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Time-Frequency Analysis of the Relationship Between EUA and CER Carbon Markets

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Abstract In this paper, interactions or co-movement between the CER and EUA futures prices are examined in order to shed light on the dependency between the European Union Emissions Trading Scheme (EU ETS) and the clean development mechanism (MDP). Our analysis uses the wavelet method to model the correlation between CER and EUA in the time-frequency domain. It highlights the impact of different investors (according to their investment horizons) on the co-movement between the CER and EUA prices, and therefore, the behavior of individual investors as speculators, arbitrageurs, and hedgers on European allowance and CDM credits cumulatively. In this vein, we analyze according to the frequency intervals, price convergence, identification of potential factors that could explain a difference in futures prices, and structural changes in the EUA and CER prices. The application is made using daily EUA's and CER's prices data.

Keywords Co-movement · Carbon market · Wavelet analysis · Co-integration · VAR · Agents heterogeneity · Spread-option

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1 Introduction

European climate policy is mainly based on the European emissions trading system says European Union Emissions Trading Scheme (EU ETS) or the European carbon market. It sets a cap on CO_2 emissions of more than 11,000 industrial sites in Europe, in the most emitting sectors: power generation (electricity and heat, refining), mineral industries (cement, lime , glass, ceramics), metallurgy (steel, iron), and paper. This ceiling is materialized by the distribution of quotas each year in industrial sites. One quota, also called European Union Allowance (EUA) = 1 tonne of CO_2. The allocation method is described for each country in a national allocation plan (NAP), approved by the European Commission. When you are emitting greenhouse gas emissions, you are forced over the years, either to reduce or to buy allowances from other issuers whose reductions exceed their quotas and thus would have a surplus. The European system of tradable quotas apply only to industrial emissions, individuals are not required to buy allowances for emissions of their cars, houses, or other sources of energy consumption, which limits these systems about 40 % of emissions by country. All carbon markets as carbon finance whose "currency," quotas or emission credits, each representing 1 tonne of greenhouse gas emissions, expressed in CO_2 equivalents. The carbon asset transactions may have a goal of compliance under regulations established by the state or a voluntary target.

The EU ETS started in the beginning of 2005. It is the main instrument of the European climate policy makers and many see it as an engine for transition to a low carbon economy. By putting a price on CO_2 emissions, the program is meant to encourage major carbon emitters to develop new technologies that emit less CO_2. In fact, the EU has committed to reduce its greenhouse gas emissions by an average of 8 % below its 1990 levels under the Kyoto Protocol. As an emission cap, some energy-intensive industrial operators receive in phases I and II of the scheme, an annual free allocation of EUAs. One EUA give to carbon emitters the right to emit 1 tonne of CO_2 in the nature. Besides the domestic reductions targeted by surrendering EUAs for energy intensive industrial operators within the EU-ETS. Linking directive phase II allows fixed installations and aircraft operators to use credits from Kyoto projects in order to comply with the obligations of EU ETS emission allowances (ETS). French banks have the right to use Kyoto credits for their compliance to 13.5 % of their allocation. Since the price of CER is less than the EUA price, they can exploit the difference in price exchanging EUA against CERs. And up to 5 years by selling their "binding capacity," institutions can generate immediate cash or buy a higher number of CERs today. By performing a swap operation, the facility will not cause any breach of its obligation to comply. It has only the price difference between EUA and CER.

Although both EUAs and CERs allow the emission of the 1 tonne of CO_2 into the atmosphere, and both can be used for compliance within the EU ETS, EUA prices nevertheless exceed CER prices. Thus, without a doubt, we can achieve arbitrage opportunities by adopting a swap carbon quotas from both EU ETS and CDM mechanisms.

The hypothesis of this paper is that there exists several important financial and structural reasons that justify for the price differential between the CER and EUA. Our assumption is realistic since the European commission has established a limit on the use of CERs (primary or secondary) up to 13.4 % of their allocation from 2008 to 2012 on average. In fact, on October 27, 2004, the EU adopted the linking-directive in order to recognize project-based credits generated through the Kyoto flexible mechanisms. However, there have been criticisms directed against this the EU adoption. Critics argue that importing the Kyoto mechanisms credit diminishes the incentives for innovations through having access to cheaper compliance options. Moreover, there are a doubts about the environmental integrity of the project-based credits [7], and therefore, in accordance with the supplementarily and the additional obligations laid down in the Marrakech Accords (under the Kyoto Protocol), the EU has prevented Member States and their operators from overusing project-based credits. According to ETS-directive, Member States must impose a limit on the maximum amount of JI-CDM credits that their covered installations are entitled to use for compliance under scheme.

On December 17, 2008, the Parliament of the European Union adopted the phase III of the EU ETS package named "The Climate and Energy Package" for the period 2013–2020 [8]. Besides extending the mechanism to certain sectors not covered by phases I and II, the mechanism of phase III imposes strict restrictions on the use of credits from the clean development mechanism (CDM). The maximum allowable utilization of CERs and ERUs during the new phase III has just increased by 20 % compared with phase II. The reason of this restriction on the import and the use of credits is reinforced, due to growing criticism of the environmental effectiveness of project-based credits. For example, the project-based credits must be accepted by all EU Member States and meet the standards of environmental integrity.

