Environment, Development and Sustainability (2023) 25:10073–10099
https://doi.org/10.1007/s10668-022-02477-2

Sustainable optimal production policies for an imperfect production system with trade credit under different carbon emission regulations

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Received: 11 November 2021 / Accepted: 27 May 2022 / Published online: 20 June 2022
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Abstract

Production systems are subject to imperfect processes among other factors that may produce items of imperfect quality. To avoid further losses, regular preventive maintenance, and system breakdowns, firms can make changes in their production planning decisions or invest in a green inventory management. Every manufacturing system generates some harmful by-products which are the causes of environmental pollution, and it is increasing day by day. As a result, one of the goals of this research is to develop a sustainable smart manufacturing model with less waste and controlled pollution. Moreover, permissible delay in payment is a popular payment method and important proportion of company finance. This research proposes a manufacturing plan for products by controlling pollution through sustainable and smart production under warranty and preventive maintenance performed with trade credits. Two different sustainable production models are presented here by considering pollution control costs. A sustainable production model with variable pollution costs is examined under the influence of three pollution control mechanisms to improve the model's applicability. The paper's novelty lies in introducing pollution control costs and pollution control mechanisms together in a flexible, sustainable production system. In comparison with the other models, the model with a variable pollution cost appears to be more sustainable as, in this case, there is a 27.76% reduction in the pollution level compared to the other models. Implementing three pollution-controlling strategies, such as pollution cap, pollution cap & trade, and pollution tax, resulted in reductions of 32.52%, 1.72%, and 0.84% in pollution levels, respectively. A sensitivity analysis of the obtained results is also carried out to show the model's strength and robustness.

Keywords Inventory · Warranty · Imperfect quality · Trade credit · Pollution control

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1 Introduction

Among other factors, raw materials may be flawed and operations are subject to human errors, and thus, items of imperfect quality are unavoidable in a production system. These items, however, influence customer service, on-hand inventory, and order frequency in an inventory system. Hsu and Hsu (2013) proposed an optimal inventory model for defective items, determining if an order should be placed and order quantity with a focus on shortage backordering, inspection errors, and sales returns. The authors presented a closed-form solution to determine optimal order size and order/reorder point as well as the maximum shortage level. Sana et al. (2007) and Sana and Chaudhuri (2010) presented two-volume flexible production models, which produce perfect and imperfect both types of items. Lin (2010) investigated the problem of stochastically integrated supplier–retailer inventory and explored the extension of an integrated periodic-review inventory model using protection intervals, backorder price discount, lead time, and the number of shipments from suppliers to retailers in a production system as control variables. Wee et al. (2013) developed an inventory model with screening and shortages, including imperfect quality items. They applied the Renewal Reward Theorem to find the optimal profit of the system. Pal et al. (2014) studied an integrated price-dependent production model with defective products and rework. Mukhopadhyay and Goswami (2014) incorporated learning and forgetting effects in a deteriorating imperfect production process considering setup cost as a variable. Examining a system with a single vendor and buyer, Lee and Kim (2014) proposed an integrated production–distribution model to decide an optimal policy for deteriorating and defective items. Roul et al. (2015) studied imperfect multi-item production inventory models, including dynamic demand and variable production rate with fuzzy budget constraints. Chang et al. (2016) presented an economic order quantity (EOQ) model considering defective items, inspection errors, and permissible payment delays from a supplier perspective. Mahata (2017) investigated the learning effect of the unit production time on optimal lot size for the imperfect production process with partial backlogging of shortage quantity in fuzzy random environments. Chang et al. (2017) further developed EOQ models with imperfect quality items and permissible payment delays in the context of retailers. Manna et al. (2017) examined a model in which defective production rate depends on the production rate. They assumed that demand is advertisement dependent. The authors recommended inspecting all items and screening defective items. Gharaei et al. (2018) estimated the optimal integrated lot sizing in a multi-level supply chain with imperfect products. The authors proposed a mathematical model with the bi-objective function with the goals of both minimizing chain inventory costs and maximizing chain total profit to determine an optimal policy. Paknejadet al. (2019) presented an imperfect economic production inventory model that accounts for partial backlogs at the beginning of the production cycle. Their model reworks all defective or imperfect items post-production, and these reworked items are considered similar to good quality products. Kang et al. (2019) emphasized the rework of defective products obtained after the screening process in a manufacturing model with backorders. Tsao et al. (2020) developed an imperfect production model with predictive maintenance and trade credits. Jamrus et al. (2020) proposed a strategy that integrates event- and period-driven methods to enhance the stability and robustness of manufacturing systems in a coordinated supply chain. Malik and Kim (2020) considered direct and indirect industrial emissions with flexible production in a bi-level supply chain model. Dey et al. (2020) incorporated automation policy in a smart production system to optimize work in process inventory. Environmentally responsible practitioners rarely touch flexible manufacturing with pollution control mechanisms. It could give a new direction to smart production.

Pollution control and sustainable manufacturing strategies have become a global responsibility for all industrialists all over the world. In business and industry, the production and development department need to incorporate sustainable manufacturing. It is mainly imposed by government policies and growing customer's environmental concerns. Sustainable manufacturing has three essential responsibilities: economic, social, and ecological. Many production models in literature did not consider environmental issues. However, before two decades, green inventory practitioners have been increased for this newly emerged research area. Cárdenas-Barrón (2009) introduced defective products and reworked them with planned backorders in an economic production quantity model. After different manufacturing stages in industries and the desired perfect outcomes, some faulty products, solid waste, and polluting gasses generate. These harmful by-products result in global warming and landfills.

Nevertheless, remanufacturing of defective products and reuse of waste obtained from factories could reduce waste and thereby help control pollution. Bonney and Jaber (2011) developed environmentally responsible inventory models. They focused on non-traditional costs related to packaging, waste management, and transportation for promoting green production. Mukhopadhyay and Goswami (2014) worked on an imperfect production model, assuming three types of defective items with constant and variable pollution costs. Khatua and Maity (2016) presented an economic production quantity model to prevent environmental pollution by applying various policies like reliability development by the current policy. Ritha and Haripriya (2017) studied an environment-sensitive inventory supply chain model using a carbon cap and trade policy to reduce harmful greenhouse gasses (GHG) emissions. Zadjafar and Gholamian (2018) developed an inventory model considering the effects of CO_2 emissions on human health. They emphasized sustainable inventory management, incorporating the effects of environmental ergonomics and pollution. Most of the classical models assume perfect manufacturing systems, which is not realistic. Raza and Faisal (2018) discussed two inventory models on greening effort and stock level dependent demand, together with pricing to maximize the industry's overall profit. Sarkar and Chung (2019) designed a flexible production system in a supply chain system with quality improvement. Gautam et al. (2019) formulated a vendor–buyer problem strategy for an imperfect manufacturing process considering carbon emissions and defect management. Mishra et al. (2020) presented a sustainable production system with a controllable carbon emission rate under a carbon tax and cap mechanism. They examined the model with or without shortages and green technology investment. Recently, Sarkar and Sarkar (2020) presented a controllable production system for multi-level biofuel generation to decrease waste. Rout et al. (2020) studied the impact of different emission control strategies for sustainable management of a model with deterioration and faulty production. Jemai et al. (2020) designed an environmentally sensitive supply chain network for blood platelets ensuring less carbon emission with efficient distribution and cost minimization. Habib et al. (2020) designed a biodiesel supply chain based on waste animal fat, which minimizes environmental impact and the supply chain's cost. They applied a possibilistic chance-constrained methodology to handle uncertain situations. Rehman et al. (2021) studied the influence of carbon dioxide emission to population growth, food production, economic growth, livestock, and energy utilization. Sarkar and Chung (2021) worked on controlling waste and emissions for maintaining the quality of production in a sustainable supply network. Cao et al. (2021) examined the influence of the financial development, stock market, globalization, economic growth, electricity, and renewable energy consumption on carbon emission. Recently, Hasan et al. (2021) analyzed optimizing inventory level and technology investment under a carbon tax, cap-and-trade and strict carbon limit regulations.

