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Adaptation and application of the fuzzy synthetic evaluation (FSE) method for characterizing the trophic state of tropical semiarid reservoirs

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Abstract Eutrophication is a recurrent problem in water bodies, especially in tropical semiarid reservoirs. The trophic state index (TSI) is an important tool for the environmental management of aquatic systems. However, determining the TSI involves uncertainties that can affect decision-making. This study aimed to adapt and apply the fuzzy synthetic evaluation (FSE) to characterize the TSI considering the uncertainties of the reference eutrophication classification system. The Castanhão reservoir, the largest in the State of Ceará, Brazil, was taken as a case study. The results showed that (i) the uncertainty of the trophic classification system can be characterized by the triangular and trapezoidal membership functions; (ii) the result matrix associates the global trophic level with a degree of certainty, providing greater confidence to the decision maker; (iii) the eutrophication index (EI) is not an adequate tool for hierarchizing the trophic degree; and (iv) the membership level of the global trophic state generated by the FSE method is a suitable alternative to the EI. It is concluded that the proposed FSE model can be a useful tool for improving water resources management, especially in drylands.

J. B. S. Raulino · I. E. Lima Neto (⊠) Department of Hydraulic Engineering and Environment, Federal University of Ceará, Fortaleza, Brazil e-mail: iran@deha.ufc.br **Keywords** Eutrophication · Fuzzy synthetic evaluation (FSE) method · Uncertainty · Trophic state index (TSI) · Degree of membership

Introduction

Reservoirs are important structures for the development of society, notably in tropical semiarid regions such as the Northeast of Brazil (NEB), where a Highdensity Reservoirs Network (HdRN) has been developed to minimize the impacts of recurrent droughts on water availability (Campos, 2015; Freire et al., 2023; Rabelo et al., 2021). However, such hydrologic infrastructures transform a lotic aquatic environment into a lentic environment which, added to the tendency of accumulation of nutrients and sediments, can favor the phenomenon of eutrophication (Moura et al., 2020; Carneiro et al., 2023).

Eutrophication is the process of increasing the production of organic materials in reservoirs due to excess of nutrients (phosphorus and nitrogen) (Le Moal et al., 2019; Vinçon-Leite & Casenave, 2019). As a consequence, reservoirs can experience growth in phytoplankton productivity and biomass (Carlson, 1977). Eutrophication can cause other important adverse effects on water bodies, such as reduced oxygen levels, as well as taste and odor problems for drinking water (Wagner & Erickson, 2017).

Anthropic activities strongly influence aquatic ecosystems, as they drastically alter nutrient fluxes from the watershed and the processes that occur within the water body (Caković et al., 2023; Rocha & Lima Neto, 2022). The main point source of pollutants that triggers the phenomenon of eutrophication is domestic sewage (Freire et al., 2021). Agriculture, on the other hand, due to the intensification of the use of fertilizers and pesticides, is considered the main diffuse source of pollutants for the receiving water bodies (Freire et al., 2021). Animal husbandry is also an important non-point source of nutrients and is particularly important when it takes place close to watercourses (Santos et al., 2014). Inside the reservoirs, the main sources of pollutants are fish farming (Lacerda et al., 2018) and the internal load from bed sediments (Moura et al., 2020; Carneiro et al., 2023).

Managing reservoir water quality to prevent or limit eutrophication is therefore of significant value (Wagner & Erickson, 2017). A useful tool to help manage eutrophication is the trophic state index (TSI). The TSI allows classifying and ranking water bodies according to the level of nutrient enrichment, in addition to being a tool for communicating the status of water quality to the public (Osgood, 1984).

Within the scope of TSI models, the trophic classification of reservoirs in semiarid regions, such as the NEB, deserves particular attention, due to their hydroclimatic and limnological particularities, which make them more prone to the phenomenon of eutrophication (Wiegand et al., 2021). The TSI introduced by Carlson (1977) modified by Toledo Jr et al. (1983) to classify reservoirs in tropical environments is commonly applied to tropical semiarid reservoirs (e.g., Chaves et al., 2013; Santos et al., 2014).

However, systems for classifying the trophic state of lakes and reservoirs (e.g., Carlson, 1977; Toledo Jr et al., 1983) have fuzzy zones (imprecise) between the trophic levels (e.g., oligotrophic-mesotrophic, mesotrophic-eutrophic, eutrophic-hypereutrophic) (Lu et al., 1999). In addition, the selection of the classification criteria is subjective and the weights of the parameters used to characterize the degree of eutrophication are equal (Chen et al., 2021). As a result, the values of water quality parameters around trophic state transitions cannot be taken accurately and deterministically for individual and global TSI calculation (Novajan et al., 2019). This is a characteristic problem that can be potentially solved by the fuzzy set theory, thus motivating the use of the fuzzy synthetic evaluation (FSE) method (Chang et al., 2001).

