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Central bank policy in a monetary union with heterogeneous member countries

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Abstract
We analyse the policy of an independent central bank in a monetary union. The monetary policy equilibrium, prevailing under either discretion or commitment, is analogous to the one country case, although the stabilization policy is less than optimal for each single country in the monetary union. The extent of optimality of the monetary rule changes with the cross-country heterogeneity in economic shocks. Heterogeneity of preferences implies, that in a dynamic setting there is variation in the incentives of each member country. A country with a low target level of output or output cost weight might not reap any benefit from a deviation from the commitment equilibrium. The commitment policy can be enforced with a proper definition of the inflation expectations rule. With homogeneous preferences the advantages and disadvantages of the monetary union commitment policy relatively to the own discretionary one, for any new candidate or existing member country, are a function of its relative size and degree of asymmetry.

Keywords Policy rules · Discretion · Credibility · Monetary union

JEL Classification C73 · E52 · F45

1 Introduction

In the Economic and Monetary Union (EMU) the responsibility for monetary policy is assigned to the European Central Bank (ECB) and to the National Central Banks (NCBs) of the European Union (EU) member countries whose currency is the euro. The heterogeneity between EMU countries requires a sound understanding of the economy of each country and of their interdependencies, for the assessment of economic and monetary developments and monetary policy decisions.

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A broad perspective for the analysis of economic policy in the EU is provided in Alesina et al. (2017) and Spolaore (2015). The advantages and disadvantages for individual member countries, from the point of view of traditional economic theory, are well-known. EU countries gain from an integrated market for goods, services and factors of production, due mostly to allocative efficiency. At the same time, the distributional consequences of the single market entail some challenging policy questions. The trade-offs regarding the integration of economic policies at the EU level have in general to do with the economies of scale, which follow from their public good features, and with preference heterogeneity between member states. Moreover, deeper language, cultural and historical traits provide additional dimensions for the evaluation of economic and political integration.

In the field of monetary theory the analysis can be specialized, to the assessment of the extent to which the EMU is an optimal currency area. For a EU member country the decision to participate in the EMU implies handing over the responsibility for the conduct of monetary policy to the Eurosystem and giving up an independent monetary policy. According to the framework originally defined by Mundell (1961), two main conditions are of relevance in an optimal currency area: the symmetry between economic and financial shocks and factor mobility across countries. Asymmetric shocks between countries lower the optimality of a currency area, since a central monetary authority would find it more demanding to carry out stabilization policy. Moreover, the necessary adjustments in good and factor markets following each economic shock might require factor mobility within the currency area. When factor mobility is restrained, for instance because of national boundaries, the advantages of a currency union are lower.

In the current work we study the policy of an independent central bank in a monetary union, from the position of the political economy on the subject. We outline a version of the Barro and Gordon (1983a, b) model and analyse monetary policy, under different assumptions regarding the central bank behavioural constraints. We assume either discretion or commitment of the independent central bank and describe the role of preference heterogeneity and asymmetric shocks in determining the optimal policy rules. We examine further the reputation problem of the central monetary authority. In a monetary union the benefits and costs of different policy rules are similar to the ones holding in the one country case, athough cross-country heterogeneity might result in different views regarding the optimal policy, in any specific institutional setting.

Finally, we consider the problem of the enlargement of a monetary union and clarify the incentives for a potential new member country to participate in the monetary union.

Section 2 specifies the monetary union model. Section 3 describes the policy equilibrium in the monetary union under discretion. Section 4 considers the commitment equilibrium. Section 5 reviews the central bank credibility problem. Section 6 outlines the incentives for a new member country to become a monetary union member, in the occurrence of asymmetric shocks. Conclusions are drawn in Sect. 7.

2 A monetary union model

We analyse the policy of a central bank in a monetary union composed of n member countries. In each country output is defined in each time period from the sum of the difference between actual and expected inflation and a random shock:

$$y_{it} = \pi_{it} - E_t(\pi_{it}) + \varepsilon_{it} \qquad i = 1, \dots, n \tag{1}$$

Equation (1) can be interpreted as an expectations augmented Phillips curve, where y_{it} and π_{it} denote the output gap and the actual inflation rate in country i in period t, $E_t(\pi_{it})$ denotes the expected inflation rate in country i conditional on information available in period t and ε_{it} is a random disturbance, which represents the effects on output of demand or supply shocks. We assume, that ε_{it} is independent and identically distributed over time and that $E(\varepsilon_{it}) = 0$, $Var(\varepsilon_{it}) = \sigma_i^2$ and $Cov(\varepsilon_{it}, \varepsilon_{jt}) = \sigma_{ij}$ for $i,j = 1, \dots, n$ and $i \neq j$.

