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Should the assembly system with direct omnichannel introduce integrated management service? A game-theoretical modelling study

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Abstract
Against the background of omnichannel retailing, this paper tries to explore the incentive conditions/regions of adopting integrated management service (IMS) and the operational strategies/performance for the assembly system with direct omnichannel (AS). Six different game-theoretical decision models, including a centralized, two decentralized and a coordination decision modelssss for the AS, and a centralized and a coordination decision models for the assembly system with direct omnichannel and IMS (ASI), are developed, analyzed and compared, respectively. Based on an electronic product case, the corresponding numerical and sensitivity analyses are conducted. On this basis, the analytical and numerical results are compared and validated to derive managerial insights. It is found that only when the dual incentive ratio indicators are in the quadrant $\{\alpha \geq 1$ and $\beta \geq 1\}$ of bidimensional incentive region matrix would the AS and IMS provider have the incentive to introduce and provide IMS. Introducing and providing IMS can effectively enhance the ability of quick response, boost the collaborative operations, and improve the operational performance of the AS. Furthermore, a revenue sharing contract-based coordination strategy can effectively improve the operational performance of ASI.

Keywords Direct omnichannel · Assembly system · Game-theoretical model · Coordination · Integrated management service (IMS)

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1 Introduction

In the era of rapid development of mobile commerce and payment, the quickly-evolving omnichannel business mode has transformed the retailing landscape in the last decade. Omnichannel mode refers to the seamless integration of online- and offline- channels to improve customer experience and thus meet the customers' demands, anytime and anywhere [10]. This mode emerged in recent years with the intention to ensure the retailer marketing strategies are geared toward tempting customers to convert on any channel [32].

According to an industry report from Shopify Plus, considering that an estimated 81% of shoppers conduct online research before making big purchases, the ability to channel even a small percentage of these customers straight from their online research to offline stores would generate a massive potential for sales [13, 24]. With the dual advantages of online information acquisition and offline product experience, the omnichannel mode could bring online customers to brick-and-mortar stores or vice versa, which provides a better shopping experience for customers and more business opportunities for retailers and their supply chains. Hence, many traditional brick-and-mortar/e-commerce retailers, manufacturers/assemblers or their parts and components suppliers are working together to transform their supply chains into an omnichannel mode.

Many industry leaders now believe that China leads the world in omnichannel development due to frequent use of mobile devices and widespread acceptance of mobile payment among the general public. For example, SUNING, a large appliances retail chain store enterprise in China committed to the integration of its online e-commerce platform and its own offline stores, actively implements the consumer-oriented omnichannel business mode and the supplier-oriented omnichannel platform. Besides, SUNING has established many self-pickup points in its offline stores while launching rebate activities of self-pickup for online shopping at the same time.

With the development of the omnichannel retailing mode, consumers' demand for product shopping experience and face-to-face consultation about price discount, product configuration and product applicability cannot be met in the traditional assembly system with direct online-channels. This calls for an omnichannel solution that provides the customer with a consistent, engaging experience across channels. As a typical assembler, DELL purchases PC components and modules from upstream suppliers, assembles PC products, and then sells PC products to end customers through its own online e-commerce platform and offline brick-and-mortar store partners, such as Walmart, CompUSA [21]. "This is not an abandonment of our direct sales model—this is intended to be additive," said Kent Cook, senior manager of consumer communications at Dell. "This is a response to customer demand. The direct model is great, but some consumers feel more comfortable going into a store to purchase electronics. We want to respond to our customers' needs." [8]. Obviously, the direct-sales channel strategy of the assembler represented by Dell necessitates an omnichannel solution that provides the customer with a consistent, engaging experience across channels. "Dell EMC believes in an omni channel strategy and leaves the choice to the customer in terms of whom they wish to do business with." Said by Anil Sethi, the vice president of channels India at Dell EMC [9].

Due to the different parts quality, process technology, cost control and delivery ability of different suppliers, the assembly system with direct omnichannel (will be abbreviated as AS) often faces the risk of supply interruption and cannot gather all the modules quickly from multiple suppliers, and quickly assemble and sell the products to the market via omnichannel, and thus resulting in non-cooperative operation and slow response of the AS and forming a "neck stuck" phenomenon in the AS. This is harmful to the healthy and sustainable operations of the AS. In order to ensure that the assembler with direct omnichannel can quickly respond to customer's needs, bring better product experience to customers, and give full play to the advantages of online information acquisition and offline product experience, the AS urgently needs to improve the collaborative operational ability of the supply chain. Against this background, the professional third-party, integrated management service (IMS) providers, came into being. For example, there are many IMS providers (IPs) in the electronics industry, such as Arrow Electronics, Avnet, WPG Holdings, Future Electronics, etc. These IPs can provide various comprehensive IMS solutions for the AS to enhance their ability of quick response, boost their collaborative operations, and improve their operational performance.