Given the increasingly strong competition, present-day companies must continually improve their products and services to sustain competitive advantage and increase market share. Suppliers generally offer trade credit to attract more buyers, who consider such credit a form of price reduction. Another effective method of increasing sales is a permissible payment delay. Balkhi (2011) proposed an optimal economic ordering policy for trade credits in a finite horizon case. The author used an inventory model with perishable items under inflation and time value of money when credits are no longer permissible. Min et al. (2012) presented a replenishment model with deteriorating items subject to trade credits and a finite replenishment rate. In Lou and Wang's (2013) economic production quantity (EPQ) model for a manufacturer (or wholesaler) with defective items, the manufacturer's supplier offers an up-stream trade credit, while simultaneously, the manufacturer supplies their buyers (or retailers) with down-stream trade credit. Tsao (2016) developed a model that accounts for the problems of joint inventory, location, and preservation decision making for non-instantaneous perishable items and delayed payments. Zhong et al. (2018) proposed an integrated model for a supply chain network considering trade credits. The model simultaneously determines warehouse locations; retailer assignments to warehouses; inventory management policy of warehouses and retailers; and payment delay permitted by warehouses for each retailer to minimize costs associated with system-wise location, transportation, multi-echelon inventory, and finance. Mahata and Mahata (2020) developed an imperfect manufacturing system with credit policies in fuzzy random environments. The supplier simultaneously offers the retailer either a permissible delay in payments or a cash discount, and retailer in turn provides its customer a permissible delay period. Cheng et al. (2020) established an inventory model for deteriorating items with demand that is price-dependent and a return period for retailers who offer customers advance sales in two phases. Chung et al. (2020) discussed two levels of trade-credit policies previously explored by Huang (2003) and Teng et al. (2006). Mahato and Mahata (2021) investigated the learning effect of the unit production time on optimal lot size for the imperfect production process with partial backlogging of shortage quantity in fuzzy random environments.

None of the research papers discussed above considered the two approaches to pollution control together under permissible delay in payment. Hence, the implementation of pollution control costs and pollution control mechanisms together is a significant research gap. To reflect real market phenomena, this study develops a sustainable EPQ model with imperfect quality items, warranty, preventive maintenance, and permissible payment delays including pollution costs. Three models have been studied with (fixed and variable) pollution control costs and three pollution control mechanisms—pollution cap, pollution cap, and trade and pollution tax are applied. The objective is to present a manufacturing plan and an easy-to-use method to derive an optimized production plan that minimizes total cost. However, the model is extended into two models with class I and class II pollution control fees. Then, three pollution control mechanisms are designed to enrich the significance of the developed model. Findings reveal better functioning of Model 3 (with class I pollution control cost) over Model 1 and Model 2 (without or with class II pollution control cost). Model 3 is successful in dropping the pollution level by 27.76% as compared to Model 2. The application of the proposed model for different pollution control mechanisms demonstrates that by implementing all three designed tools: pollution cap, pollution cap & trade, and pollution tax, the pollutants diminish by 32.52%, 1.72%, and 0.84%, respectively.

The method is applied to numerical examples to demonstrate the solution procedure, following which a sensitivity analysis is conducted to examine how changes in certain parameters influence the optimized solution.

As reviewed by the literature and analyzed in Table 1, the research gap could be examined clearly.

Many researchers have worked on pollution, imperfect manufacturing, and remanufacturing. However, there is a clear research gap on sustainable production policies for imperfect production system to control pollution with trade credit policy. This study focuses on the following work:

- Proposed research introduces sustainable production strategies. The inventory decision-maker may decide the optimal order quantity, and optimal backorder quantity to minimize total pollution, and the total cost per cycle of the complete manufacturing system.
- A popular payment method and important proportion of company finance is trade credit. This research proposes a manufacturing plan for products sold under warranty and preventive maintenance performed with trade credits.
- Three models have been studied with (fixed and variable) pollution control costs.
- Three pollution control mechanisms: pollution cap, pollution cap, and trade and pollution tax are applied.

2 The model

This study first establishes a model and then presents an easy-to-use method to design optimal production plans that can help minimize total cost. In this paper, three production models with or without pollution control costs are presented. First, a basic smart production model with imperfect production system under trade credit is designed without considering pollution control scenarios**.** Then, to gain environmental sustainability, it is extended to pollution control scenarios. Two models are made as class I and class II with and without pollution control costs.

Table 1 Review and gap analysis of existing and present research

Authors	Pollution control cost	Imperfect quality	Rework	Pollution control mechanism	Trade credit
Mukhopadhyay and Goswami (2014)	✓	✓	✓		
Pal et al. (2014)		✓	✓		
Roul et al. (2015)		✓	✓		
Manna et al. (2017)		✓	✓		
Tayyab et al. (2018)		✓	✓		
Karmakar et al. (2018)			✓		
Gautam et al. (2019)	✓	✓	✓		
Kang et al. (2019)		✓	✓		
Rout et al. (2019)		✓	✓		
Rout et al. (2020)		✓	✓	✓	
This research	✓	✓	✓	✓	✓

Here, we assume all deviant items can be immediately reworked in a parallel manufacturing system and products are sold under a free repair warranty policy where the period of warranty is W. We also assume that the rate of demand is constant which is denoted by D and allow backorder with backorder level B. With regular preventive maintenance, the probability of a manufacturing system breakdown during the production run time is zero. Here, we consider $f(\tau)$ be the probability density function of τ where τ be the time elapsed after which the production process shifts to the out-of-control state (random variable) and θ_1 be the probability of deviant items when the production process is in the in-control state and θ_2 be the probability of non-conforming items in the out-of-control state, with $0 < \theta_1 < \theta_2 < 1$. Further, we assume that if the account is not settled during the trade credit period, revenue generated from sales is deposited in an interest-bearing account at the interest rate I_e. At the end of the allowable delay, manufacturers pay off all units ordered and begin repaying the interest on raw material in stocks at interest rate I_C. We also assume that flexible production is a basic need for smart production. It may increase the setup cost, but it decreases the holding cost, pollution, and waste generation during extra production. Thus, the production rate is taken as a variable (Sarkar & Chung, 2019; Roul et al., 2015; Karmakar et al., 2018), which can vary within a prescribed interval, i.e., $\left[P_{\max}, P_{\min}\right]$. Cost of production is defined as $C = \left(a_1 + \frac{a_2}{P} + a_3 P\right)$, where, P is the Production rate, C is the Production cost per unit item including inspection and purchase costs, a_1 is the material cost (\$/unit), a_2 is the development cost (\$/unit), and a_3 is the tool/dye cost (\$/unit).

For notational convenience, we introduce the notation to be used throughout this paper. Let A be the cost of setup for each production run, S be the selling price per unit, N be the number of non-conforming items, T be the cycle length, C_h be the holding cost per unit item, per unit time excluding interest charged, C_0 be the cost per unit time for preventive maintenance, C_s be the shortage cost per unit item, per unit time, C_W be the cost of repair for warranty per unit item, C_R be the cost for reworking of each non-conforming item, N be the number of non-conforming items, τ be the time elapsed after which the production process shifts to the out-of-control state (random variable). Furthermore, since we consider an imperfect production system under different carbon emissions policies, we introduce the carbon emission parameters as follows to account for carbon emissions. Let μ_{Cap} be the pollution cap (ton/year), $\mu(t)$ be the amount of pollutant at time t (ton/year), σ be the fraction of pollutant that disappears, S_1 be unit purchasing price of pollution credit (\$/unit), S_2 be the unit selling price of pollution credit (\$/unit), and π be the pollution tax of emission (\$/ton).

2.1 Model 1: Production model without pollution control cost

The manufacturer purchases Q units of raw materials per order from the supplier and incurs a unit production cost of C. The supplier provides the manufacturer an allowable delay period M. The production rate is constant at P during the regular production uptime. Figure 1 illustrates the pattern of the production inventory system. t_1 and t_2 are the production uptime, t_3 is the production downtime, t_4 is the shortage permitted time, and T is the duration of the cycle:

$$t_1 = \frac{B}{P - D} \tag{1}$$

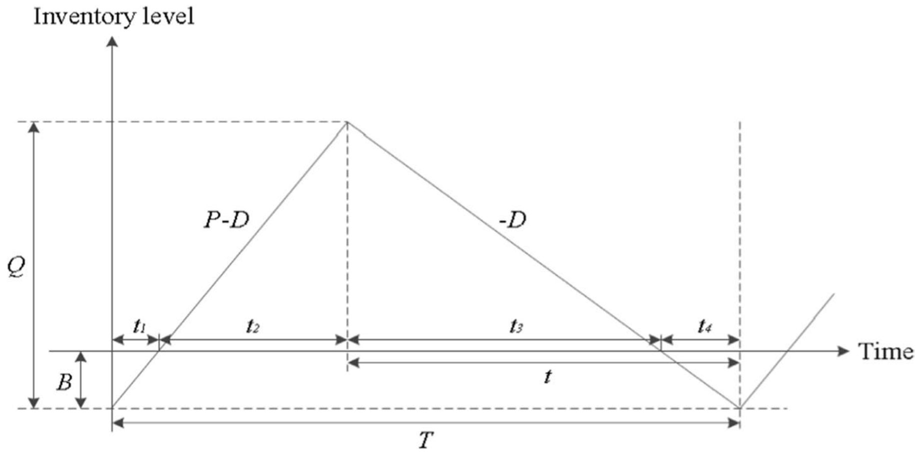

Fig. 1 Graphical representation of the inventory system

$$t_2 = \frac{Q}{P} - \frac{B}{P-D} \tag{2}$$

$$t_3 = \frac{(P-D)Q}{PD} - \frac{B}{D} \tag{3}$$

$$t_4 = \frac{B}{D} \tag{4}$$

and

$$T = t_1 + t_2 + t_3 + t_4 = \frac{Q}{D}. \tag{5}$$

For convenience,

$$t_A \equiv t_1, t_B \equiv t_1 + t_2 = \frac{Q}{P}, \text{ and } t_C = t_1 + t_2 + t_3 = \frac{Q-B}{D}. \tag{6}$$