Monetary policy in the monetary union is run by an independent central bank. In each time period the monetary authority has direct control of a number of different monetary instruments. In turn, the instruments can be used to target the monetary union average inflation rate π_t.

The actual inflation rate in each country is defined from the sum of the average inflation rate and a random disturbance, which can be interpreted either as a purchasing power parity shock or a control error:

$$\pi_{it} = \pi_t + \upsilon_{it} \qquad i = 1, \dots, n \tag{2}$$

The random disturbance υ_{it} in Eq. (2) is independent and identically distributed over time and its first and second moments are $E(\upsilon_{it}) = 0$, $Var(\upsilon_{it}) = \tau_i^2$ and $Cov(\upsilon_{it}, \upsilon_{jt}) = \tau_{ij}$ for $i,j = 1, \dots, n$ and $i \neq j$. We assume, in addition, that the random disturbances in Eqs. (1) and (2) are stochastically independent, therefore $Cov(\varepsilon_{it}, \upsilon_{jt}) = 0$ for all $i,j = 1, \dots, n$.

The monetary union central bank sets the average inflation rate π_t to minimize a loss function, defined in terms of the output gap and the inflation rate of each member country. We consider a quadratic specification, with the loss function of country i defined as:

$$L_{it} = \frac{b_i}{2}\left(y_{it} - k_i\right)^2 + \frac{1}{2}\left(\pi_{it}\right)^2 \qquad i = 1, \dots, n \tag{3}$$

where $k_i \geq 0$ is the target level of output and $b_i \geq 0$ is the output cost weight in country $i = 1, \dots, n$.[1]

The central bank objective is to minimize the expected value of a weighted average of the individual country loss functions:

[1] In a general equilibrium framework the target levels of output and the output cost weights could be defined for each member country from features of consumer preferences. A broad review of the relation between instruments, targets and objectives is provided in Woodford (2003).

$$L_t = E_t\left[\sum_{i=1}^{n} w_i L_{it}\right] \tag{4}$$

subject to (1)–(3).

In Eq. (4) $0 \le w_i \le 1$ for $i = 1, \dots, n$ is the weight of country i, $\sum_{i=1}^{n} w_i = 1$ and expectations are conditional on information available in period t.

We should note, that given Eqs. (1)–(3) several representations of the central bank objective function lead to equivalent results for the monetary policy equilibrium in the monetary union. The monetary union loss function could be defined as the conditional expected value of the sum of a term defined as a weighted average of the output gap loss in each country and a term in the monetary union average inflation rate: $L_t = E_t\left[\sum_{i=1}^{n} w_i\left(b_i/2\right)(y_{it} - k_i)^2 + (1/2)\pi_t^2\right]$.

Assuming identical target levels of output and output cost weight parameters, $b_i = b$ and $k_i = k$ for $i = 1, \dots, n$, and defining the monetary union output gap as a weighted average of the output gaps of each individual member country, $y_t = \sum_{i=1}^{n} w_i y_{it}$, the loss function could also be defined in term of the monetary union average output gap and inflation rate as: $L_t = E_t\left[(b/2)\left(y_t - k\right)^2 + (1/2)\pi_t^2\right]$.[2]

3 Equilibrium under discretion

In the above framework the monetary policy equilibrium prevailing in the monetary union is defined by the behavioural constraints imposed on the monetary authority. The monetary authority operates in conditions of discretion, when it determines the optimal average inflation rate in each time period conditional on inflation expectations.

In each country inflation expectations are formed prior to observing the demand, supply and purchasing power parity shocks. Moreover, we assume that the demand or supply shocks ε_{it} are observed before, whilst the purchasing power parity shocks υ_{it} are realized after the monetary policy decision.

With discretion the timing of events is therefore the following: a) inflation expectations are formed in each time period; b) demand or supply shocks are observed; c) the central bank determines the optimal inflation rate conditional on inflation expectations; and d) the purchasing power parity shocks are realized.

We assume, that economic agents in the monetary union have rational expectations. Since the random shocks in Eqs. (1) and (2) are stochastically independent, in each time period the expected inflation rate is equal to the average expected inflation rate in each country: $E_t\left(\pi_{it}\right) = E_t\left(\pi_t\right)$ for all $i = 1, \dots, n$.

Define the average output cost parameter $b = \sum_{i=1}^{n} w_i b_i$ and equivalent weights $0 \le \omega_i = w_i b_i/b \le 1$, $\sum_{i=1}^{n} \omega_i = 1$. The minimization of the loss function in Eq.