The study of the relationship between CER and EUA has been recently discussed in some publications. Under certain assumptions, these studies often use some conventional econometric or statistical approaches to model relation between prices or returns of CER and EUA see Nazefi [16] and some references therein. Nazefi [15] also investigates the dynamic interrelation between EUA and CER prices using Granger-causality tests as well as a generalized impulse-response analysis. Chevallier ([2], 2012) also investigate the study of inter-relationship between CER and EUA, and get results indicating that both CERs and EUAs are cointegrated and affect each other significantly.

Unfortunately, the current literature provides some conflicting and inconclusive evidence. To the best of our knowledge, to date, there has been no empirical study concentrating on the co-movement between EUA and CER prices or returns on the time-frequency domain. Such a study would exhibit the impact of investor behavior (according to their investment horizons) on the co-movement between the CER and EUA prices, and thus on the confrontation of supply and demand curves on both mechanisms. In fact, the EUA and CER carbon markets are systems of interacting agents with different term objectives. Therefore, time series prices resulting from the bid/ask of agents on some times period, are generated by processes formed by a combination of different components operating at different frequencies. Standard time series econometric methods usually consider the frequency and time components separately as in [2].

In this paper, we adopt the wavelet approach to study the correlation between EUA and CER prices indexes. The wavelet approach allows us to study the frequency components of EUA and CER time series prices without losing the time information. The powerful wavelet analysis approach is model free and its also help us uncover interactions that the other econometric model cannot easily provide. The wavelet analysis has been applied in several areas of economics. Davidson et al. [4] have applied wavelet to find semi-parametric regression for study commodity price behavior. Connor and Rossiter [3] have a precursors for estimating price correlations based on scale decomposition of time series using wavelets on commodity markets. Recently, [13] has used wavelets to studied correlation between oil prices and economic activity. More recently, [14] have also re-visited the co-movement of energy commodities using wavelet coherence analysis. Mansanet-Battaler et al. [12] study the price relationship between EUAs and CERs by investigating the price drivers of EUAs and CERs as well as the factors that can explain the spread between them. Barrieu and Fehr [1] provide arbitrage-free continuous-time model for EUA and CER price dynamics that is consistent with the compliance regulation to price spread EUA-CER options.

In summary, this paper analyzes some various interactions that may exist between the European Union Allowance (EUA) and the certificate emission reduction (CER) carbon markets. Firstly, we analyze the co-movement between EUA and CER markets prices using classical econometrics tools such as co-integration, vector autoregressive (VAR), regression...etc. Secondly, because the precede analysis does not tell us all about the contribution of heterogenous agents such as short-term investors (e.g., noisers) and long-term investors (e.g., fundamentalists) on the co-movement between EUA and CER markets, we study their co-movement by adopting the wavelet time-frequency analysis modeling. The preceding analysis take into account the change of state of the correlation between the EUA and CER prices for each frequency over time. It therefore highlight the contribution of heterogeneous agents on the co-movement and gives more precisely the impact of different agents with different term objectives (speculators, noise traders, fundamentalists, etc.) on the co-movement between CER and EUA prices returns. The application is made using daily prices data on EUA and CER period from 18 August 2008 to 21 January 2011.

The remainder of the paper is organized as follows. Section 2 presents the data used and the wavelet correlation theory framework. Section 3 studies the multi-scale analysis of the regression between EUA and CER. Section 4 finds the multi-scale correlation between EUA and CER. Section 5 studies the wavelet coherence. Section 8 analyses of multi-scales causality between EUA and CER. Section 7 concludes with some policy implications that discuss the relevance of our approach for policy markets.

2 Linking the EUA's and CER's Markets

2.1 CERs Contracts and Price Development

According to the article 12 of the Kyoto Protocol, projects under the clean development mechanism (CDM) consist in achieving greenhouse gases emissions reduction in non-Annex B countries. After validation, the CDM executive board (CDM EB) of the UNFCCC delivers credits that may be used by annex B countries for use towards their compliance position. Certified emissions reductions (CERs) from CDM projects are credits flowing into the global compliance market generated through emissions reductions. Therefore, the Kyoto Protocol's CDM provides to utilities regulated by the EU ETS the possibility to cut the costs imposed on them buying relatively cheaper carbon offsets from developing countries, funding emissions cuts these instead.

CER prices are determined on the supply side by the decisions of the CDM EB, which decides on the delivery rules. On the demand-side, CER prices are determined by various factors such as the decisions of the European Commission (EC) which determine the institutional fungibility within the European system, and the CERs demand from governments to meet their compliance within the Kyoto Protocol (such as Japan) which absorbs part of the CER demand away from compliance within the EU system.

2.2 The European Carbon Emission Trading Markets

The European Union Trading Scheme System (EU ETS) is a "cap-and-trade" system, operating under the Kyoto Protocol. Accounting to 83 % of the market value of global carbon emission markets, the EU-ETS is the most influential and successful emission trading programme in the world. The firms covered by the EU-ETS comprise approximately 12,000 installations which have a net generating capacity of more than 20 MW, located in 28 countries in the EU and 3 European countries outside of the EU (Iceland, Liechtenstein, and Norway). The sections included are power stations, mineral or oil refineries, ferrous metal, glass production, coke ovens, ceramic production, cement manufacture, and finally the aviation industry which joined in 2012.

Futures contracts for the EUAs are the dominant financial instrument in European carbon emission markets (World Bank [17]). The leading spot market for the EUA is the Bluenext exchange for the first and second commitment periods, and it will be the European Energy Exchange (EEX) for EU ETS phase III.

