle c_i, \mu_P(c_i), \nu_P(c_i) \rangle : c_i \in C \}$ , where  $\mu_P, \nu_P : C \rightarrow [0, 1]$  hold  $\sup(\mu_P(c_i)) + \sup(\nu_P(c_i)) \leq 1$ . The intervals  $\mu_P(c_i)$  and  $\nu_P(c_i)$  indicate the BD and ND of the variable  $c_i$  in  $C$ , respectively. For simplicity, suppose  $\mu_P(c_i) = [\mu_P^-(c_i), \mu_P^+(c_i)] \subset [0, 1]$  and  $\nu_P(c_i) = [\nu_P^-(c_i), \nu_P^+(c_i)] \subset [0, 1]$ .

**Table 1** Selected criteria for evaluating healthcare waste treatment technologies (HCWTT)

Criteria	Sources
Cost ( $G_1$ )	Liu et al. (2013); Dursun et al. (2011a); Liu et al. (2015); Dursun et al. (2011b); Xiao (2018); Özkan (2013); Lu, et al. (2016); Vučijak et al. (2016); Shi et al. (2017); Chauhan, Singh (2016)
Waste residuals ( $G_2$ )	Liu et al. (2013); Dursun et al. (2011a); Liu et al. (2015); Dursun et al. (2011b); Xiao (2018); Özkan (2013); Lu et al. (2016); Shi et al. (2017)
Release with health effects ( $G_3$ )	Liu et al. (2013); Dursun et al. (2011a); Liu et al. (2015); Dursun et al. (2011b); Xiao (2018); Vučijak et al. (2016); Shi et al. (2017)
Odor ( $G_4$ )	Dursun et al. (2011a); Liu et al. (2015); Dursun et al. (2011b); Liu, et al. (2014); Lu et al. (2016)
Reliability ( $G_5$ )	Liu et al. (2013); Dursun et al. (2011a); Liu et al. (2015); Dursun et al. (2011b); Xiao (2018); Liu, et al. (2014); Lu et al. (2016); Shi, et al. (2017)
Treatment effectiveness ( $G_6$ )	Liu et al. (2013); Dursun et al. (2011a); Liu et al. (2015); Dursun et al. (2011b); Xiao (2018); Liu et al. (2014); Lu et al. (2016); Shi, et al. (2017)
Level of automation ( $G_7$ )	Dursun et al. (2011a); Liu et al. (2015); Dursun et al. (2011b); Xiao (2018); Liu et al. (2014)
Adaptability to environmental policy ( $G_8$ )	Dursun et al. (2011a); Liu et al. (2015); Dursun et al. (2011b); Xiao (2018); Liu et al. (2014);
Public acceptance ( $G_9$ )	Liu et al. (2013); Liu et al. (2015); Dursun et al. (2011b); Xiao (2018); Liu et al. (2014); Lu et al. (2016); Vučijak et al. (2016); Shi, et al. (2017)

The interval  $\pi_P(c_i) = [\pi_P^-(c_i), \pi_P^+(c_i)] = [1 - \mu_P^+(c_i) - \nu_P^+(c_i), 1 - \mu_P^-(c_i) - \nu_P^-(c_i)]$  signifies the “hesitancy degree (HD)” of  $c_i$  to  $P$ . The pair  $([\mu_P^-(c_i), \mu_P^+(c_i)], [\nu_P^-(c_i), \nu_P^+(c_i)])$  is termed as an “interval-valued intuitionistic fuzzy number (IVIFN)” (Xu, 2007). For easiness, the IVIFN is commonly depicted as  $P = ([p, q], [r, s])$  such that  $[p, q] \subset [0, 1], [r, s] \subset [0, 1]$  and  $q + s \leq 1$ .

**Definition 2** (Mishra et al., 2017) Suppose  $P, Q \in IVIFSs(C)$ . Then, some operations on IVFFSs can be defined by.

- (a)  $P \subseteq Q$  iff  $\mu_P^-(c_i) \leq \mu_Q^-(c_i), \mu_P^+(c_i) \leq \mu_Q^+(c_i), \nu_P^-(c_i) \geq \nu_Q^-(c_i)$  and  $\nu_P^+(c_i) \geq \nu_Q^+(c_i), \forall c_i \in C,$
- (b)  $P = Q$  iff  $P \subseteq Q$  and  $P \supseteq Q,$
- (c)  $P^c = \{ \langle c_i, [\nu_P^-(c_i), \nu_P^+(c_i)], [\mu_P^-(c_i), \mu_P^+(c_i)] \rangle \mid c_i \in C \},$
- (d)  $P \cup Q = \left\{ \left\langle c_i, \left[ \mu_P^-(c_i) \vee \mu_Q^-(c_i), \mu_P^+(c_i) \vee \mu_Q^+(c_i) \right], \left[ \nu_P^-(c_i) \wedge \nu_Q^-(c_i), \nu_P^+(c_i) \wedge \nu_Q^+(c_i) \right] \right\rangle \mid c_i \in C \right\},$
- (e)  $P \cap Q = \left\{ \left\langle c_i, \left[ \mu_P^-(c_i) \wedge \mu_Q^-(c_i), \mu_P^+(c_i) \wedge \mu_Q^+(c_i) \right], \left[ \nu_P^-(c_i) \vee \nu_Q^-(c_i), \nu_P^+(c_i) \vee \nu_Q^+(c_i) \right] \right\rangle \mid c_i \in C \right\}.$

Bai (2013) pioneered the score value using the HD between the BD and ND of IVIFNs.

**Definition 3** Let  $P = \langle [p, q], [r, s] \rangle \in \text{IVIFN}(C)$ , then  $\mathbb{S}(P) = \frac{p+p(1-p-r)+q+q(1-q-s)}{2}$  is known as a score function of IVIFN.

**Definition 4** For an IVIFN  $P = \langle [p, q], [r, s] \rangle$  and  $\xi (> 0) \in \mathbb{R}$ , we have Xu (2007):

$$\xi P = \langle [1 - (1 - p)^\xi, 1 - (1 - q)^\xi], [r^\xi, s^\xi] \rangle. \tag{1}$$

**Definition 5** For a set of IVIFNs  $P = \{P_1, P_2, \dots, P_\ell\}$ , where  $P_k = \langle [p_k, q_k], [r_k, s_k] \rangle$ ,  $k = 1, 2, \dots, \ell$ , an ‘‘interval-valued intuitionistic fuzzy weighted averaging (IVIFWA)’’ operator as

$$\bigoplus_{k=1}^{\ell} \xi_k P_k = \left\langle \left[ 1 - \prod_{k=1}^{\ell} (1 - p_k)^{\xi_k}, 1 - \prod_{k=1}^{\ell} (1 - q_k)^{\xi_k} \right], \left[ \prod_{k=1}^{\ell} (r_k)^{\xi_k}, \prod_{k=1}^{\ell} (s_k)^{\xi_k} \right] \right\rangle. \tag{2}$$

In the similar line, an ‘‘interval-valued intuitionistic fuzzy weighted geometric (IVIFWG)’’ operator as

$$\bigotimes_{k=1}^{\ell} \xi_k P_k = \left\langle \left[ \prod_{k=1}^{\ell} (p_k)^{\xi_k}, \prod_{k=1}^{\ell} (q_k)^{\xi_k} \right], \left[ 1 - \prod_{k=1}^{\ell} (1 - r_k)^{\xi_k}, 1 - \prod_{k=1}^{\ell} (1 - s_k)^{\xi_k} \right] \right\rangle. \tag{3}$$

The DiM has been known as an efficient way to calculate the discernment degree. Firstly, Vlachos and Sergiadis (2007) developed the concept of IF-DiM. After that, Montes, et al. (2015) discussed a new axiomatic procedure of IVIF-DiM.

**Definition 6** (Montes et al., 2015) A function  $I : \text{IVIFSs}(C) \times \text{IVIFSs}(C) \rightarrow \mathbb{R}$  is called an IVIF-DiM based on the given postulates:

$$D1 \ I(P, Q) = I(Q, P),$$

$$D2 \ I(P, Q) = 0 \Leftrightarrow P = Q,$$

$$D3 \ I(P \cap R, Q \cap R) \leq I(P, Q),$$

$$D4 \ I(P \cup R, Q \cup R) \leq I(P, Q), \ \forall R \in \text{IVIFS}(C).$$

### 4 The proposed IVIF-divergence measure

Three measures have been introduced for assessing the discrimination level of IVIFSs, viz. IVIF-dissimilarity, IVIF-distance, and IVIF-divergence. The dissimilarity measure considers a different number of enviable axioms for IVIFSs. A distance measure cannot be necessarily considered as a dissimilarity measure. Essentially, a distance measure for IVIFSs cannot be necessarily helpful for applications like image processing; the reason is that if one image is signified using IVIFSs, the triangular inequality might fail to represent the enviable link. On the contrary, the IVIF-DiM can be considered as a dissimilarity



measure, and it involves a set of enviable axioms that help to assess the differences for IVIFSs (Montes et al., 2015). In this section, novel parametric IVIF-DiM is developed and various axioms are investigated.