In the context of omnichannel retailing, should the AS introduce IMS? When will the AS have the incentive to introduce IMS? When will the IMS provider (IP) have the incentive to provide IMS? How to achieve the coordinative operations for the AS and IP? These are some major issues calling for urgent consideration both in practice and in theoretical research of AS. In the theoretical research field, the available literature rarely touches upon the following critical issues in the operations management of AS: (1) the role and value of IMS in the AS; (2) the incentive conditions of introducing and providing IMS for the AS; (3) the cooperation region between AS and IP in the assembly system with direct omnichannel and IMS (will be abbreviated as ASI); (4) the operational strategies, decisions and performance for the ASI. Obviously, in the era of omnichannel retailing, these important research gaps need to be addressed urgently. However, it is a new challenge to explore the incentives of introducing and providing IMS and cooperation region between AS and IP for the ASI via game-theoretical modelling approach. Specifically, how to characterize the new demand function with IMS effort in the context of omnichannel retailing, how to formulate objective functions and design the decision structures for the game-theoretical models, and how to derive the incentive conditions and cooperation regions through comparing the profit regions, are all new challenges for our research.

From the perspective of game-theoretical modeling and comparative analysis, we try to explore the incentive conditions and cooperation regions under which the AS would have the incentive to introduce IMS and the IP would have the incentive to provide IMS. A novel and useful *bidimensional incentive region matrix* will be developed to identify the incentive conditions of introducing and providing IMS for the AS and determine the cooperation region between AS and IP in the ASI. This study aims to investigate when the IMS would be introduced and provided in the AS and how coordinative operations can be achieved across the AS, which will help AS and IP make appropriate operational decisions/strategies and improve their operational performance.

The paper consists of 7 sections. Section 2 gives an overall review of the corresponding literature. In Sect. 3, the modeling notations and assumptions of a generic ASI are defined. Then, game-theoretical decision models for a generic ASI are developed and analyzed in Sect. 4. The comparisons and discussions of analytical results are further summarized in this section. Section 5 offers numerical and sensitive analyses of an electronic product case for all developed analytical models. The comparisons and discussions of numerical results are also synthesized in this section. The managerial insights, limitations of the research and scopes for future research are discussed in Sect. 6. The final section is a summary of the research contributions and foresights drawn from this study.

2 Literature review

The omnichannel retailing mode, as a rising new business model, is quickly replacing the dual-channels. Under an omnichannel mode, the boundary between the online channel and offline channel has been removed, creating a dual advantage of online information search and offline product experience. Transition to the omnichannel mode has become an important strategic direction for both brick-and-mortar retailers and ecommerce ones as it helps to maintain a symbiotic, integrated and mutually reinforcing retail supply chain. The omnichannel mode brings both opportunities and challenges to the theoretical research as it reshapes the structure and mechanism of the competition in the retail market. Prior research on the following three streams—the operations management of the assembly system, the omnichannel retailing and that of the omnichannel supply chain—is related to our current study: However, the available literature regarding the incentive conditions, cooperation regions and operational strategies for the ASI is still very scarce, which justifies further research.

2.1 Assembly system perspective

Previous research on the first stream—the operations management of the assembly system, mainly touches upon the issues of competition, cooperation and coordination in the assembly system, such as the multi-echelon supply chains competition with an assembly network structure [4], information sharing and coordination scheme in an assembly system [37], multilateral negotiations in an assembly supply chain via Nash bargaining [22], alliance/coalition formation among multiple complementary suppliers in a decentralized assembly system [23, 35], supplier competition in a decentralized assembly system [15], and sequential contracting for the decentralized assembly systems under asymmetric demand information [17]. Nevertheless, the existing research doesn't touch upon the operational strategies/performance of the assembly system in the context of omnichannel retailing, nor does it involve the incentive regions of IMS adoption in the AS.

2.2 Omnichannel retailing perspective

Regarding the second stream concerning the operations management of omnichannel retailing, available literatures mainly focus on the impact of various new business/logistic technology on the operational decisions/strategies/performance of omnichannel retailing, such as the impact of buy online and pick up in store (BOPS) on offline store operations and consumers' channel selections [11], the impact of self-order technologies adoption on operational decisions/performances in an omnichannel restaurant [12], the route capacity sharing for an omnichannel grocery retailer [25],the impact of ship-to-store (STS) and quick response on the operational decisions/performances in the fast-fashion omnichannel retailing [34]. However, the available research neither pays attention to the operational strategies/performance of AS, nor takes into account the incentive regions of IMS adoption for the operations management of AS.