2.3 Relationship with EU Emissions Allowances

Since the EU ETS represents the world's largest emissions trading system (in terms of market activity and liquidity), it is possible that EUAs have a statistical influence on CER prices. Therefore, it is meaningful to find a relationship between EUAs and CERs prices in an economic context, since they both represent the same emissions asset that can be used for arbitrage purposes for compliance for within the ETS. The rationale behind the influence of EUAs on CERs channels through compliance mechanisms: while States directly manage their own compliance within the framework of the Kyoto Protocol, secondary CERs can be used by firms for compliance within the framework of the Kyoto Protocol, secondary CERs can be used by firms for compliance within EU ETS (up to 13 % on average). Therefore, the price path between the two assets which equally represent 1 tonne of CO_2 emitted in the atmosphere is expected to react common drivers.

2.4 Regulatory Setting

In [16], there are some background on linking the the EU ETS to the CDM. On October 27, 2004, the EU adopted directive-linking (to recognize credits based on projects generated by the Kyoto flexible mechanisms). However, there were several criticisms of this directive. Some critics argue that importing the Kyoto mechanisms credit reduces the incentives for innovation through access to cheaper compliance options. In addition, the use of international offsets in the EU ETS may reduce prices EUA, leading to few national action to reduce emissions. In addition, there are doubts about the environmental integrity of credits depending on the projects, and therefore, in accordance with the complementarity and additionally obligations under the Agreements Marrakech Accords (under the Kyoto Protocol), the EU has prevented Member States and their operators from overusing project-based credits. According to criterion 12 of Annex III of the ETS-Directive, Member States must impose a limit on the maximum amount of JI-CDM credits that their covered installations are entitled to use for compliance under the scheme. For instance, over the phase II the aggregate limit on the use of JI-CDM within the EU-ETS amounts to 13.4 % of the overall cap, which means that the maximum demand for the Kyoto mechanism's credits would be up to 278.3 MtCO2e per year, or a total of 1400 MtCO2e credits during phase II (World Bank [17]). On December 17, 2008, the European Parliament adopted "the climate and energy Package" for a third trading period of the EU ETS over 2013–2020. The main changes to be implemented in phase III are as follows: imposing more challenging emission reduction targets on installations subject to the EU ETS, expanding the scheme to more sectors and greenhouse gases, phasing-out the free allocation of allowances and replacing the auctioning system, and imposing a stricter restriction on the use and the availability of Kyoto mechanisms units. Apart from the quantitative restrictions on the import and the use of credits, there are some restrictions on the quality of credits due to growing criticism of the environmental effectiveness of project-based credits. Only credits from project types approved by all member states may be used. Moreover, amendments require additional guarantees with respect to the environmental integrity of project-based credits.

3 Data Description and Summary Statistics

In this paper, we consider the phase II EUA and CER intraday futures prices with a daily from European climate exchange (ECX) in the period from March 24, 2008 until October 19, 2012 or 1195 observations of futures quota (EUA) and (CER) carbon price. The dataset is constructed on the daily basis from various sources. It consists of daily price observations for EU allowances 2012 futures contracts (EUA Dec' 12) as well as the price of secondary CERs for futures contracts expired in December 2012 (CER Dec' 12) listed on the ECX.

3.1 EUA's and CER's Returns

To study the various relationships that may exist between CERs and EUAs prices or their returns, we use the wavelet approach. In the time period $[t-1, t]$, the return R_t that is the absolute difference of the logarithmic prices is given as follows:

$$R_t^i = Ln\left(\frac{P_t^i}{P_{t-1}^i}\right) \text{ for } i = EUA, CER. \tag{1}$$

Here, P_t^i denotes the price of i at time t.

3.2 Some Stylized Facts of CER and EUA returns

In the following table, we presents some descriptive statistics of the EUR's and EUA's returns.

Table 1 provides descriptive statistics and standard statistics for EUA and CER returns. The EUA and CER daily returns series have negative average. The average of EUA daily return series is greater than the average of CER daily

Table 1 Descriptive statistics of returns of price carbon markets

	Returns. EUA	Returns. CER
Mean	−0.000866	−0.002302
Median	0.000000	0.000000
Maximum	0.245247	0.160989
Minimum	−0.116029	−0.255347
Std. Dev	0.026732	0.030920
Skewness	0.426821	−0.889669
Kurtosis	10.22578	9.837733
Jarque-Bera	2633.798	2483.551
Probability	0.000000	0.000000
Sum	−1.034439	−2.748117
Sum Sq. Dev	0.852532	1.140538
Observations	1194	1194

returns series. The minimum and the maximum of the EUA returns series are respectively greater than those of CER returns. Moreover, the standard deviation of CER return time series is greater than that of the EUA return time series. The precede statistics comparison tell us that the performance (e.g., Sharpe ratio) of the EUA daily return is higher than that of CER daily return. In particular, the CER return is more risky than the EUA return. Because the skewness of EUA is greater than the skewness of CER which is negative, amplify the fact that EUA return is less risky than CER return. Since the Kurtosis of CER and EUA daily return are greater than 3. Thus, there are high peaks and heavy tails. This reflects that large outlying observations occur more often than can be expected under the assumption of normality. The Jarque-Bera statistics that are greater than 5.99 confirm the departure from normality for distribution probability of the process that govern the dynamic of CER and EUA daily return time series.