The random variable τ may occur within the period $t_p = t_1 + t_2$ or may occur after t_p. In this situation, the number of deviant items N can be derived as follows:

$$N = \begin{cases} \theta_1 P(t_1 + t_2), & \text{if } \tau \geq t_1 + t_2 \\ \theta_1 P\tau + \theta_2 P\left(t_1 + t_2 - \tau\right), & \text{if } \tau < t_1 + t_2 \end{cases} \tag{7}$$

The expected value of N is given by

$$\begin{aligned} E(N) &= \theta_1 P\left(t_1 + t_2\right) \int_{t_1+t_2}^{\infty} f(\tau)\mathrm{d}\tau + \int_0^{t_1+t_2} \left[\theta_1 P\tau + \theta_2 P\left(t_1 + t_2 - \tau\right) f(\tau)\mathrm{d}\tau\right] \\ &= \theta_1 P\left(t_1 + t_2\right) + \left(\theta_2 - \theta_1\right) P \int_0^{t_1+t_2} \left(t_1 + t_2 - \tau\right) f(\tau)\mathrm{d}\tau. \end{aligned} \tag{8}$$

The fraction of non-conforming items in the total produced items is estimated as

$$G = \frac{E(N)}{P\left(t_1 + t_2\right)} = \theta_1 + \frac{\theta_2 - \theta_1}{t_1 + t_2} \int\limits_0^{t_1+t_2} \left(t_1 + t_2 - \tau\right) f(\tau)\mathrm{d}\tau$$
$$= \theta_1 + \frac{\theta_2 - \theta_1}{Q/P} \int\limits_0^{Q/P} \left(\frac{Q}{P} - \tau\right) f(\tau)\mathrm{d}\tau = g(Q). \tag{9}$$

The manufacturer offers free minimal repair warranty policy on the products sold to the buyer for a specified period called warranty period. The probability of a product failure under the warranty period $[0, W]$ is

$$P_W = (1-g) \int\limits_o^W h_1(x)\mathrm{d}x + g \int\limits_0^w h_2(x)\mathrm{d}x = (1-g)W_1 + gW_2 = W_1 + \left(W_2 - W_1\right)g, \tag{10}$$

where

$$W_1 = \int\limits_0^W h_1(x)\mathrm{d}x \text{ and } W_2 = \int\limits_0^W h_2(x)\mathrm{d}x. \tag{11}$$

The components of the inventory total cost per cycle are as follows:

(1) Setup cost: A.
(2) Production cost is an essential cost in production systems. It is obtained by the cost of producing one unit by the number of units produced

$$CP\left(t_1 + t_2\right) = CQ = \left(a_1 + \frac{a_2}{P} + a_3 P\right)Q. \tag{12}$$

(3) Maintenance cost is:

$$C_0\left(t_3 + t_4\right) = C_0 \frac{(P-D)Q}{DP}. \tag{13}$$

(4) Expected free repair cost for warranty:

$$C_W P\left(t_1 + t_2\right) P_W = C_W Q\left[W_1 + \left(W_2 - W_1\right)g(Q)\right]. \tag{14}$$

(5) Expected rework cost is given as:

$$C_R g P\left(t_1 + t_2\right) = C_R g(Q)Q. \tag{15}$$

(6) Holding cost is represented as:

$$C_h\left[\frac{1}{2}(P-D)t_2^2 + \frac{1}{2}\frac{(P-D)^2}{D}t_2^2\right] = C_h \frac{(P-D)P}{2D} t_2^2. \tag{16}$$

$$= C_h \frac{[Q(P-D) - BP]^2}{2DP(P-D)} = C_h\left[\frac{(P-D)}{2DP}Q^2 + \frac{P}{2D(P-D)}B^2 - \frac{1}{D}QB\right]. \tag{17}$$

(7) Shortage cost:

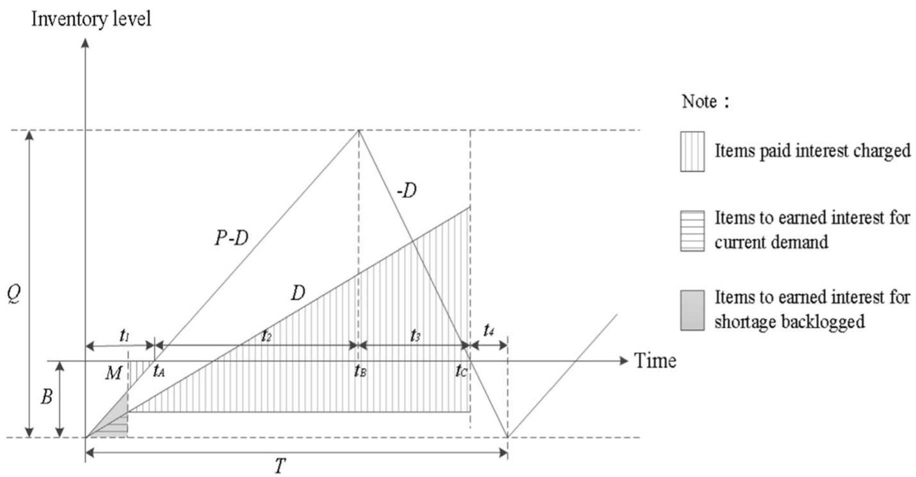

Fig. 2 Graphical representation of interest earned and interest charged when $M < t_A$

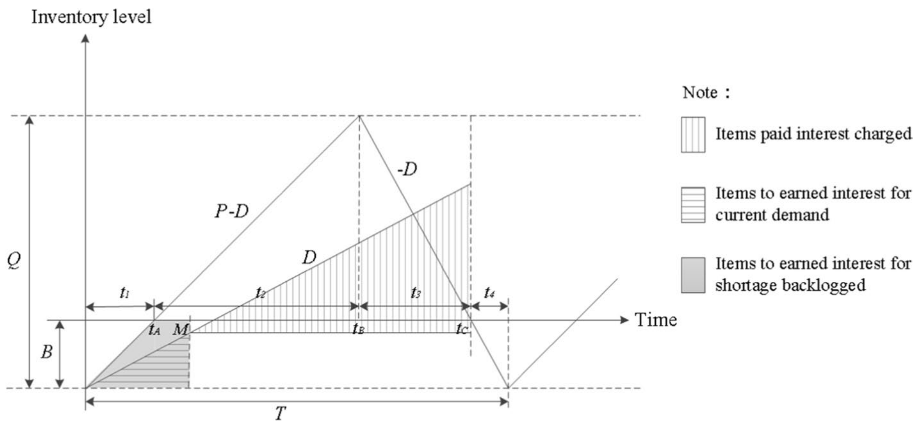

Fig. 3 Graphical representation of interest earned and interest charged when $t_A \le M < t_c$

$$C_S \frac{B}{2}\left(t_1 + t_4\right) = C_S \frac{B}{2} \times \frac{PB}{(P-D)D} = C_S \frac{PB^2}{2(P-D)D}. \tag{18}$$

(8) Interest earned and charged:

We derive the following three cases on the basis of the values for M, t_A and t_C : $M < t_A$, $t_A \le M < t_C$ and $M \ge t_C$. Figures 2, 3, 4 represent these three cases.

Case 1: $M < t_A$.

