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Personalized pricing with persuasive advertising and the value of consumer information: a duopoly framework

Lei Yan[1,2] · **Yuxiang Zhang**[1] · **Shue Mei**[1] · **Weijun Zhong**[1]

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Abstract

This paper studies the effects of persuasive advertising and personalized pricing on duopolistic firms' profits, consumer surplus and social welfare when one or two firms adopt consumer information to personalized pricing. With a game-theoretic model, the main results are summarized as follows: (1) The profits of both firms when they use consumer information are lower than those when neither of them use consumer information. (2) Consumer surplus increases with the number of firms who collect and use consumer information. Compared with the case that one or no firms have consumer information, the social welfare in the case that both firms have consumer information is the highest. (3) Given both firms adopt persuasive advertising simultaneously, the two firms will trap into "prisoner's dilemma" when they decide whether to use consumer information or not. (4) In the duopolistic competition, the optimal strategy for data intermediary is to sell information to only one firm.

Keywords Persuasive advertising · Consumer information · Personalized pricing · Consumer information value · Duopolistic competition

JEL Classification M37 · D43 · L13 · D8 · L15 · L5

✉ Yuxiang Zhang
yx3zh@hotmail.com

Lei Yan
chinayanlei@126.com

Shue Mei
meishue@seu.edu.cn

Weijun Zhong
zhongweijun@seu.edu.cn

[1] School of Economics and Management, Southeast University, Nanjing 211189, China

[2] School of Mathematics and Statistics, Pingdingshan University, Pingdingshan 467000, China

1 Introduction

Advertisings are applied widely in daily business and marketing, a significant fraction of advertising is substantially persuasive. The persuasive advertisings usually appear in newspapers, magazines, or a prominent place on the Internet website. Persuasive advertising plays a prominent promotion effect in many industries including household appliances, mobile phones, soft drinks, milk products and so on. A great number of firms invest heavily on advertising to promote sales or get more profits. For instance, Mengniu Dairy Company Limited and Yili Industrial Group, the two famous diary giant in China, spent around 8.95% and 11.34% of their operating revenue on advertising in the year of 2020,[1] respectively. Coca-Cola and Pepsi spent about 11% and 6% respectively, of their annual incomes on advertising in 2006 [4]. Persuasive advertising increases consumers' willingness to pay or/and perceived product differences [3, 36].

With the development of information technology, firms, data brokers and other related parties can record, store and analyze data about consumers as never before. Based on these collected consumers data, firms can capture plenty of information about consumers' consuming features, such as consumers' preferences, positions, gender, age, willingness to pay of a given good or service. Meanwhile, firms may offer potential consumers personalized products or service to realize personalized marketing. Moreover, firms may use personalized pricing strategies based on consumer preference simultaneously to get more profits. For instance, e-retailers could realize personalized prices based on consumer historic behavior when consumers visit the e-retailer's online store after they receive the e-retailer's persuasive advertising. The demand of consumer information has hastened the birth of data intermediaries, such as Acxiom, Experian. These data intermediaries earn profits by offering data collection and data analysis services to market participants. In fact, many e-commerce platforms, such as Taobao and eBay, achieve great profits based on consumer information data.

The use of consumer information will impact on firms' competitive persuasive advertising and pricing strategies. Indeed, most of firms cannot get these information for free, they need to buy consumer information from a data intermediaries, which increases the marketing cost. In this paper, we consider that, in a duopolistic market, two firms may compete with persuasive advertising and personalized pricing based on consumer information. The following crucial problems are discussed: will firms benefit from employing persuasive advertising and personalized pricing? Will firms' strategies of employing persuasive advertising and personalized pricing increase consumer surplus and improve social welfare when the firms use consumer information? And what is the optimal strategy for the data intermediary, selling consumer information to just one firm or both? Our research could offer effective guidance of marketing practices.

[1] The information was accessed on 2021/7/16 at https://www.sohu.com/a/467465127_120008090 .

We adopt a game theoretical model where two firms compete both with persuasive advertising and price. Persuasive advertising is considered to either increase consumers' willingness to pay or perceived product differences. The two firms may use consumer information. Our research demonstrates that the firms' profits when both firms use consumer information are lower than these when neither of the firms use consumer information. The adoption of consumer information may benefit consumers, consumer surplus increases with the number of firms who use consumer information in the competition, and social welfare is the highest when both firms use consumer information. We derive the equilibrium price of consumer information and find that the optimal strategy of the data intermediary is to sell information only to one firm.

The rest of this paper is organized as follows. In Sect. 2, we review the relevant researches. In Sect. 3, we analyze the persuasive advertising and pricing strategies in equilibrium and the impact of advertising and pricing strategy on profits, consumer surplus and social welfare. In Sect. 4, we explore the value of consumer information and the optimal strategy of data intermediary, and Sect. 5 concludes our research. Besides, all proofs are given in the Appendix.

2 Literature review

Our research relates with three aspects of literatures: persuasive advertising, consumer information and personalized pricing. We firstly present the relative literartures on persuasive advertising. VonderFehr and Stevik [37], Belleflamme and Peitz [2] present three main effects of persuasive advertising: enhancing consumer willingness to pay, changing the distribution of consumer tastes, and increasing perceived product differences. Shaffer and Zettelmeyer [31–33] examine the benefit of using targeted persuasive advertising where persuasive advertising can increase product heterogeneity. Wu et al. [38] emphatically discuss the correlative dependence between persuasive advertising competition and supply chain channels structure and find that persuasive advertising doesn't lead to channel conflict. With investigating the effects of consumer preferences and advertising efficiency on firms' persuasive advertising and pricing strategies, Jiang and Srinivasan [17] find that the firm with a lower value-added product has more incentive to increase persuasive advertising when the firm's horizontal differentiation increases, whereas its rival inclines to reduce advertising. Willis and Rogers [39], Nelson [26] study the structural relation between persuasive advertising intensity and market concentration. These literatures ignore the effect of price discrimination on persuasive advertising when firms can get consumer information.

The second string of relative literatures is consumer information. Consumer information is used in price discrimination which will influence firms' profits, consumers' surplus and social welfare. Liu and Serfes [21] find that when consumer information precision increases, consumer surplus, social welfare and equilibrium profits of the high quality firm increase monotonically, but those of the low quality firm monotonically decrease. Zhao and Ling [41] show that better quality of consumer information can help the better-informed firm to cut down advertising expense and

obtain higher expected profit in competition. Sapi [34], Liu and Shuai [22] find that improvement of consumer information quality can increase enterprise profits but decrease consumer surplus and social welfare. Colombo [8] shows that consumer information accuracy has a nonmonotonic impact on profits, consumer surplus, and social welfare. Nijs [28] shows that exchange of customer information between rival firms increases the firm's ability to realize price discrimination according to consumer's profiles and improves the efficiency of behavior-based price discrimination. Liu and Serfes [21], Zhao and Ling [41], Sapi [34], Liu and Shuai [22], Colombo [8] and Nijs [28] focus on the impact of consumer information precision, quality and information exchange on price discrimination and firms' profits.

Ouksel and Eruysal [29] analyze the impact of price discrimination and market segmentation based on consumer purchase behavior information on two asymmetric firms' competition with a game-theoretic model, and find that the game is not necessarily trapping into a prisoner's dilemma, the firm is likely to improve its profit at the expense of the rival firm, and consumer welfare will increase with market segmentation. Jentzsch et al. [16] study how consumer information shared among industry competitors influences the compeition, and find that the information-sharing can realize personalized pricing and increase social welfare, but hurt consumer surplus. Cosguner et al. [10] analyze the impacts of behavioral price discrimination on manufacturers and retailers in a distribution channel and find that behavioral price discrimination based on customers' last purchase information can improve retailers' profits and lead to lower manufacturer's profits. Colombo [9] shows that behavior-based and characteristic-based price discrimination yields higher profits than uniform pricing only if consumers are heterogeneous enough in price sensitivity, and when there is a sufficient number of high-sensitivitive consumers, the welfare under behavior-based and characteristic-based pricing is lower than under uniform pricing. Zhang et al. [40] find that when the demand elasticity is large enough, price discrimination based on customer information improves social welfare. The above literatures mainly analyze how profits, consumer surplus and social welfare are influenced by firms' pricing discrimination based on consumer information. But few of them study how the application of consumer information influences on the firms' persuasive advertising strategies.

The third aspect of literatures relative with our research is personalized pricing. Miettinen and Stenbacka [24] compare the effect of personalized pricing and history-based pricing. They find that personalized pricing harms consumer surplus and social welfare. Chen et al. [7] show that personalized pricing based on identity management of consumers can boost firm's profits at the expense of consumer surplus and social welfare. But Matsumura and Matsushima[23] show that all firms do not always employ personalized pricing because personalized pricing induces the rival firm to engage more in reducing its cost, which is more likely to harm the less-efficient firm. Lauseel[19] presents a model where an oligopoly make personalized pricing based on product quality in a hyper-segmented market. The results show that after the oligopoly traces consumers' quality preference information based on these consumers' initial purchase behavior, he could set price-quality personalized price, the personalized price-quality pricing can satisfy consumers' different demand of quality, but extract consumer surplus fully. Garella et al. [15] study duopolistic

firms' price personalization with a two period game. They find that, compared with uniform pricing, when firms set personalized price depending on consumers' historic purchase behaviors, all consumers could benefit from personalized pricing when the size of both firm's loyal consumers are fully asymmetric, and the social welfare is better. These literatures on personalized pricing mostly focus on how firms set their personalized pricing strategies after they have consumer information and how personalized pricing impacts firms' competition, consumer surplus and social welfare. But most of them (Miettinen and Stenbacka [24], Chen et al. [7], Laussel [19], Garella et al. [15]) ignore how firms set their advertising strategies when they use consumer information in personalized pricing.