2.3 Omnichannel supply chain perspective

Regarding the third stream, the available literature mainly centers around the issues of operational strategies/decisions/performance of omnichannel (or O2O) supply chain and the impact of disruptions, market power, mutual promotion, low carbon on the operational strategies/decisions/performance of omnichannel (or O2O) supply chain, such as, service competition in an O2O supply chain [38], the impact of disruptions on the O2O supply chain coordination [39], the impact of different power and decision structures on the operational decisions/performance of O2O supply chain [5], initial carbon allowance allocation rules in an O2O supply chain with the cap-and-trade regulation [14], mutual promotional effects, operational strategies and subsidy policies for the O2O supply chain [6], cooperation mechanism for the O2O consignment supply chain with complementary products [7], optimal pricing for an omnichannel supply chain with retail service [16]. Nevertheless, the existing research neither touches upon the operational strategies/performance of assembly supply chain in the omnichannel retailing environment, nor considers the incentive regions of IMS adoption for the assembly supply chain in the context of omnichannel retailing.

In brief, the available literature fails to cover the following critical issues in the operations management of AS: (1) the role and value of IMS in the AS; (2) the incentive conditions of introducing and providing IMS for the AS; (3) the cooperation region between AS and IP in the ASI; (4) the operational strategies, decisions and performance for the ASI.

Considering the critical issues aforementioned, this paper, different from previous research, intends to conduct a novel investigation into the role and economic behaviors of integrated management service (IMS) in the assembly system with direct omnichannel (AS), and explore the incentive conditions, cooperation regions and operational strategies for the ASI. This study will fill up the gap in previous research and add managerial insights for the omnichannel practitioners and their supply chain partners.

3 Modeling notations and assumptions

A generic assembly system with direct omnichannel and IMS (ASI), as shown in Fig. 1, is conceptualized for this study. This system includes one IMS provider (IP), n module or component suppliers and an assembler offering an assembled product to the market through an omnichannel mode. In this system, IP provides IMS to the assembler with direct omnichannel and gets relevant service fee as a result. Each supplier produces a module for the assembler to assemble the components into the final product for the retail market. Each supplier negotiates jointly (a centralized strategy), independently (a decentralized strategy) or collaboratively (a coordination strategy) with the assembler regarding the wholesale price that will affect the retail pricing and demands. Through the omnichannel integration, the final product will be sold at a regular retail price in the selling season, and the leftover stock will be sold at a salvage price in the clearance season.

In Fig. 1, notation i is introduced to mark the variables and parameters, $i \in N$, $N = \{1, 2, \ldots, n\}$. For the ith supplier, the unit cost of the ith module is c_i and the wholesale price of the ith module is w_i; for the assembler, the unit assembly cost of the final product is c. The final product is sold via omnichannel mode, the operational cost of the online channel is c_e and the operational cost of the offline channel is c_s. The retail price of the final product, through either the online channel or offline channel, is p and the salvage price of the final product in the clearance season is ηp, where η is the salvage discount price factor,and $0 < \eta < 1$.

The IP's IMS effort for the AS is s. Generally, the cost of effort $c(s)$ is positive and strictly convex in s, and $c(0) = 0$ [18, 27, 28]. Thus, the cost of IMS effort

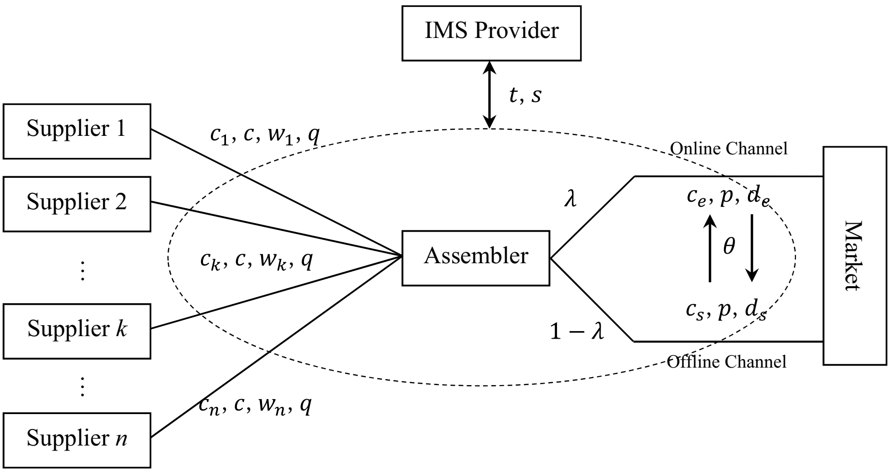


Fig. 1 A Generic Assembly System with Direct Omnichannel and IMS (ASI)

is assumed to be a quadric form: $c(s) = \frac{1}{2}gs^2$, where g is the cost coefficient of IMS effort. The assembler will pay an IMS fee t per unit product for IP. Furthermore, this fee will be shared between the assembler and suppliers via contract mechanism.