The work developed by Lu et al. (1999) was the first relevant study to apply the FSE to determine the degree of eutrophication of reservoirs. The authors demonstrated that the FSE model is more adequate than the Carlson Index to characterize the degree of eutrophication. Based on these findings, several studies began to use the FSE to determine the eutrophication level of reservoirs, considering the uncertainties of the classification system (e.g., Chen et al., 2021; Lin & Huang, 2015; Lu et al., 2010).

The FSE groups crisp input data from trophic indicators (e.g., phosphorus) into different categories (e.g., oligotrophic, mesotrophic, eutrophic) according to the classification system used (Lu et al., 2010). The FSE requires the determination of the relative influence of trophic indicators (Zou et al., 2006). The hierarchical analytical process (AHP) method is commonly used for this task (Lu et al., 1999). This method establishes subjective relative weights for the trophic indicators through a judgment matrix (Chen et al., 2021), determining those that will have greater influence in the characterization of the degree of eutrophication (Li et al., 2005).

Membership functions are designed to reflect the absence of well-defined boundaries between each pair of adjacent criteria in the ranges of trophic classification systems (Feng & Ling, 2014). Commonly used functions are the triangular and trapezoidal functions, which fit well into typical ranges of trophic degrees and can be described by using simple mathematical expressions (Akter et al., 2019; Lu et al., 2010). In addition, several studies have attested the ability of these types of membership functions to represent and characterize the uncertainties of classification systems (e.g., Lu et al., 1999). A fuzzy relation matrix constructed based on trophic indicators and trophic states is designed. From the values of the selected water quality parameters, associated membership values are determined. The product between this fuzzy relation matrix and the respective weights generates evaluation result matrix, indicating the degrees of membership for the trophic levels used (Lu et al., 1999). Usually, the Max operator is applied to this matrix to determine the global trophic level (Chang et al., 2001).

The present study aims to adapt and apply the FSE method to characterize the degree of eutrophication of tropical semiarid reservoirs. The Castanhão reservoir, the largest in the State of Ceará, Brazil, is used

as a case study. In the view of the authors, this is the first study of this nature on a reservoir located in a tropical semiarid region.

Methodology

Study area

Located in the municipality of Alto Santo, Ceará, Brazil, the Castanhão reservoir (6700 hm³) was selected as a case study for the use of the methodology developed in this work (see Fig. 1). It is the most important reservoir in the state of Ceará, being used, for example, for flood protection, drinking water supply, and irrigation (Lacerda et al., 2018). It covers an area of 325 km² and is 48 km long, with the depth exceeding 50 m in some areas (Lacerda et al., 2018). This water body belongs to the Middle Jaguaribe subbasin, which is located in the eastern part of the State of Ceará and covers an area of approximately 10,355 km² (13% of Ceará's territory). The main inflow to Castanhão comes from the Jaguaribe River (610 km), whose source is located in the Upper Jaguaribe region and its mouth on the state coast. The climate is considered hot/semi-arid (BSw'h', according to the Köppen classification) (Lacerda et al., 2018), with an annual average temperature of 28 °C and annual average rainfall of 700 mm (Raulino et al., 2021). As an effect, the stored volume varies significantly (Rocha & Lima Neto, 2022). In addition, the region is characterized by rainfall concentrated in a few months (February to May), which often reach the sub-basins after long periods of drought or absence of rain (Santos et al., 2014).

The Water Resources Management Company of the State of Ceará (COGERH) monitors the Castanhão reservoir. Although this reservoir started operating in 2004, qualitative monitoring only started in 2011, on a quarterly basis (Raulino et al., 2021). Among the monitored water quality parameters are total phosphorus concentration (TP), chlorophylla concentration (Chl-a), and the transparency of the water, measured by the Secchi disk depth (SD).

Data source

In this study, water quality data (TP, Chl-a, and SD) were obtained from the Ceará Hydrological Portal (http://www.hidro.ce.gov.br/), which was developed jointly by the Water Resources of the State of Ceará Secretary (SRH), by the Foundation of Meteorology and Water Resources of Ceará (FUNCEME) and by COGERH. Only data collected at the point of withdrawal for water supply were used (CTN-20) (see Fig. 1), focusing on the



Fig. 1 Location of the Castanhão reservoir (Ceará, Brazil), indicating the measurement point considered in this study (CTN-20)

period from April 2014 to December 2021, due to the simultaneous availability of data for determination of the three trophic indicators used in this study. Note that CTN-20 is the point with the most complete dataset. Moreover, COGERH uses this point as a reference to provide the official trophic classification of the reservoir.