[2] A condition of identical parameters across countries could follow from the assumption, that loss functions are assigned to each country and the monetary authority at an initial institutional design stage, as for instance in Rogoff (1985).

(4) subject to (1)–(3) and conditional on inflation expectations yields the following solution for the average inflation rate:

$$\pi_t = \frac{b}{1+b}\left[k + E_t\left(\pi_t\right) - \varepsilon_t\right] \tag{5}$$

where $\varepsilon_t = \sum_{i=1}^{n} \omega_i \varepsilon_{it}$ and $k = \sum_{i=1}^{n} \omega_i k_i$ are weighted averages of the shocks and target levels of output of each country.[3]

The assumption of rational expectations and Eq. (5) in turn imply, that $E_t\left(\pi_t\right) = bk$ and hence:

$$\pi_t = bk - \frac{b}{1+b}\varepsilon_t \tag{6}$$

From Eq. (6) it follows, that in each time period the inflation rate and the output gap in each country are:

$$\pi_{it} = bk - \frac{b}{1+b}\varepsilon_t + \upsilon_{it} \qquad i = 1, \dots, n \tag{7}$$

and:

$$y_{it} = \varepsilon_{it} - \frac{b}{1+b}\varepsilon_t + \upsilon_{it} \qquad i = 1, \dots, n \tag{8}$$

In the discretionary rational expectations equilibrium the average output gap in each country is equal to zero, whilst the average inflation rate is equal to $bk \geq 0$: $E\left(y_{it}\right) = 0$ and $E\left(\pi_{it}\right) = bk$ for all $i = 1, \dots, n$. The greater are the average target level of output, compiled on the basis of the corrected set of weights, or the average output cost parameter, the greater the inflation bias of the discretionary equilibrium.

The quadratic loss function implies in addition, that it is optimal for the central bank to implement stabilization policy. Because the optimal discretionary rule in Eq. (6) is a function of a weighted average of the shocks observed in each country, the stabilization policy is less than optimal from the perspective of each individual country. From Eq. (7) and the definition of ε_t:

$$Var\left(\pi_{it}\right) = \left(\frac{b}{1+b}\right)^2 Var\left(\varepsilon_t\right) + \tau_i^2 \qquad i = 1, \dots, n \tag{9}$$

where $Var\left(\varepsilon_t\right) = (\sum_{i=1}^{n} \omega_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{j=1,\ j \neq i}^{n} \omega_i \omega_j \sigma_{ij})$.

As by the Cauchy-Schwarz's inequality $\sigma_{ij}^2 \leq \sigma_i^2 \sigma_j^2$ for all $i,j = 1, \dots, n$ and $i \neq j$, assuming equal variances of both the demand or supply and the purchasing power parity shocks across countries, $\sigma_i^2 = \sigma^2$ and $\tau_i^2 = \tau^2$, it follows that $\tau^2 \leq Var\left(\pi_{it}\right) \leq \left[b/(1+b)\right]^2 \sigma^2 + \tau^2$ for all $i = 1, \dots, n$. The upper bound of the

[3] We provide an explicit derivation of the solution to the monetary union optimization problem, under different behavioural constraints, in "Appendix". Advanced treatments of the monetary policy optimization theory are provided, for instance, in Fischer (1990) and Persson and Tabellini (1999, 2000).

individual inflation rate variance is attained, when the observable shocks of each country are perfectly positively correlated. In the case of perfect symmetry across countries, with identical output cost weights $b_i = b$ and target levels of output $k_i = k$ for $i = 1, \dots, n$, the stabilization policy implemented by the central bank is optimal for each single country in the monetary union.

The lower bound of the individual inflation rate variance is reached, when the observable shocks are perfectly asymmetric across countries and in each time period their weighted average is identically equal to zero. In this circumstance the central bank does not actually implement any stabilization policy.[4]

4 Commitment

The equilibrium in the monetary union can be improved, when the monetary authority can make a binding commitment on the monetary rule to be implemented in each time period, conditional on the observed demand or supply shocks. We might assume, that the central bank announces at an *ex ante* stage a monetary policy rule of the form $\pi_t = \pi\left(\varepsilon_{1t}, \dots, \varepsilon_{nt}\right)$. The announced central bank rule should minimize the loss function (4) subject to (1)–(3) and to the rational expectations constraint $E_t\left(\pi_t\right) = E_t\left[\pi\left(\varepsilon_{1t}, \dots, \varepsilon_{nt}\right)\right]$. We assume as before, that in each time period the purchasing power parity shocks are realized after the monetary policy decision. The timing of events is the same as with discretion, though at stage c) the central bank determines the inflation rate following the announced monetary rule.