Following Chen et al. [6] and Chen and Su [7], let $d_e(p,s)$ and $d_s(p,s)$ denote the online demand function and the offline demand function respectively. They can be defined as $d_e(p) = \lambda v(p,s)x$ and $d_s(p) = (1-\lambda)v(p,s)x$. Thus, the total market demand function $d(p,s) = d_e(p,s) + d_s(p,s) = v(p,s)x$. In all the demand functions, λ is the market demand share of the online channel and $(1-\lambda)$ is the market demand share of the offline channel, $0 < \lambda < 1$. $v(p,s)$ is a deterministic function of price p and IMS effort s, decreasing of price p and increasing of IMS effort s. $v(p,s) = ap^{-b}p^{\theta}s^{\kappa} = ap^{-(b-\theta)}s^{\kappa}$, where a is the positive constant number, b is the price-elasticity index of the expected demand, $\theta \in (0,1)$ is the mutual fusion coefficient between channels, and $\kappa \in (0,1)$ is the IMS effort-elasticity index of the expected demand, and $b > n > 1$. Let $y(p) = ap^{-(b-\theta)}$, then $v(p,s) = y(p)s^{\kappa}.x$ is a random factor defined in the range $[A,B]$ with $B > A > 0$. The CDF (Cumulative distribution function) and PDF (Probability density function) of x are $F(\cdot)$ and $f(\cdot)$, and the mean value and standard deviation of x are μ and σ. Following Petruzzi and Dada [26], Wang et al. [33], and Wang [30], $z = \frac{q}{v(p,s)}$ is defined as the 'stock factor' where q is the production quantity. In this study, the 'stock factor' is used to model the equilibrium and coordination conditions of supply chain.

When the distribution of x in the demand function satisfies the Increasing Generalized Failure Rate (IGFR) condition, i.e., $\frac{dg(x)}{dx} = h(x) + \frac{xdh(x)}{dx} > 0$, where $g(x) = xh(x)$, and $h(x) = \frac{f(x)}{[1-F(x)]}$ is the classical failure rate [19, 20, 33], the first order conditions of the expected profit function with respect to p and z provide a unique solution to the problem of maximizing the expected profit function. The decision variables are the retail price p and the stock factor z of the final product, the wholesale price of the ith module w_i.

On this basis, the profit functions of the suppliers, assembler, and assembly system under omnichannel (OMO) mode can be formulated as follows:

$$\Pi_{S_i}(w_i) = (w_i - c_i)q(p,z) = (w_i - c_i)y(p)z$$

$$\Pi_A(p,z) = py(p)\cdot E[\min\{z,x\}] + \eta py(p)\cdot E\left[(z-x)^+\right] - \left[c + \lambda c_e + (1-\lambda)c_s + \sum_{i=1}^{n} w_i\right]y(p)z$$

$$\Pi_{SC}(p,z) = py(p)\cdot E[\min\{z,x\}] + \eta py(p)\cdot E\left[(z-x)^+\right] - \left[c + \lambda c_e + (1-\lambda)c_s + \sum_{i=1}^{n} c_i\right]y(p)z$$

Furthermore, the profit functions of the suppliers, assembler, and assembly system and IMS provider under omnichannel with IMS (OMI) mode can be formulated as follows:

$$\Pi^{s}_{S_i}\left(w_i\right) = \left(w_i - c_i\right)q(p,z,s) = \left(w_i - c_i\right)v(p,s)z$$

$$\Pi^{s}_{A}(p,z,s) = pv(p,s)\cdot E[\min\{z,x\}] + \eta pv(p,s)\cdot E\left[(z-x)^{+}\right] \\ -\left[c + \lambda c_e + (1-\lambda)c_s + \sum\nolimits_{i=1}^{n} w_i + t\right]v(p,s)z$$

$$\Pi^{s}_{SC}(p,z,s) = pv(p,s)\cdot E[\min\{z,x\}] + \eta pv(p,s)\cdot E\left[(z-x)^{+}\right] \\ -\left[c + \lambda c_e + (1-\lambda)c_s + \sum\nolimits_{i=1}^{n} c_i + t\right]v(p,s)z$$

$$\Pi^{s}_{IP}(s) = tv(p,s)z - \frac{1}{2}gs^2$$

Table 1 provides a framework of the game-theoretical decision models developed and analyzed in Sect. 4.

Although the ideal centralized decision mode is difficult to realize in an AS in practice, it can be used as an important benchmark for decentralized decision mode in Sect. 4.1.2 and coordination decision mode in Sect. 4.1.3, which is worthy of analysis and discussion. Therefore, Sect. 4.1.1 will develop and analyze a centralized decision model for the AS.