Trophic state index (TSI)

COGERH considers the TSI model of Toledo Jr et al. (1983) to characterize the degree of eutrophication of the reservoirs. It is a deterministic model introduced by Carlson (1977) and modified by the abovementioned authors to indicate the degree of eutrophication of tropical reservoirs. This model is widely used for tropical semiarid reservoirs (e.g., Chaves et al., 2013; Santos et al., 2014). In this sense, it is used in this study for the deterministic characterization of the degree of eutrophication of the studied reservoir and as a reference for the results generated by the FSE method.

This index is calculated by the mathematical expressions described below.

$$TSI (TP) = 10 \left\{ 6 - \frac{\left[\ln \left(\frac{80.32}{TP} \right) \right]}{\ln(2)} \right\}$$
(1)

TSI (Chl – a) =
$$10 \left\{ 6 - \frac{[2.04 - 0.695(\ln{(Chla)})]}{\ln(2)} \right\}$$
(2)

TSI (SD) =
$$10 \left[6 - \frac{(0.64 + \ln(SD))}{\ln(2)} \right]$$
 (3)

where SD = Secchi disk depth (m); TP = total phosphorus concentration ($\mu g L^{-1}$); and Chl-a = chlorophyll-a concentration ($\mu g L^{-1}$).

Hence, the average TSI (TSI_a) was calculated by:

$$TSI_{a} = \frac{TSI(TP) + TSI(Chl - a) + TSI(SD)}{3}$$
(4)

According to the results of TSI_a (Eq. (4)), the trophic states presented in Table 1 were considered to perform the trophic classification of the studied reservoir.

 Table 1
 Trophic
 classification
 system
 proposed
 by
 Toledo
 Júnior (1990)

Estado trófico	Criterion	TP (µg/L)	Chl-a (µg/L)	SD (m)
Oligotrophic	≤ 44	≤ 26	≤ 3.81	> 1.7
Mesotrophic	44 < TSI ≤ 54	27–52	3.82-10.34	1.1–1.7
Eutrophic	$54 < TSI \le 74$	53–211	10.35-76.06	0.8–1.1
Hypere- utrophic	TSI > 74	> 211	> 76.06	< 0.8

Fuzzy synthetic evaluation (FSE) method

The FSE method was employed in this study to consider uncertainties in trophic state classification. The methodology was organized into seven steps in order to facilitate its understanding and application.

Step 1: Determination of trophic indicators

The selection of trophic indicators is crucial (Lu et al., 2010). The trophic indicators used in this study were three, namely TP, Chl-a, and SD. The criteria used to select these water quality parameters combined characteristics of the tropical semiarid reservoirs (reasons i and ii) and justifications of the literature (reasons iii and iv), as detailed as following:

i). Phosphorus is considered the limiting nutrient for eutrophication of tropical semiarid reservoirs, according to numerous studies carried out in the region (e.g., Lima Neto et al., 2022; Raulino et al., 2021; Rocha & Lima Neto, 2022). Therefore, it will be considered the causative agent of the eutrophication phenomenon. Chl-a concentration is adopted as a measure of the response of the water body to the causative agent, representing the algal biomass (Carlson, 1977). The adoption of water transparency, evaluated by the SD, intends to represent the effect of increasing the density of planktonic organisms on the reduction of light penetration in the water column (Carlson, 1977). It is important to mention that these justifications are also the basis for ranking the relative importance of these trophic indicators in the definition of the judgment matrix of step 5;

- ii). These three water quality parameters are periodically monitored by COGERH;
- iii). The selection of these water quality parameters allowed the comparison with the results of the Toledo index; and
- iv). These trophic indicators are commonly used in trophic classification systems.

Thus, the trophic indicators, also called evaluation indicators, constitute the fuzzy subset $U = \{TP, Chl-a, SD\}$ in the FSE method.

Step 2: Selection of trophic state classification criteria

In the present study, the same trophic levels of the classification system proposed by Toledo Júnior (1990) were used for two main reasons: (i) to allow an adequate comparison between the results of the models (proposed FSE model and the Toledo index) and (ii) to offer a more adequate representation of the trophic condition of the reservoir since it adds a trophic level (hypereutrophic) to the commonly used ones.

According to the trophic classification system established by Toledo Júnior (1990), the trophic state of a reservoir is characterized according to four trophic levels, namely oligotrophic (O), mesotrophic (M), eutrophic (E), and hypereutrophic (H) (Table 1). Thus, the evaluation criteria set is the fuzzy subset V = $\{O, M, E, H\}$.

Step 3: Determination of membership functions (fuzzification)

In this study, the selection criterion for the type of membership function for the trophic indicators was the by Lu et al. (1999) and Lu & Lo (2002). These key points consist of mean values and extreme values of the trophic ranges and are shown in Table 2 along with their respective levels of pertinence.