Following Persson and Tabellini (1999, 2000), in order to determine the optimal monetary policy rule, we note that the assumption of a quadratic loss function implies, that the optimal rule is linear. The optimal monetary rule therefore minimizes the monetary union loss function in the class of linear monetary policy rules.

The optimal rule is:

$$\pi_t = -\frac{b}{1+b}\varepsilon_t \tag{10}$$

From Eq. (10) it follows, that in each time period the equilibrium inflation rate and output gap in each country are:

$$\pi_{it} = -\frac{b}{1+b}\varepsilon_t + v_{it} \qquad i = 1, \dots, n \tag{11}$$

and:

$$y_{it} = \varepsilon_{it} - \frac{b}{1+b}\varepsilon_t + v_{it} \qquad i = 1, \dots, n \tag{12}$$

[4] The perfectly asymmetric case obtains, when the monetary union member countries can be partitioned in an even number of groups, with equal sets of equivalent country weights between pairs of groups and with observable shocks perfectly positively correlated within each group and perfectly negatively correlated across groups in each pair.

Equations (11) and (12) imply, that the equilibrium average output gaps and inflation rates are equal to zero: $E(y_{it}) = 0$ and $E(\pi_{it}) = 0$ for all $i = 1, \dots, n$.

Since the monetary authority can make a binding commitment on the monetary policy rule, in equilibrium the inflation bias is equal to zero. The assumption of a quadratic loss function implies, that at the same time it is optimal for the central bank to implement stabilization policy. The stabilization term in Eq. (10) is the same as for the discretionary equilibrium case.

5 Central bank credibility

The monetary policy equilibrium solution obtained for the case of commitment is subject to the well-known dynamic inconsistency problem described by Kydland and Prescott (1977). Given the optimal monetary policy rule (10) and the equilibrium equations for inflation and output (11)–(12), the expected inflation rate conditional on information available in each time period is equal to zero in each country: $E_t(\pi_t) = 0$. Conditional on inflation expectations, at stage c) it is not optimal for the central bank to implement the monetary rule. This result follows from a comparison of the discretionary solution for the average inflation rate (5) and the commitment solution (10), under the assumption that $E_t(\pi_t) = 0$.

In order to analyse monetary policy credibility further, we reformulate the central bank optimization problem in an explicit dynamic setting, assuming that the objective is to minimize the expected discounted sum of the loss function (4) over an infinite time horizon:

$$L = E_t\left[\sum_{\tau=0}^{+\infty} \beta^{\tau} L_{t+\tau}\right] \tag{13}$$

where $0 < \beta < 1$ is the intertemporal discount factor.

We assume that, at the *ex ante* institutional stage, the monetary authority announces the commitment policy rule (10) and suppose inflation expectations are formed in each time period according to the rule:

$$\begin{aligned} &E_t(\pi_t) = 0 \\ &E_{t+\tau}(\pi_{t+\tau}) = 0 \text{ if } \pi_{t+\tau-1} = \pi^c \text{ or } \pi_{t+\tau-T} = \dots = \pi_{t+\tau-1} = \pi^d \\ &E_{t+\tau}(\pi_{t+\tau}) = bk \text{ otherwise, for } \tau > 0 \end{aligned} \tag{14}$$

where $\pi^c = -[b/(1+b)]\varepsilon_{t+\tau-1}$ and $\pi^d = bk - [b/(1+b)]\varepsilon_{t+\tau-k}$ for $k = 1, \dots, T$ are the average inflation rates prevailing under the announced and discretionary monetary policy rules.

Following Barro and Gordon (1983a, b) we note, that the assumptions about inflation expectations in Eq. (14) provide an explicit intertemporal dimension to the central bank optimization problem. According to Eq. (14) expected inflation is initially set in each country following the announced monetary rule. In each period subsequent to the initial period the expected inflation in each country is set to the commitment level

$E_{t+\tau}\left(\pi_{t+\tau}\right)=0$, if the central bank followed the announced monetary policy rule in the previous period, and to the discretionary level $E_{t+\tau}\left(\pi_{t+\tau}\right)=bk$ otherwise. In the case of a deviation of the central bank policy from the announced monetary policy rule, the expectation of an average inflation rate corresponding to the discretionary equilibrium is supposed to last for T periods, the average expected inflation rate returns to the commitment level thereafter.