In the real-world scenario, the AS is generally operated under a decentralized decision mode in the absence of contractual coordination. Under the decentralized decision mode, multiple suppliers may make decisions simultaneously or sequentially, i.e., there are two decentralized decision modes for the AS. Since the operational performance under decentralized decision mode is lower than that under the centralized decision mode, it is necessary to analyze and discuss operational decisions and performances under the decentralized decision mode, which can serve as the lower boundary reference for coordination decision mode. Therefore, Sect. 4.1.2 will develop and analyze two decentralized decision models for the AS.

Table 1 Framework of game-theoretical decision models

Section	Model scenarios	Game-theoretical decision models	Theories applied
4.1	Omnichannel mode (OMO mode)	4.1.1 Centralized Decision Model	OT & BC
		4.1.2 Decentralized Decision Model	SG & BC
		4.1.2.1 Assembler's Decision	SG & BC
		4.1.2.2 Suppliers' Simultaneous Decision	SG & BC
		4.1.2.3 Suppliers' Sequential Decision	SG & BC
		4.1.3 Coordination Decision Model	RSC & BC
4.2	Omnichannel mode with IMS (OMI mode)	Centralized/Coordination Decision Models under OMI mode	OT + SG + RSC + BC

OT optimization theory; *BC* bertrand competition; *SG* stackelberg game; *RSC* revenue sharing contract

In the case of contractual mechanism introduction, taking the centralized decision mode as the benchmark and taking decentralized decision mode as the lower boundary reference, the AS may achieve coordination decision and realize Pareto improvement of the operational performance for all the stakeholders. Therefore, Sect. 4.1.3 will develop and analyze a coordination decision model for the AS, based on the centralized decision model in Sect. 4.1.1 and decentralized decision models in Sect. 4.1.2.

Nevertheless, it is still quite challenging to guarantee effective implementation of the contract. Due to the complicated structure and multiple participants in the AS, the assembler has to deal with the complicated collaborative business relationships with multiple suppliers, resulting in non-cooperative operations and slow-response ability in the AS. Meanwhile, for lack of effective demand information sharing and collaborative planning, forecasting and replenishment mechanism across the AS, suppliers are often unable to guarantee the perfect matching supply of parts and components for the assembly process of the assembler, which may lead to assembly disruption. Therefore, it is necessary to introduce the IMS provider (IP) as it provides a unified logistics and information flow management platform and corresponding comprehensive IMS solutions for the AS, which enhances the demand information sharing between the assembler and multiple suppliers and helps multiple suppliers collaboratively carry out matching collection and distribution of the corresponding parts and components to the assembler in accordance with the assembly requirements of product. The corresponding goal is to enhance the ability of collaborative operation and quick response, strengthen the mutual fusion effect between online- and offline- channels, raise the value added for customers and improve operational performance in the AS. Therefore, Sect. 4.2 will develop and analyze centralized and coordination decision models for the ASI, based on the centralized, decentralized and coordination decision models for the AS in Sect. 4.1.

4 Model setup and discussion

Based on the modeling notations and assumptions discussed in Sect. 3, this section conducts an extensive game-theoretical modeling of the equilibrium and coordination conditions for the ASI. In the models to follow, the superscript or subscript c represents centralized decision and coordination decision under omnichannel mode (i.e., OMO mode); the superscript or subscript d: decentralized decision with suppliers' simultaneous action under OMO mode; the superscript or subscript d': decentralized decision with suppliers' sequential action under OMO mode; the superscript or subscript sc: centralized decision and coordination decision under omnichannel mode with IMS (i.e., OMI mode).

4.1 Game-theoretical decision models under OMO mode

Under OMO mode, the AS does not introduce IMS, i.e., $t = 0$, $g = 0$, $s = 1$ and $\kappa = 0$. A centralized decision model, two decentralized decision models and a coordination decision model will be developed, analyzed and compared for the AS in this section.

4.1.1 Centralized decision model

Under the centralized decision model, the detailed decision sequences are as follows: the assembly system will first decide the retail price p, and then the stock factor z. The optimal problem for the assembly system under the centralized decision can be formulated as follows:

$$\max_{p,z} \quad \Pi_{SC}(p,z) \tag{1}$$

Solving this optimal problem, we can get the optimal retail price p_c, the distribution function of the centralized optimal stock factor $F(z_c)$ and the optimal production quantity q_c. Furthermore, the optimal profit of the assembly system can be calculated as Π_{SC}^{c}. (See Table 2 for the detailed analytical modeling results and their derivations can be seen in "Appendix").