Several literatures study the advertising strategies when firms adopt price discrimination. Villas-Boas [36] studies the advertising strategy of a firm launches a product line with vertically differentiated products (high-quanlity, low-quality). He finds that the firm should advertise under general conditions because the firm can get a greater proportion of sales of the low-quality product compared to the scenario that advertising has no cost. Because when advertising has no cost the firm has to charge a lower price of the high-quality product and improve the quality of the low-quality product which decreases the marginal profit. Anderson et al.[1] studies personalized price competition with costly advertising among several quality-cost differentiated firms, and they show that only the top two firms advertise, and earn "Bertrand-like" profits, and that social welfare initially falls then rises with the inceasing ad cost. Esteves [11] explores the impacts of price discrimination on the advertising efficiency. When there is no advertising cost or low advertising cost, price discrimination leads firms to provide too little advertising, which is bad for consumers and overall welfare. Price discrimination brings firms excessive advertising, which is bad for consumers and overall welfare but good for firms with high advertising costs. Nijs [27] examines two-stage advertising and pricing strategies and show that price discrimination can restore symmetry in equilibrium advertising decisions, and that price discrimination increases (resp. decreases) profits and total welfare but hurts (resp. benefits) consumers when advertising cost is high (resp. low). Esteves and Resende [12] examine the impact of price discrimination on advertising strategies and firm's profits, showing that when price discrimination is allowed, firms can increase or reduce the intensity of advertising targeted to each segment compared with uniform pricing. When the attractiveness of the weak market is high and advertising cost is sufficiently high, price discrimination reduces firms' profits, and the reverse happens when advertising cost is low. Esteves and Cerqueira [13] show that firms send targeted advertisements with different prices when they recognize customers with different purchasing histories. In comparison to no discrimination, firms reduce their advertising efforts, behavior-based price discrimination increase industry profits and reduces consumer welfare. Esteves and Resende [14] analyze how firms choose advertising and pricing modes, they conclude that, compared with the scenario that firm choose massive advertising and uniform pricing, when firm chooses targeted advertising and personalized pricing, he can earn more profit, but consumer surplus will decrease even social welfare may increase. Pepall and Richards [30] show that value-enhancing advertising is beneficial to consumers but harmful to firms' profits compared with no targeted advertising in the standard uniform-pricing case. When firms adopt targeted advertising and price discrimination simultaneously, profitability

his same as the scenario when they can only use price discriminatation. Several literatures strudy the topic in the two-sided market. Busse and Rysman [5] examize how purchasers buy the display advertising position in Yellow Pages directiories when the Yellow Pages adopt second-degree price discrimination, they find that purchasers of the largest ads pay less per ad size than purchasers of small ads in more-competitive directories. Kodera [18] describes a spatial model of price discrimination in two-sided media markets and shows that price discrimination is harmful to both consumer's welfare and media platform's profit if consumers dislike advertising, while price discrimination benefits media platform if consumers have a deep aversion to advertising. Lin [20] investigates advertising and price strategy of media platform within a two-sided market game, and show that price discrimination on one side of the media platform can strengthen the incentive to discriminate on the other side. Kodera [18] and Lin [20] focus on the advertising and pricing discrimination strategy of media platform. The above papers analyze the advertising and price discrimination strategies and relative impacts on the competition, but few of them consider the role of advertising, while we study persuasive advertising in this paper.

Indeed, advertising and pricing strategy are the important tools that help firms to gain competition advantages. In the marketing process, firms usually make advertising strategy firstly, and then set pricing strategy. The collection and application of consumer information influence firms' pricing strategy, and then advertising strategy. In contrast to the above researches, we study the competitive firms' pricing strategy when they use consumer information or not. Furthermore, we analyze the firms' persuasive advertising strategy in competition. This paper relies on the two functions of persuasive advertising in VonderFehr and Stevik [37], Belleflamme and Peitz [2], where persuasive advertising could increase consumers' willingness to pay or increase perceived product differences. We separately study the duopolistic firms' pricing strategy with/without using consumer information and persuasive advertising strategy when the persuasive advertising plays the two different functions. Then, we analyze the impact of firms' pricing and advertising strategies on profits, consumer surplus and social welfare. Furthermore, we explore the optimal strategy for data intermediary and the value of consumer information.

In this paper, we firstly focus on how the consumer information influences the competition between the duopolistic firms which follows Taylor and Wagman [35]. Indeed, in real world, some firms could use consumer information for free or with a very limited cost, such as Tencent, JD, Amazon. In the main analysis, we assume firms will use consumer information when they have consumer information. In this paper, we emphasize the role of consumer information in a duopolistic competition like Taylor and Wagman [35] and exclude consumer's arbitrage behavior.

3 The model

As in Belleflamme and Peitz [2], persuasive advertising "may affect consumers' preferences by enhancing the value of the product in the eyes of the consumer", or "may finally lead consumers to attach more importance to those differences that already exist between the two products". We consider two different functions of

persuasive advertising on consumers: (1) Enhancing consumer's willingness to pay, (2) Increasing perceived product differences. (Belleflamme and Peitz [2]). We analyze three scenarios (1) Neither of them use consumer information; (2) Only one firm uses consumer information; (3) Both of them use consumer information. We propose a two-stage game model to analyze firms' pricing and persuasive advertising strategies, and further explain whether the adoption of consumer information influences advertising and pricing decisions or not.

Consider in a Hotelling model two firms who sell horizontal differentiated goods with price $p_i(i = 1, 2)$, the two firms locate at the endpoints of the unit line [0,1] separately. That is, firm 1 locates at 0 and firm 2 locates at 1. Both firms' products are produced at a constant marginal cost c. There is a unit of mass consumers with same valuation v of the products distributed on the unit line uniformly, the demand of the individual consumers is normalize to 1. In order to ensure the market is always covered, we assume that the v is sufficiently large that all consumers always buy one unit of product from either of the two firms. Without advertising and personalized price, the utilities of the consumer locates at x buys from firm 1 and firm 2 are: $v - tx - p_1$ and $v - t(1 - x) - p_2$. The parameter t represents unit transportation cost which is a measure of degree of consumer heterogeneity preferences over the two firms' product. Firms may launch persuasive advertising to influence consumers' purchase decisions. Firm $i(i = 1, 2)$ chooses its advertising intensity ϕ_i, and the advertising cost is $\frac{a\phi_i^2}{2}$, where a depicts the unit advertising cost. When consumers receive the advertising, their willingness to pay of the advertised product increases with $\beta\phi_i$ in Sect. 3.1, or the perceived product differentiation is enhanced with $\beta\phi_i$, where β is a positive parameter which measures the level of consumers' valuation in Sect. 3.1 or the degree of perceived product differentiation (i.e. the degree of competition alleviation between the two firms) in Sect. 3.2 increased by the advertising. Furthermore, we assume, that in Sect. 3.1, β^2 depicts the positive effects of advertising which increase the consumer's valuation, and in Sect. 3.2, β^2 shows the positive effect that advertising moderates competition.

The timing sequence of our game is: firms set persuasive advertising intensities firstly, then set their pricing strategies based on consumer information. Finally, consumers make their purchase decisions.

3.1 Advertising increases willingness to pay

As Kodera [18], Belleflamme and Peitz [2], in order to satisfy the second-order conditions of the equilibrium, the following assumption is needed, and the proof is given in Appendix. We define that the positive effect of advertising is that advertising increases consumers' willingness to pay.

Assumption 1 $\beta^2 < 2at$.

Even when the positive effect of advertising is low, advertising increases the sales but the cost simultaneously. When advertising cost becomes negligible, firms would invest in advertising as much as possible, that is, $\phi_i = \infty$. In other words, in this case

firms need not to balance the benefit of advertising and the cost, meaning there does not exist optimal advertising intensity. To ensure the optimal advertising intensity and related study significance, we should impose the condition of Assumption 1.

3.1.1 Equilibrium when neither firms use consumer information

Our analysis process is similar to Belleflamme and Peitz [2]. If consumer's preference is unknown then no firm employs a personalized pricing policy, that is, two firms charge a standard uniform price p_i. Consumer utility is $v+\beta\phi_1-tx-p_1$ if consumer locates at x purchases from firm 1, while consumer utility is $v+\beta\phi_2-t(1-x)-p_2$ if consumer purchases form firm 2. The marginal indifferent consumer locates at $\bar{x}$ is given as $\bar{x}=1/2+p_2-p_1/2t+\beta\phi_1-\beta\phi_2/2t$. Therefore, in the price competition, the expected profit of firm i are given as Eq. (1).

$$\pi_1=(p_1-c)\left(\frac{1}{2}+\frac{p_2-p_1}{2t}+\frac{\beta\phi_1-\beta\phi_2}{2t}\right),\ \pi_2=(p_2-c)\left(\frac{1}{2}+\frac{p_1-p_2}{2t}+\frac{\beta\phi_2-\beta\phi_1}{2t}\right). \tag{1}$$

The first-order conditions yield the firms' equilibrium prices shown as Eq. (2).

$$p_i=\frac{1}{2}(c+t+p_j+\beta\phi_i-\beta\phi_j) \tag{2}$$

So, the firms' profit functions are

$$\pi_1(\phi_1,\phi_2)=\frac{(3t+\beta\phi_1-\beta\phi_2)^2}{18t},\ \pi_2(\phi_2,\phi_1)=\frac{(3t+\beta\phi_2-\beta\phi_1)^2}{18t}. \tag{3}$$

In the advertising competition, the profit-maximization problem of the firms are:

$$\begin{aligned}\max_{\phi_1}\Pi_1&=\pi_1(\phi_1,\phi_2)-\frac{a}{2}\phi_1^2=\frac{(3t+\beta\phi_1-\beta\phi_2)^2}{18t}-\frac{a}{2}\phi_1^2,\\ \max_{\phi_2}\Pi_2&=\pi_2(\phi_2,\phi_1)-\frac{a}{2}\phi_2^2=\frac{(3t+\beta\phi_2-\beta\phi_1)^2}{18t}-\frac{a}{2}\phi_2^2\end{aligned} \tag{4}$$

Solving the two firms' profit-maximization problems simultaneously, we could obtain the equilibrium results, including ϕ_i^{VZ}, Π_i^{VZ},CS^{VZ} and SW^{VZ}, shown in the first column of Table 1, where the subscript VZ indicates that advertising increases consumer's willingness to pay and no firm uses consumer information.