Here, the manufacturer begins production and replenishing shortage at the time of initialization. The manufacturer accumulates revenue that earns I_e per dollar, per year starting from 0 to M. The interest earned per cycle IE_1 is SI_e times the sum of the areas of two triangles (Fig. 2) as given as

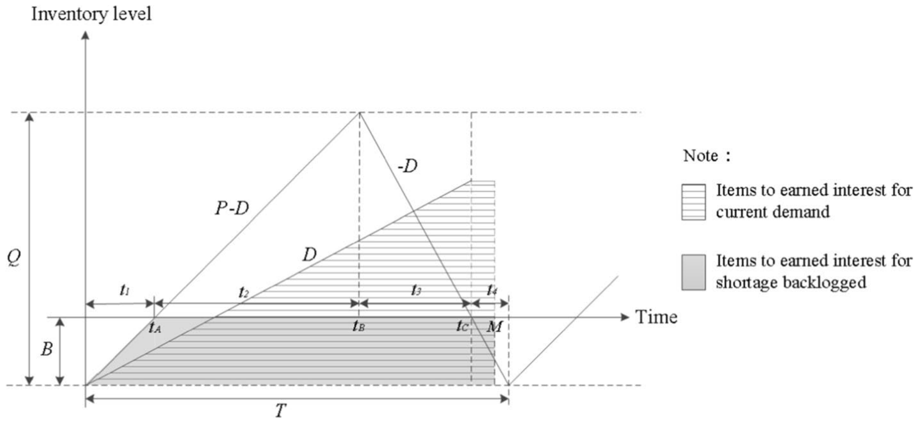

Fig. 4 Graphical representation of interest earned when $M \geq t_c$

$$\mathrm{IE}_1 = \mathrm{SI}_e\left[\frac{DM^2}{2} + \frac{(P-D)M^2}{2}\right] = \mathrm{SI}_e\frac{PM^2}{2}. \tag{19}$$

The manufacturer pays off all units sold by supplier at time M. The manufacturer retains the profits and pays interest charged on items sold after time M. Interest charged per cycle IC_1 is CI_C multiplied by the sum of the areas of two triangles (Fig. 2).

$$\mathrm{IC}_1 = \mathrm{CI}_C\left[\frac{(P-D)\left(t_A - M\right)^2}{2} + \frac{D\left(t_C - M\right)^2}{2}\right] = \mathrm{CI}_C\left[\frac{Q^2}{2D} + \frac{PB^2}{2D(P-D)} - \frac{QB}{D} - MQ + \frac{P}{2}M^2\right]. \tag{20}$$

Case 2: $t_A \leq M < t_C$.

In this case, interest earned per cycle IE_2 is SI_e times the sum of the areas of a trapezoid and triangle (Fig. 3).

$$\mathrm{IE}_2 = \mathrm{SI}_e\left\{\frac{DM^2}{2} + \frac{\left[\left(M - t_A\right) + M\right)]B}{2}\right\} = \mathrm{SI}_e\left[\frac{DM^2}{2} + MB - \frac{B^2}{2(P-D)}\right]. \tag{21}$$

Interest charged per cycle IC_2 is CI_C multiplied by the sum of the area of a triangle (Fig. 3).

$$\mathrm{IC}_2 = \mathrm{CI}_C\left[\frac{D}{2}\left(t_C - M\right)^2\right] = \mathrm{CI}_C\left[\frac{Q^2}{2D} + \frac{B^2}{2D} - \frac{QB}{D} + \frac{DM^2}{2} - MQ + MB\right]. \tag{22}$$

Case 3: $M \geq t_C$.

Interest earned per cycle IE_3 is SI_e times the sum of the areas of two trapezoids (Fig. 4).

$$\mathrm{IE}_3 = \mathrm{SI}_e\left\{\frac{\left[\left(M - t_A\right) + M\right]B}{2} + \frac{\left[\left(M - t_C\right) + M\right]Dt_C}{2}\right\} = \mathrm{SI}_e\left[MQ + \frac{QB}{D} - \frac{Q^2}{2D} - \frac{PB^2}{2(P-D)D}\right]. \tag{23}$$

Interest charged per cycle IC_3 is zero, i.e., $\mathrm{IC}_3 = 0$.

Drawing on the abovementioned arguments, the total inventory cost per cycle is obtained as:

$$\mathrm{TC}_i(Q,B) = \text{setup cost} + \text{production cost} + \text{maintenance cost} + \text{warranty cost} + \text{rework cost} + \text{holding cost} + \text{shortage cost} + \text{interest charged} - \text{interest earned}, \; i = 1,2,3.$$

Case 1: $M < t_A$

$$\begin{aligned}\mathrm{TC}_1(Q,B) = {} & A + CQ + C_0\frac{(P-D)Q}{DP} + C_WQ\left[W_1 + \left(W_2 - W_1\right)g(Q)\right] + C_Rg(Q)Q \\ & + C_h\left[\frac{(P-D)}{2DP}Q^2 + \frac{P}{2D(P-D)}B^2 - \frac{1}{D}QB\right] + C_S\frac{PB^2}{2(P-D)D} \\ & + \mathrm{CI}_C\left[\frac{Q^2}{2D} + \frac{PB^2}{2D(P-D)} - \frac{QB}{D} - MQ + \frac{P}{2}M^2\right] - \mathrm{SI}_e\frac{PM^2}{2}.\end{aligned} \tag{24}$$

Therefore, the total inventory cost per unit time is

$$\begin{aligned}\mathrm{TCU}_1(Q,B) = {} & \mathrm{TC}_1(Q,B)/T = A\frac{D}{Q} + CD + C_0\frac{(P-D)}{P} \\ & + C_WD\left[W_1 + \left(W_2 - W_1\right)g(Q)\right] + C_Rg(Q)D \\ & + C_h\left[\frac{(P-D)}{2P}Q + \frac{P}{2Q(P-D)}B^2 - B\right] + C_S\frac{PB^2}{2(P-D)Q} \\ & + \mathrm{CI}_C\left[\frac{Q}{2} + \frac{PB^2}{2Q(P-D)} - B - MD + \frac{PD}{2Q}M^2\right] - \mathrm{SI}_e\frac{PDM^2}{2Q}.\end{aligned} \tag{25}$$

Case 2: $t_A \le M < t_C$

$$\begin{aligned}\mathrm{TC}_2(Q,B) = {} & A + CQ + C_0\frac{(P-D)Q}{DP} + C_WQ\left[W_1 + \left(W_2 - W_1\right)g(Q)\right] + C_Rg(Q)Q \\ & + C_h\left[\frac{(P-D)}{2DP}Q^2 + \frac{P}{2D(P-D)}B^2 - \frac{1}{D}QB\right] + C_S\frac{PB^2}{2(P-D)D} \\ & + \mathrm{CI}_C\left[\frac{Q^2}{2D} + \frac{B^2}{2D} - \frac{QB}{D} - MQ + \frac{DM^2}{2} + MB\right] - \mathrm{SI}_e\left[\frac{DM^2}{2} + MB - \frac{B^2}{2(P-D)}\right].\end{aligned} \tag{26}$$

Thus, the total inventory cost per unit time is

$$\begin{aligned}\mathrm{TCU}_2(Q,B) = {} & \mathrm{TC}_2(Q,B)/T = A\frac{D}{Q} + CD + C_0\frac{(P-D)}{P} \\ & + C_WD\left[W_1 + \left(W_2 - W_1\right)g(Q)\right] + C_Rg(Q)D + C_h\left[\frac{(P-D)}{2P}Q + \frac{P}{2Q(P-D)}B^2 - B\right] \\ & + C_S\frac{PB^2}{2(P-D)Q} + \mathrm{CI}_C\left[\frac{Q}{2} + \frac{B^2}{2Q} - B + \frac{D^2M^2}{2Q} - MD + \frac{DMB}{Q}\right] - \mathrm{SI}_e\left[\frac{D^2M^2}{2Q} + MB\frac{D}{Q} - \frac{DB^2}{2(P-D)Q}\right].\end{aligned} \tag{27}$$

Case 3: $M \ge t_C$

$$\begin{aligned}\mathrm{TC}_3(Q,B) = {} & A + CQ + C_0\frac{(P-D)Q}{DP} + C_WQ\left[W_1 + \left(W_2 - W_1\right)g(Q)\right] + C_Rg(Q)Q \\ & + C_h\left[\frac{(P-D)}{2DP}Q^2 + \frac{P}{2D(P-D)}B^2 - \frac{1}{D}QB\right] + C_S\frac{PB^2}{2(P-D)D} - \mathrm{SI}_e\left[MQ + \frac{QB}{D} - \frac{Q^2}{2D} - \frac{PB^2}{2(P-D)D}\right].\end{aligned} \tag{28}$$

Accordingly, the total inventory cost per unit time is

$$\mathrm{TCU}_3(Q,B)=\frac{\mathrm{TC}_3(Q,B)}{T}=A\frac{D}{Q}+CD+C_0\frac{(P-D)}{P}+C_WD\left[W_1+\left(W_2-W_1\right)g(Q)\right]$$
$$+C_Rg(Q)D+C_h\left[\frac{(P-D)}{2P}Q+\frac{P}{2Q(P-D)}B^2-B\right]+C_S\frac{PB^2}{2(P-D)Q}-\mathrm{SI}_e\left[MD+B-\frac{Q}{2}-\frac{PB^2}{2(P-D)Q}\right]. \tag{29}$$

Based on the above arguments, the relevant total inventory cost per unit time is

$$\mathrm{TCU}(Q,B)=\begin{cases}\mathrm{TCU}_1(Q,B),\ M<t_A\\ \mathrm{TCU}_2(Q,B),\ t_A\le M<t_C\\ \mathrm{TCU}_3(Q,B),\ M\ge t_C\end{cases} \tag{30}$$

where $\mathrm{TCU}_i(Q,B)$, $i=1,2,3$ are given by (25), (27), and (29), respectively.