The central bank incentives following from the dynamic optimization setting in Eqs. (13)–(14) can be described considering the consequences of a deviation from the announced monetary policy rule in period t. Conditional on the average expected inflation rate $E_t\left(\pi_t\right)=0$ Eq. (5) implies, that it is optimal for the monetary authority to settle for the average inflation rate:

$$\pi_t=\frac{b}{1+b}\left(k-\varepsilon_t\right) \tag{15}$$

Equation (15) implies, that following a deviation from the announced monetary policy rule actual inflation and the output gap in each country in period t are:

$$\pi_{it}=\frac{b}{1+b}\left(k-\varepsilon_t\right)+\upsilon_{it} \qquad i=1,\dots,n \tag{16}$$

and:

$$y_{it}=\frac{b}{1+b}\left(k-\varepsilon_t\right)+\varepsilon_{it}+\upsilon_{it} \qquad i=1,\dots,n \tag{17}$$

The single period loss function (3), the commitment equilibrium conditions (11)–(12) and Eqs. (16)–(17) imply that for each single country the expected benefit resulting from a deviation from the announced rule in period t is:

$$B_i=\frac{b_i}{2}\left[\frac{2bk}{1+b}k_i-\left(\frac{bk}{1+b}\right)^2\right]-\frac{1}{2}\left(\frac{bk}{1+b}\right)^2 \qquad i=1,\dots,n \tag{18}$$

As inflation expectations are defined in Eq. (14), in the T periods following the deviation from the announced monetary policy rule the equilibrium corresponds to the discretionary one. The central bank sets the average inflation rate according to Eq. (6) and the actual inflation rate and output gap in each country are defined by Eqs. (7)–(8). From the single period loss function and the commitment equilibrium conditions it follows, that the expected cost in period t of the deviation from the announced monetary policy rule is:

$$C_i=\frac{\beta\left(1-\beta^T\right)}{(1-\beta)}\frac{(bk)^2}{2} \qquad i=1,\dots,n \tag{19}$$

For each country the benefit of a deviation from the optimal monetary rule is a function of the target level of output and of the output cost weight. Following Eq. (18) $B_i=-(1/2)\left[bk/(1+b)\right]^2<0$ when $b_i=0$ and $B_i=-\left[\left(1+b_i\right)/2\right]\left[bk/(1+b)\right]^2<0$ when $k_i=0$, for $i=1,\dots,n$. Because the output expansion following the deviation from the optimal monetary rule is obtained at the cost of an higher average inflation rate, a country with a low output cost weight does not have an incentive to deviate. Similarly, a country with a low target level of output does not benefit from an expansion of output. A necessary condition for a positive benefit is $k_i > bk/[2(1+b)]$.

The cost term in Eq. (19) is a function of the expected inflation rate in the discretionary equilibrium, it is discounted for T periods and equal across member countries.

Equations (18) and (19) imply that preferences are single-peaked, either the benefit of a deviation from the optimal monetary rule is lower than or equal to the cost or the converse holds in each single country. The application of majority rule across the monetary union member countries could therefore mean, that the decision about whether or not to deviate would be determined by the preferences of the median voter country. Since the term $\beta\left(1-\beta^T\right)/(1-\beta) \rightarrow T$ for $\beta \rightarrow 1$, it is possible to set the discount factor β in the loss function (13) and the number of discretionary equilibrium periods T in the expectations rule (14) in order to make the period t benefit from deviation lower than the expected cost in each member country.[5]

When the target levels of output and the output cost weights are equal across countries, Eq. (18) simplifies to $B_i = (bk)^2/[2(1+b)]$, for $i = 1, \dots, n$. In order to make the optimal monetary rule preferred in each member country, in this case it is sufficient to assume a discount factor $1/(1+b) < \beta < 1$ and $T = 1$ discretionary periods in the expectations rule (14).[6]

6 Enlargement of the monetary union

We next consider the possibility of enlargement of the monetary union, due to the acceptance of the union common currency unit by a new member country. We assume, that aggregate supply and inflation in the candidate member country are defined according to Eqs. (1) and (2) and that the single period loss function is described by Eq. (3). For the purpose of this analysis we suppose, that target levels of output and output cost weights are equal across countries.

[5] In the field of game theory trigger strategies of the type in Eq. (14) have been used by Friedman (1971), Fudenberg and Maskin (1986) and Abreu et al. (1994), in order to prove the existence of subgame perfect equilibria as the one above described. We note, that the application of choice by majority rule is not a desirable solution in the present model, since the target levels of output and the output cost weights of each member country are common knowledge at the *ex ante* stage. The benefits of majority voting as a social choice rule require in general a condition of anonymity to be satisfied, a review can be found in Dasgupta and Maskin (2008). An interesting alternative perspective is provided by the Dixit (2000) model, where the optimal commitment rule is derived taking into account of the incentive compatibility constraints of each country.