3.1.2 Equilibrium when only one firm uses consumer information

Since the two firms are symmetric, without loss of generality, we assume that firm 1 doesn't use consumer information, and firm 2 know the potential consumer information and they compete for every individual consumer locates at x. In this scenario, firm 1 can only employ standard uniform price p_1 but firm 2 can employ personalized price $p_2(x)$. Consumer's utility when he buys from firm 1 is $v+\beta\phi_1-tx-p_1$, and his utility buying from firm 2 is

Table 1 Equilibrium results when advertising increases willingness to pay

	VZ	*VO*	*VT*
ϕ_1	$\frac{\beta}{3a}$	$\frac{2at\beta-\beta^3}{8a^2t-3a\beta^2}$	$\frac{\beta}{2a}$
ϕ_2	$\frac{\beta}{3a}$	$\frac{3at\beta-\beta^3}{8a^2t-3a\beta^2}$	$\frac{\beta}{2a}$
Π_1	$\frac{t}{2}-\frac{\beta^2}{18a}$	$\frac{(2at-\beta^2)^2(4at-\beta^2)}{2a(8at-3\beta^2)^2}$	$\frac{t}{4}-\frac{\beta^2}{8a}$
Π_2	$\frac{t}{2}-\frac{\beta^2}{18a}$	$\frac{(3at-\beta^2)^2(8at-\beta^2)}{2a(8at-3\beta^2)^2}$	$\frac{t}{4}-\frac{\beta^2}{8a}$
CS	$v-c-\frac{5t}{4}+\frac{\beta^2}{3a}$	$v-c-t+\frac{5at\beta^2-2\beta^4}{2a(8at-3\beta^2)}$	$v-c-\frac{3t}{4}+\frac{\beta^2}{2a}$
SW	$v-c-\frac{t}{4}+\frac{2\beta^2}{9a}$	$v-c-t+\frac{88a^3t^3-37a^2t^2\beta^2-9at\beta^4+4\beta^6}{2a(8at-3\beta^2)^2}$	$v-c-\frac{t}{4}+\frac{\beta^2}{4a}$

$v+\beta\phi_2-t(1-x)-p_2(x)$. Firm 2 captures the consumer locates at x if and only if $p_2(x)<p_1+t(2x-1)+\beta\phi_2-\beta\phi_1$. The optimal price of firm 2 at position x is given by

$$p_2(x)=\begin{cases} p_1+t(2x-1)+\beta\phi_2-\beta\phi_1 & p_1+t(2x-1)+\beta\phi_2-\beta\phi_1\geq c,\\ c & p_1+t(2x-1)+\beta\phi_2-\beta\phi_1<c. \end{cases} \tag{5}$$

The marginal indifferent consumer x is given by

$$\bar{x}=\frac{1}{2}+\frac{c-p_1+\beta\phi_1-\beta\phi_2}{2t}. \tag{6}$$

So in the price competition, the profit functions are:

$$\begin{aligned} \pi_1(\phi_1,\phi_2)&=(p_1-c)\left(\frac{1}{2}+\frac{c-p_1+\beta\phi_1-\beta\phi_2}{2t}\right)=\frac{(t+\beta\phi_1-\beta\phi_2)^2}{8t},\\ \pi_2(\phi_2,\phi_1)&=\int_{\frac{1}{2}+\frac{c-p_1+\beta\phi_1-\beta\phi_2}{2t}}^{1}(p_1+t(2x-1)+\beta\phi_2-\beta\phi_1-c)dx\\ &=\frac{(\beta\phi_2-\beta\phi_1)^2}{16t}+\frac{3(\beta\phi_2-\beta\phi_1)}{8}+\frac{9t}{16}. \end{aligned} \tag{7}$$

In the advertising competition, the profit-maximization problems are:

$$\begin{aligned} \max_{\phi_1}\Pi_1&=\pi_1(\phi_1,\phi_2)-\frac{a}{2}\phi_1^2=\frac{(t+\beta\phi_1-\beta\phi_2)^2}{8t}-\frac{a}{2}\phi_1^2,\\ \max_{\phi_2}\Pi_2&=\pi_2(\phi_2,\phi_1)-\frac{a}{2}\phi_2^2=\frac{(\beta\phi_2-\beta\phi_1)^2}{16t}+\frac{3(\beta\phi_2-\beta\phi_1)}{8}+\frac{9t}{16}-\frac{a}{2}\phi_2^2. \end{aligned} \tag{8}$$

Solving the two firms' profit-maximization problems simultaneously, we obtain the equilibrium results, including advertising level (ϕ_i^{VO}), firm's profit (Π_i^{VO}), consumers' surplus (CS^{VO}) and (SW^{VO}), which are shown in the second column of Table 1, the

subscript *VO* indicates that advertising increases consumer's willingness to pay and only one firm use consumer information.

3.1.3 Equilibrium with two firms use consumer information

Based on the consumer information, the firms employ personalized pricing policies. When both firms use consumer information to personalized pricing, we assume firm i charges the consumer locates at x a personalized price $p_i(x)$. When consumer locates at x buys from firm 1, his utility is $v + \beta\phi_1 - tx - p_1(x)$, and when he buys from firm 2, his utility is $v + \beta\phi_2 - t(1-x) - p_2(x)$. When personalized pricing strategy is employed by both firms, the lowest price of firm i is c for each consumer. Firm 2 captures the consumer locates at x if and only if

$$p_2(x) < c + t(2x-1) + \beta\phi_2 - \beta\phi_1. \tag{9}$$

The optimal price of firm 2 at point x is given by

$$p_2(x) = \begin{cases} c + t(2x-1) + \beta\phi_2 - \beta\phi_1 & t(2x-1) + \beta\phi_2 - \beta\phi_1 \geq 0, \\ c & t(2x-1) + \beta\phi_2 - \beta\phi_1 < 0. \end{cases} \tag{10}$$

Analogously, the optimal price of firm 1 at point x is given by

$$p_1(x) = \begin{cases} c + t(1-2x) + \beta\phi_1 - \beta\phi_2 & t(1-2x) + \beta\phi_1 - \beta\phi_2 \geq 0, \\ c & t(1-2x) + \beta\phi_1 - \beta\phi_2 < 0. \end{cases} \tag{11}$$

The marginal indifferent consumer x is given by

$$\bar{x} = \frac{1}{2} + \frac{\beta\phi_1 - \beta\phi_2}{2t}. \tag{12}$$

So in the price competition, the profit functions are:

$$\begin{aligned} \pi_1(\phi_1, \phi_2) &= \int_0^{\frac{1}{2} + \frac{\beta\phi_1 - \beta\phi_2}{2t}} [t(1-2x) + \beta\phi_1 - \beta\phi_2]dx = \frac{t}{4} + \frac{(\beta\phi_1 - \beta\phi_2)^2}{4t} + \frac{\beta\phi_1 - \beta\phi_2}{2}, \\ \pi_2(\phi_2, \phi_1) &= \int_{\frac{1}{2} + \frac{\beta\phi_1 - \beta\phi_2}{2t}}^{1} [t(2x-1) + \beta\phi_2 - \beta\phi_1]dx = \frac{t}{4} + \frac{(\beta\phi_2 - \beta\phi_1)^2}{4t} + \frac{\beta\phi_2 - \beta\phi_1}{2}. \end{aligned} \tag{13}$$

In the advertising competition, firm i has the following profit-maximization problem:

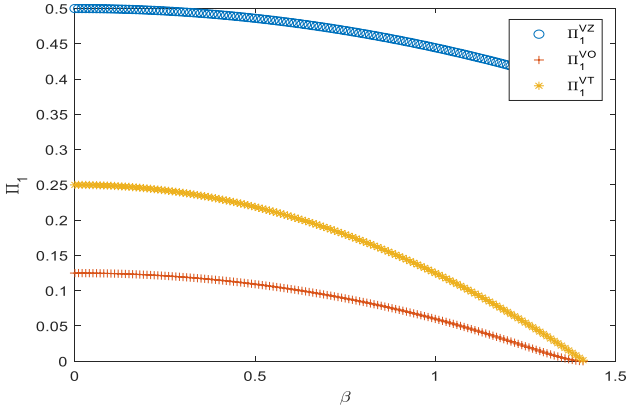

Fig. 1 $\Pi_1^{VO} < \Pi_1^{VT} < \Pi_1^{VZ}$

$$\begin{aligned}
\max_{\phi_1} \Pi_1 &= \pi_1(\phi_1, \phi_2) - \frac{a}{2}\phi_1^2 = \frac{t}{4} + \frac{(\beta\phi_1 - \beta\phi_2)^2}{4t} + \frac{\beta\phi_1 - \beta\phi_2}{2} - \frac{a}{2}\phi_1^2, \\
\max_{\phi_2} \Pi_2 &= \pi_2(\phi_2, \phi_1) - \frac{a}{2}\phi_2^2 = \frac{t}{4} + \frac{(\beta\phi_2 - \beta\phi_1)^2}{4t} + \frac{\beta\phi_2 - \beta\phi_1}{2} - \frac{a}{2}\phi_2^2.
\end{aligned} \tag{14}$$

Solving the two firms' problems simultaneously, we obtain the equilibrium results, including ϕ_i^{VT}, Π_i^{VT},CS^{VT} and SW^{VT}, as shown in the third column of Table 1, where the subscript VT indicates that advertising increases consumer's willingness to pay and both firms have consumer information. All the derivations of the results listed in Table 1 are given in Appendix.

3.1.4 Comparison and analysis

In this section, we explain how the firms' profits, consumer surplus and social welfare are affected by the advertising competition within three different scenarios when persuasive advertising increases consumer's willingness to pay. The comparison results (see Table 1) of the advertising levels, firms' profits, consumer surplus and social welfare are shown in Proposition 1 and Proposition 2, respectively.

Proposition 1 *When persuasive advertising increases consumer's willingness to pay, in equilibrium, (a) firm*1*'s advertising level is the lowest under VO, medium under VZ,and the highest under VT, i.e.,* $\phi_1^{VO} < \phi_1^{VZ} < \phi_1^{VT}$; *(b) firm* 2*'s advertising level is the lowest under VZ, medium under VO,and the highest under VT, i.e.,* $\phi_2^{VZ} < \phi_2^{VO} < \phi_2^{VT}$; *(c) firm* 1*'s profit is the lowest under VO, medium under VT, and the highest under VZ, i.e.,*$\Pi_1^{VO} < \Pi_1^{VT} < \Pi_1^{VZ}$; *(d) firm* 2*'s profit is the lowest under VT, medium under VZ, and the highest under VO, i.e.,*$\Pi_2^{VT} < \Pi_2^{VZ} < \Pi_2^{VO}$.