The chosen functions were triangular (mesotrophic and eutrophic levels) and trapezoidal (oligotrophic and hypereutrophic levels). These functions are represented by simple expressions, which are generically represented as follows (Lu et al., 2010):

$$\mu_{O}(x) = \begin{cases} 0, x \ge b \\ \frac{b-x}{b-a}, a < x < b \\ 1, x \le a \end{cases}$$
(5)

$$\mu_{M}(x) = \begin{cases} 0, x \ge c \\ \frac{c-x}{c-b}, b < x < c \\ \frac{x-a}{b-a}, a < x \le b \\ 0, x \le a \end{cases}$$
(6)

$$\mu_{M}(x) = \begin{cases} 0, x \ge d \\ \frac{d-x}{d-c}, c < x < d \\ \frac{x-b}{c-b}, b < x \le c \\ 0, x \le b \end{cases}$$
(7)

$$\mu_{\rm H}(x) = \begin{cases} 1, x \ge d \\ \frac{x-c}{d-c}, c < x < d \\ 0, x \le c \end{cases}$$
(8)

Trophic indicator	Oligotrophic	Mesotrophic	Eutrophic	Hypertrophic
TP (μg L ⁻¹)	μ (26) = 1 μ (39.50) = 0	$\mu (26) = 0 \mu (39.50) = 1 \mu (53) = 0$	$\mu (53) = 0 \mu (132) = 1 \mu (211) = 0$	$\mu (211) = 1 \mu (132) = 0$
Chl-a (µg L ⁻¹)	$\mu (3.81) = 1 \mu (7.08) = 0$	$\mu (3.81) = 0 \mu (7.075) = 1 \mu (10.34) = 0$	$\mu (10.34) = 0 \mu (43.21) = 1 \mu (76.06) = 0$	$\mu (76.06) = 1 \mu (43.205) = 0$
SD (m)	μ (26) = 1 μ (39.5) = 0			$\begin{array}{l} \mu \left(0.80 \right) = 1 \\ \mu \left(0.95 \right) = 0 \end{array}$

Table 2Key values fortrophic indicators

in which x is the evaluated trophic indicator, μ is the membership level, and the constants a, b, c, and d are the key points.

Step 4: Construction of the fuzzy relation matrix

The fuzzy relation matrix used for the application of the FSE is constructed from the relation between the subset U = {TP, Chl-a, SD} and the evaluation criteria subset V = {oligotrophic, mesotrophic, eutrophic, hypereutrophic}. In the 4 \times 3 matrix formed, the lines are the three trophic indicators and the columns are the four trophic levels

$$R = \begin{bmatrix} \mu_{TP,H} & \mu_{TP,E} & \mu_{TP,M} & \mu_{TP,O} \\ \mu_{Chl-a,H} & \mu_{Chl-a,E} & \mu_{Chl-a,M} & \mu_{Chl-a,O} \\ \mu_{SD,H} & \mu_{SD,E} & \mu_{SD,M} & \mu_{SD,O} \end{bmatrix}$$
(9)

Step 5: Weighting of trophic indicators

The attribution of weights defines the relative importance and influence of trophic indicators in the assessment of eutrophication of the water body (Taheriyoun et al., 2010). In this study, the analytic hierarchy process (AHP) method was used to determine the subjective importance of the indicators (Chen et al., 2021)

The ranking of importance of the most relevant factors for eutrophication consists in increasing order of relevance: TP, Chl-a, and SD. The justifications for this ranking are the same as those described in step 1 (see reason 1). Hence, this study assumes an initial judgment matrix J for these three parameters as listed in Table 3.

The verification of the adequacy measure of weight assignment is necessary in the subjective choice of

 Table 3 Initial judgment matrix J for trophic indicators

	TP	Chl-a	SD
TP	1	2	3
Chl-a	1/2	1	2
SD	1/3	1/2	1

weights. In this sense, the inconsistency ratio (CR) of the matrix J is calculated according to the Eq. (10) (Chen et al., 2021):

$$CR = \frac{CI}{RI}$$
(10)

where RI = mean random inconsistency coefficient (Table 4) and CI = coefficient of inconsistency, calculated according to Eq. (11):

$$CI = \frac{\lambda_{max} - n}{n - 1}$$
(11)

in which n = dimension of the judgment matrix and = maximum eigenvalue of this pairwise matrix. When CR < 0.10, the consistency of the judgment matrix J is good; otherwise, the element values of matrix J need to be adjusted to meet the consistency requirements (Chen et al., 2021).

After verifying the consistency of the matrix J, the relative weights of the trophic indicators are obtained by normalizing two matrices consecutively. The matrix J is the first to be normalized. This first normalization is obtained by dividing each element by the average of the values in its respective column, generating a new 3×3 matrix. This matrix is again normalized through the average of each row. The resulting matrix is the weighted vector W.

$$W = \begin{bmatrix} w_{TP} & w_{Chl-a} & w_{SD} \end{bmatrix}$$
(12)

where w_{TP} , w_{Chl-a} , and w_{SD} are the weights of the parameters TP, Chl-a, and SD, respectively.