2.1.1 Theoretical results and optimal solution

Here, we explain the solution procedure and derive the optimal solution for the aforementioned case. However, we are unable to prove that the inventory total cost per unit time is a joint convex or joint quasi-convex in B and Q given the complexity of the problem. Nevertheless, it is proved that the inventory total cost per unit time is pseudo-convex if either Q or B is given.

The current theoretical results in convex fractional programming are applied. According to Cambini and Martein (2009), the real-value function,

$$q(x)=\frac{u(x)}{v(x)} \tag{31}$$

is (strictly) pseudo-convex if $u(x)$ is differentiable, non-negative, and (strictly) convex and if $v(x)$ is concave, differentiable, and positive. For simplicity,

$$K=D\left[2g'(Q)-Qg''(Q)\right]\left[C_W\left(W_2-W_1\right)+C_R\right]. \tag{32}$$

For any given B, Theorem 1 is obtained by applying (31).

Theorem 1 *For any given backorder level B, if $K>0$, then $\mathrm{TCU}_i(Q,B)$, for $i=1,2,3$, is a strictly pseudo-convex function in Q, and thus, there is a unique minimum solution Q_i^*, where $i=1,2$, and 3.*

Proof See "Appendix 1".

For any given backorder level B, we derive the necessary and sufficient condition for the optimal production lot size Q_i^*, where $i=1,2$, and 3, by applying Theorem 1; taking the first-order derivative of $\mathrm{TCU}_i(Q,B)$, where $i=1,2$, and 3 with respect to Q; letting the result to zero; and re-arranging terms:

$$
\begin{aligned}
&Q\times\Big\{CD+C_0\frac{P-D}{P}+C_WD\big[W_1+(W_2-W_1)g(Q)\big]+C_WQD\big(W_2-W_1\big)g'(Q)\\
&\quad+C_RDg(Q)+C_RDQg'(Q)+C_h\Big[\frac{P-D}{P}Q-B\Big]+\mathrm{CI}_C[Q-B-MD]\Big\}\\
&\quad=AD+CDQ+C_0\frac{(P-D)}{P}Q+C_WQD\big[W_1+\big(W_2-W_1\big)g(Q)\big]+C_Rg(Q)QD\\
&\quad+C_h\Big[\frac{(P-D)}{2P}Q^2+\frac{P}{2(P-D)}B^2-QB\Big]+C_S\frac{PB^2}{2(P-D)}\\
&\quad+\mathrm{CI}_C\Big[\frac{Q^2}{2}+\frac{PB^2}{2(P-D)}-QB-MDQ+\frac{PD}{2}H^2\Big]-\frac{\mathrm{SI}_ePDM^2}{2},\quad\text{for }Q_1^*;
\end{aligned}
\tag{33}
$$

$$
\begin{aligned}
&Q\times\Big\{CD+C_0\frac{P-D}{P}+C_WD\big[W_{1+}\big(W_2-W_1\big)g(Q)\big]+C_WDQ\big(W_2-W_1\big)g'(Q)+C_RDg(Q)\\
&\quad+C_RDg'(Q)Q+C_h\Big[\frac{P-D}{P}Q-B\Big]+CI_C[Q-B-MD]\Big\}\\
&\quad=AD+CDQ+C_0\frac{P-D}{P}Q+C_WDQ\big[W_1+\big(W_2-W_1\big)g(Q)\big]+C_RDg(Q)Q\\
&\quad+C_h\Big[\frac{P-D}{2P}Q^2+\frac{P}{2(P-D)}B^2-QB\Big]+C_S\frac{PB^2}{2(P-D)}\\
&\quad+\mathrm{CI}_C\Big[\frac{Q^2}{2}+\frac{B^2}{2}-QB+\frac{D^2M^2}{2}-MDQ+MDB\Big]-\mathrm{SI}_e\Big[\frac{D^2M^2}{2}+DMB-\frac{D}{2(P-D)}B^2\Big],\quad\text{for }Q_2^*;
\end{aligned}
\tag{34}
$$

and

$$
\begin{aligned}
&Q\times\Big\{CD+C_0\frac{P-D}{P}+C_WD\big[W_1+\big(W_2-W_1\big)g(Q)\big]+C_WDQ\big(W_2-W_1\big)g'(Q)\\
&\quad+C_RDg(Q)+C_RDg'(Q)Q+C_h\Big[\frac{P-D}{P}Q-B\Big]-\mathrm{SI}_e[MD+B-Q]\Big\}\\
&\quad=AD+CDQ+C_0\frac{P-D}{P}Q+C_WDQ\big[W_1+\big(W_2-W_1\big)g(Q)\big]+C_RDg(Q)Q\\
&\quad+C_h\Big[\frac{P-D}{2P}Q^2+\frac{P}{2(P-D)}B^2-QB\Big]+C_S\frac{P}{2(P-D)}B^2-\mathrm{SI}_e\Big[MDQ+QB-\frac{Q^2}{2}-\frac{PB^2}{2(P-D)}\Big],\quad\text{for }Q_3^*.
\end{aligned}
\tag{35}
$$

Theorem 2 *For any production lot size Q, $\mathrm{TCU}_i(Q,B)$, where $i = 1, 2,$ and 3 is a strictly pseudo-convex function in B. Thus, there exists a unique minimum solution B_i^*, where $i = 1, 2,$ and 3.*

Proof See “Appendix 2”.

For any given production lot size Q, we obtain the necessary and sufficient condition for the optimal production lot size B_i^*, where $i = 1, 2,$ and 3 by adopting Theorem 2;taking the first-order derivative of $\mathrm{TCU}_i(Q,B)$, where $i = 1, 2,$ and 3 with respect to B; letting the result to zero; and re-arranging terms:

$$
C_h\Big[\frac{P}{P-D}B-Q\Big]+C_S\frac{P}{P-D}B+CI_C\Big[\frac{P}{P-D}B-Q\Big]=0,\ \text{for }B_1^*
\tag{36}
$$

$$
C_h\Big[\frac{P}{P-D}B-Q\Big]+C_S\frac{P}{P-D}B+CI_C[B-Q+MD]-\mathrm{SI}_e\Big[DM-\frac{D}{P-D}B\Big]=0,\ \text{for }B_2^*
\tag{37}
$$

and

$$C_h\left[\frac{P}{P-D}B-Q\right]+C_S\frac{P}{P-D}B-\mathrm{SI}_e\left[Q-\frac{P}{P-D}B\right]=0,\ \text{for } B_3^*. \tag{38}$$

2.2 Production model with pollution control cost

The pollution factor represents the value that correlates the number of pollutants delivered in the environment during the process related to the discharge of that pollutant. It is defined as the mass of pollutant divided by a unit mass, volume, distance, or period of action that outcomes a pollutant. According to IPCC (2006) guidelines for National Greenhouse Gas inventories, estimate of pollutants is given by the formula

$$\text{Total emission} = (\text{amount of product produced}) \times (\text{pollution factor}),$$

where the amount of activity is calculated in tones/year and the emission factor is dimensionless.

The pollution control cost consists of two essential cost factors, i.e., capital costs and operating costs. The capital cost is like inventory models' setup covering the cost of space (own or rented), setting up a pollution control plant, and machinery to rework before starting production. These costs are assumed independent of the number of production cycles and invested one time. Operation and maintenance costs are the costs related to the upkeep of treatment facilities, record keeping, and obsolescence costs.

The new notation is used for mathematical modeling of pollution control scenarios:

- μ_0 Pollution factor
- C_1 Pollution control cost (\$/unit)
- C_2 Capital cost for pollution control per production run (\$/unit)
- C_3 Operating and maintenance cost for pollution control per unit of production quantity (\$/unit)

The following assumptions are made to build the model:

1. The pollution control cost is taken as constant. It is the sum of two pollution control costs, setup cost and maintenance cost. Setup cost is generally stable.
2. The operating and maintenance cost of pollution per unit of production is taken constant and further assumed to be independent of time.

2.2.1 Model 2: Production model with class I pollution control cost

This model assumes that the pollution removal/treatment cost is independent of time but depends on quantity like scraps, junks, and sewage. All the amount of pollutants produced during the production process is under control and usable for the treatment process (Mukhopadhyay and Goswami 2014).