4.1.2 Decentralized decision model

4.1.2.1 Stackelberg game model (suppliers' simultaneous decisions) Under this scenario, the detailed decision sequences are as follows: all the module suppliers decide their wholesale price w_i simultaneously, and then, the assembler with direct omnichannel decides the retail price p and stock factor z. The two-stage Stackelberg game model for the decentralized AS can be formulated as:

$$\begin{cases} \begin{cases} \max\limits_{w_1} \Pi_{S_1}\left(w_1, q_d\left(w_1,\dots,w_i,\dots,w_n\right)\right) \\ \vdots \\ \max\limits_{w_i} \Pi_{S_i}\left(w_i, q_d\left(w_1,\dots,w_i,\dots,w_n\right)\right) \\ \vdots \\ \max\limits_{w_n} \Pi_{S_n}\left(w_n, q_d\left(w_1,\dots,w_i,\dots,w_n\right)\right) \end{cases} \\ s.t. \begin{cases} p_d\left(w_1,\dots,w_i,\dots,w_n\right), F\left(z_d\right), q_d\left(w_1,\dots,w_i,\dots,w_n\right) \\ \textit{are derived from solving the following problem} \\ \max\limits_{p,z} \Pi_A(p,z) \end{cases} \end{cases} \tag{2}$$

Solving this two-stage Stackelberg game problem, we get the equilibrium wholesale price w_i^d, the equilibrium retail price p_d, the distribution function of the equilibrium stock factor $F(z_d)$ and the equilibrium production quantity q_d. Furthermore, the equilibrium profits of the supplier i, the assembler and the assembly system can be calculated as $\Pi_{S_i}^{d}$, Π_{A}^{d} and Π_{SC}^{d} (see Table 2 for detailed analytical modeling results).

Table 2 Analytical results under OMO mode and OMI mode

Scenarios	OMO mode		OMI mode	
Out-comes	Decentralized strategy		Coordination strategy (CO)	Coordination strategy (COI)
	Suppliers' simultaneous decisions (SI)	Suppliers' Sequential Decisions (SE)		
s_*	NA	NA	NA	$s_c = \left(\frac{g}{\kappa t \rho^{b-\theta} q_c}\right)^{\frac{1}{\kappa-2}}$
$F(z_*)$	$F(z_d) = F(z_c)$	$F(z_{d'}) = F(z_c)$	$F(z_c) = \frac{1}{(1-\eta)(b-\theta)} + \frac{(b-\theta-1)\Lambda(z_c)}{(b-\theta)z_c}$	$F(z_c^s) = F(z_c)$
p_*	$p_d = \frac{b-\theta}{b-\theta-n}p_c$	$p_{d'} = \left(\frac{b-\theta}{b-\theta-1}\right)^n p_c$	$p_c = \frac{b-\theta}{b-\theta-1}\frac{[c+\lambda c_e+(1-\lambda)c_s+\sum_{i=1}^n c_i]z_c}{z_c-(1-\eta)\Lambda(z_c)}$	$p_c^s = \frac{1}{\rho}p_c$
q_*	$q_d = \left(\frac{b-\theta-n}{b-\theta}\right)^{b-\theta} q_c$	$q_{d'} = \left(\frac{b-\theta-1}{b-\theta}\right)^{n(b-\theta)} q_c$	$q_c = y(p_c)z_c$	$q_c^s = \rho^{b-\theta}s_c^\kappa q_c$
w_i^*	$w_i^d = \frac{1}{b-\theta-n}\left[c + \lambda c_e + (1-\lambda)c_s + \sum_{i=1}^n c_i\right] + c_i$	$w_i^{d'} = \frac{(b-\theta)^{i-1}}{(b-\theta-1)^i}\left[c + \lambda c_e + (1-\lambda)c_s + \sum_{i=1}^n c_i\right] + c_i$	$w_i^c = \phi^* c_i$	$w_i^{sc} = \delta^* c_i$
$\Pi_{S_i}^*$	$\Pi_{S_i}^d = \frac{b-\theta-1}{b-\theta}\left(\frac{b-\theta-n}{b-\theta}\right)^{b-\theta-1}\Pi_{SC}^c$	$\Pi_{S_i}^{d'} = \left(\frac{b-\theta-1}{b-\theta}\right)^{n(b-\theta)-i+1}\Pi_{SC}^c$	$\Pi_{S_i}^c = \frac{c_i}{\sum_{i=1}^n c_i}(1-\phi^*)\Pi_{SC}^c$	$\Pi_{S_i}^{sc} = \frac{c_i}{\sum_{i=1}^n c_i}(1-\delta^*)\Pi_{SC}^{sc}$
Π_A^*	$\Pi_A^d = \left(\frac{b-\theta-n}{b-\theta}\right)^{b-\theta-1}\Pi_{SC}^c$	$\Pi_A^{d'} = \left(\frac{b-\theta-1}{b-\theta}\right)^{n(b-\theta-1)}\Pi_{SC}^c$	$\Pi_A^c = \phi^* \Pi_{SC}^c$	$\Pi_A^{sc} = \delta^* \Pi_{SC}^{sc}$
Π_{SC}^*	$\Pi_{SC}^d = \left[(n+1) - \frac{n}{b-\theta}\right]\left(\frac{b-\theta-n}{b-\theta}\right)^{b-\theta-1}\Pi_{SC}^c$	$\Pi_{SC}^{d'} = \left[(b-\theta)\left(\frac{b-\theta-1}{b-\theta}\right)^{n(b-\theta-1)} - (b-\theta-1)\left(\frac{b-\theta-1}{b-\theta}\right)^{n(b-\theta)}\right]\Pi_{SC}^c$	$\Pi_{SC}^c = \frac{1}{b-\theta-1}\left[c + \lambda c_e + (1-\lambda)c_s + \sum_{i=1}^n c_i\right]q_c$	$\Pi_{SC}^{sc} = \rho^{b-\theta-1}s_c^\kappa \Pi_{SC}^c$
Π_{IP}^*	NA	NA	NA	$\Pi_{IP}^{sc} = tq_c^s - \frac{1}{2}gs_c^2$
ϕ^*, δ^*	NA	NA	$\phi^* \in \left[\underline{\phi}, \bar{\phi}\right]$	$\delta^* \in \left[\underline{\delta}, \bar{\delta}\right]$
Note	$\Lambda(z_c) = \int_A^{z_c}(z_c - x)f(x)dx$, $\underline{\phi} = \max\left\{\left(\frac{b-\theta-n}{b-\theta}\right)^{b-\theta-1}, \left(\frac{b-\theta-1}{b-\theta}\right)^{n(b-\theta-1)}\right\}$, $\bar{\phi} = \min_{i\in N}\left\{1 - \frac{\sum_{i=1}^n c_i}{c_i}\max\left\{\frac{b-\theta-1}{b-\theta-n}\left(\frac{b-\theta-n}{b-\theta}\right)^{b-\theta}, \left(\frac{b-\theta-1}{b-\theta}\right)^{n(b-\theta)-i+1}\right\}\right\}$, $\underline{\delta} = \frac{\phi^* \Pi_{SC}^c}{\Pi_{SC}^{sc}}$, $\bar{\delta} = \frac{\Pi_{SC}^{sc} - (1-\phi^*)\Pi_{SC}^c}{\Pi_{SC}^{sc}}$, $\rho = \frac{c + \lambda c_e + (1-\lambda)c_s + \sum_{i=1}^n c_i}{c + \lambda c_e + (1-\lambda)c_s + \sum_{i=1}^n c_i + t}$, $\alpha \equiv \rho^{b-\theta-1}s_c^\kappa$, $\beta \equiv 2tg^{-1}\rho^{b-\theta}q_c s_c^{\kappa-2}$			