Step 6: Determination of the evaluation result matrix (aggregation)

Aggregation is necessary to obtain the degree of certainty for each trophic status. The aggregation process is performed by the expression (Akter et al., 2019):

$$\mathbf{B} = \mathbf{W} \mathbf{X} \mathbf{R} = \begin{bmatrix} \mathbf{b}_{\mathrm{H}} & \mathbf{b}_{\mathrm{E}} & \mathbf{b}_{\mathrm{M}} & \mathbf{b}_{\mathrm{O}} \end{bmatrix}$$
(13)

in which b_H , b_E , b_M , and b_O indicate the level of belonging of the reservoir to the hypereutrophic,

Table 4 Random inconsistency coefficients of a n x n matrix (Taheriyoun et al., 2010).

n	1	2	3	4	5	6	7	8	9
RI	0	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45

eutrophic, mesotrophic, and oligotrophic states, respectively.

Step 7: Conversion of fuzzy results into a crisp number (defuzzification)

All values representing water quality are then synthesized into a single crisp value based on a particular fuzzy operator. This is defuzzification, which aims to interpret the fuzzy results through a crisp number, which facilitates decision-making (Lu et al., 2010). In this study, the fuzzy operator used is the maximum membership value generated by the matrix B, according to the following expression (Akter et al., 2019):

$$E max = (b_H, b_E, b_M, b_O)$$
(14)

where E represents the predominant trophic level of the reservoir, that is, the highest membership value determines the global trophic classification (Lu et al., 2010).

The results of the FSE can also be synthesized by means of a eutrophication index (IE), whose value is calculated by the following expression (Lu et al., 2010).

$$EI = b_0 + 2b_M + 3b_E + 4b_H$$
(15)

2/2014

)6/2015)9/2015 [2/2015

3/2015

Results and discussion

Evolution of the water quality of the Castanhão Reservoir

The temporal evolution of the trophic state of the Castanhão reservoir from April 2014 to December 2021 is shown in Fig. 2. This characterization of eutrophication was determined by the index of Toledo Jr et al. (1983).

The results indicated the condition of eutrophication of the reservoir most of the time, with few dates presenting the condition of non-eutrophication. This scenario is mainly a reflection of the climatic conditions of the period, as the reservoir is located in a region that experienced a prolonged drought during the study period (Pontes Filho et al., 2020). Wiegand et al. (2021) observed that the drought was responsible for raising the TSI of reservoirs in NEB during this period. This was mainly due to the gradual reduction in the stored volume, as this increases the internal concentration of pollutants (Rocha Júnior et al., 2018), and the occurrence of heavy rains after dry periods (Santos et al., 2014), which mobilize the nutrients stored in the basin during the absence of precipitation, increasing the phosphorus loads to the reservoirs (Lisboa et al., 2020). These impacts are

06/2020

)9/202([2/2020] (3/202] (6/202] (9/202] [2/202]

---- Toledo index



Mesotrophic

03/2017

12/2017 03/2018 06/2018

09/2018

Month/Year

Eutrophic

03/2019 06/2019 09/2019 12/2019 03/2020

Hipertrophic

7102/60

)9/2016 12/2016

3/2016

Oligotrophic

consistent with the results of Raulino et al. (2021), who also pointed to the decline in the volume of the Castanhão reservoir as the main mechanism for the deterioration of its water quality in the studied period, according to Fig. 3.

In general, this trend of degradation of water quality due to climatic conditions is commonly observed in the tropical semiarid reservoirs (Chaves et al., 2013; Lima et al., 2015; Lima Neto et al., 2022; Raulino et al., 2021, 2022; Rocha & Lima Neto, 2022).

Membership function

Figure 4 shows the representation of the trophic classification system of Toledo Júnior (1990) using triangular and trapezoidal fuzzy numbers. It is possible to observe perfect compatibility between the triangular and trapezoidal membership functions and the key points defined from the trophic classification system used, confirming the choice of the respective types of membership functions and the suitability of the selection criterion.

Feng and Ling (2014) suggest the conditions of the study area as a criterion for formulating the membership functions. However, this is an inadequate criterion since, as the central objective of the FSE is to consider the uncertainties of the trophic classification system, it is essential to consider as a criterion information on trophic ranges (average values of ranges and transitions), according to the vast majority of studies on this subject (e.g., Lu & Lo, 2002; Taheriyoun et al., 2010; Lin & Huang, 2015; Chen et al., 2021) and the results found in this study.