**Definition 7** A parametric symmetric IVIF-DiM between IVIFSs  $P$  and  $Q$  with  $\gamma > 0 (\gamma \neq 2)$  is given as follows:

$$I_1(P, Q) = \frac{1}{2n(2^{(1-\gamma/2)} - 1)} \sum_{i=1}^n \left[ \begin{aligned} & \left( \frac{(\mu_P^-(c_i))^2 + (\mu_Q^-(c_i))^2}{2} \right)^{\gamma/2} - \frac{(\mu_P^-(c_i))^\gamma + (\mu_Q^-(c_i))^\gamma}{2} + \left( \frac{(\mu_P^+(c_i))^2 + (\mu_Q^+(c_i))^2}{2} \right)^{\gamma/2} \\ & - \frac{(\mu_P^+(c_i))^\gamma + (\mu_Q^+(c_i))^\gamma}{2} + \left( \frac{(v_P^-(c_i))^2 + (v_Q^-(c_i))^2}{2} \right)^{\gamma/2} - \frac{(v_P^-(c_i))^\gamma + (v_Q^-(c_i))^\gamma}{2} \\ & + (v_P^+(c_i))^2 + (v_Q^+(c_i))^{\gamma/2} - \frac{(v_P^+(c_i))^\gamma + (v_Q^+(c_i))^\gamma}{2} - \left( \frac{(\pi_P^-(c_i))^2 + (\pi_Q^-(c_i))^2}{2} \right)^{\gamma/2} \\ & - \frac{(\pi_P^-(c_i))^\gamma + (\pi_Q^-(c_i))^\gamma}{2} + \left( \frac{(\pi_P^+(c_i))^2 + (\pi_Q^+(c_i))^2}{2} \right)^{\gamma/2} - \frac{(\pi_P^+(c_i))^\gamma + (\pi_Q^+(c_i))^\gamma}{2} \end{aligned} \right] \tag{4}$$

**Theorem 1** The function  $I_1(P, Q)$  is a valid IVIF-DiM. This meets the following properties:

P1  $I_1(P, Q) = I_1(Q, P)$ ,

P2  $I_1(P, Q) = 0 \Leftrightarrow P = Q$ ,

P3  $I_1(P, P \cap Q) = I_1(Q, P \cup Q)$ ,

P4  $I_1(P, Q) = I_1(P^c, Q^c)$ ,

P5  $I_1(P, Q^c) = I_1(P^c, Q)$ ,

P6  $I_1(P, P^c) = 1$  iff  $P$  is a crisp set.

To express the advantage of the DiM, we compare it with the extant DiMs. The comparison is discussed with the broadly utilized counter-intuitive cases. First, the existing IVIF-DiMs are reviewed as.

(i) Zhang, Yao, and Zhang (2011):

$$I_Z(P, Q) = \sum_{i=1}^n \left[ \mu_P(c_i) \ln \frac{\mu_P(c_i)}{\frac{1}{2}(\mu_P(c_i) + \mu_Q(c_i))} + (1 - \mu_P(c_i)) \ln \frac{1 - \mu_P(c_i)}{\frac{1}{2}(2 - \mu_P(c_i) - \mu_Q(c_i))} \right],$$

where  $\mu_P(c_i)$  and  $\mu_Q(c_i)$  stand for the average of the BDs of  $c_i$  to IVIFSs  $P$  and  $Q$ , respectively, with

$$\mu_P(c_i) = \frac{1}{2} \left[ \frac{\mu_P^+(c_i) + \mu_P^-(c_i)}{2} + 1 - \frac{v_P^+(c_i) + v_P^-(c_i)}{2} \right] = \frac{\mu_P^+(c_i) + \mu_P^-(c_i) + 2 - v_P^+(c_i) - v_P^-(c_i)}{4}$$

$$\mu_Q(c_i) = \frac{1}{2} \left[ \frac{\mu_Q^+(c_i) + \mu_Q^-(c_i)}{2} + 1 - \frac{v_Q^+(c_i) + v_Q^-(c_i)}{2} \right] = \frac{\mu_Q^+(c_i) + \mu_Q^-(c_i) + 2 - v_Q^+(c_i) - v_Q^-(c_i)}{4}$$

(ii) Ye (2011):

$$I_Y(P, Q) = \sum_{i=1}^n \left[ \left( \frac{\mu_P^+(c_i) + \mu_P^-(c_i) + 2 - v_P^+(c_i) - v_P^-(c_i)}{4} \right) \right. \\ \left. \log_2 \frac{\mu_P^+(c_i) + \mu_P^-(c_i) + 2 - v_P^+(c_i) - v_P^-(c_i)}{(1/2) \cdot \left[ (\mu_P^+(c_i) + \mu_P^-(c_i) + 2 - v_P^+(c_i) - v_P^-(c_i)) + (\mu_Q^+(c_i) + \mu_Q^-(c_i) + 2 - v_Q^+(c_i) - v_Q^-(c_i)) \right]} \right. \\ \left. + \sum_{i=1}^n \left( \frac{v_P^+(c_i) + v_P^-(c_i) + 2 - \mu_P^+(c_i) - \mu_P^-(c_i)}{4} \right) \right. \\ \left. \log_2 \frac{v_P^+(c_i) + v_P^-(c_i) + 2 - \mu_P^+(c_i) - \mu_P^-(c_i)}{(1/2) \cdot \left[ (v_P^+(c_i) + v_P^-(c_i) + 2 - \mu_P^+(c_i) - \mu_P^-(c_i)) + (v_Q^+(c_i) + v_Q^-(c_i) + 2 - \mu_Q^+(c_i) - \mu_Q^-(c_i)) \right]} \right]$$

(iii) Meng et al. (2013):

$$I_{MT}(P, Q) = \frac{1}{n} \sum_{i=1}^n \left[ \frac{\mu_P^o(c_i)}{\mu_P^o(c_i) + v_P^o(c_i)} \log_2 \frac{1 + \frac{2\mu_P^o(c_i)}{\mu_P^o(c_i) + v_P^o(c_i)}}{\left( 1 + \frac{\mu_P^o(c_i)}{\mu_P^o(c_i) + v_P^o(c_i)} + \frac{\mu_Q^o(c_i)}{\mu_Q^o(c_i) + v_Q^o(c_i)} \right)} + \frac{v_P^o(c_i)}{\mu_P^o(c_i) + v_P^o(c_i)} \log_2 \frac{1 + \frac{2v_P^o(c_i)}{\mu_P^o(c_i) + v_P^o(c_i)}}{\left( 1 + \frac{v_P^o(c_i)}{\mu_P^o(c_i) + v_P^o(c_i)} + \frac{v_Q^o(c_i)}{\mu_Q^o(c_i) + v_Q^o(c_i)} \right)} \right]$$

where  $\mu_E^o(c_i) = \mu_E^+(c_i) + \mu_E^-(c_i)$  and  $v_E^o(c_i) = v_E^+(c_i) + v_E^-(c_i)$  for  $E = P, Q$ .

(iv) Mishra et al. (2020b):

$$I_{M1}(P, Q) = 1 - \log_2 \left[ 1 + \frac{1}{2n} \left\{ \begin{aligned} &\min \{ \mu_P^-(c_i), \mu_Q^-(c_i) \} + \min \{ \mu_P^+(c_i), \mu_Q^+(c_i) \} \\ &+ \min \{ v_P^-(c_i), v_Q^-(c_i) \} + \min \{ v_P^+(c_i), v_Q^+(c_i) \} \\ &+ \min \{ \pi_P^-(c_i), \pi_Q^-(c_i) \} + \min \{ \pi_P^+(c_i), \pi_Q^+(c_i) \} \end{aligned} \right\} \right]$$

**Example 1** Let  $P_i$  and  $Q_i$  be two IVIFNs as shown in Table 2. We evaluate the degree of discrimination between  $P_i$  and  $Q_i$  by various IVIF-DiMs. The outcomes are portrayed in Table 2.