We can illustrate Proposition 1 with numerical simulation, where $\beta \in \left(0, \sqrt{2}\right)$, $a = 1$ and $t = 1$. The comparison simulation results are shown as Figs. 1 and 2.

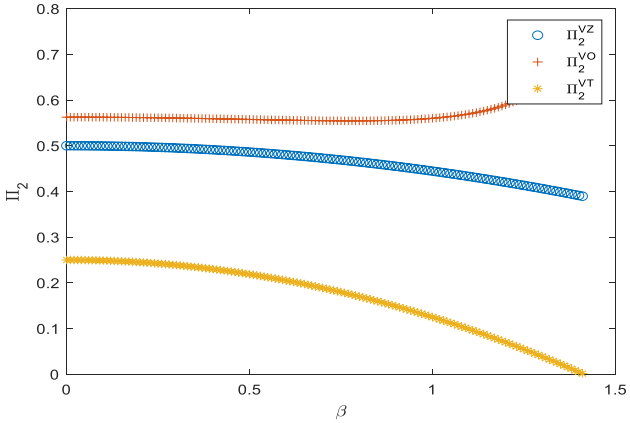

Fig. 2 $\Pi_2^{VT} < \Pi_2^{VZ} < \Pi_2^{VO}$

Proposition 1 shows that when persuasive advertising increases consumer's willingness to pay, in equilibrium, the advertising level is the highest when both firms use consumer information, but the lowest if the rival use consumer information exclusively. The firm using consumer information exclusively can get the highest profit; in contrast, his rival gets the lowest profit. When both firms use consumer information, both firms' profits are lower than the scenario both firms do not use consumer information.

In order to earn more profits, both firms will increase advertising intensity and adopt personalized pricing strategy base on consumer information, and the market competition is the fiercest at this time. In the advertising competition stage, both firms increase the advertising intensify to keep respective demand, resulting in the increment of advertising cost. On the other hand, they adopt personalized price which may trap into price competition and lead to a lower profit. Consequently, the amount of advertising is the largest, but the profit is the lowest when both firms use consumer information. When both firms do not use consumer information, they set a unified price. Compared with the personalized pricing strategy, the market competition is more relaxed, and the firms are less intensive to increase advertising intensity. At this time, the advertising intensity is the lowest.

Compared with the rival firm, the firm using consumer information exclusively will increase the advertising intensity and employs personalized pricing, meanwhile, he extracts more surplus from consumers, i.e., it takes strategic advantage. We conclude the consumer surplus and social welfare within three different scenarios as follows.

Proposition 2 *When persuasive advertising increases consumer's willingness to pay, in equilibrium,* (*a*) *the consumer surplus is the lowest under VZ, medium under VO, and the highest under VT, i.e.,*$CS^{VZ} < CS^{VO} < CS^{VT}$; (*b*) *when* $\beta^2 < 59 - \sqrt{457}\Big/21at$, *the social welfare level is the lowest under VO, medium*

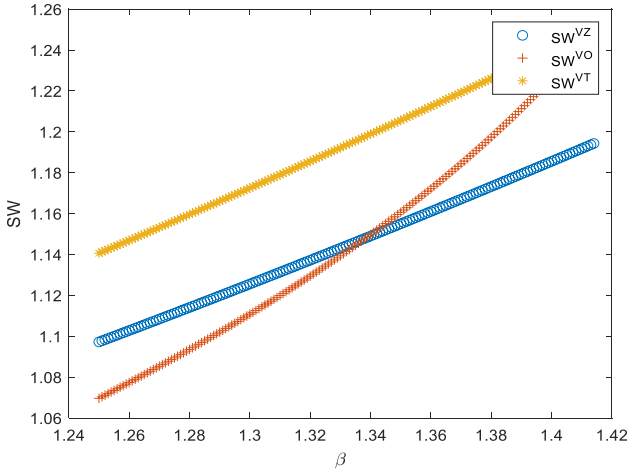

Fig. 3 $SW^{VZ} < SW^{VO} < SW^{VT}$

under VZ, and the highest under VT, i.e., $SW^{VO} < SW^{VZ} < SW^{VT}$; *when* $59 - \sqrt{457}\Big/21at < \beta^2 < 2at$, *the social welfare level is the lowest under VZ, medium under VO, and the highest under VT, i.e.,*$SW^{VZ} < SW^{VO} < SW^{VT}$.

To comprehend Proposition 2(b, c) more easily, we conduct a numerical simulation with $\beta \in \left(0, \sqrt{2}\right)$, $a = 1$, $t = 1$, $v = 2$, $c = 1$, the comparesive result of social welfare in three different scenarios is given as Fig. 3.

Proposition 2demonstrates that when persuasive advertising increases consumer's willingness to pay, the consumer surplus increases with the number of firms who use consumer information. The result indicates more consumer information disclosure and utilization will help to the improvement of consumer surplus. The reason may be that when more firms use consumer information to set personalized price, price competition intensifies. The fiercer of the price competition, the less consumer surplus the firms can capture from consumers. On the other hand, firms increase their advertising intensity to get more demand which increases advertising cost, consumer's perceived utility of the advertised product will increase, and thus consumer surplus increases (Fig. 4).

Proposition 2 demonstrates that when both firms use consumer information, the social welfare is the highest. The reason is that the increment of consumer surplus caused by two firms using consumer information is greater than the decrease of profits caused by the intense market competition. If the positive effect of advertising is smaller, the social welfare when only one firm uses consumer information is smaller than that when neither firm use consumer information. Because if the positive effect of advertising is smaller, the function of advertising which increases consumer's willingness to pay is weakened, the gain of consumer surplus caused by the positive effect of advertising is smaller than the loss of the firm profits caused by the fierce competition when one firm uses consumer information exclusively, so the social

Firm 2 \ Firm 1	Use	Not use
Use	Π_1^{VT}, Π_2^{VT}	Π_2^{VO}, Π_1^{VO}
Not use	Π_1^{VO}, Π_2^{VO}	Π_1^{VZ}, Π_2^{VZ}

Fig. 4 The relationships that firms use or not use consumer information

welfare increases. On the contrary, the social welfare when one firm uses consumer information is larger than that when neither firm uses consumer information.

Based on Proposition 1 and Proposition 2, we can derive:

Corollary 1 *When both firms adopt persuasive advertising simultaneously, the two firms will trap into "prisoner's dilemma" when they choose whether they need to use consumer information or not. The uniqueness of pure Nash equilibrium is both firms use consumer information.*

Corollary 1 indicates that, when persuasive advertising increases consumers' willingness to pay in competition, no matter whether the firm's competitor use consumer information to personalized price or not, the optimal strategy of the firm is use consumer information to personalize price.

3.2 Advertising Increases Perceived Product Differences

3.2.1 Equilibrium when neither firms use consumer information

Our analysis process is similar to Belleflamme and Peitz[2]. Consider no firm uses consumer information, both firms will employ a uniform pricing policy. Assume firm i charges a standard uniform price p_i. If consumer purchases from firm 1, his utility is $v-(t+\beta\phi_1+\beta\phi_2)x-p_1$, if consumer purchases from firm 2, his utility is $v-(t+\beta\phi_1+\beta\phi_2)(1-x)-p_2$. The marginal indifferent consumer $\bar{x}$ is given by $\bar{x}=1/2+p_2-p_1/2(t+\beta\phi_1+\beta\phi_2)$. Therefore, in the price competition, firms' profits are:

$$\begin{aligned}\pi_1&=(p_1-c)\left(\frac{1}{2}+\frac{p_2-p_1}{2(t+\beta\phi_1+\beta\phi_2)}\right),\\ \pi_2&=(p_2-c)\left(\frac{1}{2}+\frac{p_1-p_2}{2(t+\beta\phi_1+\beta\phi_2)}\right).\end{aligned}\tag{15}$$

The first-order conditions yield firms' equilibrium prices:

$$p_1=p_2=c+t+\beta\phi_1+\beta\phi_2.\tag{16}$$

So in the price competition, the profit function is:

Table 2 Equilibrium results when advertising increases perceived product differences

et	*TZ*	*TO*	*TT*
ϕ_1	$\frac{\beta}{2a}$	$\frac{\beta}{8a}$	$\frac{\beta}{4a}$
ϕ_2	$\frac{\beta}{2a}$	$\frac{9\beta}{16a}$	$\frac{\beta}{4a}$
Π_1	$\frac{t}{2}+\frac{3\beta^2}{8a}$	$\frac{t}{8}+\frac{5\beta^2}{64a}$	$\frac{t}{4}+\frac{3\beta^2}{32a}$
Π_2	$\frac{t}{2}+\frac{3\beta^2}{8a}$	$\frac{9t}{16}+\frac{117\beta^2}{512a}$	$\frac{t}{4}+\frac{3\beta^2}{32a}$
CS	$v-c-\frac{5t}{4}-\frac{5\beta^2}{4a}$	$v-c-t-\frac{11\beta^2}{16a}$	$v-c-\frac{3t}{4}-\frac{3\beta^2}{8a}$
SW	$v-c-\frac{t}{4}-\frac{\beta^2}{2a}$	$v-c-\frac{5t}{16}-\frac{195\beta^2}{512a}$	$v-c-\frac{t}{4}-\frac{3\beta^2}{16a}$

$$\pi_1(\phi_1,\phi_2)=\pi_2(\phi_2,\phi_1)=\frac{1}{2}(t+\beta\phi_1+\beta\phi_2). \tag{17}$$

Therefore, in the advertising competition, firms' profit-maximization problems are:

$$\begin{aligned}\max_{\phi_1}\Pi_1&=\pi_1(\phi_1,\phi_2)-\frac{a}{2}\phi_1^2=\frac{t+\beta\phi_1+\beta\phi_2}{2}-\frac{a}{2}\phi_1^2,\\ \max_{\phi_2}\Pi_2&=\pi_2(\phi_2,\phi_1)-\frac{a}{2}\phi_2^2=\frac{t+\beta\phi_1+\beta\phi_2}{2}-\frac{a}{2}\phi_2^2.\end{aligned} \tag{18}$$

Solving the profit-maximization problems simultaneously, we obtain the equilibrium results, including ϕ_i^{TZ},Π_i^{TZ},CS^{TZ} and SW^{TZ}, which are shown in the first column of Table 2, where the subscript *TZ* indicates that advertising increases perceived product differences and no firm has consumer information.

