Sensitivity analysis assessment of remotely based vegetation indices to improve water resources management

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Abstract Normalized Difference Vegetation Index (NDVI) is estimated from Landsat 8 sensor acquired in June 2013 to drive four different water-related indices calculated as NDVI derivatives. Different vegetation indices (VIs) have been extracted exclusively in estimation of different VIs: Leaf Area Index, Water Supply Vegetation Index, Crop Water Shortage Index, and Drought Severity Index in addition to estimation of daily evapotranspiration (ET). Sensitivity analysis assesses the contributions of the inputs to the total uncertainty in the analysis outcomes. Vegetation indices are complex and intercepted, therefore the interceptions of the five different vegetation indices are considered in the current study. A comparative analysis of Gaussian process emulators for performing global sensitivity analysis was used to conduct a variance-based sensitivity analysis to identify which uncertain inputs are driving the output uncertainty. The results showed that the interconnections between different VIs vary, but the extent of the features sensitivity is uncertain. Findings from the current work conducted are anticipated to contribute decisively toward an inclusive VIs assessment of its overall verification. Daily ET is the less sensitive and more certain index followed by Drought Vegetation Index.

Keywords Global sensitivity analysis · Remote sensing · Uncertainty · Spatial decision support system · Suitability map · Vegetation indices · Water resources management

1 Introduction

Vegetation indices (VIs) have been widely used to monitor terrestrial precision farming by satellite sensors and have been highly successful in assessing vegetation condition, foliage, cover, and phenology (Pettorelli et al. [2005;](#page-12-0) Kerr and Ostrovsky [2003;](#page-12-0) Huete et al. [2008\)](#page-12-0). VIs are robust satellite data products computed the same way across all pixels in time and space,

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regardless of surface conditions. As ratios, they can be easily cross-calibrated across sensor systems, ensuring continuity of data sets for long-term monitoring of the land surface and climate-related processes (Edward et al. [2008](#page-12-0)). Studies carried out by Pettorelli et al. [\(2005](#page-12-0)), Kerr and Ostrovsky ([2003\)](#page-12-0), and Huete et al. [\(2008](#page-12-0)) demonstrated that VIs have been employed exclusively in estimation of vegetation parameters such as fractional vegetation cover (FC), Leaf Area Index (LAI), Water Supply Vegetation Index (WSVI), Crop Water Shortage Index (CWSI), and Drought Severity Index (DSI).

Spatial decision support system process inaugurates with the recognition of the problem to be decided. In the intelligence phase, a situation is examined for conditions calling for a decision (Dragan et al. [2003\)](#page-12-0). In the design phase, decision makers develop alternative solutions to the decision problem already identified. Typically, a formal model is used to support a decision maker in determining the set of alternatives. In the choice phase, decision makers evaluate the decisions and choose the best alternative (Accorsi et al. [1999\)](#page-11-0). In the context of decision problems with a spatial connotation, the potential for application of spatial decision support system has already been examined (Elhag [2010](#page-12-0)). While the intelligence and design activities can mostly be covered by multi-purpose spatial analysis methods, the choice phase requires specific thresholds still absent from most of GIS models (Focht et al. [1999](#page-12-0); Gregory and Wellman [2001](#page-12-0)).

Global sensitivity analysis (GSA) methods assign the output inconsistency to the inconsistency of the input parameters when they vary over their whole uncertainty domain (Petropoulos et al. [2009\)](#page-12-0). Sensitivity of the input parameters is basically examined based on the generation of samples distributed across the parameter domain of interest. Comprehensive review of the available GSA methods and their applications is provided, for example, by Saltelli et al. [\(2000](#page-12-0), [2004\)](#page-12-0), Saltelli [\(2002\)](#page-12-0). GSA is a powerful tool due to its ability to integrate the influence of the input parameters over their whole range of discrepancy (Saltelli et al. [1999\)](#page-12-0). GSA techniques are able to deliver quantitative estimates not only of the most sensitive model inputs, but also of the model input parameter interactions (Schwieger [2004\)](#page-12-0), yielding quantitative information on the degree of complexity of the model input–output interactions (Petropoulos et al. [2009\)](#page-12-0).