In Table 2, from the first and fourth columns,  $I_Z(P_1, Q_1) = I_Z(P_4, Q_4)$ ,  $I_Y(P_1, Q_1) = I_Y(P_4, Q_4)$ , ( $i = 1, 4$ ), when  $P_1 \neq P_2, Q_1 \neq Q_4$ . In the similar way, from the first, third and fourth columns,  $I_{M1}(P_1, Q_1) = I_{M1}(P_2, Q_2) = I_{M1}(P_4, Q_4)$ , ( $i = 1, 2, 4$ ), when  $P_1 = P_2 \neq P_4, Q_1 \neq Q_2 = Q_4$ . From the third column,  $I_Z(P_3, Q_3) = I_Y(P_3, Q_3) = 0$ , when  $P_3 \neq Q_3$ . On a similar line, regarding the DiM  $I_{MT}(P, Q)$ , all entries are negative for

**Table 2** Comparison of various IVIF-DiMs with counter-intuitive sets

$P_i$	([0.2, 0.3], [0.4, 0.6])	([0.2, 0.3], [0.4, 0.6])	([0.2, 0.3], [0.3, 0.5])	([0.2, 0.3], [0.3, 0.5])
$Q_i$	([0.3, 0.4], [0.4, 0.6])	([0.3, 0.4], [0.3, 0.5])	([0.3, 0.4], [0.4, 0.6])	([0.3, 0.4], [0.3, 0.5])
$I_Z(P, Q)$	0.001	0.005	0.000	0.001
$I_Y(P, Q)$	0.001	0.005	0.000	0.001
$I_{MT}(P, Q)$	-0.420	-0.411	-0.420	-0.414
$I_{MI}(P, Q)$	0.074	0.074	0.152	0.074
$I_1(P, Q)$	0.075	0.015	0.122	0.023

$P_i$  and  $Q_i$  ( $i = 1, 2, 3, 4$ ). Therefore, we can establish that the similarity measures  $I_Z(P, Q)$ ,  $I_Y(P, Q)$ ,  $I_{MT}(P, Q)$  and  $I_{MI}(P, Q)$  are not reasonable. For now, we observe that  $I_Z(P_3, Q_3) = I_Y(P_3, Q_3) = 0$ , when  $P_3 \neq Q_3$ , which does not hold the axiom (D2) of Definition 5. Therefore, we determine that the developed IVIF-DiM  $I_1(P, Q)$  tackles these flaws. Hence, the proposed IVIF-DiM is more practical and sensible than the existing ones.

### 5 The divergence measure-based IVIF-DNMA method

Since the IVIFSs have been broadly used to address the uncertain decision-making problems, therefore, we investigate the DNMA framework with IVIFSs to treat the concerns. The procedural steps for IVIF-DNMA framework are presented (graphically illustrated in Fig. 2) by.

*Step 1* Originate the “decision-matrix.”

For MCDM, Let consider  $B = \{b_1, b_2, \dots, b_p\}$  be set of options and organize attribute set  $A = \{a_1, a_2, \dots, a_q\}$ . The DE offers his/her evaluation degree  $\lambda_{ij}$  of options  $b_i$  ( $i = 1, 2, \dots, p$ ) over criteria  $a_j$  ( $j = 1, 2, \dots, q$ ) by “linguistic values (LVs).” Then, we convert the LVs into IVIFNs, which creates a “linguistic decision-matrix (LDM)”  $M = (\lambda_{ij})_{p \times q}$  with  $\lambda_{ij} = ([p_{ij}, q_{ij}], [r_{ij}, s_{ij}])$ .

*Step 2* Computing the weights of experts.

Consider  $\ell$  experts with importance weight vector  $\psi_k = (\psi_1, \psi_2, \dots, \psi_\ell)^T$ . The ratings are taken into LVs and described into IVIFNs. Let  $e_k = ((\mu_k^-, \mu_k^+), [v_k^-, v_k^+])$  be a rating of the  $k^{th}$  expert and it is evaluated by

$$\psi_k = \frac{(\mu_k^- + \mu_k^+)(2 + \pi_k^- + \pi_k^+)}{\sum_{k=1}^{\ell} ((\mu_k^- + \mu_k^+)(2 + \pi_k^- + \pi_k^+) )}, \quad k = 1, 2, \dots, \ell \tag{5}$$

Clearly,  $\psi_k \geq 0$  and  $\sum_{k=1}^{\ell} \psi_k = 1$ .

*Step 3* Obtaining the AIVIF-DM.

Assume that  $Z = (z_{ij}^{(k)})$  is the LDM of the  $k^{th}$  expert, where  $z_{ij}^{(k)} = \langle [p_{ijk}^-, q_{ijk}^+], [r_{ijk}^-, s_{ijk}^+] \rangle$ ,  $k = 1, 2, \dots, \ell$ . Here, to combine all the distinct decisions and construct an aggregate decision, we must form an AIVIF-DM. To this end, let

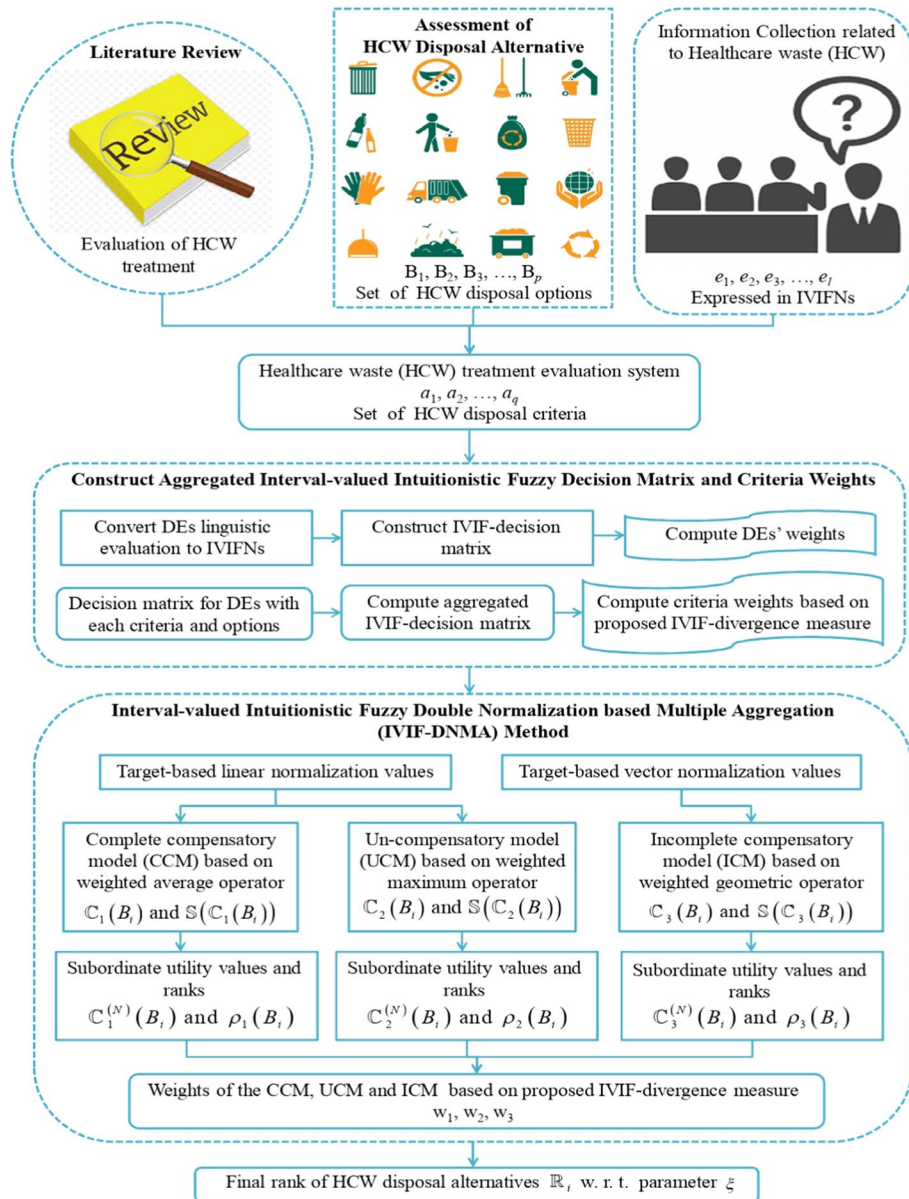


Fig. 2 Procedure of the IVIF-DNMA approach

$Z = [z_{ij}]_{p \times q}$  be the AIVIF-DM, where  $z_{ij} = \langle [p_{ij}, q_{ij}], [r_{ij}, s_{ij}] \rangle, i = 1, 2, \dots, p, j = 1, 2, \dots, q$ . Then,  $Z = \sum_{k=1}^{\ell} \psi_k z_{ij}^{(k)}$  and

$$z_{ij} = \left\langle \left[ 1 - \prod_{k=1}^{\ell} (1 - p_{ijk}^-)^{\psi_k}, 1 - \prod_{k=1}^{\ell} (1 - q_{ijk}^+)^{\psi_k} \right], \left[ \prod_{k=1}^{\ell} (r_{ijk}^-)^{\psi_k}, \prod_{k=1}^{\ell} (s_{ijk}^+)^{\psi_k} \right] \right\rangle \tag{6}$$

*Step 4* Computing the attribute weights.