3.2.2 Equilibrium with one firm uses consumer information

Without loss of generality, we assume that firm 1 does not use consumer information, firm 2 uses consumer information which can help him to set personalized price, two firms compete for each consumer locates at x. In this scenario, firm 1 can only employ standard uniform price p_1, but firm 2 can employ personalized price $p_2(x)$. If consumer purchases from firm 1, his utility is $v-(t+\beta\phi_1+\beta\phi_2)x-p_1$. If consumers purchases from firm 2, his utility is $v-(t+\beta\phi_1+\beta\phi_2)(1-x)-p_2(x)$. Firm 2 captures consumer locates at x if and only if $p_2(x)\le p_1+(t+\beta\phi_1+\beta\phi_2)(2x-1)$. The optimal price of firm 2 at x is given as

$$p_2(x)=\begin{cases}p_1+(t+\beta\phi_1+\beta\phi_2)(2x-1) & p_1+(t+\beta\phi_1+\beta\phi_2)(2x-1)\ge c,\\ c & p_1+(t+\beta\phi_1+\beta\phi_2)(2x-1)<c.\end{cases} \tag{19}$$

The marginal indifferent consumer locates at $\bar{x}$ is given by

$$\bar{x}=\frac{1}{2}+\frac{c-p_1}{2(t+\beta\phi_1+\beta\phi_2)}. \tag{20}$$

So in the price competition, the two firms' profits are:

$$\pi_1(\phi_1,\phi_2)=(p_1-c)\left(\frac{1}{2}+\frac{c-p_1}{2(t+\beta\phi_1+\beta\phi_2)}\right)=\frac{t+\beta\phi_1+\beta\phi_2}{8},$$
$$\pi_2(\phi_2,\phi_1)=\int\limits_{\frac{1}{2}+\frac{c-p_1}{2(t+\beta\phi_1+\beta\phi_2)}}^{1}(p_1+(t+\beta\phi_1+\beta\phi_2)(2x-1)-c)dx=\frac{9(t+\beta\phi_1+\beta\phi_2)}{16}. \tag{21}$$

Therefore, in the advertising competition, firm i has the following profit maximization problem:

$$\begin{aligned}\max_{\phi_1}\Pi_1&=\pi_1(\phi_1,\phi_2)-\frac{a}{2}\phi_1^2=\frac{t+\beta\phi_1+\beta\phi_2}{8}-\frac{a}{2}\phi_1^2,\\ \max_{\phi_2}\Pi_2&=\pi_2(\phi_2,\phi_1)-\frac{a}{2}\phi_2^2=\frac{9(t+\beta\phi_1+\beta\phi_2)}{16}-\frac{a}{2}\phi_2^2.\end{aligned} \tag{22}$$

Solving the two profit-maximization problems simultaneously, we obtain the equilibrium results, including ϕ_i^{TO}, Π_i^{TO}, CS^{TO} and SW^{TO}, as shown in the second column of Table 2 where the subscript TO indicates that advertising increases perceived product differences and only one firm uses consumer information.

3.2.3 Equilibrium when two firms uses consumer information

If firms uses consumer information, and they employ personalized pricing policy. Firm i charges consumer located at x with personalized price $p_i(x)$. If consumer purchases from firm 1, his utility is $v-(t+\beta\phi_1+\beta\phi_2)x-p_1(x)$, if consumer purchases from firm 2, his utility is $v-(t+\beta\phi_1+\beta\phi_2)(1-x)-p_2(x)$. Analogous to Chapter 3.1.3, where personalized pricing strategy are employed by both firms, firm 2 may capture the consumer at x if and only if $p_2(x)\le c+(t+\beta\phi_1+\beta\phi_2)(2x-1)$. The optimal price of firm 2 at point x is given as

$$p_2(x)=\begin{cases}c+(t+\beta\phi_1+\beta\phi_2)(2x-1) & (t+\beta\phi_1+\beta\phi_2)(2x-1)\ge 0,\\ c & (t+\beta\phi_1+\beta\phi_2)(2x-1)<0.\end{cases} \tag{23}$$

Similarly, the optimal price of firm 1 at x is given as

$$p_1(x)=\begin{cases}c+(t+\beta\phi_1+\beta\phi_2)(1-2x) & (t+\beta\phi_1+\beta\phi_2)(1-2x)\ge 0,\\ c & (t+\beta\phi_1+\beta\phi_2)(1-2x)<0.\end{cases} \tag{24}$$

The marginal indifferent consumer locates at $\bar{x}$ is given by $\bar{x}=\frac{1}{2}$. So in the price competition, the firms' profit functions are:

$$\begin{aligned}\pi_1(\phi_1,\phi_2) &= \int_0^{\frac{1}{2}} (t+\beta\phi_1+\beta\phi_2)(1-2x)dx = \frac{1}{4}(t+\beta\phi_1+\beta\phi_2),\\ \pi_2(\phi_2,\phi_1) &= \int_{\frac{1}{2}}^{1} (t+\beta\phi_1+\beta\phi_2)(2x-1)dx = \frac{1}{4}(t+\beta\phi_1+\beta\phi_2).\end{aligned} \tag{25}$$

Therefore, in the advertising competition, firm i has the following profit-maximization problem:

$$\begin{aligned}\max_{\phi_1}\Pi_1 &= \pi_1(\phi_1,\phi_2)-\frac{a}{2}\phi_1^2 = \frac{t+\beta\phi_1+\beta\phi_2}{4}-\frac{a}{2}\phi_1^2,\\ \max_{\phi_2}\Pi_2 &= \pi_2(\phi_2,\phi_1)-\frac{a}{2}\phi_2^2 = \frac{t+\beta\phi_1+\beta\phi_2}{4t}-\frac{a}{2}\phi_2^2.\end{aligned} \tag{26}$$

Solving the two equations simultaneously, we obtain the symmetric equilibrium results, including ϕ_i^{TT}, Π_i^{TT}, CS^{TT} and SW^{TT}, shown in the third column of Table 2. The subscript TT indicates that persuasive advertising increases perceived product differences and both firms use consumer information.

3.2.4 Comparison and analysis

Similar with Chapter 3.1.4, we investigate how the firms' profits, consumer surplus and social welfare are affected by advertising competition when the firms use consumer information or not. In this Section, the persuasive advertising is considered to increase consumer's perceived product differences. Comparing the equilibrium results among the three scenarios (see Table 2), the comparison results of firms' advertising levels, profits, consumer surplus and social welfare are given in proposition 3 and proposition 4, respectively.

Proposition 3 *When persuasive advertising increases consumer's perceived product differences, in equilibrium, (a) firm* 1*'s advertising intensity is the lowest under TO, medium under TT, and the highest under TZ,i.e.,* $\phi_1^{TO}<\phi_1^{TT}<\phi_1^{TZ}$; *(b) firm* 2*'s advertising intensity is the lowest under TT, medium under TZ, and the highest under TO, i.e.,* $\phi_2^{TT}<\phi_2^{TZ}<\phi_2^{TO}$; *(c) firm* 1*'s profit is the lowest under TO, medium under TT, and the highest under TZ, i.e.,*$\Pi_1^{TO}<\Pi_1^{TT}<\Pi_1^{TZ}$; *(d) When* $\beta^2<\frac{32}{75}at$, *firm* 2*'s profit is the lowest under TT, medium under TZ, and the highest under TO,i.e.,*$\Pi_2^{TT}<\Pi_2^{TZ}<\Pi_2^{TO}$. *When* $\beta^2>\frac{32}{75}at$, *firm* 2*'s profit is the lowest under TT, medium under TO, and the highest under TZ, i.e.,*$\Pi_2^{TT}<\Pi_2^{TO}<\Pi_2^{TZ}$.

We can illustrate Proposition 3with numerical simulation, where $\beta\in\left(0,\sqrt{2}\right)$, $a=1$ and $t=1$. The comparison simulation results of firms' equilibrium profits are shown as Figs. 5 and 6.

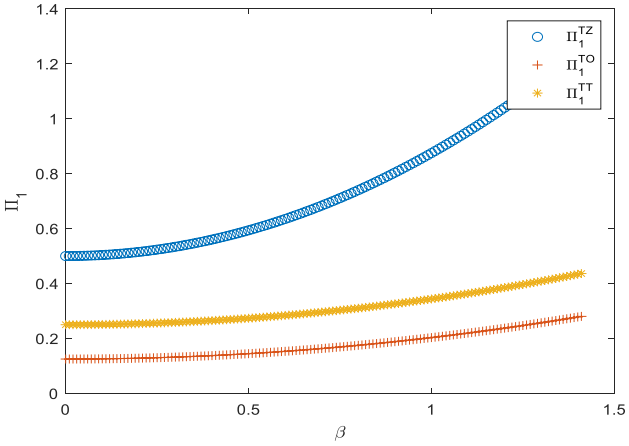

Fig. 5 The comparision results of firm 1's profit

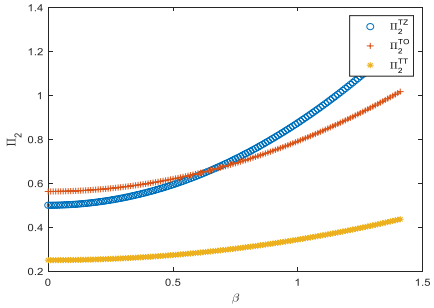

Fig. 6 The comparision results of firm 2's profit

Proposition 3shows that when persuasive advertising increases consumer's perceived product differences, the advertising intensity of firm 2 who use consumer information exclusively is the highest and his rival firm 1's advertising intensity is the lowest in the three different scenarios. The two firms' advertising intensities and profits when both firms use consumer information are lower than these when neither firm uses consumer information. Meanwhile, if the positive effects of advertising is smaller, the profit of firm 2 when he uses consumer information exclusively is higher than its profit in the scenario that neither firm uses consumer information. We can derive that when advertising increases perceived product differences, it alleviates competition between firms. Compared with firm 1, firm 2 will employ personalized price to get more demand and increase advertising intensity to gain competitive advantage when only he uses consumer information, and he chooses the highest advertising intensity.