Now,

$$C_1 = C_2 + \mu_0 C_3 Q. \tag{39}$$

Total annual cost per cycle, including pollution prevention cost, is

$$\mathrm{TCUP}_1(Q,B) = \mathrm{TCU}(Q,B) + \frac{C_1}{T}. \tag{40}$$

2.2.2 Model 3: Production model with class II pollution control cost

In this model, it is assumed that pollution can be controlled partially. It is observed that a fraction σ of the pollutant quantity automatically becomes less intense and disappears gradually. The reason for this is evaporation, decay, chemical reaction, biological decomposition (Mukhopadhyay & Goswami, 2014). Thus, the remaining fraction of the pollutant, i.e., $(1-\sigma)$ is considered only for treatment. In this model, it is assumed that treatment policy cannot be applied to all parts of the pollutant as the quantity of the contaminants is not under control. Therefore, the treatment can only be done on the remaining portion of the contaminants.

Let $\rho(t)$ denote the number of pollutants accumulated at time t. Variations in the number of contaminants with time can be represented with the following equation's help (Mukhopadhyay & Goswami, 2014).

$$\dot{\rho} = \rho_0 - \sigma\rho, \text{ with the initial condition: } \rho(0) = 0. \tag{41}$$

With the help of the initial state, the solution of the above equation can be represented as

$$\rho(t) = \frac{\rho_0}{\sigma}\left(1 - e^{-\sigma t}\right). \tag{42}$$

The above equation gives the number of pollutants accumulated at time t. Thus, the amount of contaminants produced during the whole production process is $\sigma\left(t_1 + t_2\right)$, where t_p is the duration of the production.

Now, the pollution prevention cost is given by

$$C_1' = C_2 + \rho_0 C_3\left(\frac{Q}{P} - \frac{\sigma}{2}\left(\frac{Q}{P}\right)^2\right). \tag{43}$$

Hence, the total cost per cycle including pollution control

$$\mathrm{TCUP}_2(Q,B) = \mathrm{TCU}(Q,B) + \frac{C_1'}{T}. \tag{44}$$

2.3 Implementation of Model 3 for different pollution control mechanisms

Three pollution control mechanisms are applied to Model 3, and the optimization models under these mechanisms are developed.

2.3.1 Implementation of Model 3 for pollution cap mechanism

A pollution cap is a constraint imposed on the company through the government to a certain amount of pollutants. Its motive is to compel companies to find innovative methods of pollution control. Suppose μ_{Cap} (ton/unit time) denote a pollution cap. The optimization model under this mechanism can be defined as:

$$\left.\begin{array}{l}\text{Minimize } \text{TCUP}_2(Q,B)\\ \text{subject to } \mu\left(t_p\right) \le \mu_{\text{Cap}}\end{array}\right\} \tag{45}$$

2.3.2 Implementation of the model for pollution cap and trade mechanism

Cap and trade is a top rated government regulatory program applied to control, or cap, the number of pollutants generated as a by-product of industrial activities. Companies that surplus the cap are taxed, while companies that cut their contaminants may sell or purchase unused credits. In 2005, the European Union (EU) started the world's first international cap and trade policy to reduce pollutants.

Suppose s_1 and s_2 denote selling price and purchasing price per unit of emission generated. The optimization model under this mechanism can be defined as:

$$\text{Minimize}\begin{cases}\text{TCUP}_2(Q,B) + s_1\left\{\mu\left(t_p\right) - \mu_{\text{Cap}}\right\}, & \text{when } \mu\left(t_p\right) > \mu_{\text{Cap}}\\ \text{TCUP}_2(Q,B) - s_2\left\{\mu_{\text{Cap}} - \mu\left(t_p\right)\right\}, & \text{when } \mu\left(t_p\right) < \mu_{\text{Cap}}\end{cases} \tag{46}$$

2.3.3 Implementation of the model for pollution tax mechanism

A pollution tax is a type of fixed price imposed by the government on the number of pollutants generated in the industries' production process. It aimed to control the consumption of fossil fuels and emphasize initiatives to adopt environmentally friendly alternates. Pollution tax has been executed in various countries all over the world. The first country which implemented a pollution tax was Finland in 1990. Suppose π (\$/ton) denote a pollution taxper unit of emission generated. The optimization model under this mechanism can be defined as:

$$\text{Minimize } \text{TCUP}_2(Q,B) + \pi\mu\left(t_p\right). \tag{47}$$

3 Numerical examples

In this section, we provide some numerical examples to demonstrate the solution procedure of all the three models and carbon control mechanisms designed in the previous area and examine the effect of changes in certain parameter values on the optimized solution.

Example 1 The parameters are as follows: $P = 10000$, $D = 2000$, $A = 750$, $C = 2/\text{unit}$, $C_0 = 500$, $S = 4/\text{unit}$, $C_h = 0.2$, $C_S = 0.25$, $C_W = 1$, $C_R = 0.2$, $\theta_1 = 0.1$, $\theta_2 = 0.25$, $W = 2$, $M = 0.5$, $I_c = 0.09$, $I_e = 0.05$, $\lambda_1 = 1/36$, $\lambda_2 = 1/6$, $\beta_1 = 2$, and $\beta_2 = 2$,

Table 2 Optimal results for the models with or without pollution control costs

Model	Q^*	B^*	Total cost per unit time	Total amount of pollutant
Model 1	4305.68	2089.79	4953.98	–
Model 2	4464.81	2166.09	5002.52	2.0538
Model 3	4525.90	2195.38	4992.64	1.4835

$f(\tau) = 0.5e^{-0.5\tau}$, $h_1(x) = \sigma_1^{\rho_1}\rho_1 x^{\rho_1 - 1}, \sigma_1 = \frac{1}{36}$, $\rho_1 = 2$, $h_2(x) = \sigma_2^{\rho_2}\rho_2 x^{\rho_2 - 1}$, $\sigma_2 = \frac{1}{12}$, $\rho_2 = 2$, $C_2 = 3, C_3 = 0.5$, $\mu_0 = 0.02$, $\pi = 6$, $\mu_{\text{cap}} = 1.5$, $s_2 = 2$, $\sigma = 0.02$, $\rho = 0.2$.

Based on the three models' optimal policies and the amount of pollution generated, a comparison among all three models and all three pollution control policies is made to analyze the appropriate environmental and economic sustainability procedure.

From Table 2, it is observed that.

- The total annual costs per cycle for Model 2 and Model 3 are 0.98%and 0.78% greater than the total yearly cost per cycle for Model 1, which is due to extra pollution control costs in Model 2 and Model 3.
- Optimal backlogging quantity remains almost the same in all three models.
- The amount of pollutants in Model 3 is 27.76% less than in Model 2, which is remarkable.

On observing Table 2, it is concluded that Model 2 and Model 3 are excellent for sustainable production and a better approach towards a green environment. Moreover, Model 3 appeals more towards the environment and fulfills all three aspects of sustainability. Therefore, Model 3 is recommended for further application of pollution control scenarios.

From Table 3, it is observed that.

- All three pollution control mechanisms: pollution cap, pollution cap & trade, and pollution tax, reduce pollutants by 32.52%, 1.72%, and 0.84%. It proves the significance of these mechanisms.
- The pollution cap mechanism reduces the pollution significantly, but the total inventory cost per cycle of the system increases due to this mechanism. However, this policy can

Table 3 Comparison table for optimal results of different pollution control mechanisms concerning Model 3

	Total cost per cycle	Q^*	B^*	Total cycle time ((T^*)	The total amount of pollutant
Model 3	4992.64	4525.90	2195.38	2.2605	1.4835
Model 3.1 (cap)	4968.26	4391.70	2131.04	2.1958	1.0010
Model 3.2 (cap and trade)	5003.50	4566.04	2214.63	2.2830	1.4579
Model 3.3 (tax)	5000.16	4539.33	2201.82	2.2696	1.4710

be applied because customers nowadays are very responsible and ready to spend more money buying eco-friendly goods.

- The pollution cap and trade mechanism is a highly preferable policy for controlling pollution. Because in comparison with all the three pollution control policies, the inventory cost corresponding to this mechanism is minimum, also it reduces the number of pollutants effectively.

4 Sensitivity analysis

To obtain more insights, the robustness of the proposed work is examined concerning the crucial inventory parameters of Model 1.

This section demonstrates the effects of changes in parameters such as A, C_0, C_h, C_S, C_W,C_R, W, M, I_c, and I_e on the optimal production lot size, backorder level and cycle length, and the minimum total inventory cost per unit time. A sensitivity analysis is conducted by changing each parameter by −50%, −20%, +20%, and +50% and by examining each parameter while keeping others constant. Table 4 presents the results.

We derive the following observations from the results in Table 4.

First, a higher setup cost for each production run A positively changes the optimal backorder level B^*, production time t_C^*, cycle length T^*, and production lot size Q^*, and the minimum total inventory cost per unit time $\mathrm{TCU}(Q^*, B^*)$. Thus, manufacturers should produce larger quantities to reduce production and setup costs.