Let  $\varpi = (\varpi_1, \varpi_2, \dots, \varpi_q)^T$  be the attribute weights, where  $\sum_{j=1}^q \varpi_j = 1, \varpi_j \in [0, 1]$ . Successively for the purpose of obtaining  $\varpi_j$ , the following equation is applied as

$$\varpi_j = \frac{\frac{1}{p-1} \left( \sum_{i=1}^p \sum_{k=1, k \neq i}^p I_1(z_{ij}, z_{kj}) \right)}{\sum_{j=1}^q \left( \frac{1}{p-1} \left( \sum_{i=1}^p \sum_{k=1, k \neq i}^p I_1(z_{ij}, z_{kj}) \right) \right)}, \quad j = 1, 2, \dots, q \tag{7}$$

*Step 5* Assessment of the normalized AIVIF-DM.

With the use of an appropriate normalization method, the efficiency of the final decision can be improved (Belton & Gear, 1985). Here, we propose linear and vector normalization formulae. Both the numerical values and IVIFNs are managed by these formulae. A linear normalization approach is given by

$$\mathbb{N}^{(1)} = \left( \eta_{ij}^{(1)} \right)_{p \times q}, \quad \text{where} \quad \eta_{ij}^{(1)} = \left( \left[ \bar{p}_{ij}^{(1)}, \bar{q}_{ij}^{(1)} \right], \left[ \bar{r}_{ij}^{(1)}, \bar{s}_{ij}^{(1)} \right] \right) = \begin{cases} \frac{z_{ij}}{\max_i \mathbb{S}(z_{ij})}, & j \in a_b \\ 1 - \frac{z_{ij}}{\max_i \mathbb{S}(z_{ij})}, & j \in a_n, \end{cases} \tag{8}$$

where  $\mathbb{S}(\cdot)$  is a score degree of IVIFNs.

Next, we utilize vector normalization to obtain the normalizes AIVIF-DM  $Z = [z_{ij}]_{p \times q}$  with  $z_{ij} = \langle [p_{ij}, q_{ij}], [r_{ij}, s_{ij}] \rangle$  into  $\mathbb{N}^{(2)} = \left( \eta_{ij}^{(2)} \right)_{p \times q}$ , where

$$\eta_{ij}^{(2)} = \begin{cases} \left( \left[ \bar{p}_{ij}^{(2)}, \bar{q}_{ij}^{(2)} \right], \left[ \bar{r}_{ij}^{(2)}, \bar{s}_{ij}^{(2)} \right] \right), & j \in a_b, \\ \left( \left[ \bar{r}_{ij}^{(2)}, \bar{s}_{ij}^{(2)} \right], \left[ \bar{p}_{ij}^{(2)}, \bar{q}_{ij}^{(2)} \right] \right), & j \in a_n, \end{cases} \tag{9}$$

$$\bar{p}_{ij}^{(2)} = \frac{p_{ij}}{\left( \sum_{i=1}^p \left\{ (p_{ij})^2 + (q_{ij})^2 \right\} \right)^{1/2}}, \quad \bar{q}_{ij}^{(2)} = \frac{q_{ij}}{\left( \sum_{i=1}^p \left\{ (p_{ij})^2 + (q_{ij})^2 \right\} \right)^{1/2}}, \tag{10}$$

$$\bar{r}_{ij}^{(2)} = \frac{r_{ij}}{\left( \sum_{i=1}^p \left\{ (r_{ij})^2 + (s_{ij})^2 \right\} \right)^{1/2}}, \quad \bar{s}_{ij}^{(2)} = \frac{s_{ij}}{\left( \sum_{i=1}^p \left\{ (r_{ij})^2 + (s_{ij})^2 \right\} \right)^{1/2}}, \quad i = 1, 2, \dots, p, j = 1, 2, \dots, q. \tag{11}$$

*Step 6* Implement the the “subordinate aggregation models (SAMs).”

Here, we present three different SAMs with diverse normalization models.

*Step 6.1* The “complete compensatory method (CCM).”

Zeleny (1982) discussed a function  $r(d; p)$  which is an AO applicable to measuring the discrimination of option  $B_i$  to the “Ideal Solution (IS)”  $B^*$ :

$$r(d;p) = \left\{ \sum_{j=1}^q \left( \varpi_j |d^{ij} - r_j| \right)^p \right\}^{1/p}, \quad 1 \leq p \leq \infty,$$

where  $\varpi_j$  signifies the criteria weight.

The functions  $r(d;p)$  of  $p = 1$  and  $p = \infty$  have been applied to the VIKOR approach and the function  $r(d;p)$  of  $p = 2$  to the conventional TOPSIS approach. Each criterion holds a weight; as a result, we do not have any reason for the addition of another weight to a bigger one. Therefore, we make use of the function  $r(d;p)$  of  $p = 1$  as the primary AO of the developed IVIF-DNMA approach. For more details, see (Shi et al., 2017). Therefore, the CCM is defined using the IVIFWAO as follows:

$$C_1(B_i) = \left( \left[ \hat{p}_{ij}^{(1)}, \hat{q}_{ij}^{(1)} \right], \left[ \hat{r}_{ij}^{(1)}, \hat{s}_{ij}^{(1)} \right] \right) = \bigoplus_{j=1}^q \varpi_j \eta_{ij}^{(1)} \tag{12}$$

The options can be prioritized using the  $C_1(B_i) : i = 1, 2, \dots, p$  in a decreasing manner, and we obtain the ranking outcomes  $\rho_1(B_i) : i = 1, 2, \dots, p$ .

*Step 6.2* The “un-compensatory method (UCM).”

For the avoidance of a situation in which the chosen solution has a very improper performance in some criterion, the function  $r(d;p)$  with  $p = \infty$ , termed the weighted maximum operator is used for the purpose of composing the second aggregation function, as shown below:

$$C_2(B_i) = \left( \left[ \hat{p}_{ij}^{(2)}, \hat{q}_{ij}^{(2)} \right], \left[ \hat{r}_{ij}^{(2)}, \hat{s}_{ij}^{(2)} \right] \right) = \max_j \varpi_j \left( \eta_{ij}^{(1)} \right)^c \tag{13}$$

The options can be ranked using the  $C_2(B_i) : i = 1, 2, \dots, p$  in a descending way, and we get the ranks as  $\rho_2(B_i) : i = 1, 2, \dots, p$ .

*Step 6.3* The “incomplete compensatory method (ICM).”

The vector normalization process with the IVIFWGO is used as follows:

$$C_3(B_i) = \left( \left[ \hat{p}_{ij}^{(3)}, \hat{q}_{ij}^{(3)} \right], \left[ \hat{r}_{ij}^{(3)}, \hat{s}_{ij}^{(3)} \right] \right) = \bigotimes_{j=1}^n \varpi_j \eta_{ij}^{(2)} \tag{14}$$

The options s can be ranked by arranging  $C_3(B_i) : i = 1, 2, \dots, p$  in a descending mode, and we get the ranks as  $\rho_3(B_i) : i = 1, 2, \dots, p$ .

*Step 7* Obtain of “subordinate utility degrees (SUDs).”