The practice shows the lack of proper incentives discourages the promotion of water saving technologies and changes in land management. Subsidies are responsible for ''failure to value water at anything to its true worth'' and underpricing assists in anchoring believes that water resources are plenty and abundant (Postel [1997\)](#page-12-0). Heavy subsidies for irrigation are not only a luxury the developed countries' farmers benefit from, but the practice of undercharging is widely spread in developing countries as well, especially in Egypt. The most challenging is the fact that the free of charge approach for irrigation service is used in countries, which have high risks of acute water shortages in the near future. (Postel [1997;](#page-12-0) Myers and Kent [1998](#page-12-0)).

The aim of sensitivity analysis was to determine how sensitive the output of remotely sensed VIs is, with respect to the elements of the practiced water resources management in the study area, which are subject to uncertainty or variability. This is useful as a guiding tool when the model is under development as well as to understand model behavior when it is used for prediction or for decision support.

2 Materials and methods

2.1 Study area

The Nile Delta was selected for this study because it is representative of farming scenarios in the whole Egypt (different agricultural systems, different soil types, different systems of fertilizer application, irrigation and drainage systems); consequently, the research in this study area can be applied to farms in other regions of Egypt. Also, the problems affecting agriculture in this area are a miniature of that of the entire territory (salinity, alkalinity, and water logging).

The huge triangle of the Nile Delta extends to the north of Cairo between Lake Mareotis in the west and the Suez Canal in the east, forming a wide arc along the Mediterranean coast bordered by lagoons and sand spits. Formed over millions of years by the deposits of mud brought down by the regular annual inundation and sediment transport and deposition of Nile, it marks the end of the river's long journey; when emerging from its narrow bed at the edge of the desert plateau, it breaks up into separate arms which pursue their meandering courses toward the sea. This study was carried out in the one of the main agricultural regions of Egypt represented by several Governorates located centrally at 30.07°N, 30.57E. The Governorates are as follows: Alexandria, Buhayra, Cairo, Daqahliya, Damietta, Gharbiya, Ismailia, Kafr El-Sheikh, Minufiya, Port Said, Qalyubiya, Sharqiya and Suez and cover around $25{,}000 \text{ km}^2$ in total representing 2.5 % of the total area of Egypt (Fig. [1\)](#page-3-0).

The most significant factors of land degradation are as follows: (a) wind, (b) water erosion, (c) water logging, (d) salinization, and (e) soil compaction. On the other hand, land reclamation processes, enclosing the wider Delta region, are very pronounced due to human activities. The land use and land cover categories are as follows: (a) agriculture, (b) bare soil, (c) sand area, (d) salt flat, (e) swamps, (f) salt, (g) fish farms, (h) water bodies, and (i) urban areas (Elhag et al. [2013\)](#page-12-0).

2.2 Vegetation indices

In principle, Normalized Difference Vegetation Index (NDVI) was obtained from Landsat 8 satellite sensor acquired in July 2013 to drive four different water-related indices calculated as NDVI derivatives. Spectral enhancement is the changing of the values of each pixel in the original image by transforming the values of each pixel along a multi-band basis. Spectral enhancement allows different features that have specific reflective characteristics in different bands of the electromagnetic spectrum to be compressed if data are similar. It also allows modifying of the pixels of an image independent of the values of surrounding pixels. Spectral enhancement creates new bands of data that are more interpretable to the eye. NDVI derivatives are useful to recognize other VIs including, LAI and WSVI. Meanwhile, soil–water relation in term of evapotranspiration is useful to calculate other VIs including, CWSI and DSI. Estimation of the aforementioned VIs is previously described in Elhag ([2014\)](#page-12-0) and estimation of daily evapotranspiration (ET) is described in Elhag et al. ([2011\)](#page-12-0).