4.1.2.2 Stackelberg game model (suppliers' sequential decisions) Under this scenario, the detailed decision sequences are as follows: all the module suppliers first make their wholesale price w_i sequentially: module supplier 1 first decides his wholesale price w_1, and then, module supplier 2 decides his wholesale price w_2 based on supplier 1's decision…, and finally, module supplier n decides his wholesale price w_n based on decisions of suppliers 1, 2,…, $n-1$; and then, the assembler with direct omnichannel decides the retail price p and stock factor z. The $(n+1)$-stage Stackelberg game model for the decentralized AS can be formulated as:

$$\begin{cases} \begin{cases} \max\limits_{w_1} \Pi_{S_1}\left(w_1, q_{d\prime}\left(w_1, \ldots, w_i, \ldots, w_n\right)\right) \\ s.t. \max\limits_{w_2} \quad \Pi_{S_2}\left(w_2|w_1, q_{d\prime}\left(w_1, \ldots, w_i, \ldots, w_n\right)\right) \\ \vdots \\ s.t.\ \max\limits_{w_i} \quad \Pi_{S_i}\left(w_i|w_1, w_2, \ldots, w_{i-1}, q_{d\prime}\left(w_1, \ldots, w_i, \ldots, w_n\right)\right) \\ \vdots \\ s.t.\ \max\limits_{w_n} \quad \Pi_{S_n}\left(w_n|w_1, w_2, \ldots, w_{n-1}, q_{d\prime}\left(w_1, \ldots, w_i, \ldots, w_n\right)\right) \end{cases} \\ s.t. \begin{cases} p_{d\prime}\left(w_1, \ldots, w_i, \ldots, w_n\right), F\left(z_{d\prime}\right), q_{d'}\left(w_1, \ldots, w_i, \ldots, w_n\right) \\ \textit{are derived from solving the following problem} \\ \max\limits_{p,z} \quad \Pi_A(p, z) \end{cases} \end{cases} \tag{3}$$

Solving this $(n+1)$-stage Stackelberg game problem, we get the equilibrium wholesale price $w_i^{d'}$, the equilibrium retail price $p_{d'}$, the distribution function of the equilibrium stock factor $F\left(z_{d'}\right)$ and the equilibrium production quantity $q_{d'}$. Furthermore, the equilibrium profits of the supplier i, the assembler and the assembly system can be calculated as $\Pi_{S_i}^{d'}$, $\Pi_A^{d'}$ and $\Pi_{SC}^{d'}$. (See Table 2 for detailed analytical modeling results).