The three considered models are represented as CCM ( $\mathbb{Q}_1$ ), UCM ( $\mathbb{Q}_2$ ) and ICM ( $\mathbb{Q}_3$ ). Each alternative  $B_i$  has two types of aspects: the SUDs  $C_\tau(B_i) : i = 1, 2, \dots, p$  and the preference order  $\rho_\tau(B_i) : i = 1, 2, \dots, p$  with each attribute  $\mathbb{Q}_\tau : \tau = 1, 2, 3$ . Clearly, we define two “decision-matrices (DMs)”: the SUD-DM  $\mathcal{A}(\mathbb{C}) = [C_\tau(B_i)]_{m \times 3}$  and the ranking-DM  $\mathcal{A}(\rho) = [\rho_\tau(B_i)]_{m \times 3}$ .

$$\mathcal{A}(\mathbb{C}) = \begin{bmatrix} C_1(B_1) & C_2(B_1) & C_3(B_1) \\ \vdots & \vdots & \vdots \\ C_1(B_i) & C_2(B_i) & C_3(B_i) \\ \vdots & \vdots & \vdots \\ C_1(B_p) & C_2(B_p) & C_3(B_p) \end{bmatrix} \text{ and } \mathcal{A}(\rho) = \begin{bmatrix} \rho_1(B_1) & \rho_2(B_1) & \rho_3(B_1) \\ \vdots & \vdots & \vdots \\ \rho_1(B_i) & \rho_2(B_i) & \rho_3(B_i) \\ \vdots & \vdots & \vdots \\ \rho_1(B_p) & \rho_2(B_p) & \rho_3(B_p) \end{bmatrix}.$$

To reserve the innovativeness of the SUDs  $C_\tau(B_i) : \tau = 1, 2, 3$ , the normalized values are defined as

$$C_{\tau}^{(N)}(B_i) = ([\hat{p}_{\tau}^{(N)}, \hat{q}_{\tau}^{(N)}], [\hat{r}_{\tau}^{(N)}, \hat{s}_{\tau}^{(N)}]); \tau = 1, 2, 3,$$

where  $\hat{p}_{ij}^{(N)} = \frac{\hat{p}_{ij}^{(\tau)}}{\left(\sum_{i=1}^p \left\{ \left(\hat{p}_{ij}^{(\tau)}\right)^2 \right\}\right)^{1/2}}, \hat{q}_{ij}^{(N)} = \frac{\hat{q}_{ij}^{(\tau)}}{\left(\sum_{i=1}^p \left\{ \left(\hat{q}_{ij}^{(\tau)}\right)^2 \right\}\right)^{1/2}},$

$$\hat{r}_{ij}^{(N)} = \frac{\hat{r}_{ij}^{(\tau)}}{\left(\sum_{i=1}^p \left\{ \left(\hat{r}_{ij}^{(\tau)}\right)^2 \right\}\right)^{1/2}}, \hat{s}_{ij}^{(N)} = \frac{\hat{s}_{ij}^{(\tau)}}{\left(\sum_{i=1}^p \left\{ \left(\hat{s}_{ij}^{(\tau)}\right)^2 \right\}\right)^{1/2}}, \tau = 1, 2, 3, i = 1, 2, \dots, p, j = 1, 2, \dots, q$$

(15)

Step 8 Assess the “overall utility degree (OUD)” of each alternative.

A coefficient  $\xi \in [0, 1]$  is considered to illustrate the SUDs and the prioritizations of alternatives. Here, we assume  $\xi = 0.5$ . The OUD of alternative is defined as

$$\mathbb{R}_i = \left[ \begin{array}{l} \left[ \begin{array}{l} w_1 * \sqrt{\xi \left( C_1^{(N)}(B_i) / \max_i C_1^{(N)}(B_i) \right)^2 + (1 - \xi) \left( \frac{p - \rho_1(B_i) + 1}{p} \right)^2} \\ -w_2 * \sqrt{\xi \left( C_2^{(N)}(B_i) / \max_i C_2^{(N)}(B_i) \right)^2 + (1 - \xi) \left( \frac{\rho_2(B_i)}{p} \right)^2} \\ +w_3 * \sqrt{\xi \left( C_3^{(N)}(B_i) / \max_i C_3^{(N)}(B_i) \right)^2 + (1 - \xi) \left( \frac{p - \rho_3(B_i) + 1}{p} \right)^2} \end{array} \right] + 1 \end{array} \right] \tag{16}$$

where  $w_1, w_2$  and  $w_3$  are the weights of three models, respectively, with  $w_1 + w_2 + w_3 = 1$ . Here, the weight  $w_1, w_2$  and  $w_3$  is obtained using the developed IVIF-divergence measure. The ultimate preference set  $\rho = \{\rho(B_1), \rho(B_2), \rho(B_3), \dots, \rho(B_p)\}$  is found in descending value of  $\mathbb{R}_i : i = 1, 2, \dots, p$ .

### 6 Case study about the healthcare waste treatment selection

Here, we present a case study carried a sample of six hospitals in Himachal Pradesh (India) to illustrate how the proposed framework can be employed effectually in selecting the optimal HCWT method.

In the first step, for the purpose of this study, we prepare the information concerning ten hospitals that usage HCWT technologies. Only six hospitals have been allowed to collect the information and other related information to assist this study. Out of these six hospitals, three are from the public domain, and the remaining is from the private domain. All considered hospitals have the necessary structures about the HCWT methods; though, after the duration of data collection, it was observed that the considered hospitals lacked advanced high-tech facilities associated with HCWT technologies. The considered hospitals’ details due to the ethical standards of the work cannot be revealed. For data collection regarding the HCW in these hospitals, there was required to get the approval of the administrator because of the ethical approval from the hospitals, though the data collection was approximately three months in the period of 2019 September and 2019 December. There are numerous criteria that are essential to choosing the optimal HCWT procedures.

**Table 3** LVs for the importance ratings of attributes and DEs

LVs	IVIFNs
Extremely important (EI)	([0.9,0.95], [0, 0.05])
Very important (VI)	([0.75, 0.85], [0.1, 0.15])
Important (I)	([0.6, 0.7], [0.15, 0.25])
Moderate (M)	([0.45, 0.6], [0.3, 0.4])
Unimportant (U)	([0.3, 0.45], [0.4,0.55])
Very unimportant (VU)	([0.15, 0.3], [0.6, 0.7])
Extremely unimportant (EU)	([0, 0.1], [0.8, 0.9])

**Table 4** LTs for the rating of criteria and options

LVs	IVIFNs
Extremely poor (EP)	([0.05, 0.1], [0.85, 0.9])
Very poor (VP)	([0.1, 0.2], [0.7, 0.75])
Poor (P)	([0.2, 0.3], [0.55, 0.65])
Medium poor (MP)	([0.30, 0.40], [0.45, 0.55])
Medium (M)	([0.45, 0.55], [0.35, 0.4])
Medium good (MG)	([0.55, 0.65], [0.3, 0.35])
Good (G)	([0.65, 0.75], [0.2, 0.25])
Very good (VG)	([0.75, 0.8], [0.15, 0.2])
Very very good (VVG)	([0.8, 0.85], [0.1, 0.15])
Extremely good (EG)	([0.85, 0.9], [0.05, 0.1])

An analysis (see Table 1) was given with an interview, feedback, and literature review to evaluate the critical attributes for assessing the appropriate “HCW treatment (HCWT)” technologies.

To choose the best HCWT technology, an expert committee from various departments or institutions was created consisting of four experts ( $e_1, e_2, e_3, e_4$ ). The committee comprised two fields DEs from a waste disposal firm, an environmental engineer, a professor of industrial engineering, and an HCW management expert. Next, the procedure of implementing the IVIF-DNMA approach to HCW treatment method selection is given.

Each expert performs his/her knowledge concerning the rating of HCWT alternative under the considered attributes. Tables 3 and 4 considered from Khoshnava et al. (2020), Alrasheedi et al. (2021), Mishra et al. (2023) signify the LVs in the IVIFNs to calculate the weights of considered criteria and the HCWT options by DEs. Based on Table 3 and Eq. (5), the DEs’ weights are obtained and given in Table 5. Table 6 expresses the DMs of each DE  $e_k$ : 1,2,3,4, regarding the performance of suppliers.

By Eq. (6), Tables 5 and 6, the AIVIF-DM is obtained and presented in Table 7. Next, by Eq. (7) and Table 6, the attribute weights for HCWT selection problem are calculated as:  $w_j = (0.096, 0.047, 0.177, 0.089, 0.053, 0.108, 0.115, 0.155, 0.16)$ .

According to Eqs. (8), (9), (10) (11) and Table 7, the linear and vector normalized AIVIF-DM is assessed and presented in Tables 8 and 9.

The SUDs of the CCM, UCM and ICM are obtained by Eqs. (12), (13), (14) and portrayed in Table 10. Corresponding to Eq. (15), the normalized SUDs of these models are computed and are presented in Table 11. Next, from Eq. (3) and normalized SUDs, the weights of SUDs are obtained and given in Table 11.