Figure 4 also shows that the trophic indicators TP and Chl-a have the largest tolerance interval corresponding to the eutrophic state, whereas for the trophic indicator SD corresponds to the mesotrophic state. This is because these trophic degrees show the greatest imprecision in the trophic ranges of the respective trophic indicators (Chang et al., 2001). Chang et al. (2001) point out that the adequate representation of these inaccuracies is essential for the fuzzy mechanism for generating the global TSI to be consistent with reality.

All trophic states are characterized by transitions that simultaneously consider the two trophic states that generate them. As a result, there is a classification mechanism that considers inputs from each trophic indicator as belonging to all trophic levels to a certain degree. In this sense, the triangular and trapezoidal membership functions were able to characterize the uncertainties of the trophic state transitions (Chang et al., 2001). This result confirms the ability of the membership function to represent uncertainties of the classification system (Lu et al., 1999) and avoid the use of personal language (I, you, we, us, etc.)

It is important to point out that the SD indicator decreases its trophic degree from left to right, contrary to the TP and Chl-a parameters. This behavior, however, does not generate divergences in the classification mechanism, as it considers independent inputs (Taheriyoun et al., 2010).

Another important point is that the triangular function is particularly useful in situations of sparse data and that have a wide range of variation (Wang & Ran, 2019), as is the case of poor monitoring of water quality of tropical semiarid reservoirs (Wiegand









et al., 2021). It should be noted that, from the results, the trapezoidal function also has this capability. Therefore, such functions are useful tools to represent the TP concentration, being fundamental for the performance of the FSE model. Thus, the membership functions demonstrated the capacity to adequately represent the uncertainties of the transitions of the trophic classification of Toledo Júnior (1990).

Judgment matrix and weighted vector

The results of the consistency analysis of the J initial judgment matrix are (Table 3): = 3.0092, CI = 0.0046, RI = 0.5800, and CR = 0.79%. Based on the criteria of Chen et al. (2021) (CR < 10%), the initial judgment matrix meets the required consistency. Therefore, there was no need to adjust the values of the J matrix.

After the normalization processes, the result of the weighted vector was reached through the AHP process:

$$W = \begin{bmatrix} 0.5390 \ 0.2972 \ 0.1638 \end{bmatrix}$$
(16)

Equation (1) expresses the relative importance of the trophic indicators used in this study. It is observed that the highest weight corresponds to the TP concentration, so the level of eutrophication is determined mainly as a function of this trophic indicator (Taheriyoun et al., 2010). On the other hand, SD has the lowest relative importance for eutrophication in this study. These results are consistent with studies that used the AHP method to weigh the effects of water quality parameters on the eutrophication phenomenon (Lu & Lo, 2002; Chen et al., 2021).

The vast majority of studies on water quality in tropical semiarid reservoirs demonstrates that the phenomenon of eutrophication is conditioned by phosphorus (e.g., Lima Neto et al., 2022; Moura et al., 2020; Raulino et al., 2021). Therefore, the relative importance of TP concentration for the phenomenon is consistent with reality. On the other hand, studies indicate that the SD can be influenced by the high turbidity resulting from mineral material in suspension and not only by the density of planktonic organisms, which would impair the trophic classification of the reservoir (Cunha et al., 2013). Thus, its lower relative importance of the SD tends to reduce

the errors in the characterization of eutrophication (Toledo Júnior, 1990).

In general, these results are consistent with other studies of this nature, which attested to the AHP method to determine the relative importance of the trophic indicators on the eutrophication (Chen et al., 2021; Lin & Huang, 2015).

Comparative analysis of reservoir trophic state modeling

Table 5 shows the comparison of the trophic states generated by the Toledo index and by the FSE. The objective of this comparison was to validate the FSE for tropical semiarid reservoirs. Although differences are observed between the TSI and the FSE (highlighted in red), in general, the results are compatible.

The trophic states indicated by the two models coincided in 54% of the years. In addition, the eutrophication condition of the reservoir, essential information for managing the water quality of the water body, coincided in 85% of the years.

The two models indicated different classifications in eight dates of the historical series without relevant impacts. Two of these dates showed negligible divergences regarding decision-making because they indicated no eutrophication (\leq mesotrophic). On the other hand, although the two models converged in terms of indicating eutrophication, the FSE indicated eutrophication to a greater degree than the TSI on the other six of these. Furthermore, the FSE was the only one that pointed out the hypereutrophic level.