Similar with Proposition 1, when both firms use consumer information, they employ personalized prices. This competitive strategy intensifies market competition

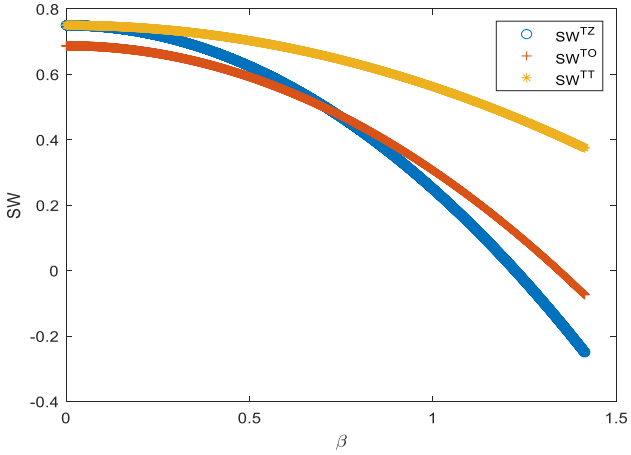

Fig. 7 The comparision results of social welfare

and decreases both firms' profits, so that both firms' equilibrium profits are the lowest among the three scenarios. If the positive effects of advertising is smaller, the function of advertising to alleviate competition is weakened. When only firm 2 uses consumer information exclusively in the market, the increment of profit caused by employing personalized price is higher than the increment of profit caused by the positive effect of advertising which alleviates price competition, firm 2's profit when only firm 2 uses consumer information exclusively is higher than that when neither firms use consumer information. If the positive effect of advertising is larger, the function of advertising to alleviate competition is strengthened, and similarly, firm 2's profit when only firm 2 uses consumer information exclusively is lower than that when neither firms use consumer information.

Proposition 4 *When persuasive advertising increases consumer's perceived product differences, in equilibrium, (a) the consumer surplus is the lowest under TZ, medium under TO, and the highest under TT, i.e.,*$CS^{TZ} < CS^{TO} < CS^{TT}$; *(b) when* $\beta^2 < 32/61at$, *the social welfare level is the lowest under TO, medium under TZ, and the highest under TT, i.e.,*$SW^{TO} < SW^{TZ} < SW^{TT}$; *when* $\beta^2 < 32/61at$, *the social welfare level is the lowest under TZ, medium under TO, and the highest under TT, i.e.,* $SW^{TZ} < SW^{TO} < SW^{TT}$.

We can show Proposition 4 with numerical simulation, where $\beta \in \left(0, \sqrt{2}\right)$, $a = 1$ and $t = 1$. The comparison simulation results of social welfare is shown as Fig. 7.

Similar with Proposition 2, Proposition 4 demonstrates that when persuasive advertising increases consumer's perceived product difference, the consumer surplus increases with the number of firms who use consumer information.

Proposition 4 also shows that the social welfare is highest when both firms use consumer information. When only firm 2 uses consumer information, he will

increase his advertising intensity which also increases consumer' perceived utility, thus, the consumer surplus increases. Meanwhile, he will adopt personalized price, which leads to fierce market competition, consequently, his profit reduces. If the positive effect of advertising is smaller or the function of advertising to alleviate competition is weakened, the increment of consumer surplus is still less than the decrease of firms' profits, the social welfare when only firm 2 uses consumer information is lower than the social welfare when neither firms use consumer information. Similarly, we can draw the opposite conclusion that the social welfare when only firm 2 uses consumer information is higher than the social welfare when neither firms uses consumer information.

Based on Proposition 3 and Proposition 4, we can derive:

Corollary 2 *When persuasive advertising increases consumers' perceived product differences, it migrates competition. If* $\beta^2 < 32/61at$, *a unique pure strategy Nash equilibrium exists, that is, both firms should use consumer information and trap into "Prisoner's dilemma". If* $\beta^2 < 32/61at$, *there are two pure strategy Nash equilibrium solutions, that is, both firms use consumer information or neither firms use consumer's information.*

The product diffrentation includes subjective product differentiation and objective product differentiation. Objective product differentiation is the differentiation of quality, performance, texture and so on, which is objective existence. Subjective product differentiation is caused by consumers' subjective feeling of the products, such as the differentiation between a famous brand handbag and a common handbag which is the same of objective characteristics. We define the positive effect of persuasive advertising as persuasive advertising enhancing consumers' perceived product differentation. Corollary 2 shows that, when persuasive advertising enhances consumers' perceived product differentation, it will migrate competition, but firms may choose to use consumer information to set discrimination price depending on the positive effect of persuasive advertising. If the firm's competitior use consumer information, the optimal strategy of the firm is to use consumer information. It is common in practice, after consumers browse the product without any purchase decisions, they will receive advertising from the retailers which enhances consumers' perceived product differentation and then increases consumers' willingness to pay. But if the positive effect of persuasive advertising is great, the optimal strategy of the firm is to choose the same strategy as her competitior. That is, if the firm's competitor doesn't use consumer information, the firm should not use consumer information, because it will not suffer from advertising efficiency.

4 Extension: the value of consumer information

Like Braulin and Valletti [6] and Montes et al. [25], we further analyze the effect of the value of consumer information on the data intermediary's marketing strategy. Following the analytical clue, we firstly study when persuasive advertising

increases consumers' willingness to pay. Assume the data intermediary owns the data of consumer information, he gets the bargaining power dominantly and can choose different marketing strategies, such as selling consumer information to only one or both firms.

When the data intermediary is willing to sell information to firm 2 exclusively, the maximum price I_1 that data intermediary can charge is the profit gap between firm 1 and firm 2 when only firm 2 get the consumer's information from the data intermediary. Hence, we can have

$$I_1 = \Pi_2^{VO} - \Pi_1^{VO} = \frac{(3at-\beta^2)^2(8at-\beta^2)}{2a(8at-3\beta^2)^2} - \frac{(2at-\beta^2)^2(4at-\beta^2)}{2a(8at-3\beta^2)^2} = \frac{7at^2-2t\beta^2}{16at-6\beta^2}. \tag{27}$$

Similarly, if the data intermediary sells consumer information to both firms then the price I_2 is given by the profit gap between firm i's profit when the consumer information data is sold to both firms and firm i's profit when the consumer's information data is only sold to firm i. Hence, we have

$$\begin{aligned} I_2 &= \Pi_i^{VT} - \Pi_i^{VO} = \frac{t}{4} - \frac{\beta^2}{8a} - \frac{(2at-\beta^2)^2(4at-\beta^2)}{2a(8at-3\beta^2)^2} \\ &= \frac{(2at-\beta^2)(32a^2t^2-24at\beta^2+5\beta^4)}{8a(8at-3\beta^2)^2}. \end{aligned} \tag{28}$$

It can be easily proved $I_1 > 2I_2$. Proposition 5 presents the equilibrium information price and the data intermediary's strategies.

Proposition 5 *When persuasive advertising increases consumer's willingness to pay, the data intermediary prefers to sell consumer information uniquely to one firm, and the optimal information price is* $7at^2 - 2t\beta^2 \big/ 16at - 6\beta^2$.

The data intermediary should choose between two strategies when the firms compete. The first strategy is that consumer information is sold only to one firm. The second strategy is consumer information is sold to both firms. When the data intermediary choose the first strategy, she can earn I_1. Similarly, when the data intermediary choose the second strategy, she charges I_2 from both firms and gets $2I_2$. Hence, when $I_1 > 2I_2$, the optimal choice of data intermediary is the first strategy, and the optimal information price is I_1。

Secondly, when persuasive advertising increases consumer's perceived product differences, similar with the analysis where persuasive advertising increases consumer's willingness to pay, we derive the equilibrium information price and the data intermediary's strategies in Proposition 6.

Proposition 6 *When persuasive advertising increases perceived product differences, the data intermediary prefers to sell consumer information to only one firm, and the optimal information price is* $\frac{7t}{16} + \frac{77\beta^2}{512a}$.

5 Conclusions

The issue of advertising and price competition is a hot research topic in marketing science. Previous studies have demonstrated that persuasive advertising increases willingness to pay or increases perceived product differences. In this paper, we analyze competitive advertising and pricing strategy, considering firms uses consumer information or not under the different advertising functions. Our research show that the firms' profits when both firms use consumer information are lower than the firms' profits in the case that both firms do not use consumer information. The firms' adoption of consumer information provides firms the opportunities to change the performance of its products to better meet the needs of consumers. We conclude that the consumer surplus increasess with the number of firms who use consumer information in the competition, the social welfare is the highest in the case that both firms use consumer information. The equilibrium consumer information price is given in this paper when the source of consumers data comes from data intermediary, and we also find that the optimal strategy for data intermediary is selling information uniquely to one firm.

There are a few limitations and some issues which need to be further studied. Firstly, persuasive advertising may increase consumers' willingness to pay and perceived product differentiation simultaneously, and meanwhile, consumers' willingness to pay may interact with consumers' perceived product differentiation. We fail to consider this case to study how the adoption of consumer information influences the competition, consumer surplus and social welfare. Secondly, we do not consider the impact of consumers' privacy concern, which may be caused by the adoption of their information, on firms' competition. Finally, the amount of consumer information may influence firms' competitive advertising and pricing strategy, how to enhance the advertising accuracy and effectiveness based on consumer information can be a research direction in the future.

Appendix 1

Chapter 3.1.1 The first order condition is

$$\frac{\partial \Pi_1}{\partial \phi_1} = \frac{\beta(3t + \beta\phi_1 - \beta\phi_2)}{9t} - a\phi_1 = 0,\ \frac{\partial \Pi_2}{\partial \phi_2} = \frac{\beta(3t + \beta\phi_2 - \beta\phi_1)}{9t} - a\phi_2 = 0.$$

The second-order condition is $\frac{\partial^2 \Pi_i}{\partial \phi_i^2} = \frac{\beta^2}{9t} - a < 0$ require that $\beta^2 < 9at$.