**Table 5** DEs' weight assessment

DEs	$e_1$	$e_2$	$e_3$	$e_4$
LVs/ratings	I	M	VI	VI
IVIFNs	([0.60, 0.70], [0.15, 0.25])	([0.45, 0.60], [0.30, 0.40])	([0.75, 0.85], [0.10, 0.15])	([0.75, 0.85], [0.10, 0.15])
Weights ( $\psi_k$ )	0.245	0.193	0.281	0.281

**Table 6** LDM of options presented by the DEs

	$b_1$	$b_2$	$b_3$	$b_4$
$a_1$	(G, VG, G, VG)	(F, P, P, P)	(MP, F, F, F)	(F, F, P, F)
$a_2$	(P, F, P, MG)	(F, F, G, F)	(P, P, P, P)	(P, VP, F, VP)
$a_3$	(VG, G, G, G)	(G, G, G, MG)	(P, MP, P, P)	(P, P, VP, P)
$a_4$	(M, M, M, M)	(P, P, P, G)	(P, G, M, P)	(VG, VG, VG, VG)
$a_5$	(VG, G, G, VG)	(P, F, F, G)	(F, MG, F, F)	(F, F, G, VG)
$a_6$	(G, G, G, MG)	(VP, MP, P, P)	(MG, F, F, F)	(G, VG, MG, F)
$a_7$	(G, G, G, M)	(G, G, VG, G)	(G, G, G, G)	(M, P, P, VP)
$a_8$	(P, P, VL, M)	(G, VG, VG, G)	(M, M, G, G)	(P, P, VP, P)
$a_9$	(VG, G, G, MG)	(VG, G, VG, G)	(P, MP, P, P)	(P, P, P, F)

**Table 7** AIVIF-DM for the HCWT options

	$b_1$	$b_2$	$b_3$	$b_4$
$a_1$	([0.702, 0.775], [0.174, 0.225])	([0.270, 0.372], [0.492, 0.577])	([0.417, 0.517], [0.372, 0.432])	([0.389, 0.49], [0.397, 0.458])
$a_2$	([0.367, 0.471], [0.425, 0.497])	([0.516, 0.619], [0.299, 0.35])	([0.20, 0.30], [0.55, 0.65])	([0.239, 0.341], [0.543, 0.61])
$a_3$	([0.678, 0.763], [0.186, 0.237])	([0.624, 0.725], [0.224, 0.275])	([0.220, 0.320], [0.529, 0.629])	([0.173, 0.273], [0.589, 0.677])
$a_4$	([0.450, 0.550], [0.350, 0.400])	([0.366, 0.476], [0.414, 0.497])	([0.386, 0.493], [0.398, 0.471])	([0.75, 0.80], [0.15, 0.20])
$a_5$	([0.707, 0.778], [0.172, 0.222])	([0.469, 0.575], [0.334, 0.395])	([0.471, 0.57], [0.340, 0.390])	([0.612, 0.697], [0.236, 0.289])
$a_6$	([0.624, 0.725], [0.224, 0.275])	([0.198, 0.298], [0.561, 0.652])	([0.476, 0.577], [0.337, 0.387])	([0.60, 0.689], [0.248, 0.30])
$a_7$	([0.603, 0.705], [0.234, 0.285])	([0.682, 0.765], [0.184, 0.235])	([0.650, 0.750], [0.200, 0.250])	([0.245, 0.348], [0.527, 0.600])
$a_8$	([0.256, 0.360], [0.518, 0.590])	([0.702, 0.775], [0.174, 0.225])	([0.573, 0.677], [0.255, 0.307])	([0.173, 0.273], [0.589, 0.677])
$a_9$	([0.654, 0.74], [0.210, 0.260])	([0.707, 0.778], [0.172, 0.222])	([0.220, 0.320], [0.529, 0.629])	([0.28, 0.382], [0.484, 0.567])

From Eq. (20), the normalized SUDs, the OUDs, and the ranks of HCWT options are estimated and shown in Table 11. Therefore, the ranking of HCWT is  $B_2 > B_3 > B_1 > B_4$  and the option  $B_2$  is the optimal HCWT options.

**Table 8** Linear normalization values for the HCWT options

	$b_1$	$b_2$	$b_3$	$b_4$
$a_1$	([0.255, 0.311], [0.612, 0.689])	([0.575, 0.651], [0.218, 0.305])	([0.462, 0.519], [0.344, 0.434].)	([0.486, 0.543], [0.32, 0.41])
$a_2$	([0.586, 0.646], [0.248, 0.328])	([0.47, 0.52], [0.364, 0.452])	([0.689, 0.764], [0.13, 0.20])	([0.683, 0.732], [0.156, 0.23])
$a_3$	([0.276, 0.33], [0.58, 0.669])	([0.318, 0.37], [0.528, 0.628])	([0.614, 0.70], [0.174, 0.256])	([0.666, 0.74], [0.136, 0.217])
$a_4$	([0.426, 0.475], [0.385, 0.477])	([0.488, 0.566], [0.309, 0.408])	([0.473, 0.543], [0.327, 0.424])	([0.214, 0.27], [0.676, 0.73])
$a_5$	([0.618, 0.693], [0.251, 0.307])	([0.392, 0.489], [0.423, 0.482])	([0.393, 0.486], [0.428, 0.477])	([0.524, 0.608], [0.321, 0.377])
$a_6$	([0.507, 0.606], [0.34, 0.393])	([0.147, 0.225], [0.659, 0.734])	([0.373, 0.462], [0.456, 0.504])	([0.484, 0.57], [0.366, 0.42])
$a_7$	([0.508, 0.609], [0.327, 0.38])	([0.585, 0.672], [0.273, 0.328])	([0.554, 0.656], [0.29, 0.344])	([0.195, 0.28], [0.61, 0.676])
$a_8$	([0.206, 0.293], [0.598, 0.662])	([0.611, 0.689], [0.255, 0.311])	([0.486, 0.586], [0.344, 0.397])	([0.138, 0.221], [0.661, 0.737])
$a_9$	([0.565, 0.652], [0.292, 0.347])	([0.618, 0.693], [0.251, 0.307])	([0.178, 0.262], [0.607, 0.695])	([0.227, 0.314], [0.566, 0.641])

### 6.1 Comparison with other models

Here, comparison analyses between the results assessed from the IVIF-DNMA with different extant models are given. To clarify the usefulness and exhibit the distinctive benefits of the IVIF-DNMA framework, the IVIF-WASPAS (Mishra & Rani, 2018a, 2018b) and IVIF-COPRAS (Wang et al., 2016) models are applied to tackle aforementioned MCDM problem.

#### 6.1.1 Comparing with the IVIF-WASPAS model

Steps 1–4 Same as those in the developed method.

Step 5 Obtain the “normalized-AIVIF-DM”  $Z = [z_{ij}]_{p \times q}$  with  $z_{ij} = \langle [a_{ij}, b_{ij}], [c_{ij}, d_{ij}] \rangle$ , into  $\mathbb{N}^{(1)} = (\eta_{ij}^{(1)})_{p \times q}$ , where  $\eta_{ij}^{(1)}$  is given by Eq. (8).

Step 6 Compute the degree  $S_i^{(1)} = \bigoplus_{j=1}^n \varpi_j \eta_{ij}^{(1)}$  of the weighted sum method (WSM).

Step 7 Compute the degree  $S_i^{(2)} = \bigotimes_{j=1}^n \varpi_j \eta_{ij}^{(1)}$  of the weighted product method (WPM).

Step 8 Calculate the combined degree  $S_i = \lambda S_i^{(1)} + (1 - \lambda) S_i^{(2)}$  of the WASPAS measure for each option, where  $\lambda \in [0, 1]$  stands for the coefficient of strategic accuracy.

Step 9 Prioritization of options by decreasing score values of  $S_i$ .

Next, the results of HCWT assessment are discussed in Table 12.

Therefore, the preference order of the HCWT options is  $B_2 > B_3 > B_1 > B_4$  and the option  $B_2$  is with the highest suitability degree of the HCWT options.