The more pessimistic results of the FSE model are justified by the high TP concentrations, which have greater weight in determining the trophic state of the water body in the structure of the developed model. This result is consistent with studies carried out in the tropical semiarid reservoirs, which point to phosphorus as the causative agent of the eutrophication (e.g., Moura et al., 2020; Wiegand et al., 2021). Taheriyoun et al. (2010) reported that one of the most important characteristics of the FSE method is precisely the consideration of different weights for the trophic indicators that make up the fuzzy relation matrix. The conventional TSI model has equal weights for its trophic indicators, not being able to properly weight the relative influence of each water quality parameter on the final trophic degree (Chen et al., 2021). As a consequence, the effects of Chl-a concentration

Date	TP (μg.L ⁻¹)	Chl-a (µg.L ⁻¹)	SD (m)	TSI	FSE	
12/04/2014	48.00	4.49	3.80	Oligotrophic	Mesotrophic	
03/04/2015	64.00	13.63	1.60	Mesotrophic	Mesotrophic	
05/21/2015	87.00	2.84	3.00	Mesotrophic	Oligotrophic	
08/11/2015	74.00	18.88	1.70	Mesotrophic	Mesotrophic	
01/21/2016	90.00	19.70	1.70	Eutrophic	Eutrophic	
07/27/2016	136.00	52.33	1.20	Eutrophic	Eutrophic	
10/19/2016	102.00	34.38	1.30	Eutrophic	Eutrophic	
01/26/2017	236.00	22.68	1.20	Eutrophic	Hipereutrophic	
04/27/2017	64.00	54.08	1.20	Eutrophic	Mesotrophic	
07/25/2017	120.00	51.80	1.00	Eutrophic	Eutrophic	
11/07/2017	103.00	45.00	0.90	Eutrophic	Eutrophic	
01/10/2018	92.00	45.92	0.70	Eutrophic	Eutrophic	
04/10/2018	190.00	56.14	0.80	Eutrophic	Hipereutrophic	
07/11/2018	154.00	24.63	1.30	Eutrophic	Eutrophic	
10/16/2018	154.00	7.09	0.60	Eutrophic	Eutrophic	
01/08/2019	189.00	93.56	0.60	Eutrophic	Hipereutrophic	
04/02/2019	225.00	78.72	0.50	Eutrophic	Hipereutrophic	
07/09/2019	205.00	29.50	1.00	Eutrophic	Hipereutrophic	
10/08/2019	83.00	33.26	0.90	Eutrophic	Eutrophic	
05/19/2020	219.00	5.61	1.60	Eutrophic	Hipereutrophic	
08/18/2020	103.00	12.78	2.20	Mesotrophic	Mesotrophic	
11/19/2020	66.00	9.64	2.40	Mesotrophic	Mesotrophic	
03/17/2021	46.00	23.59	1.20	Mesotrophic	Mesotrophic	
06/15/2021	93.00	15.80	1.40	Eutrophic	Mesotrophic	
09/14/2021	64.00	25.20	1.30	Eutrophic	Mesotrophic	
12/21/2021	70.00	23.57	1.45	Eutrophic	Mesotrophic	

 Table 5
 Comparison of the trophic state generated by the Toledo index and the FSE

and SD are amplified in the model by Toledo Jr et al. (1983). These results are in agreement with the study by Lu et al. (1999). These authors also mentioned that the unlimited influence that an individual factor can exert on the index and the failure to weight each factor, which can create inconsistent results, are disadvantages of traditional TSI models.

In May 2017 and in the last three dates in Table 5, important divergences are observed in the trophic state, with the Toledo index indicating eutrophication of the water body and the FSE pointing no eutrophication. It should be noted that other studies also reported divergences between the FSE and the traditional TSI model regarding the possibility of eutrophication (e.g., Lu et al., 1999; Wang & Ran, 2019).

The four dates present, respectively, the matrices of results [0.10 0.41 0.49 0], [0 0.38 0.62 0], [0 0.38 0.62 0], and [0 0.31 0.69 0]. Although divergences are observed between the trophic states indicated by the two models, the levels of membership of the eutrophic state in the three dates above 0.30 indicate that the FSE model is also able to capture the possibility of eutrophication of the reservoir with a relatively high degree of certainty. These results and similar observations were pointed out in studies conducted by Wang and Ran (2019). In this sense, FSE model offers broader results and compensates for the

deficiency inherent in traditional (deterministic) TSI models.

Eutrophication index (EI) vs result matrix B

Table 6 highlights the eutrophication index (EI) values and their respective trophic levels at which trophic status and EI are incoherent. It is observed, for example, that the EI of the oligotrophic state of the date 05/21/2015 is greater than the EI of the mesotrophic state of the date 12/04/2014. This problem was also observed in the studies of Lu et al. (1999) and Lu & Lo (2002). This type of situation points to the failure of the eutrophication index to rank eutrophication levels and, consequently, as an inadequate tool to help manage eutrophication. Apparently, the reason for the inadequacy of the EI to quantify the degree of eutrophication is the arbitrary choice of its weights (Lu et al., 1999; Lu & Lo, 2002; Taheriyoun et al., 2010).