Solving for the Nash equilibrium, we obtain

$$\phi_1^{VZ} = \phi_2^{VZ} = \frac{\beta}{3a},\ p_1^{VZ} = p_2^{VZ} = c + t,\ \bar{x}^{VZ} = \frac{1}{2},\ \Pi_1^{VZ} = \Pi_2^{VZ} = \frac{t}{2} - \frac{\beta^2}{18a}.$$

Consumer surplus is given by

$$CS^{VZ} = \int_0^{\bar{x}^{VZ}} (v + \beta\phi_1^{VZ} - p_1^{VZ} - tx)dx + \int_{\bar{x}^{VZ}}^1 (v + \beta\phi_2^{VZ} - p_2^{VZ} - t(1-x))dx$$

$$= \int_0^{\frac{1}{2}} (v + \frac{\beta^2}{3a} - c - t - tx)dx + \int_{\frac{1}{2}}^1 (v + \frac{\beta^2}{3a} - c - t - t(1-x))dx = v - c - \frac{5t}{4} + \frac{\beta^2}{3a}.$$

Social welfare is given by

$$SW^{VZ} = CS^{VZ} + \Pi_1^{VZ} + \Pi_2^{VZ} = v - c - \frac{t}{4} + \frac{2\beta^2}{9a}.$$

Chapter 3.1.2 The first order condition is

$$\frac{\partial\Pi_1}{\partial\phi_1} = \frac{\beta(t + \beta\phi_1 - \beta\phi_2)}{4t} - a\phi_1 = 0,\ \frac{\partial\Pi_2}{\partial\phi_2} = \frac{\beta(\beta\phi_2 - \beta\phi_1)}{8t} + \frac{3\beta}{8} - a\phi_2 = 0.$$

The second-order condition is $\frac{\partial^2\Pi_1}{\partial\phi_1^2} = \frac{\beta^2}{4t} - a < 0,\ \frac{\partial^2\Pi_2}{\partial\phi_2^2} = \frac{\beta^2}{8t} - a < 0$ and it requires that $\beta^2 < 4at$.

Solving for the Nash equilibrium, we obtain

$$\phi_1^{VO} = \frac{2at\beta - \beta^3}{8a^2t - 3a\beta^2},\ \phi_2^{VO} = \frac{3at\beta - \beta^3}{8a^2t - 3a\beta^2},\ \bar{x}^{VO} = \frac{2at - \beta^2}{8at - 3\beta^2},$$
$$\Pi_1^{VO} = \frac{(2at - \beta^2)^2(4at - \beta^2)}{2a(8at - 3\beta^2)^2},\ \Pi_2^{VO} = \frac{(3at - \beta^2)^2(8at - \beta^2)}{2a(8at - 3\beta^2)^2}.$$

Consumer surplus is given by

$$CS^{VO} = \int_0^{\bar{x}^{VO}} (v + \beta\phi_1^{VO} - tx - p_1)dx + \int_{\bar{x}^{VO}}^1 (v + \beta\phi_2^{VO} - t(1-x) - p_2(x))dx$$

$$= \int_0^{\bar{x}^{VO}} (v + \beta\phi_1^{VO} - tx - p_1)dx + \int_{\bar{x}^{VO}}^1 (v + \beta\phi_1^{VO} - tx - p_1)dx$$

$$= \int_0^1 (v + \beta\phi_1^{VO} - tx - p_1)dx = v + \beta\phi_1^{VO} - \frac{t}{2} - \left(\frac{t}{2} + c + \frac{\beta\phi_1^{VO} - \beta\phi_2^{VO}}{2}\right)$$

$$= v - c - t + \frac{\beta\phi_1^{VO} + \beta\phi_2^{VO}}{2} = v - c - t + \frac{5at\beta^2 - 2\beta^4}{2a(8at - 3\beta^2)}.$$

$$SW^{VO} = CS^{VO} + \Pi_1^{VO} + \Pi_2^{VO} = v - c - t + \frac{88a^3t^3 - 37a^2t^2\beta^2 - 9at\beta^4 + 4\beta^6}{2a(8at - 3\beta^2)^2}.$$

Chapter 3.1.3 The first order condition is

$$\frac{\partial \Pi_1}{\partial \phi_1} = \frac{\beta(\beta\phi_1 - \beta\phi_2)}{2t} + \frac{\beta}{2} - a\phi_1 = 0, \ \frac{\partial \Pi_2}{\partial \phi_2} = \frac{\beta(\beta\phi_2 - \beta\phi_1)}{2t} + \frac{\beta}{2} - a\phi_2 = 0.$$

The second-order condition is $\frac{\partial^2 \Pi_i}{\partial \phi_i^2} = \frac{\beta^2}{2t} - a < 0$ require that $\beta^2 < 2at$.

Solving for the Nash equilibrium, we obtain

$$\phi_1^{VT} = \phi_2^{VT} = \frac{\beta}{2a}, \ \bar{x}^{VT} = \frac{1}{2}, \ \Pi_1^{VT} = \Pi_2^{VT} = \frac{t}{4} - \frac{\beta^2}{8a}.$$

Consumer surplus is given by

$$\begin{aligned} CS^{VT} &= \int_0^{\bar{x}^{VT}} (v + \beta\phi_1^{VT} - p_1(x) - tx)dx + \int_{\bar{x}^{VT}}^1 (v + \beta\phi_2^{VT} - p_2(x) - t(1-x))dx \\ &= \int_0^{\frac{1}{2}} (v + \frac{\beta^2}{2a} - tx - (c + t(1-2x)))dx + \int_{\frac{1}{2}}^1 (v + \frac{\beta^2}{2a} - t(1-x) - (c + t(2x-1)))dx \\ &= v - c - \frac{3t}{4} + \frac{\beta^2}{2a}. \end{aligned}$$

Social welfare is given by.

$SW^{VT} = CS^{VT} + \Pi_1^{VT} + \Pi_2^{VT} = v - c - \frac{t}{4} + \frac{\beta^2}{4a}$.

To compare the equilibrium solutions and equilibrium profits under the common and available parameter setting among Chapter 3.1.1 ($\beta^2 < 9at$), Chapter 3.1.2 ($\beta^2 < 4at$) and Chapter 3.1.3 ($\beta^2 < 2at$), we need to impose the strongest condition $\beta^2 < 2at$.

Appendix 2

Proof Obviously, $\phi_1^{VO} < \phi_1^{VZ} < \phi_1^{VT}$, $\phi_2^{VZ} < \phi_2^{VT}$. Because $\beta^2 < 2at$,

$$\beta^2 < 2at \Rightarrow \frac{2a^2t\beta - a\beta^3}{2a(8a^2t - 3a\beta^2)} > 0 \Rightarrow \frac{\beta}{2a} - \frac{3at\beta - \beta^3}{8a^2t - 3a\beta^2} > 0 \Rightarrow \phi_2^{VO} < \phi_2^{VT},$$

$$\frac{-a^2t\beta}{3a(8a^2t - 3a\beta^2)} < 0 \Rightarrow \frac{\beta}{3a} - \frac{3at\beta - \beta^3}{8a^2t - 3a\beta^2} < 0 \Rightarrow \phi_2^{VZ} < \phi_2^{VO},$$

Therefore, we have $\phi_2^{VZ} < \phi_2^{VO} < \phi_2^{VT}$.

Compare the profits in the three scenarios:

$$\Pi_2^{VZ} - \Pi_2^{VT} = \frac{t}{2} - \frac{\beta^2}{18a} - \frac{t}{4} + \frac{\beta^2}{8a} > 0,$$

$$\Pi_2^{VZ} - \Pi_2^{VO} = \frac{t}{2} - \frac{\beta^2}{18a} - \frac{(3at-\beta^2)^2(8at-\beta^2)}{2a(8at-3\beta^2)^2}$$

$$= \frac{(9at-\beta^2)(8at-3\beta^2)^2 - 9(3at-\beta^2)^2(8at-\beta^2)}{18a(8at-3\beta^2)^2}, = \frac{3at\beta^4 + 17a^2t^2\beta^2 - 72a^3t^3}{18a(8at-3\beta^2)^2}$$

$$= \frac{a^3t^3[3(\frac{\beta^2}{at})^2 + 17(\frac{\beta^2}{at}) - 72]}{18a(8at-3\beta^2)^2}.$$

It is easy to verify that $f(\frac{\beta^2}{at}) = 3(\frac{\beta^2}{at})^2 + 17(\frac{\beta^2}{at}) - 72$, which increases with the $\frac{\beta^2}{at}$ in $[0, 2]$, the maximum value for $f(\frac{\beta^2}{at})$ is $f(\frac{\beta^2}{at})_{\max}\big|_{x=2} = -26 < 0$. So $f(\frac{\beta^2}{at}) < 0$ in $[0, 2]$. Moreover,$3at\beta^4 + 17a^2t^2\beta^2 - 72a^3t^3 \big/ 18a(8at-3\beta^2)^2 < 0$, we have $\Pi_2^{VZ} < \Pi_2^{VO}$. Finally, we get $\Pi_2^{VT} < \Pi_2^{VZ} < \Pi_2^{VO}$.

Similarly, it is easy to verify that $\Pi_1^{VO} < \Pi_1^{VT} < \Pi_1^{VZ}$.

Appendix 3

Proof Obviously, $CS^{VZ} < CS^{VT}$, $CS^{VZ} < CS^{VO}$. Moreover,

$$\beta^2 < 2at \Rightarrow \frac{t}{4} + \frac{3at\beta^2 - \beta^4}{2a(8at-3\beta^2)} > 0 \Rightarrow v - c - \frac{3t}{4} + \frac{\beta^2}{2a} > v - c - t + \frac{5at\beta^2 - 2\beta^4}{2a(8at-3\beta^2)}$$

$\Rightarrow CS^{VO} < CS^{VT}$. Therefore, we have $CS^{VZ} < CS^{VO} < CS^{VT}$.