[6] With equal target levels of output and output cost weights the expressions for the expected benefit and cost of a deviation from the announced monetary policy rule in period t are equal to the corresponding ones provided by Alesina and Stella (2011), for the single country case without exogenous shocks. This finding is due to the assumption, that the demand or supply and purchasing power parity shocks have zero means and identical variances and covariances over time.

Following Alesina and Barro (2002), the incentives for the endorsement of the monetary union currency for the new country can be described comparing the discretionary equilibrium, which would result with an own currency, to the commitment one, in the expanded monetary union.

The monetary equilibrium in the candidate member country with an own currency follows the usual relations for the one country case. The discretionary monetary policy rule of the candidate member country central bank would be defined as:

$$\pi_t = bk - \frac{b}{1+b}\varepsilon_{n+1t} \tag{20}$$

The monetary policy rule (20) in turn leads to the actual inflation rate:

$$\pi_{n+1t} = bk - \frac{b}{1+b}\varepsilon_{n+1t} + \upsilon_{n+1t} \tag{21}$$

and the output gap:

$$y_{n+1t} = \frac{1}{1+b}\varepsilon_{n+1t} + \upsilon_{n+1t} \tag{22}$$

In each time period, the expected loss for the candidate member country resulting from the application of the discretionary monetary policy rule (20) can be compiled as:

$$\begin{aligned} L_{Dn+1} = {} & \frac{b}{2}\left[\left(\frac{1}{1+b}\right)^2 \sigma^2_{n+1} + \tau^2_{n+1} + k^2\right] \\ & + \frac{1}{2}\left[(bk)^2 + \left(\frac{b}{1+b}\right)^2 \sigma^2_{n+1} + \tau^2_{n+1}\right] \end{aligned} \tag{23}$$

Conversely, suppose that the candidate member country anchors its currency to the monetary union one, receiving a weight equal to $0 \le \omega \le 1$ in the single period monetary union loss function. Following the decision to participate in the monetary union by the new member country, the common monetary authority announces a monetary policy rule of the form:

$$\pi_t = -\frac{b}{1+b}\left[\omega\varepsilon_{n+1t} + (1-\omega)\varepsilon_t\right] \tag{24}$$

We should note, that the monetary policy rule in Eq. (24) has the same form as the one in Eq. (10). The equivalence can be established by a suitable definition of the country weights and of the weighted average of the demand or supply shocks, for the enlarged monetary union with $n + 1$ countries. The single period monetary union loss function is then defined accordingly, as a sum of $n + 1$ terms following Eq. (4).

The announced monetary policy rule implies, that the actual inflation rate in the new country is:

$$\pi_{n+1t} = -\frac{b}{1+b}\left[\omega\varepsilon_{n+1t} + (1-\omega)\varepsilon_t\right] + \upsilon_{n+1t} \tag{25}$$

and the output gap is:

$$y_{n+1t} = \varepsilon_{n+1t} - \frac{b}{1+b}\left[\omega\varepsilon_{n+1t} + (1-\omega)\varepsilon_t\right] + \upsilon_{n+1t} \tag{26}$$

In each time period, the expected loss for the candidate member country following from the application of the monetary union announced policy rule (24) is:

$$\begin{aligned} L_{MUn+1} = {} & \frac{b}{2}\left\{ \left[\frac{1+b(1-\omega)}{1+b}\right]^2 \sigma^2_{n+1} + \tau^2_{n+1} + k^2 \right\} \\ & + \frac{b}{2}\left\{ \left[\frac{b(1-\omega)}{1+b}\right]^2 Var\left(\varepsilon_t\right) - 2\frac{[1+b(1-\omega)]b(1-\omega)}{(1+b)^2} Cov\left(\varepsilon_{n+1t}, \varepsilon_t\right) \right\} \\ & + \frac{1}{2}\left[\left(\frac{b\omega}{1+b}\right)^2 \sigma^2_{n+1} + \tau^2_{n+1} \right] \\ & + \frac{1}{2}\left\{ \left[\frac{b(1-\omega)}{1+b}\right]^2 Var\left(\varepsilon_t\right) + 2\left(\frac{b}{1+b}\right)^2 \omega(1-\omega) Cov\left(\varepsilon_{n+1t}, \varepsilon_t\right) \right\} \end{aligned} \tag{27}$$