Second, a higher cost per unit time for preventive maintenance C_0 positively changes the minimum total inventory cost per unit time $\mathrm{TCU}(Q^*, B^*)$. Thus, a higher cost per unit time for preventive maintenance increases the minimum total inventory cost per unit time.

Third, a greater holding cost per unit item per unit time C_h increases the optimal backorder level B^* and the minimum total inventory cost per unit time $TCU(Q^*, B^*)$. However, it causes negative changes in the optimal cycle length T^*, production time t_C, and production lot size Q^*, indicating that manufacturers must reduce cycle length and produce smaller quantities to avoid higher holding costs.

Fourth, a higher shortage cost per unit item per unit time C_S and repair cost for warranty per unit item C_W positively changes the minimum total inventory cost per unit time TCU, although it induces negative changes in the optimal backorder level B^*, cycle length T^*, and production lot size Q^*. Thus, manufacturers should increase the cycle length to avoid higher shortage cost.

Fifth, a higher reworking cost for each deviant item C_R and a higher warranty period W reduces the optimal B^*, Q^*, t_C^*, and T^* yet induces positive changes in the minimum total inventory cost per unit time $\mathrm{TCU}(Q^*, B^*)$. Manufacturers should, therefore, produce smaller quantities to avoid higher reworking costs, particularly if the reworking cost is higher for each non-conforming item C_R. Higher reworking costs for non-conforming items and a higher warranty period increase the minimum total inventory cost per unit time.

Sixth, a higher permissible delay period by supplier M and a higher interest earned per dollar per unit time I_e cause negative changes in the optimal production lot size Q^*, production time t_C^*, cycle length T^*, and the minimum total inventory cost per unit time

Table 4 Sensitivity analysis corresponding to Model 1

Parameter	% Change	Q^*	B^*	T^*	t_C	TCU
A	+50%	5280.92	2557.41	2.6404	1.3617	5110.45
	+20%	4719.92	2288.42	2.3599	1.2157	5020.45
	−20%	3847.28	1869.99	1.9236	0.9886	4880.38
	−50%	3032.91	1479.50	1.5164	0.7767	4749.57
C_0	+50%	4305.68	2089.79	2.1584	1.1079	5153.98
	+20%	4305.68	2089.79	2.1584	1.1079	5033.98
	−20%	4305.68	2089.79	2.1528	1.1079	4873.98
	−50%	4305.68	2089.79	2.1528	1.1079	4753.00
C_h	+50%	4164.27	2200.38	2.0821	0.9819	4976.59
	+20%	4242.33	2138.61	2.1211	1.0518	4963.92
	−20%	4381.13	2033.11	2.1905	1.1740	4942.50
	−50%	4525.93	1928.48	2.2629	1.2987	4921.53
C_S	+50%	3952.18	1604.03	1.9760	1.1740	5017.40
	+20%	4137.48	1862.27	2.0687	1.1376	4982.78
	−20%	4531.51	2386.27	2.2657	1.0726	4918.71
	−50%	5070.22	3059.62	2.5351	1.0053	4847.42
C_W	+50%	4303.53	2088.76	2.1517	1.1073	4959.90
	+20%	4304.82	2089.38	2.1524	1.1077	4956.35
	−20%	4306.55	2090.20	2.1532	1.1081	4951.60
	−50%	4307.84	2090.82	2.1539	1.1085	4948.05
C_R	+50%	4288.33	2081.47	2.1441	1.1034	4976.98
	+20%	4298.72	2086.45	2.1493	1.1061	4963.18
	−20%	4312.68	2093.14	2.1563	1.1097	4944.77
	−50%	4323.23	2098.20	2.1616	1.1125	4930.96
W	+50%	4300.31	2087.21	2.1501	1.1065	4968.79
	+20%	4303.79	2088.88	2.1518	1.1074	4959.19
	−20%	4307.24	2090.53	2.1536	1.1083	4949.71
	−50%	4308.92	2091.34	2.1544	1.1087	4945.08
M	+50%	4287.10	2093.50	2.1435	1.0968	4856.20
	+20%	4299.15	2091.70	2.1495	1.1037	4915.01
	−20%	4311.02	2087.30	2.1555	1.1118	4992.75
	−50%	4226.19	2000.00	2.1130	1.1130	5050.85
I_C	+50%	4096.26	2102.25	2.0481	0.9970	4966.92
	+20%	4211.14	2095.15	2.1055	1.0579	4959.71
	−20%	4419.91	2083.87	2.2099	1.1680	4947.27
	−50%	4644.48	2073.88	2.3222	1.2853	4934.77
I_e	+50%	4148.72	2075.24	2.0743	1.0367	4905.71
	+20%	4243.84	2084.31	2.1219	1.0797	4934.86
	−20%	4366.31	2094.82	2.1831	1.1357	4972.84
	−50%	4455.01	2101.50	2.2275	1.1767	5000.70

Table 5 Sensitivity analysis corresponding to Model 2 class I

Parameter	% Change	Q^*	B^*	T^*	t_C	TCUP	Amount of pollutant
C_2	+50%	4542.30	2203.24	2.2711	1.1695	5014.95	2.0894
	+20%	4495.97	2181.03	2.2479	1.1574	5007.51	2.0681
	−20%	4433.44	2151.05	2.2167	1.1411	4997.48	2.0393
	−50%	4385.97	2128.29	2.1929	1.1288	4989.86	2.0175
C_3	+50%	4464.81	2166.09	2.2324	1.1493	5014.02	2.0538
	+20%	4464.81	2166.09	2.2324	1.1493	5007.12	2.0538
	−20%	4464.81	2166.09	2.2324	1.1493	4997.92	2.0538
	−50%	4464.81	2166.09	2.2324	1.1493	4991.02	2.0538
μ_0	+50%	4464.81	2166.09	2.2324	1.1493	5014.02	3.0807
	+20%	4464.81	2166.09	2.2324	1.1493	5007.12	2.4645
	−20%	4464.81	2166.09	2.2324	1.1493	4997.92	1.6430
	−50%	4464.81	2166.09	2.2324	1.1493	4991.02	1.0269

TCU(Q^*, B^*). Thus, manufacturers should decrease the cycle length and produce smaller quantities to more frequently make use of permissible delays.

Finally, a higher I_c that is interest charged per dollar per unit time in stocks by the supplier leads to negative changes in the optimal cycle length T^*, production time $t_{C,}$ and production lot size Q^*. However, a higher I_c positively changes the optimal backorder level B^* and the minimum total inventory cost per unit time TCU(Q^*, B^*), and thus, manufacturers should decrease the cycle length and produce smaller quantities in the case of higher interest charged by the supplier.

4.1 Sensitivity analysis corresponding to Model 2 class I

It is observed from Table 5 that.

- On increasing the pollution control cost C_2, the system's total cost per cycle increases from 4989.86 to 5014.95.
- On increasing another pollution control costs C_3, the system's total cost per cycle increases from 4991.02 to 5014.02, which is quite apparent to C_2, but this increment is significantly less. Therefore, it is advised to apply pollution control policies to ensure cleaner production practices.

Table 6 Optimal policies corresponding to Model 4.1

Pollution tax (π)	Total cost per cycle	Q	B	Amount of pollutant
6	5000.16	4539.33	2201.82	1.4710
14	5003.03	4522.53	2193.77	1.4669
22	5005.88	4505.91	2185.80	1.4628
30	5008.73	4489.46	2177.91	1.4587

Table 7 Optimal policies corresponding to Model 4.2

Selling price (s_2)	Total cost per cycle	Q	B	Amount of pollutant
4	5003.50	4566.04	2214.63	1.8573
6	5003.72	4561.82	2212.61	1.7563
8	5003.94	4557.62	2210.59	1.6553
10	5004.15	4553.43	2208.58	1.5543
12	5004.37	4549.24	2206.58	1.4533
14	5004.58	4545.07	2204.57	1.3523
16	5004.80	4540.91	2202.58	1.2513
18	5005.01	4536.76	2200.59	1.1503

Table 8 Optimal policies corresponding to Model 4.3

Pollution cap (μ_{Cap})	Total cost per cycle	Q	B	Amount of pollutant
0.25	5191.19	9365.13	4000.00	0.8333
0.30	5115.03	8286.75	3998.69	0.8666
0.35	5057.01	7218.22	3486.34	0.9000
0.40	5008.95	6159.37	2978.62	0.9337
0.45	4976.36	5110.01	2475.46	0.9666

- The rise in the value of pollution factor (μ_0) increases the total cost per cycle. It is because an increase in the pollution factor needs to increase pollution control cost, resulting in an increased overall cost of the organization. Hence, it is suggested that inventory managers focus on rework and waste-reducing practices.