2.3 Spatial decision support system

Spatial decision support system methods make the options and their contribution to the different criteria explicit, and all require the exercise of judgment. They differ, however, in how they combine the data. Formal SDSS techniques usually provide an explicit relative weighting system for the different criteria. SDSS techniques can be used to identify a single most preferred option, to rank options, and to set a limited number of options for subsequent detailed evaluation, and SDSS techniques adopted in the current research are explained in Elhag [\(2014](#page-12-0)). The evaluation of land in term of rice cultivation was based on the methods described in Food and Agriculture Organization (FAO [1976](#page-12-0), [1978,](#page-12-0) [1983](#page-12-0),

Fig. 1 Location map of the study area (Elhag et al. [2011\)](#page-12-0)

[1984,](#page-12-0) [1985,](#page-12-0) [2005](#page-12-0)) guideline for land evaluation and rice cultivation preferences concerning the study area.

2.4 Generating the analysis matrix

The analysis matrix $(M \times N: M$ options and N criteria) is to be built from the environmental indicators identified in the conceptual phase. The cells of the matrix relate to the option–criterion pairs and contain the outcomes or consequences for a set of options and a set of evaluation criteria. Construction of the reciprocal matrix and matrix analysis is exemplified in Elhag ([2014\)](#page-12-0).

2.5 Basics of variance-based sensitivity analysis

Sensitivity analysis approaches are categorized according to the outcome of the related sensitivity procedures into local or global methods. Sensitivity analysis methods may depend on or are independent of the model characteristics (Schwieger [2004](#page-12-0)). To find out which of the uncertain input factors are more important in determining the uncertainty in the output of interest, GSA is required (Sobol [2001](#page-13-0)).

2.5.1 Global sensitivity analysis concept

Consistent with Saltelli et al. ([2000\)](#page-12-0), GSA is the study about the relations between the input and the output of a model. Basically, sensitivity analysis is dealing with the variation correspondingly the uncertainties of the input magnitudes. Moreover, input parameters introduced to uncertainties of the model parameters and to the overall model structure. The discrepancy of the input parameters encounters discrepancies of the output magnitudes. The interconnections between speckled input and output are measured by different sensitivity measures that are the basis for model validation and optimization (Schwieger [2004\)](#page-12-0). Broad practice of sensitivity analysis is shown in Fig. [2.](#page-4-0) GSA is emphasis on variance-based techniques to estimate global, quantitative, and model independent sensitivity measures.

Fig. 2 General procedure for sensitivity analysis (Schwieger [2004](#page-12-0))

2.5.2 Global sensitivity analysis procedures

Based on Monte Carlo methods, sensitivity analysis methods are regression and correlation analysis as well as analysis of rank transformed data. The general procedure to estimate global sensitivity measures is founded on the following equations:

$$
S_i = \frac{\sigma_{E(Y/X_i)}^2}{\sigma_Y^2} \tag{1}
$$

where $\sigma_{E(Y|X_i)}^2$ is the conditional variance, and σ_Y^2 is the unconditional variance. For noncorrelated input additive models:

$$
\sum_{i=1}^{n} S_i = 1 \tag{2}
$$

According to Schwieger [\(2004\)](#page-12-0), this leads to an easy quantitative interpretation of the sensitivity indices, because each S_i delivers a direct measure for the portion of X_i on the output variance σ_Y^2 . For non-additive models, the interactions among the input quantities within the model have to be taken into account. Non-additive models need a complete decomposition of the function Y into summands of increasing order:

$$
\sum_{i=1}^{n} S_i + \sum_{i=1}^{n} \sum_{j=i+1}^{n} S_{i,j} + \sum_{i=1}^{n} \sum_{j=i+1}^{n} \sum_{k=j+1}^{n} S_{i,j,k} + \dots + \sum \dots S_{i,j,k,\dots,n} = 1
$$
 (3)

The terms of higher order are estimated by holding more than one input quantity fixed:

$$
S_{i,j} = \frac{\sigma_{E(Y/X_i, X_j)}^2}{\sigma_Y^2} - S_i - S_j.
$$
 (4)