In Table 2, we observe a positive correlation (74,47 %) between the return of the allowance price and the price of carbon credits. Both yields move in the same direction. In other words, when the price of carbon quota increase (respectively decrease) on the European carbon market, the price of carbon credits from the clean development mechanism for increasing (respectively decreasing).

4 Multi-scale Analysis of the Regression Between EUA and CER

This section proposes a wavelet analysis to examine the relationship between yields of the quota and credits carbon price on different time scales. We investigate that the decision-making of investors depends on their investment horizons. Given this heterogeneity, behavioral true dynamic structure of the relationship between the yields of carbon price varies on different time scales. This study is therefore based on the discrete wavelet transform (DWT), in particular the manual overlap discrete wavelet transform (MODWT) in a multi-resolution analysis (MRA) is a property of rebuilding linear wavelet decomposition. For a time series X with an arbitrary sample size N, the jth level wavelet ($\widetilde{W}_j$) and scaling ($\widetilde{V}_j$) are defined as:

$$\widetilde{W}_{j,t} = 2^{-\frac{j}{2}} \sum_{l=0}^{L-l} \widetilde{h_{j,l}} X_{t-l}, \ t = 1, 2, \ldots, N \tag{2}$$

$$\widetilde{V}_{j,t} = 2^{-\frac{j}{2}} \sum_{l=0}^{L-l} \widetilde{g_{j,l}} X_{t-l}, \ t = 1, 2, \ldots, N \tag{3}$$

where h is a discrete scaling low-pass filter and g is a discrete scaling high-pass filter associated with the wavelet function. Each scale j corresponds to a frequency interval given by $[2^{-j-1}, 2^{-j}]$ for $j = 1, \ldots, J$; inverting the frequency range enables us to obtain the corresponding time periods $[2^j, 2^{j+1}]$. L is the length of the data. For a time series X with N samples, the MRA provides an additive decomposition through MODWT which is as follows:

$$X = \widetilde{S}_J + \sum_{j=1}^{J} \widetilde{D}_j \tag{4}$$

where $L_j = (2^j - 1)(L - 1)$ is the width of each MODWT filter, $\widetilde{D}_{j,t} = \sum_{l=0}^{L_j} \widetilde{h}_{j,l} \widetilde{W}_{j,t+l}$ and $\widetilde{S}_{j,t} = \sum_{l=0}^{L_j} \widetilde{g}_{j,l} \widetilde{W}_{j,t+l}$.

According to Eq. 4 at a scale j, we obtain a set of coefficient D_j, each with the same number of samples N as in the original signal X. These coefficients capture the details at each scale local fluctuations over the period of a time series. The set of values S_J smooth or provided overall trend of the original signal.

Adding D_j to S_J for $j = 1, 2, 3, \ldots, J$ gives an approximation more accurate to the original signal X. This additive form of reconstruction allows us to validate each of these subsets (D_j, S_J) separately and add validation to generate a single overall deduction. In addition, each scale j, the correlation between two wavelet time series X and Y coefficients in the decomposition of *details* and *scaling* can be obtained using the simple correlation method.

Price series Returns of EUA and CER are decomposed into the sum of orthogonal signals as follows:

$$S = T(-j) + \sum_{i=1}^{j} D(-i) \tag{5}$$

where j is a positive integer suitably selected with respect to the objectives of description, $T(-j)$ is the trend level $-j$, containing the components of S, period in excess of 2^j days. $D(-k)$ is the detail level $-k$ containing signal components of period between 2^{k-1} and 2^k days.

Table 2 The correlation matrix of EUA and CER log-returns log-returns

	R^{EUA}	R^{CER}
R^{EUA}	100 %	74.47 %
R^{CER}	74.47 %	100 %

TLocal scans will be carried out by choosing $j = 10$ (since 2^{10} = 1024) in order to study the components of the signal period of less than 1195 days.

Our study will be based on Daubechies filter to eight nonzero coefficients taking into account the limits of periodic boundary conditions while using the invariant discrete wavelet transform with a number of scales ($j = 10$) and different from the filter coefficients v_1, w_1, w_2, w_3, w_4, w_5, w_6, w_7, w_8, w_9, w_{10}.

Table 3 translates the wavelet scales into appropriate time horizons, providing an overview of the relationship between the levels MODWT and time scales for time series. Each level corresponds to a frequency interval and is associated with a range of different lengths that range from a few days to several years. Linear regression between returns of credit and quota carbon at different time horizons is presented in Tables 4 and 5.

As can be seen in Table 5, the wavelet regression has estimating the coefficients β, the values of test statistics are in brackets student and the coefficients of determination R^2. Firstly, the values of R^2 are not stable with increasing time scales. Band d_1 (equivalent to a period of 2 to 4 days) shows that the relationship between yields carbon quota prices and the carbon credit is significantly positive, which means that carbon credits (CER) Following the mechanism for the clean development mechanism (CDM) is a good cover to acquit their compliance obligations on the European market of carbon.

All scales have a significance and estimated coefficients positive, showing a positive relationship in most time horizons, based on the wavelet regression, except for the band d_9 (equivalent to a period of 256 to 512 days), which leaves appear an estimated coefficient (β) negative. Decomposed wavelet series can be classified as follows: Short-term $LT = D_1 + D_2 + D_3$, medium-term $MT = D_4 + D_5 + D_6 + D_7$, and long-term $LT = D_8 + D_9 + D_{10}$.