It is easily seen that $SW^{VZ} < SW^{VT}$.

$$SW^{VZ} - SW^{VO} = \left(v - c - \frac{t}{4} + \frac{2\beta^2}{9a}\right) - \left(v - c - t + \frac{88a^3t^3 - 37a^2t^2\beta^2 - 9at\beta^4 + 4\beta^6}{2a(8at-3\beta^2)^2}\right)$$

$$= \frac{144a^3t^3 - 118a^2t^2\beta^2 + 21at\beta^4}{36a(8at-3\beta^2)^2} = \frac{a^3t^3\left[21\left(\frac{\beta^2}{at}\right)^2 - 118\left(\frac{\beta^2}{at}\right) + 144\right]}{36a(8at-3\beta^2)^2}.$$

It is easy to verify that.
when $\frac{\beta^2}{at} < \frac{59-\sqrt{457}}{21}$, $f(\frac{\beta^2}{at}) = 21(\frac{\beta^2}{at})^2 - 118(\frac{\beta^2}{at}) + 144 > 0$;
when $\frac{59-\sqrt{457}}{21} < \frac{\beta^2}{at} < 2$, $f(\frac{\beta^2}{at}) = 21(\frac{\beta^2}{at})^2 - 118(\frac{\beta^2}{at}) + 144 < 0$.

$$SW^{VT} - SW^{VO} = (v - c - \frac{t}{4} + \frac{\beta^2}{4a}) - \left(v - c - t + \frac{88a^3t^3 - 37a^2t^2\beta^2 - 9at\beta^4 + 4\beta^6}{2a(8at-3\beta^2)^2}\right)$$

$$= \frac{\beta^6 - 3at\beta^4 - 6a^2t^2\beta^2 + 16a^3t^3}{4a(8at-3\beta^2)^2} = \frac{(\frac{\beta^2}{at})^3 - 3(\frac{\beta^2}{at})^2 - 6(\frac{\beta^2}{at}) + 16}{4a(8at-3\beta^2)^2}.$$

When $\beta^2 < 2at$, it is easy to verify that $(\frac{\beta^2}{at})^3 - 3(\frac{\beta^2}{at})^2 - 6(\frac{\beta^2}{at}) + 16 > 0$.
We can obtain $SW^{VT} > SW^{VO}$.
Finally, we obtain that when $\beta^2 < \frac{59-\sqrt{457}}{21}at$, $SW^{VO} < SW^{VZ} < SW^{VT}$; when.

$\frac{59-\sqrt{457}}{21}at < \beta^2 < 2at, SW^{VZ} < SW^{VO} < SW^{VT}.$

Appendix 4

Chapter 3.2.1 The first order condition is $\frac{\partial \Pi_i}{\partial \phi_i} = \frac{\beta}{2} - a\phi_i = 0$, the second-order condition is $\frac{\partial^2 \Pi_i}{\partial \phi_i^2} = -a < 0$.

Solving for the Nash equilibrium, we obtain

$$\phi_1^{TZ} = \phi_2^{TZ} = \frac{\beta}{2a},\ p_1^{TZ} = p_2^{TZ} = c + t + \frac{\beta^2}{a},\ \bar{x}^{TZ} = \frac{1}{2},\ \Pi_1^{TZ} = \Pi_2^{TZ} = \frac{t}{2} + \frac{3\beta^2}{8a}.$$

Consumer surplus is given by

$$\begin{aligned} CS^{TZ} &= \int_0^{\bar{x}^{TZ}} (v - (t + \beta\phi_1^{TZ} + \beta\phi_2^{TZ})x - p_1^{TZ})dx + \int_{\bar{x}^{TZ}}^1 (v - (t + \beta\phi_1^{TZ} + \beta\phi_2^{TZ})(1-x) - p_2^{TZ})dx \\ &= \int_0^{\frac{1}{2}} (v - (t + \frac{\beta^2}{a})x - (c + t + \frac{\beta^2}{a}))dx + \int_{\frac{1}{2}}^1 (v - (t + \frac{\beta^2}{a})(1-x) - (c + t + \frac{\beta^2}{a}))dx \\ &= v - c - \frac{5t}{4} - \frac{5\beta^2}{4a}, \end{aligned}$$

social welfare is given by.
$SW^{TZ} = CS^{TZ} + \Pi_1^{TZ} + \Pi_2^{TZ} = v - c - \frac{t}{4} - \frac{\beta^2}{2a}$.

Chapter 3.2.2 The first order condition is

$$\frac{\partial \Pi_1}{\partial \phi_1} = \frac{\beta}{8} - a\phi_1 = 0,\ \frac{\partial \Pi_2}{\partial \phi_2} = \frac{9\beta}{16} - a\phi_2 = 0.$$

The second-order condition is $\frac{\partial^2 \Pi_i}{\partial \phi_i^2} = -a < 0$.

Solving for the Nash equilibrium, we obtain

$$\phi_1^{TO} = \frac{\beta}{8a},\ \phi_2^{TO} = \frac{9\beta}{16a},\ \bar{x}^{TO} = \frac{1}{4},\ \Pi_1^{TO} = \frac{t}{8} + \frac{5\beta^2}{64a},\ \Pi_2^{TO} = \frac{9t}{16} + \frac{117\beta^2}{512a}.$$

Consumer surplus is given by

$$CS^{TO} = \int_0^{\bar{x}^{TO}} (v - (t + \beta\phi_1^{TO} + \beta\phi_2^{TO})x - p_1^{TO})dx + \int_{\bar{x}^{TO}}^1 (v - (t + \beta\phi_1^{TO} + \beta\phi_2^{TO})(1-x) - p_2(x))dx$$

$$= \int_0^{\frac{1}{4}} (v - p_1^{TO})dx - (t + \beta\phi_1^{TO} + \beta\phi_2^{TO}) \int_0^{\frac{1}{4}} xdx + \int_{\frac{1}{4}}^1 (v - p_1^{TO})dx - (t + \beta\phi_1^{TO} + \beta\phi_2^{TO}) \int_{\frac{1}{4}}^1 xdx$$

$$= v - c - t - \frac{11\beta^2}{16a}$$

Social welfare is given by
$SW^{TO} = CS^{TO} + \Pi_1^{TO} + \Pi_2^{TO} = v - c - \frac{5t}{16} - \frac{195\beta^2}{512a}$.

Chapter 3.2.3 The first order condition is $\frac{\partial\Pi_i}{\partial\phi_i} = \frac{\beta}{4} - a\phi_i = 0$, the second-order condition is $\frac{\partial^2\Pi_i}{\partial\phi_i^2} = -a < 0$.

Solving for the Nash equilibrium, we obtain

$$\phi_1^{TT} = \phi_2^{TT} = \frac{\beta}{4a},\ \bar{x}^{TT} = \frac{1}{2},\ \Pi_1^{TT} = \Pi_2^{TT} = \frac{t}{4} + \frac{3\beta^2}{32a}.$$

Consumer surplus is given by.

$$CS^{TT} = \int_0^{\bar{x}^{TT}} (v - (t + \beta\phi_1^{TT} + \beta\phi_2^{TT})x - p_1(x))dx$$

$$+ \int_{\bar{x}^{TT}}^1 (v - (t + \beta\phi_1^{TT} + \beta\phi_2^{TT})(1-x) - p_2(x))dx$$

$$= \int_0^{\frac{1}{2}} (v - c - (t + \beta\phi_1^{TT} + \beta\phi_2^{TT})(1-x))dx + \int_{\frac{1}{2}}^1 (v - c - (t + \beta\phi_1^{TT} + \beta\phi_2^{TT})x)dx$$

$$= v - c - \frac{3t}{4} - \frac{3\beta^2}{8a}.$$

Social welfare is given by

$$SW^{TT} = CS^{TT} + \Pi_1^{TT} + \Pi_2^{TT} = v - c - \frac{t}{4} - \frac{3\beta^2}{16a}$$

Appendix 5

Proof Obviously, $\phi_1^{TO} < \phi_1^{TT} < \phi_1^{TZ}, \phi_2^{TT} < \phi_2^{TZ} < \phi_2^{TO}$.

Since $\Pi_1^{TO} = \frac{t}{8} + \frac{5\beta^2}{64a} = \frac{2t}{16} + \frac{40\beta^2}{512a}, \Pi_1^{TT} = \frac{t}{4} + \frac{3\beta^2}{32a} = \frac{4t}{16} + \frac{48\beta^2}{512a}, \Pi_1^{TZ} = \frac{8t}{16} + \frac{192\beta^2}{512a}$,
we have $\Pi_1^{TO} < \Pi_1^{TT} < \Pi_1^{TZ}$. Obviously,$\Pi_2^{TT} < \Pi_2^{TO}, \Pi_2^{TT} < \Pi_2^{TZ}$,

$\beta^2 < \frac{32}{75}at \Rightarrow \frac{t}{16} - \frac{75\beta^2}{512a} > 0 \Rightarrow (\frac{9t}{16} + \frac{117\beta^2}{512a}) - (\frac{t}{2} + \frac{3\beta^2}{8a}) > 0 \Rightarrow \Pi_2^{TZ} < \Pi_2^{TO}$

On the contrary, if $\beta^2 > \frac{32}{75}at$, $\Pi_2^{TO} < \Pi_2^{TZ}$.

Finally, if $\beta^2 < \frac{32}{75}at$, $\Pi_2^{TT} < \Pi_2^{TZ} < \Pi_2^{TO}$; else if $\Pi_2^{TT} < \Pi_2^{TO} < \Pi_2^{TZ}$.

Appendix 6

Proof Since $v - c - \frac{5t}{4} - \frac{5\beta^2}{4a} < v - c - t - \frac{11\beta^2}{16a} < v - c - \frac{3t}{4} - \frac{3\beta^2}{8a}$, we obtain the result of comparison that $CS^{TZ} < CS^{TO} < CS^{TT}$.

Obviously, $SW^{TO} < SW^{TT}$, $SW^{TZ} < SW^{TT}$.

Since $\beta^2 < \frac{32}{61}at \Rightarrow \frac{t}{16} - \frac{61\beta^2}{512a} > 0 \Rightarrow (v - c - \frac{t}{4} - \frac{\beta^2}{2a}) - (v - c - \frac{5t}{16} - \frac{195\beta^2}{512a}) > 0.$

$\Rightarrow SW^{TZ} > SW^{TO}$,

On the contrary, if $\beta^2 > \frac{32}{61}at$, $SW^{TO} > SW^{TZ}$.

Finally, if $\beta^2 > \frac{32}{61}at$, $SW^{TZ} < SW^{TO} < SW^{TT}$; else if $SW^{TO} < SW^{TZ} < SW^{TT}$.

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