Estimation of higher order terms leads to the estimation of total effects S_{T_i} with respect to an input quantity X_i to be computed as follows:

$$
S_{Ti} = S_i + \sum_{i=1}^{n} S_i + \sum_{i=1}^{n} \sum_{j=i+1}^{n} S_{i,j}
$$

+
$$
\sum_{i=1}^{n} \sum_{j=i+1}^{n} \sum_{k=j+1}^{n} S_{i,j,k} + \dots + \sum \dots S_{i,j,k,...,n} \text{ mit } j, k \neq i, \quad j \neq k \tag{5}
$$

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Fig. 3 LAI suitability map for designated study area in Nile Delta region

Corresponding total effect is computed as following:

$$
S_{Ti} = 1 - \frac{\sigma_{E(Y/X \sim i)}^2}{\sigma_Y^2} \tag{6}
$$

Consistently, a judgment between S_i and S_{Ti} leads to a conclusion concerning the additivity of models with non-correlated input:

 $S_{Ti} = S_i$ for additive model

 $S_{Ti} > S_i$ fornon-additivemodel

3 Results and discussion

Evaluation of different VIs importance regarding to rice cultivation water requirements in the designated study area is carried out to find out valuable parametric values in terms of crop–water relations.

Fig. 4 WSVI suitability map for designated study area in Nile Delta region

Fig. 5 CWSI Suitability map for designated study area in Nile Delta region

Reciprocal matrix is performed using five different factors. Table [1](#page-5-0) demonstrated the eigenvector weight of the five used factors and its importance were estimated through the reciprocal matrix. The interception of the LAI and ET is the strongest among the rest of the used factors according to Elhag ([2014](#page-12-0)). The results of the matrix pointed out that the ET is

Fig. 6 DSI suitability map for designated study area in Nile Delta region

Fig. 7 ET_{daily} suitability map for designated study area in Nile Delta region

very important and had the biggest eigenvector weight followed by the distance to the LAI (Malczewski [1999;](#page-12-0) Afify et al. [2011\)](#page-11-0). Such a correlation need to be considered in water conservation plans regarding to rice cultivation (Diker and Unlu [1999](#page-12-0)). Only ratios less than 0.1 are accepted for a proper spatial decision support system, the importance and the

Fig. 8 Overlaid suitability map for rice cultivation in designated study area

Fig. 9 Sensitivity variances of Crop Water Shortage Index main effects

Fig. 10 Sensitivity variances of Drought Severity Index main effects

Fig. 11 Sensitivity variances of Daily Evapotranspiration Index main effects

Fig. 12 Sensitivity variances of Leaf Area Index main effects

Fig. 13 Sensitivity variances of Water Shortage Vegetation Index main effects

	Variance $(\%)$	SD.	Total effect
Crop Water Shortage Index	23.8	24.9	25.48
Drought Severity Index	21.5	22.1	22.9
Daily Evapotranspiration	9.8	10.1	10.6
Leaf Area Index	13.9	14.3	16.8
Water Supply Vegetation Index	22.9	22.7	24.21

Table 2 Vegetation indices sensitivity analysis

Fig. 14 Vegetation indices total effect sensitivity analysis

weights of the used factor need to be reconsider in case of consistency ratios higher than 0.1 (Dragan et al. [2003](#page-12-0); Elhag [2010](#page-12-0)).

Vegetation indices including CWSI, Water Vegetation Supply Index, and DSI need to be integrated together for satisfactory results (Bouman and Tuong [2001](#page-12-0)). Each one of the eigenvector weights was multiply to its corresponded layer, and then the layers were overlaid all together to be introduced to a final suitability map (Dragan et al. [2003\)](#page-12-0).

Rice cultivation suitability values were ranging from zero to one and demonstrated in Figs. [3](#page-5-0), [4](#page-6-0), [5](#page-6-0), [6](#page-7-0), [7](#page-7-0), and [8](#page-8-0) according to the corresponding factor. The spatial distribution of rice cultivation suitable areas varies in the designated study area (Elhag [2014](#page-12-0)).