Table 6 presents the results of the simple linear regression between the performance of the quotas and the credit carbon for different so-called time horizons:

- The long term (LT) that refers to the structure of high frequency.
- The medium term (MT) refers to the structure of intermediate frequency.
- The short term (ST) that refers to the structure of low-frequency

These results clearly show that whatever the time horizon (short, medium, and long term), there is a strong significant positive relationship between both price returns. The performance of credit carbon is taken into account in explaining of the carbon quota (vice versa). In other term, there is a positive relationship on different investment horizons between the price of carbon from ETS and the carbon credit from the clean development mechanism (CDM). The CDM is therefore an effective tool of assistance to compensate for the different agents involved in the European market.

Table 3 Using daily data on the first level represents the dynamics of the 2–4 day period, the second scale represents the dynamics of the 4–8 day period, while the scales and 3, 4, 5, 6, 7, 8, and 9 represent dynamic 8 to 16, 16 to 32, 32 to 64, 64 to 128, 128 to 256, 256 to 512, and finally 512 to 1024 daily periods

Time horizons	Wavelet scales	Days
	D_1	2−4
	D_2	4−8
	D_3	8−16
	D_4	16−32
	D_5	32−64
	D_6	64−128
	D_7	128−256
	D_9	256−512
	D_{10}	512−1024

Table 4 $Return.EUA_t = \alpha + \beta * Return.CER_t + \epsilon_t$

	D_1	D_2	D_3	D_4	D_5
β	0.799	0.914	0.970	0.651	1.044
	(35.454)	(44.285)	(46.609)	(47.179)	(36.148)
R^2	0.513	0.621	0.645	0.651	0.522

Wavelet regression between $Return.EUA_t$ and $Return.CER_t$

Table 5 $Return.EUA_t = \alpha + \beta * Return.CER_t + \epsilon_t$

	D_6	D_7	D_8	D_9	D_{10}
β	0.857	1.270	1.085	-2.441	17.536
	(33.579)	(14.046)	(15.359)	(8.990)	(567.298)
R^2	0.486	0.142	0.156	0.063	0.983

Wavelet regression between $Return.EUA_t$ and $Return.CER_t$

5 Multi-scale Correlation Between EUA and CER

We proceed to a decomposition of the two transformed series into different components using the time scale of the maximum overlap discrete wavelet transform (MODWT) which is a variant of the non-orthogonal wavelet discrete transform versus conventional processing orthogonal

Table 6 Estimation results of equation $Return.EUA_t = \alpha + \beta * Return.CER_t + \epsilon_t$ for different time horizons

	Short term	Medium term	Long term
α	0.000033	−0.000564	−0.000402
β	0.659763	0.475907	0.035126
Standard deviation$_\alpha$	0.000477	0.000197	0.0000834
Standard deviation$_\beta$	0.016661	0.015796	0.005396
T − statistic$_\alpha$	0.069332	−2.8659	−4.825
T − statistic$_\beta$	39.59830	30.12903	6.509
R^2	0.5681	0.432	0.034
F − statistic($Prob$)	1568.026 (0.0000)	907.75 (0.0000)	42.377 (0.0000)

discrete wavelet which is invariant under translation. The wavelet approach we do, allows us to disentangle the correlation on a consideration of scale by scale basis and allows us to determine that it is the scale that contributes most to the overall relationship between the two prices yields. Variances at different scales after the maximum overlap discrete wavelet transform (MODWT) yields quota and credit price carbon are shown below (Fig. 1), where straight lines indicate the variances and the dotted lines denote the confidence intervals at 95 %.

After analyzing Fig. 1 above, we find that there is a linear relationship between the wavelet variances and wavelet scales respectively. There is an increase of the variance of wavelet with wavelet scales forward. The performance of carbon allowance prices are more volatile than the price of carbon credits, which confirms the operation of the carbon trading market on which activity has a great exchange of emission permits.

The correlation between the performance of the quota and the credit carbon by different time horizons is shown in Fig. 2 below:

The correlation coefficients of wavelet shown in Fig. 2 below show that there is a strong relationship between increasing returns of EUA and CER at the first level (that is, say between second and fourth days). However, this relationship tends to decrease at level 2 to level 8 (between fourth and fifth days). Finally, we observe a positive correlation increasing, a growing trend common between the two yields starting from level 8 to level 16 is a period from 512th days to 1024th days functioning of our carbon markets. Overall, the result of the wavelet correlation is consistent with the regression analysis wavelet performs above. This result indicates that, according to our data, the relationship between EUA and CER returns yields is positive and moving in the same direction in the short, medium, and long term. But it is more detailed than that obtained when calculating the linear correlation coefficient (0744) calculated in the time domain.

6 Wavelet Coherence

Expenditures on compliance obligations from kyoto protocol play a crucial role in the decision-making industrial enterprises and entrepreneurs. A good understanding of the dynamics and interconnections between the yields of carbon prices is important. The calculation of the unconditional correlation we have made has provided evidence of a high dependence between the returns of the EUA and CER carbon price. However, the time of our study is long enough, it can be interesting to see how the correlations can develop over time.

In addition, taking into account the heterogeneity of behavioral agents on carbon markets refers to a complex interaction between our price returns. Therefore, the series resulting from this process are formed by a combination of different components operating at different frequencies.