From Eqs. (23) and (27) it follows, that the loss for the new member country resulting from the application of the own discretionary policy rule is lower than or equal to the one arising from the anchor to the monetary union commitment rule if and only if $L_{Dn+1} \leq L_{MUn+1}$ or:

$$\frac{[b(1-\omega)]^2}{1+b} Var\left(\varepsilon_{n+1t} - \varepsilon_t\right) \geq (bk)^2 \tag{28}$$

In order to interpret Eq. (28) notice, that the term on the right hand side is a function of the expected costs resulting from the average inflation bias of the own discretionary rule. The term on the left hand side is proportional to the variance of the difference between the candidate new member country demand or supply shock and the average demand or supply shock in the monetary union, before the decision of the new member country to apply. It represents the benefit from the stabilization term of the own currency discretionary rule, or the cost of anchoring the own currency to the monetary union one. Since $Var\left(\varepsilon_{n+1t} - \varepsilon_t\right) = \sigma^2_{n+1} + Var\left(\varepsilon_t\right) - 2Cov\left(\varepsilon_{n+1t}, \varepsilon_t\right)$ an increase of the own demand or supply variance, or of the average demand or supply variance in the monetary union before the decision of the new member country to participate, increases the benefit of the stabilization term of the own discretionary rule, or the cost of the corresponding term of the monetary union commitment one. Similarly, a decrease in the covariance between the own and the average monetary union demand or supply shock, before the decision of the new member country to anchor its currency, increases the benefit of the own discretionary rule, or the cost of the monetary union commitment one.

Moreover, the factor of proportionality on the left hand side of Eq. (28) is decreasing with the weight of the new member country in the single period monetary union loss function. For $\omega = 0$ the new member country receives a weight equal to zero and the monetary policy rule (24) is identical to the rule (10), prevailing in the monetary union before the decision of the new member country to apply. For $\omega = 1$ the monetary union rule is equivalent to an own commitment rule for the new member country. In this case the left hand side of Eq. (28) is equal to zero.[7]

Assuming that for $\omega = 0$ the expected single period loss of the own discretionary rule is lower than or equal to the one resulting from the anchor to the monetary union currency, we might conclude that there exists a level of the weight parameter $0 \leq \omega \leq 1$ which makes the new member country indifferent between retaining its own currency and participating in the union.

Finally, since $Cov\left(\varepsilon_{n+1}, \varepsilon_t\right) = \sum_{j=1}^{n} w_j \sigma_{n+1j}$, assuming equal variance of the demand or supply shock across countries and applying the Cauchy-Schwarz's inequality, in the case of perfect symmetry the term on the left hand side of Eq. (28) is equal to zero. With perfect symmetry the expected loss resulting from the anchor to the monetary union rule is lower than or equal to the one of the own discretionary rule for all values of the weight parameter $0 \leq \omega \leq 1$.

7 Concluding remarks

For the evaluation of a monetary union and of the incentives of any individual country to participate it is important to distinguish between *ex ante* and *ex post* costs and benefits. We have been mostly concerned with *ex ante* valuations. The criteria for a country to participate in a monetary union has been defined along two main dimensions: the symmetry between demand or supply shocks and its relative size.

As in our model in each time period demand or supply shocks are observed before the monetary policy decision, an *ex post* analysis of the commitment policy can also be provided. For this type of analysis an important contribution is the model of currency crises provided by Obstfeld (1996). In the model, subsequently to the realization of a negative demand or supply shock, a country might find it optimal to withdraw from a monetary union. This circumstance might hold, when taking into account of the eventual additional costs, which would have to be incurred in the case of withdrawal. The *ex post* incentives might in turn change the *ex ante* ones. The monetary policy outcome could be determined by the existence of sunspot equilibria with self-fulfilling properties. The expectation of a currency crisis might lead the decision of a country to leave a monetary union, prior to the actual realization of the crisis.[8]

[7] The above analysis also implies, that a country able to enforce the optimal own commitment policy would not have an advantage to participate in the monetary union. The expected cost of the own monetary rule is equal to zero in this case and the expected benefit of the own stabilization term or the expected cost of the monetary union one are the same.

[8] Currency crises have historically taken several forms, ranging from banking to public debt crises. In the context of the EMU we should recall, that the euro conversion rate is irrevocable for any participating

The current developments in the EU can be rationalized by our model, as for instance the recent decision of the United Kingdom (UK) to leave the EU. Since the beginning of the monetary union the number of EMU countries increased, it follows that the size of the EMU has grown relatively to any participating or candidate member country. The increase in relative size of the EMU in turn implies, that the relative size of the UK has decreased. For other historical reasons, we might also hold there is a fair degree of asymmetry between the two economies.