Suitable area for rice cultivation, based on LAI factor, is distributed on both east and west side of the study area (Fig. [3](#page-5-0)) in contradiction with the spatial distribution of WSVI (Fig. [4](#page-6-0)). Crop Water Shortage Index suitable area distribution (Fig. [5\)](#page-6-0) showed no definite pattern and is distributed all over the command area. DSI (Fig. [6\)](#page-7-0) expressed the same pattern of CWSI. Daily ET suitable area (Fig. [7\)](#page-7-0) demonstrated marginal spatial distribution pattern surrounding the lake and keep most of the designated study area with less rice cultivation suitable values.

The results from the sensitivity analysis are presented in Figs. [9](#page-8-0), [10,](#page-8-0) [11](#page-9-0), [12,](#page-9-0) and [13](#page-9-0), focusing specifically on the decomposition of variance (%) of the mean total variance in emulator output, when input parameters have been assumed non-correlated, normally distributed and varying within their whole range. Red lines in the following figures represent the mean and the standard deviation from the mean total effects according to Saltelli et al. ([2000\)](#page-12-0). The relative sensitivity of the model input parameters with respect to the sensitivity of the VIs estimated in the study area can be found in Table [2](#page-9-0), and sensitivity total effect is shown in pie chart representation (Fig. 14).

In the following table, parameters variances estimated to be deterministically sensitive at the Crop Water Shortage Index (23.8 %), the second in order sensitive vegetation index is Water Supply Vegetation Index (22.9 %), the third in order is Drought Severity Index with estimated vaurince of 21.5 $\%$. The sum of the above three VIs total effect is exceeding the value of 70 % which implies presence of interactions in term of dependency on the rest VIs (Holvoet et al. [2005](#page-12-0)). The least sensitive vegetation index is daily ET (9.8 %). Daily ET has the smallest individual contribution to the total variances (Saltelli [2002](#page-12-0)).

4 Conclusions

Multi-criteria decision analysis was constructed with the five important VIs, and the relative importance of each one was measured through the calculation of weights. The importance is given to the value of the daily ET as the key factor for rational water resources management in arid areas. Once the factor maps and the weight of composite layers were obtained, and then the physical suitability map was evaluated for the rice cultivation by weighted overlay (Yager [1988](#page-13-0); Turban et al. [2005\)](#page-13-0).

Identification of Rice crop suitable areas by realizing of Remote Sensing techniques and Spatial Decision Support System model were successful in the designated study area. The results obtained from this study indicate that the integration of RS–GIS and application of multi-criteria evaluation using pairwise comparison matrix could provide a superior database and guide map for decision makers considering crop substitution in order to achieve better agricultural production (Allam and Allam 2007). However, in Egypt, this approach is a new and original application in agriculture, because it has not been used to identify suitable areas for rice crop intensively. Spatial distribution of rice crop derived from Remote Sensing data in conjunction with evaluation of biophysical variables of soil and topographic information in GIS context is helpful in crop management options for intensification or diversification.

Global sensitivity analysis of five different VIs delivered a quantitative and model independent sensitivity measure of each of the input factors and of groups of them to the simulated outputs under consideration. Results evidenced the model concept to be sufficiently sensitive to represent the natural systems' behavior. The sensitivity analysis confirmed that daily ET is consecutively less sensitive among the different VIs based on water management hypothesis.

Input parameters are related directly to the estimated variables derived from the uncertainty analysis (Holvoet et al. [2005\)](#page-12-0). The GSA is used to identify the portions of the variance related to different measured input quantities. This method for sensitivity analysis is independent of the characteristics of the analyzed model. The results may be used for future model optimization, e.g., by comparing different VIs from different locations.

This study has been done considering crop water consumption and five different VIs to find out the common thread in rice cultivation water requirements to achieve greater accuracy from remotely sensed data. Therefore, it gives primary results, but for further study, additional factors such as soil, climate, irrigation facilities, and socioeconomic factors that influence the sustainable use of the land are required. Such conclusions are a robust evidence of a defective water resources management planes used by either the government or the farmers, and all of these management planes need to reformed, and estimation and forecasting of daily ET values must be concentric.

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