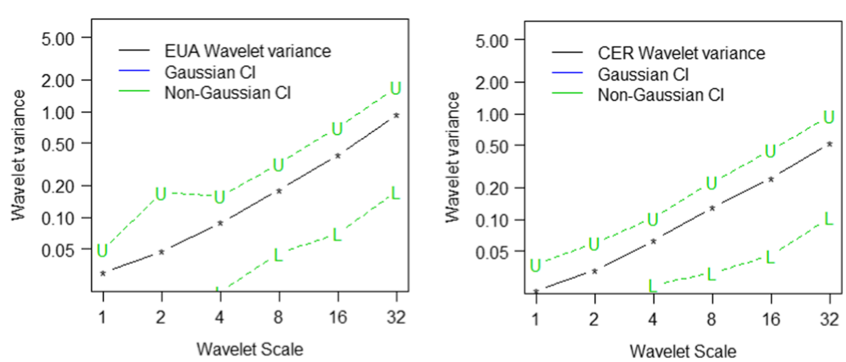

Fig. 1 Wavelet variance

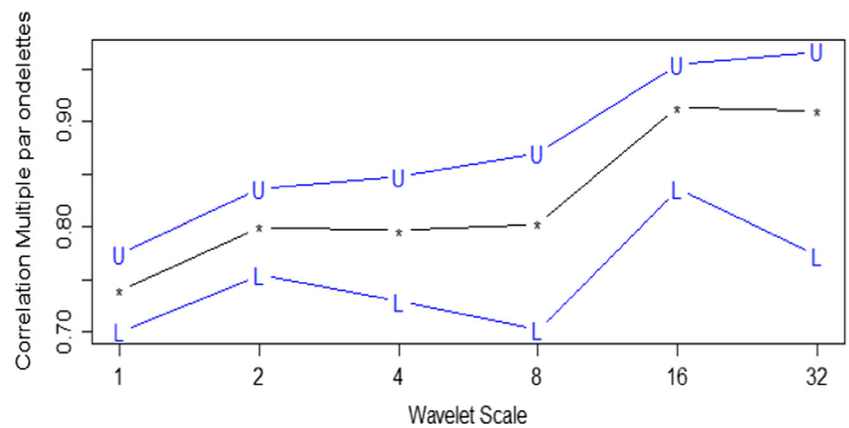

Fig. 2 Correlation between returns EUA and CER

Traditional methods of time series econometric generally consider the frequency components and time separately. For unifiers, we will involve the wavelet coherence to deepen our analysis. We focus on the wavelet analysis and more precisely on the wavelet coherence (WTC). Wavelets have recently become a very frequent method in finance.

Wavelet coherence is used in this article as a tool to study the simultaneous dependence (co-movement) between the two sets of yields in the time domain and the frequency. It can be interpreted as a measure of local correlation calculated on a non-parametric. It is interesting to know about the functioning of the carbon market, if there are strong dependencies between EUA and CER returns in investment horizons shorter or longer. The evaluation of the local correlation of each frequency over time will be analyzed after Carlo simulations mounted. Figure 3 shows the estimated wavelet coherence and phase difference for both price returns examined from a scale of 1 (1 day) at a scale of 256 (about 1 year of the carbon market).

Time is represented by the horizontal axis, while the frequency on the vertical axis. The wavelet coherence can be found in the time frequency regions, the space in which the two series move together yield:

- Regions that leave dark lines appear in which we observe red colors represent areas with high dependency.
- Areas containing the color blue are those in which the two series are weakly dependent. More blue color becomes darker, the dependence between the return series down.
- The dark blue regions for each frequency over time show no dependence of the two yields.

Wavelet coherence allows us to see how the two returns move together significantly both for each frequency and for each time interval. Our results are acquired by using Matlab package, which was written by Grinsted et al. [10]. As a result of this, we can obtain very detailed results based on the time domain and the frequency domain at the same time.

Another thing that helps us to interpret results are so called phase arrows, which show the relative phasing of time series at given scale. If arrows are pointing to the right that means that time series are in phase, opposite direction means anti-phase. If they are pointing down then the first variable is leading the second one, and if they are pointing up then the second variable is leading the first one.

The wavelet coherence analysis can be observed very interesting results. A first observation in Fig. 3 shows results that confirm our previous findings. Most of the time they are in phase (phase arrows pointing to the right) at all frequencies for the whole period and that means that there is not a leading market, but the two markets have returns that evolve the same direction over time.

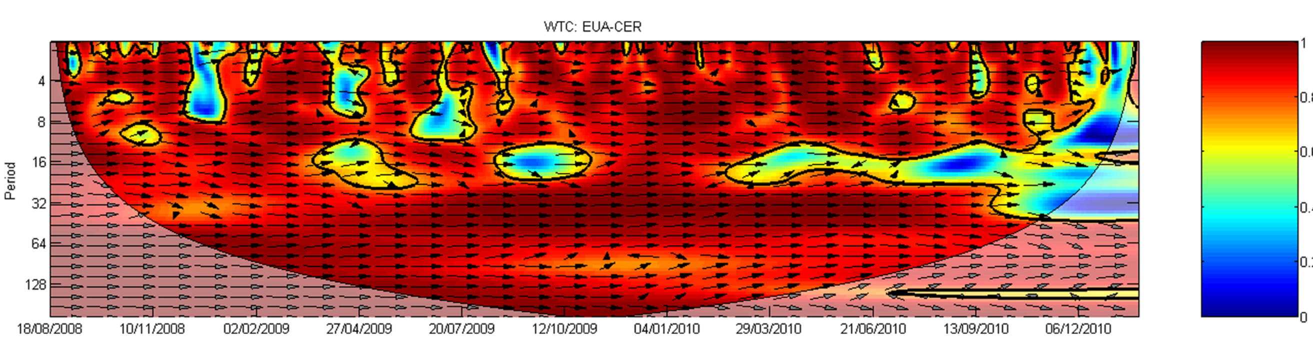

Fig. 3 Wavelet coherence

Based on the analysis of the wavelet coherence yields, we observe a strong dependence between EUA and CER returns over multiple periods and multiple frequencies. This dependence has existed since the creation of the two markets, it was confirmed in the high frequencies (short term) that are related to the behavior of speculators, where we observe the 18/08/2008 to 21/01/2011 total dependence.