**Table 9** Vector normalization values for the HCWT options

	$b_1$	$b_2$	$b_3$	$b_4$
$a_1$	[(0.15, 0.193], [0.478, 0.53])	[(0.424, 0.497], [0.185, 0.254])	[(0.32, 0.372], [0.285, 0.354])	[(0.342, 0.394], [0.266, 0.335])
$a_2$	[(0.298, 0.349], [0.321, 0.412])	[(0.21, 0.246], [0.541], 0.541])	[(0.386, 0.456], [0.175, 0.262])	[(0.381, 0.426], [0.209, 0.298])
$a_3$	[(0.143, 0.182], [0.455, 0.513])	[(0.172, 0.211], [0.42, 0.487])	[(0.406, 0.483], [0.148, 0.215])	[(0.452, 0.519], [0.116, 0.184])
$a_4$	[(0.327, 0.374], [0.287, 0.351])	[(0.387, 0.464], [0.233, 0.303])	[(0.372, 0.441], [0.246, 0.314])	[(0.14, 0.187], [0.478, 0.51])
$a_5$	[(0.404, 0.444], [0.198, 0.256])	[(0.268, 0.329], [0.385, 0.455])	[(0.269, 0.326], [0.392, 0.449])	[(0.35, 0.398], [0.272, 0.332])
$a_6$	[(0.40, 0.464], [0.198, 0.243])	[(0.126, 0.191], [0.496, 0.576])	[(0.305, 0.369], [0.298, 0.342])	[(0.384, 0.441], [0.219, 0.265])
$a_7$	[(0.344, 0.402], [0.238, 0.29])	[(0.389, 0.436], [0.188, 0.239])	[(0.37, 0.428], [0.203, 0.254])	[(0.14, 0.198], [0.536, 0.611])
$a_8$	[(0.173, 0.243], [0.402, 0.458])	[(0.475, 0.525], [0.135, 0.174])	[(0.389, 0.458], [0.198, 0.238])	[(0.117, 0.185], [0.457, 0.525])
$a_9$	[(0.417, 0.472], [0.175, 0.218])	[(0.451, 0.496], [0.144, 0.186])	[(0.14, 0.205], [0.444, 0.528])	[(0.179, 0.244], [0.406, 0.475])

**6.1.2 Comparing with the IVIF-COPRAS method**

Steps 1–4 Same as those in the aforesaid method.

Step 5 Add the benefit and cost-type criteria values. In IVIF-COPRAS model, each alternative is exemplified with its sum of maximum degree of  $\alpha_i$  and minimum degree  $\beta_i$ . Let  $\Delta = \{1, 2, \dots, m\}$  be the benefit-type attributes, then the aggregate value for each alternative is given by

$$a_i^{(1)} = \bigoplus_{j=1}^m \varpi_j \eta_{ij}^{(2)}, \tag{17}$$

Let  $\nabla = \{m + 1, m + 2, \dots, q\}$  be the cost-type attribute. Then, the aggregated value is obtained as

$$a_i^{(2)} = \bigoplus_{j=m+1}^q \varpi_j \eta_{ij}^{(2)}. \tag{18}$$

where  $m$  and  $q$  are numbers of benefit-type and overall attributes, respectively.

Step 7 Calculate the “relative degree (RD)” ( $\gamma_i$ )

$$\gamma_i = \mathbb{S}(a_i^{(1)}) + \frac{\sum_{i=1}^p \mathbb{S}(a_i^{(2)})}{\mathbb{S}(a_i^{(2)}) \sum_{i=1}^p \frac{1}{\mathbb{S}(a_i^{(2)})}}. \tag{19}$$

where  $\mathbb{S}(a_i^{(1)})$  and  $\mathbb{S}(a_i^{(2)})$  signifies the score values of  $a_i^{(1)}$  and  $a_i^{(2)}$ , respectively.

Step 8 Assess the “utility degree (UD).” The UD is obtained by relating the entire obtained options over the best option. The UD ranges from 0 to 100%. The UD ( $\lambda_i$ ) is

**Table 10** SUDs for the HCWT options

HCWT	CCM(Q <sub>1</sub> )		UCM(Q <sub>2</sub> )		ICM(Q <sub>3</sub> )	
	C <sub>1</sub> (B <sub>i</sub> )	S*(C <sub>1</sub> (B <sub>i</sub> ))	C <sub>2</sub> (B <sub>i</sub> )	S*(C <sub>2</sub> (B <sub>i</sub> ))	C <sub>3</sub> (B <sub>i</sub> )	S*(C <sub>3</sub> (B <sub>i</sub> ))
b <sub>1</sub>	([0.425,0.507], [0.411, 0.481])	0.504	([0.143,0.178], [0.796, 0.822])	0.165	([0.256,0.311], [0.327, 0.383])	0.384
b <sub>2</sub>	([0.49,0.574], [0.339, 0.413])	0.58	([0.125,0.161], [0.816, 0.839])	0.146	([0.302,0.36], [0.287, 0.352])	0.445
b <sub>3</sub>	([0.475,0.565], [0.325, 0.402])	0.577	([0.138,0.172], [0.7587, 0.807])	0.164	([0.309,0.378], [0.269, 0.333])	0.463
b <sub>4</sub>	([0.414,0.495], [0.38, 0.466])	0.506	([0.154,0.187], [0.735, 0.790])	0.181	([0.233,0.30], [0.349, 0.415])	0.358

obtained by  $\lambda_i = \frac{\gamma_i}{\gamma_{max}} \times 100\%$ , where  $\gamma_i$  and  $\gamma_{max}$  are the importance of alternatives given by Eq. (19).

Here, the assessment of HCWT technologies is portrayed in Table 13.

Hence, the rank of the HCWT alternatives is  $B_3 > B_2 > B_1 > B_4$  and the option  $B_3$  is the best HCWT options.

Next, we present a comparison with existing works aiming at demonstrating the superiority of the proposed MCDM method. We have applied the similar case study using various extant models for making comparisons with the developed method.

- The proposed approach, Zavadskas, et al. (2014) approach, Wang et al. (2016) method and Mishra and Rani (2018b) method developed the methods using the IVIFSSs, unlike Hinduja & Pandey (2018) wherein the IFSs were used.
- In the proposed method, the two normalization approaches are implemented. The superiority of a developed framework depends on the sensible combination of normalization (for more details see (Liu & Wu, 2019), while Mishra and Rani (2018b) and Hinduja and Pandey (2018) used the linear normalization and Wang et al. (2016) applied the vector normalization method.
- To address vagueness in MCDM concerns, all the inputs, such as the assessments of options on attributes by various experts, expert and criteria weights, were measured uncertain and were characterized by IVIFNs, while Hinduja and Pandey (2018) considered uncertain and articulated by IFNs.

Moreover, we exemplify an organized comparison of the presented method with several methods using various parameters discussed in the MCDM procedures (see Table 14).

### 6.2 Sensitivity investigation

The sensitivity over the parameter  $\xi$  values of presented method is discussed. The changing of  $\xi$  is a assisting way to assess the sensitivity level of the IVIF-DNMA model by varying f from SUDs to the prioritizations of options. Table 15 and Fig. 3 signify the variation in the HCWT alternatives for different values of  $\xi$ . Based on the investigations, we get the same prioritization as  $B_2 > B_3 > B_1 > B_4$ , which implies steam sterilization ( $B_2$ ) is at the top of the ranking, while the microwave ( $B_3$ ) is ranked the second. Incineration ( $B_1$ ) is the

**Table 11** Normalized SUDs and OUDs of the HCWT options

HCWT	CCM( $\mathbb{Q}_1$ )		UCM( $\mathbb{Q}_2$ )		ICM( $\mathbb{Q}_3$ )		$\mathbb{R}_i(\xi = 0.5)$	Final Ranking
	$C_1^{(N)}(B_i)$	$\rho_1(B_i)$	$C_2^{(N)}(B_i)$	$\rho_2(B_i)$	$C_3^{(N)}(B_i)$	$\rho_3(B_i)$		
$b_1$		0.464 4		0.499 3		0.463 3	0.333	3
$b_2$		0.534 1		0.445 1		0.536 2	0.462	1
$b_3$		0.531 2		0.499 2		0.559 1	0.41	2
$b_4$		0.466 3		0.551 4		0.431 4	0.28	4
Weight value of model	$w_1 = 0.238$		$w_2 = 0.665$		$w_3 = 0.097$			

third, while Landfill disposal ( $B_4$ ) has the fourth rank. From Table 15, the presented IVIF-DNMA model is proficient of producing stable and, simultaneously, flexible prioritizations in variation in parameter.