Table 6 demonstrates that the membership levels correct the divergences between the EI and their respective trophic level. Therefore, as an alternative to the IE, the membership levels associated with the trophic states can be used in the FSE, specifically the result matrix B. Other studies, although not explicitly discussing this problem, pointed to solutions in this direction (Lu et al., 2010; Taheriyoun et al., 2010; Wang & Ran, 2019; Zou et al., 2006).

Zou et al. (2006) characterized the eutrophication of reservoirs based on the Max operator, making it possible to perform a hierarchy between the different reservoirs studied based on the values of the levels of membership associated with the trophic degrees. Wang and Ran (2019) emphasized that membership levels can prevent the occurrence of inconsistent values of the degree of eutrophication.

Therefore, it is confirmed that the global classification of the trophic level must be performed with the Max operator applied to the result matrix B (Eq. 14). However, the level of pertinence associated with the global trophic state must be adopted as an additional tool instead of the EI, especially for hierarchical purposes of the degree of eutrophication of the water body or between water bodies.

Traditional trophic state index vs fuzzy synthetic evaluation

The convergence of the results, in general, indicates that the traditional TSI model is satisfactory to characterize the eutrophication. However, it is necessary to highlight two additional advantages of the FSE as a tool for managing reservoir eutrophication.

Initially, the FSE indicates how much the condition of the water quality of the reservoir fits the concept of eutrophication in terms of a fuzzy set, and the specific membership value generated by the Max operator allows an easy quantitative hierarchy of a set of reservoirs in terms of the degree of eutrophication. A direct effect, for example, is on the allocation of resources for the management of the reservoir and priorities for the application of measures to mitigate eutrophication (Chen et al., 2021).

Second, the results of the evaluation matrix B express how much the water body is contained in

Table 6 EI values and their corresponding trophic degrees

Data	$TP (\mu g L^{-1})$	Chl-a ($\mu g L^{-1}$)	SD (m)	EI	FSE	b _H	b _E	b_{M}	b _O
12/04/2014	48.00	4.49	3.80	1.65	Mesotrophic	0	0.05	0.55	0.40
05/21/2015	87.00	2.84	3.00	1.82	Oligotrophic	0	0.28	0.26	0.46
08/11/2015	74.00	18.88	1.70	2.63	Mesotrophic	0	0.30	0.54	0.16
10/19/2016	102.00	34.38	1.30	2.63	Eutrophic	0	0.63	0.37	0
01/21/2016	90.00	19.70	1.70	2.90	Eutrophic	0.08	0.51	0.44	0.16
07/27/2016	136.00	52.33	1.20	3.02	Eutrophic	0.11	0.80	0.09	0
07/25/2017	120.00	51.80	1.00	2.99	Eutrophic	0.08	0.83	0.09	0
11/07/2017	103.00	45.00	0.90	2.90	Eutrophic	0.07	0.76	0.17	0
01/10/2018	92.00	45.92	0.70	2.96	Eutrophic	0.19	0.58	0.23	0
10/16/2018	154.00	7.09	0.60	3.02	Eutrophic	0.31	0.39	0.30	0
05/19/2020	219.00	5.61	1.60	2.84	Hypereutrophic	0.54	0	0.22	0.24

the different trophic states of the adopted classification system. This indication is crucial to highlight the importance of using the FSE, as it consists of quantifying the uncertainties generated by the transitions of pairs of trophic levels, which is the major inconsistency generator of traditional TSI models (Lu et al., 2010). Furthermore, reservoirs have a large degree of variability in their response to a trophic indicator, such as nutrient concentration, and this leads to uncertainties in the trophic state response (Novajan et al., 2019) since the membership levels in terms of the different trophic levels also face this uncertainty (Zou et al., 2006).

Therefore, the information generated by the FSE method is more reliable and adequate for the proper management of water quality in reservoirs due to the uncertainties that permeate the eutrophication classification system of water bodies.

Conclusions

The trophic state index (TSI) is a traditional tool for determining the degree of eutrophication of a reservoir. Nevertheless, the TSI does not consider the uncertainties of the transitions of the trophic states of the classification systems, which can negatively affect the determination of the degree of eutrophication and, consequently, the management of the water quality. This deficiency can be circumvented by the fuzzy synthetic evaluation (FSE) method. Thus, the referred method was used to evaluate the eutrophication pattern of the Castanhão reservoir, a large tropical semiarid reservoir, and the results were compared with those obtained by the TSI reference. The main results were the following:

- The uncertainties of classification systems can be represented by triangular and trapezoidal membership functions;
- 2. The FSE method is able to quantify the uncertainty of classification systems and provide more reliable results for managing eutrophication of the water body;
- 3. The eutrophication index (EI) is inadequate to hierarchize the degree of eutrophication;
- 4. The levels of membership of the result matrix indicate adequately the degree of uncertainty of the trophic states for the reservoir;

5. The adaptation of the FSE method is an important tool with the potential to improve the management of water quality, not only in drylands.

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Declarations

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