4.1.1 Sensitivity corresponding to Model 4.1 concerning a pollution tax

From Table 6, the following analysis is done:

- On increasing the pollution tax (π) from 6 to 30, the total amount of pollution of the system decreases from 1.4710 to 1.4587.
- Apart from this, on increasing the pollution tax (π) from 6 to 30, the total cost per cycle increases from 5000.16 to 5008.73.
- It is also noted that an increase in pollution tax (π) resulted in a decrease in both Q and B.

4.2 Sensitivity corresponding to Model 4.2 concerning the selling price of pollution credit

From Table 7, the following analysis is done:

When the selling price $\left(s_2\right)$ of pollution credit increases from 4 to 18;

- The total cost per cycle slowly increases from 5003.50 to 5005.01.
- Apart from this, the total order quantity Q and the backorder level B decrease from 4566.04 to 4536.76 and 2214.63 to 2200.59, respectively.
- The total amount of pollution of the system decreases from 1.8573 to 1.1503.

So, it is observed that this mechanism shows better performance than the pollution tax mechanism.

4.3 Sensitivity corresponding to Model 4.3 concerning pollution cap

From Table 8, the following analysis is done:

When the pollution cap $\left(\mu_{\text{Cap}}\right)$ increases from 0.25 to 0.45,

- The total cost per cycle decreases remarkably from 5191.19 to 4976.36 and Q decreases from 9365.13 to 5110.01. So it is concluded that this mechanism could lower the number of pollutants significantly, but it creates extra monetary liabilities for the organizations.
- The amount of pollutant increases from 0.8333 to 0.9666, but the backorder level B decreases from 4000.00 to 2475.46.
- By comparing and analyzing three mechanisms, it is found that this mechanism reduces pollutants fast as μ_{Cap} is highly sensitive to the number of pollutants.

Although this mechanism increases the total cost per cycle while imposing a pollution cap, it is expected that decision-makers should be responsible enough/ethical to restrict the amount of pollution.

4.4 Managerial insights and industry implication

The results present the strategy for properly managing defectives and pollution control in the manufacturing system with trade credit and rework. The design with minimum cost and minimum emission and least waste generation presents the optimal solution. This study has the following insights:

- Trade credit helps industrialists to ensure improved customer satisfaction.
- The implementation of various pollution prevention mechanisms and pollution control costs is a step towards sustainability. It also helps industries get their trade license to renew easily.
- Rework of defectives decreases energy usage and also lowers the amount of waste that causes landfills. It could be beneficial to the industry economically.
- This study provides three models with or without pollution control cost. However, the system's total cost is minimal in the model without pollution controls cost, even if it is

not economical to use. As it has no pollution check parameter, some industry adopts this model. It may have to face the penalty for emitting pollution.

- The model with variable pollution control cost is suggested for the industrial managers, as it lowers the emission significantly and economical.
- Although the implementation of pollution control policies raises the system's total cost, results prove that carbon cap and trade policy is best for the industry's economic and environmental sustainability.

5 Conclusions

This study incorporates real market phenomena to develop a *EPQ* model that considers imperfect quality items, preventive maintenance, warranty, and permissible payment delays defined by suppliers. Here, we develop a green inventory system under various carbon emissions policies, as well as considering the impact of trade credit policy. As per Theorem 1, the inventory total cost per unit time is pseudo-convex for any given backorder level B, and thus, a unique minimum solution exists. Theorem 2 proves that the inventory total cost per unit time is pseudo-convex for any given production lot size Q, thus giving a unique minimum solution. With this study, we present an easy and useful method to determine an optimal production plan for manufacturers to minimize total cost. Then two different sustainable production models are presented here by considering pollution control costs. A sustainable production model with variable pollution costs is examined under the influence of three pollution control mechanisms to improve the model's applicability. The paper's novelty lies in introducing pollution control costs and pollution control mechanisms together in a sustainable production system with trade credit. We demonstrate how environmental regulations can be incorporated into a trade credit policy and inventory order decision making problem for an imperfect production system.

There are several interesting extensions to this work for future research. We can extend the proposed model with a stochastic demand and default risk rates, and allow for partial backlogging. Future research also can extend this model to consider hidden inventory costs incurred by manufacturers such as transportation costs and effective investment in low-carbon technologies to cut emissions. The effects of inflation, recycling, and error in screening could also be studied in the future.

Appendix 1: Proof of Theorem 1

From Eq. (25), for any given B, we have.

$$\mathrm{TCU}_1(Q,B)=\frac{u_1(Q)}{v_1(Q)},$$

where

$$u_1(Q) = AD + CDQ + C_0\frac{(P-D)}{P}Q + C_W QD\left[W_1 + \left(W_2 - W_1\right)g(Q)\right] + C_R g(Q)Q + C_h\left[\frac{(P-D)}{2P}Q^2 + \frac{P}{2(P-D)}B^2 - QB\right] + C_S\frac{PB^2}{2(P-D)} + \mathrm{CI}_C\left[\frac{Q^2}{2} + \frac{PB^2}{2(P-D)} - QB - MDQ + \frac{PD}{2}M^2\right] - \frac{\mathrm{SI}_e PDM^2}{2},$$

and

$$v_1(Q) = Q > 0.$$

Taking the first-order and second-order derivative of $u_1(Q)$, we have

$$u_1'(Q) = CD + C_0\frac{P-D}{P} + C_W D\left[W_1 + \left(W_2 - W_1\right)g(Q)\right] + C_W QD\left(W_2 - W_1\right)g'(Q) + C_R Dg(Q) + C_R DQg'(Q) + C_h\left[\frac{P-D}{P}Q - B\right] + \mathrm{CI}_C[Q - B - MD],$$

and

$$u_1''(Q) = D\left[2g'(Q) - Qg''(Q)\right]\left[C_W\left(W_2 - W_1\right) + C_R\right] + C_h\frac{P-D}{P} + \mathrm{CI}_C.$$

As a result, if $K > 0$, then $u_1''(Q) > 0$, and hence, $u_1(Q)$ is non-negative, differentiable, and strictly convex. Thus, if $K > 0$, then $\mathrm{TCU}_1(Q, B)$ as in (25) is a strictly pseudo-convex function in Q and exists a unique optimal solution. By using an analogous argument, we can prove that if $K > 0$, then $\mathrm{TCU}_i(Q, B), i = 2, 3$, is a strictly pseudo-convex function in Q and exists a unique minimum solution $Q_i^*, i = 2, 3$. Consequently, we have completed the proof of Theorem 1.

Appendix 2: Proof of Theorem 2

From Eq. (25), for any given Q, we have.

$$\mathrm{TCU}_1(Q, B) = \frac{f_1(B)}{Q},$$

where

$$
\begin{aligned}
f_1(B) &= AD + CDQ + C_0\frac{(P-D)}{P}Q + C_W QD\left[W_1 + \left(W_2 - W_1\right)g(Q)\right] + C_R g(Q)QD \\
&+ C_h\left[\frac{(P-D)}{2P}Q^2 + \frac{P}{2(P-D)}B^2 - QB\right] + C_S\frac{PB^2}{2(P-D)} \\
&+ \mathrm{CI}_C\left[\frac{Q^2}{2} + \frac{PB^2}{2(P-D)} - QB - MDQ + \frac{PD}{2}M^2\right] - \frac{\mathrm{SI}_e PDM^2}{2} \\
&\frac{\mathrm{d}}{\mathrm{d}B}\mathrm{TCU}_1(Q,B) = \frac{1}{Q}\frac{\mathrm{d}f_1(B)}{\mathrm{d}B}, \text{ and } \frac{\mathrm{d}^2}{\mathrm{d}B^2}\mathrm{TCU}_1(Q,B) = \frac{1}{Q}\frac{\mathrm{d}^2 f_1(B)}{\mathrm{d}B^2}
\end{aligned}
$$

Taking the first-order and second-order derivative of $f_1(B)$, we have

$$
f_1'(B) = C_h\left[\frac{P}{P-D}B - Q\right] + C_S\frac{P}{P-D}B + \mathrm{CI}_C\left[\frac{P}{P-D}B - Q\right],
$$

and

$$
f_1''(B) = C_h\frac{P}{P-D} + C_S\frac{P}{P-D} + \mathrm{CI}_C\frac{P}{P-D} = \frac{P}{P-D}\left[C_h + C_S + CI_C\right] > 0,
$$

respectively.

As a result, $f_1(B)$ is non-negative, differentiable, and strictly convex. Thus, $\mathrm{TCU}_1(Q,B)$ as in (25) is a strictly pseudo-convex function in B and exists a unique optimal solution. By using an analogous argument, we can prove that $\mathrm{TCU}_i(Q,B), i = 2, 3$, is a strictly pseudo-convex function in B and hence exists a unique minimum solution $B_i^*, i = 2, 3$. Consequently, we have completed the proof of Theorem 2.

Acknowledgements The authors are grateful to the editor and anonymous reviewers for their valuable comments and suggestions to improve the quality of this article.

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