### 7 Conclusions

For most of the developing countries, HCW management is an important challenge. This is because of the fact that any HCW mismanagement may lead to severer problems in public health and environmental pollutions through transmitting dangerous diseases by a variety of micro-organisms and viruses, and add to the list the risk of groundwater contamination that may take place by the untreated medical waste in landfills. As a result, to choose an appropriate treatment strategy for HCW is of a high significance in order to finally establish an effective HCW management system. This has been known as a complex MCDM problem since it requires taking into consideration numerous optional solutions and a variety of criteria in terms of benefit aspects, cost aspects, and target aspects. Furthermore, the experts generally make available inaccurate linguistic terms as their judgments, which is because of the fuzzy nature of structure constraints, imprecise information, and hesitancy. In the present paper, a novel ranking framework named the double normalization-based multi aggregation approach with IVIFSs (IVIF-DNMA) has been developed. A suitable combination of two normalization methods, three subordinate utility models, and a parametric divergence measure for criteria weights have been discussed to strengthen their benefits but evade their weaknesses. For criteria weights, a new parametric divergence measure for IVIFSs has been introduced and then compared with extant measures to illustrate the reasonability of proposed one. In addition, empirical research has been carried out in this paper, aiming at validating the proposed method. The empirical research has illustrated the implementation of the presented approach to the HCWT selection. Comparative discussion and sensitivity analysis have been confirmed its accuracy, adaptability, and efficacy. From the empirical work, it has been recognized that the developed approach is capable of efficiently handling the concern of HCW treatment method selection in IVIFSs.

The results of the presented IVIF-DNMA have been expressed that the ranks of HCWT technology assessment is consistent, and “steam sterilization (SS)” is the best HCWT technology in India. Also, results show that “incineration (IN)” is an operative HCWT method, but because of its high cost and adverse outcome on environment and public health; thus, it is not chosen as a suitable HCWT technology.

**Table 12** Implementation of IVIF-WASPAS for the HCWT options

HCWT	WSM	WPM			WASPAS		Ranking
	$S_i^{(1)}$	$\mathbb{S}(S_i^{(1)})$	$S_i^{(2)}$	$\mathbb{S}(S_i^{(2)})$	$S_i$	$\mathbb{S}(S_i)$	
$b_1$	([0.425,0.507], [0.411, 0.481])	0.504	([0.378,0.457], [0.453, 0.527])	0.453	([0.402,0.482], [0.432, 0.504])	0.479	3
$b_2$	([0.496,0.574], [0.338, 0.413])	0.58	([0.437,0.517], [0.384, 0.465])	0.521	([0.467,0.546], [0.36, 0.438])	0.551	1
$b_3$	([0.475,0.565], [0.325, 0.402])	0.577	([0.425,0.517], [0.372, 0.448])	0.523	([0.45,0.541], [0.347, 0.424])	0.551	2
$b_4$	([0.414,0.495], [0.38, 0.466])	0.506	([0.32,0.409], [0.478, 0.553])	0.404	([0.368,0.453], [0.426, 0.508])	0.457	4

**Table 13** Overall results of IVIF-COPRAS model for the HCWT options

$\alpha_i$	$\mathbb{S}(a_i^{(1)})$	$\beta_i$	$\mathbb{S}(a_i^{(2)})$	$\gamma_i$	$\lambda_i$	Ranking
$b_1$	([0.218, 0.261], [0.423, 0.481])	0.312	([0.197, 0.232], [0.505, 0.55])	0.269	0.438 81.81%	3
$b_2$	([0.247, 0.285], [0.391, 0.447])	0.348	([0.155, 0.194], [0.576, 0.62])	0.213	0.508 94.83%	2
$b_3$	([0.189, 0.234], [0.475, 0.527])	0.271	([0.09, 0.124], [0.669, 0.716])	0.128	0.535 100%	1
$b_4$	([0.133, 0.174], [0.565, 0.621])	0.192	([0.114, 0.144], [0.629, 0.674])	0.156	0.409 76.30%	4

Though the presented IVIF-DNMA model has offered significant insights into MCDM process, still there are several limitations that permit advance study and model improvement. The presented approach could be enhanced using the linguistic assessments of criteria with an aggregated decision-matrix. Only objective weight of attribute is used that determines from the IVIF-DM and derives based on information provided by DEs.

In future work, the extension of developed framework can be introduced to handle other MCGDM problems, such as human resource management, healthcare recycling partner assessment, sustainable supplier selection, medical supplier selection, and sustainable biomass crop selection. Also, we will extend the proposed method using different objective criteria-weight-determining methods.

Additionally, the policy implications to improve observance to HCWT guidelines are discussed as.

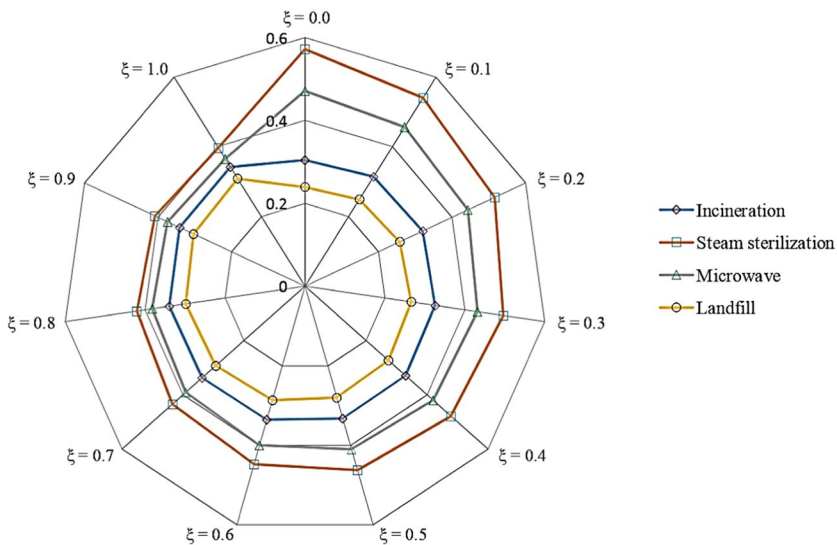
- The “government hospitals (GHs)” need compact support assistances to protect sufficient funding from the administration to begin operating the services of “common waste management facilities (CWMFs).” The “state health structures development project (SHSDP)” can illustrate a main influence in the application of “bio-medical waste (BMW)” recommendations by the GHs.

**Table 14** Comparison of the presented with various extant approaches

Parameters	Zavadskas, et al. (2014) method	Wang et al. (2016) method	Mishra and Rani (2018b) method	Hinduja and Pandey (2018) method	Proposed Method
Standards	IVIF-WAPAS method	IVIF-COPRAS method based on IVIF-ANP method	Information measures-based IVIF-WASPAS method	An combined approach with DEMATEL, IF-ANP, and IF-AHP	IVIF-DNMA method based on divergence measure
MCDM procedure	Scoring model (utility based method)	compromising model-based method	Scoring model (utility based method)	Scoring model (utility based method)	utility value-based methods
Aggregation procedure	Arithmetic, geometric	Arithmetic	Arithmetic, geometric	Arithmetic, geometric	Arithmetic, max, geometric
Criteria weights	Assumed	Computed (IVIF-ANP model)	Computed	Computed (IF-ANP method)	Computed (Divergence measure)
MCDM procedure	Single	Group	Group	Single	Group
HD in assessments	Excluded	Excluded	Included	Excluded	Included
DEs' weights	Not Applicable	Computed (Normal distribution-based method)	Evaluated	Not applicable	Computed (Score degree based method)
Normalization type	Linear	Vector	Linear	Linear	Linear, vector
Optimal HCW disposal option	$B_2$	$B_3$	$B_2$	$B_2$	$B_2$
correlation coefficient ( $r_B$ )	1.00	0.80	1.00	0.80	-

**Table 15** OUDs of the IVIF-DNMA model with different values of  $\xi$

$\xi$	$b_1$	$b_2$	$b_3$	$b_4$
0.0	0.305	0.572	0.471	0.239
0.1	0.314	0.541	0.457	0.249
0.2	0.321	0.516	0.444	0.258
0.3	0.326	0.496	0.432	0.266
0.4	0.323	0.478	0.421	0.273
0.5	0.333	0.462	0.41	0.28
0.6	0.335	0.447	0.40	0.286
0.7	0.337	0.434	0.391	0.292
0.8	0.339	0.421	0.382	0.298
0.9	0.341	0.409	0.374	0.304
1.0	0.342	0.397	0.366	0.309



**Fig. 3** OUDs ( $R_i$ ) over the utility parameter  $\xi$  values

- The “pollution control board (PCB)” also needs to act as a initiator between the GHs and the CWMFs more proficiently so as to quick settlement can be warmed between the two structures.
- Various types of advertisements are employed by two CMFWs in the government. These can create misunderstanding among GHs workers, particularly among those who restructured their CMFW. The PCB should verify that various memorandums are not discussed by the CMFWs to the GHs.



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