The medium term linked to intermediate frequencies (i.e., agents investors) appear non total dependence on 29/03/2010 to 21/01/2011. In this period, the level of investment had declined due to the European crisis that originates from the U.S. subprime crisis. There are observation of a decrease in the activity of agents whose behavior temiste middle. The latter since 29/03/2010 no longer intervene frequently in the two markets to offset carbon déficite quota. Economic activity since the subprime crisis in Europe has declined significantly, which resulted in a reduction of carbon emissions.

We can see in Fig. 3 that changes in co-movement can be observed not only in time but also in various investment horizons. For example on 04/01/2010, there was a sharp increase in the correlation between the returns of the EUA and CER taking values starting from 0.8 to 1. This strong dependence was present for all investment horizons (short, medium, and long term). We observe from the 29/03/2010, in lower rapid correlation between the two yields in the medium and high frequencies.

Wavelet coherence shows that since the 12/09/2009, yields EUA and CER are still connected to low-frequency but low, and this is confirmed with a correlation coefficient of 0.7 to 0.8 variety.

The results of our study indicate that the co-movements of yields prices in the EU ETS and the CDM market are much stronger when we consider that the agents are homogeneous; this could be due to the strong relationship that develops rapidly between the two business executives (see also [12]) while taking into account the heterogeneity of agents shows that some horizons of investment markets are independent.

After seeing that a relationship exists between our two carbon markets, we will proceed to the analysis of the causal wavelet to deepen our study to see their causal relationship to each frequency level over time.

7 Analysis of Multi-scales Causality Between EUA and CER

The causality analysis that we performed in the time domain to verify the results obtained by [2] and Nazifi [15], mask disparities behavioral agents operating on carbon markets. Our study will be to examine the causal relationship between the performance of carbon allowance prices and the credit carbon while integrating the heterogeneous behavior of individual agents.

Table 7 Multi-scales Granger causality test

	CER⇒EUA		EUA⇒CER		
Trading horizon	F-stat.	P-val.	F-stat.	P-val.	Lags
D_1 : 1 à 2 days	2.46477	0.0119	1.43884	0.1758	8
D_2 : 2 à 4 days	2.43969	0.0128	5.45767	9.E-07	8
D_3 : 4 à 8 days	0.44167	0.8762	1.91799	0.0634	7
D_4 : 8 à 16 days	1.16783	0.3153	0.07999	0.9997	7
D_5 : 16 à 32 days	0.42875	0.8847	0.10466	0.9981	7
D_6 : 32 à 64 days	0.06784	0.2862	1.22408	0.9995	7
D_7 : 64 à 128 days	0.19610	0.9863	2.10766	0.0401	7
D_8 : 128 à 256 days	0.07206	0.7149	0.64961	0.9994	7
D_9 : 256 à 512 days	1.19944	0.3000	0.40304	0.5581	7
D_{10} : 512 à 1024 days	1.1445	0.3513	0.25110	0.9008	7

In this section, we perform causality tests at each frequency band. That, in order to compare with the results we obtained previously. The objective will be to see if the results found in prior persists or whether the behavioral difference of agents involved in the carbon market has an impact on the relations of causality.

All tests of Granger causality repaired in between pairs of selected frequencies are presented in Table 7 below:

We observe an instability of causality in different bands. There is a two-dimensional causal in high frequency (D_2) between the two markets. In the remaining cases, if causal, it is uni-dimensional starting credit market to quotas carbon market.

In carrying a unification of our different bands, it means seeking three frequency bands. A high-frequency band which corresponds to the behavior of the short-term agents.

This band is the sum of the first three bands (D_1, D_2, D_3). A medium-frequency band which corresponds to the behavior of agent in the medium term is the sum of four bands (D_4, D_5, D_6, D_7), and finally a low-frequency band which corresponds to the behavior of agents with long-term carbon markets and is the sum of the last three bands (D_8, D_9, D_{10}).

Table 8 above shows the existence of a causal bi-dimensional between our two high-frequency returns[1] (associated probabilities are less than 5 %). Returns fluctuations of the quotas carbon price cause those carbon credits and vice versa. Both carbon markets are interconnected at high frequency, i.e., in the short term. These agents are thus involved in the formation of different price components in these two markets.

[1]The agents that have trading strategies periodicity of 1 to 8 days.