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Appendix: Proofs of statements

We provide the basis for the monetary policy equilibrium in the monetary union model, with alternative assumptions about the institutional constraints on the behaviour of the monetary authority.

Conditional on the institutional constraints, the monetary policy equilibrium is determined under the assumption of rational expectations. In each model the monetary policy rule is defined as a function of the realized values of the demand or supply shocks.

In the discretionary equilibrium the monetary authority chooses in each time period the average inflation rate π_t, in order to minimize the loss function (4) subject to (1)–(3) and to the rational expectations constraint that the expected inflation rate in each country is equal to the average expected inflation rate: $E_t(\pi_{it}) = E_t(\pi_t)$ for all $i = 1, \dots, n$. In addition, under discretion the optimal monetary rule is determined in each period conditional on inflation expectations.

The first order condition for the central bank optimization problem is the following:

$$E_t \sum_{i=1}^{n} w_i \{ b_i [\pi_t - E_t(\pi_t) + \varepsilon_{it} + \upsilon_{it} - k_i] + \pi_t + \upsilon_{it} \} = 0 \tag{29}$$

Since the expected value of the purchasing power parity shock υ_{it} for $i = 1, \dots, n$, conditional on information available in period t, is by assumption equal to zero, solving Eq. (29) for the average inflation rate π_t yields Eq. (5). The assumption of a quadratic loss function implies moreover, that the second order condition is satisfied.

In the commitment equilibrium the central bank selects a monetary policy rule of the form $\pi_t = \pi(\varepsilon_{1t}, \dots, \varepsilon_{nt})$, to minimize the loss function (4) subject to (1)–(3) and to the rational expectations constraint $E_t(\pi_t) = E_t[\pi(\varepsilon_{1t}, \dots, \varepsilon_{nt})]$. In order to derive the optimal commitment rule, we use the linear properties of the conditional expectation operator. Denote respectively with $F(\varepsilon_{1t}, \dots, \varepsilon_{nt})$ and $G(\upsilon_{1t}, \dots, \upsilon_{nt})$ the

Footnote 8 (continued)
member country. Following the Lisbon Treaty in 2007 any EU country has the right to withdraw from the union.

period t distributions of the demand or supply shocks and of the purchasing power parity shocks. Since the demand or supply shocks and the purchasing power parity shocks are assumed to be stochastically independent, the central bank loss function resulting from Eqs. (1)–(4) takes the form:

$$\int \sum_{i=1}^{n} w_i \left\{ \frac{b_i}{2} \left[\pi_t - E_t(\pi_t) + \varepsilon_{it} + \upsilon_{it} - k_i\right]^2 + \frac{1}{2}\left(\pi_t + \upsilon_{it}\right)^2 \right\} dFdG \tag{30}$$

and the rational expectations constraint is:

$$E_t(\pi_t) = \int \pi\left(\varepsilon_{1t}, \dots, \varepsilon_{nt}\right) dF \tag{31}$$

Substituting the rational expectations constraint (31) in the loss function (30), the first order condition for the central bank optimization problem can be represented as follows:

$$\begin{aligned} &\int \sum_{i=1}^{n} w_i \left\{ b_i[\pi_t - E_t(\pi_t) + \varepsilon_{it} + \upsilon_{it} - k_i] + \pi_t + \upsilon_{it} \right\} dG \\ &\quad - \int \sum_{i=1}^{n} w_i b_i [\pi_t - E_t(\pi_t) + \varepsilon_{it} + \upsilon_{it} - k_i] \, dFdG = 0 \end{aligned} \tag{32}$$

Computing the integrals in Eq. (32) leads to the condition:

$$(1 + b)\pi_t = b\left[E_t(\pi_t) - \varepsilon_t\right] \tag{33}$$

where as before $b = \sum_{i=1}^{n} w_i b_i$ is the weighted average of the output cost weights, $\varepsilon_t = \sum_{i=1}^{n} \omega_i \varepsilon_{it}$ is the weighted average of the demand or supply shocks in period t, compiled using the equivalent set of weights, and the equivalent weights are defined as $0 \le \omega_i = w_i b_i / b \le 1$, $\sum_{i=1}^{n} \omega_i = 1$.

The rational expectations constraint and Eq. (33) in turn imply, that in equilibrium $E_t(\pi_t) = 0$ in each period t. Substituting this result in Eq. (33) yields the optimal commitment monetary policy rule (10). We note again, that the assumption of a quadratic loss function implies, that the second order condition for the central bank optimization problem is fulfilled.

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