Table 8 Multi-scales Granger causality test

Trading horizon	CER⇒EUA F-stat.	CER⇒EUA P value	EUA⇒CER F-stat.	EUA⇒CER P value	Lags
High frequency: 1 to 8 days	17.7386	3.E-08	4.29233	0.0139	8
Medium frequency: 8 to 128 days	1.96363	0.1408	23.0790	1.E-10	8
Low frequency: 128 to 1195 days	0.88912	0.4113	16.4958	9.E-08	7

Based on the short-term behavior of the agents involved in our markets, it appears that changes in market prices of carbon quota exert a significant influence on those carbon credit market and vice versa (fluctuations of CERs returns have an impact on the functioning of the European carbon market). This causes the level of speculation on the ETS is driven by the characteristic priori volatile of credits market carbon from CDM and vice versa.

Regarding medium and low frequencies, the Table 8 shows that there is one-dimensional causal from carbon quota price to carbon credits. In other words, operators medium and low frequency trading participate in the construction of components low and medium frequency quota carbon price and carbon credit price. Taking into account both strucutres frequency, it seems that changes in the EUA market have a significant influence on those of the CER in the medium term. This change of direction shows an instability of the causal relationship between the two markets. This finding uni-dimensional causal can be explained by other factors such as the maturity of the market quota of carbon relative to carbon credits. The volume of allowances offered to ETS and non clarity of the CDM market and post-2012 uncertainty may also explain the causality.

8 Conclusion and Policy Implications

Traditional econometric models often fail to accurately model the co-movement of financial time series because of the complex structure and irregularities of the underlying data. This paper provides fresh new insights into the relationship between the CER and the EUA returns from 2008 to 2012, applying the novel wavelet analysis. Wavelet analysis allows a simultaneous assessment of co-movement and causality between the two returns in both the time and frequency domains.

Using daily data for CER and EUA returns, our results demonstrate the importance of wavelet analysis in better encircling the underlying characteristics of the the co-movement between CER and EUA markets. Adopting wavelet approach allows us to study the frequency components of EUA and CER time series returns and their relationships without losing the time information. The powerful wavelet analysis approach is model free, and it also help us uncover interactions that the other econometric models cannot easily provide. Our analysis uses the wavelet method to model the correlation between CER and EUA returns in the time frequency domain. It highlights the impact of different investors ’according to their investment horizons) on the co-movement (correlation, Granger causality, coherence) between CER and EUA returns, and therefore, the behavior of individual investors such as speculators, arbitrageurs, and hedgers on European Allowance and CDM credits cumulatively.

The identification of the co-movement between the two markets at different frequencies is clearly pivotal for investors, since it suggests that investors with different investment horizons should pay more attention to the co-movement at corresponding frequencies so as to allocate their assets more effectively. More specifically, if investors prefer the short-term investment horizon, then they should focus on the co-movement at higher frequencies and, hence, the economic factors driving such co-movement between CER and EUA markets. On the other hands, if investors prefer the long-term investment horizon, then they should focus on the co-movement at lower frequencies and the corresponding driving factors. For instance, in Section 5, the multi-scale relationship between CER and EUA returns allows us to graphically identify different bi-directional cross correlation patterns for a convenient window size. There is an increase of the variance of wavelet with wavelet scales forward. The performance of carbon allowance prices are more volatile than the price of carbon credits, which confirms the operation of the carbon trading market on which activity has a great exchange of emission permits.

We do find that the co-movement between the CER and EUA returns varies across frequencies and evolves with time.

One serious economic policy implication is given as follows:

In page 242 (Section 4), [1] consider the question of the valuation of a spread option that allows its owner to exchange one CER for one phase II EUA. To do so, they prove that considering log-normally driven emission market model is complete and then derive the spread option price when the situation of noncompliance is unlikely to take place. Finally, they analyze the sensitivity of this option price on some key parameters that control the EUA and CER price response.

As we can see in the proposition 7 of page 243 of [1], the spread option price depend on the standard deviation of the difference between ther CER's and EUA's return. Hence, the spread option price depend on the coefficient of correlation $\rho_{a,c}$ between CER and EUA returns. The latter induces some serious implication due to our wavelet analysis.

In fact, according to their investment horizons and therefore their horizon hedge against the risk of price differential between CER and EUA with a use of spread options, investors and market agents take into account the analysis of the correlation between the two returns in relation with horizon of investment. Our analysis confirm in Table 5 that the coefficients of the regression β, the coefficient of determination R^2, and their statistical test are different depending on the frequency bands D_i for $i = 1, \ldots, 10$. Depending on the investment horizon, investors do not have the same consideration for the correlation between the CER and EUA returns. For other ideas on policy implications, we recommend [16], and [1] with some references there in.

8.1 Limits of Wavelet and Possible Extension

Currently, most wavelet decomposition software packages require that the original set of data have sample size n equal to a power of two in order to achieve an exact orthogonal wavelet transform. In statistical data analysis, such is rarely the case, so in an effort to broaden the applicability of such methods, various ways of preconditioning data not meeting this restriction are discussed and compared. These results illustrate the important point that wavelet coefficients resulting from preconditioned data should never be thrown blindly into a threshold selection procedure which depends on the coefficients being independent with equal variance. Such procedures can still be used, but great care must be taken to choose an appropriate preconditioning method. Also, the resulting wavelet vector can certainly be variance-corrected (with only rather light computational burden) before a thresholding procedure is applied to it. Some of the correlation can also be removed, though this is certain to be quite computationally expensive.

The use of length of data equal to a power of 2, strongly limit the application of wavelet method. Thus, a serious alternative to analyze the causality between two series CER and EUA would be to use the methodology comparable to the causality analysis in the short and long-term developed in [5, 6] and some subsequences references there in.

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