4.1.3 Coordination decision model

Under this scenario, the detailed decision sequences are as follows: the suppliers simultaneously offer the assembler a revenue sharing contract in which suppliers charge a lower wholesale price w_i from the assembler; if the assembler accepts the contract, he will place an order of quantity q with the module suppliers, after the final product is assembled by modules or components, he will sell the final product through omnichannel at regular retail price p and decide the stock factor z when the selling season starts, and sell the leftover stock through omnichannel at salvage price ηp in the clearance season. Finally, the assembler will share a fraction $1-\phi$ of his net revenue with the suppliers (supplier i will get a fraction $\frac{c_i}{\sum_{i=1}^{n} c_i}(1-\phi)$ of the assembler's sharing revenue), where ϕ is the revenue keeping fraction of the assembler, and $0 \le \phi \le 1$. The revenue shared by the assembler with supplier i is as follows:

$$T_i = \frac{c_i}{\sum_{i=1}^{n} c_i}(1-\phi)\left\{ \begin{array}{r} py(p)\cdot E[\min\{z,x\}] + \eta py(p)\cdot E\left[(z-x)^+\right] \\ -\left[c + \lambda c_e + (1-\lambda)c_s\right]y(p)z \end{array} \right\}$$

Thus, the profit functions of the supplier i and assembler under revenue sharing contract are as follows:

$$\Pi_{S_i}^c\left(w_i\right) = \Pi_{S_i}\left(w_i\right) + T_i$$

$$\Pi_A^c(p,z) = \Pi_A(p,z) - \sum_{i=1}^{n} T_i$$

The optimal problem for the AS under the revenue sharing contract is as follows:

$$\begin{array}{l} \textit{Feasible domain of } \phi^* \textit{ is derived from solving } \Pi_A^c(\phi) \ge \max\left\{\Pi_A^d, \Pi_A^{d'}\right\} \textit{ and } \Pi_{S_i}^c(\phi) \ge \max\left\{\Pi_{S_i}^d, \Pi_{S_i}^{d'}\right\} \\ s.t. \left\{ \begin{array}{l} w_i^c(\phi), \Pi_{S_i}^c(\phi) \textit{ and } \Pi_A^c(\phi) \textit{ are derived from solving the following problem} \\ s.t. \left\{ \begin{array}{l} p_r\left(w_1,\ldots,w_i,\ldots,w_n\right) = p_c,\ F\left(z_r\right) = F\left(z_c\right) \\ s.t. \left\{ \begin{array}{l} p_r\left(w_1,\ldots,w_i,\ldots,w_n\right), F\left(z_r\right) \textit{ are derived from solving } \max\limits_{p,z}\ \Pi_A^c(p,z) \\ p_c, F\left(z_c\right),\ q_c \textit{ and } \Pi_{SC}^c \textit{ are derived from solving } \max\limits_{p,z}\ \Pi_{SC}(p,z) \end{array} \right. \end{array} \right. \end{array} \right. \end{array} \tag{4}$$

Solving this two-stage Stackelberg game problem, we get the feasible domain of revenue keeping rate ϕ^*, the coordinated wholesale price w_i^c, the optimal retail price p_c, the distribution function of the optimal stock factor $F\left(z_c\right)$ and the optimal production quantity q_c. Furthermore, the coordinated profits of the supplier i, the assembler and the assembly system can be calculated as $\Pi_{S_i}^c$, Π_A^c and Π_{SC}^c. (See Table 2 for detailed analytical modeling results).

The analytical results of Sect. 4.1 are summarized in Table 2. The centralized strategy neglects the roles of the suppliers in making crucial pricing and production quantity decisions and therefore is inferior to the coordination strategy regarding the derived solutions. Thus, the centralized decision results are not shown in Table 2 and will be ruled out in the coming discussions.

4.2 Game-theoretical decision models under OMI mode

Under OMI mode, the AS introduces IMS, i.e., $t > 0$, $g > 0$, $s > 1$ and $\kappa \in (0,1)$. With the help of integrated management services (IMS) provided by the IP, the demand information sharing between the assembler and multiple suppliers will be enhanced, on which basis, the matching collection and distribution of the corresponding parts and components to the assembler in accordance with the assembly requirements of product will be carried out collaboratively, and thus, the AS can achieve effective coordinative management and improve operational performance.

Therefore, the decentralized decision scenario does not exist under OMI mode and will not be considered in this section. On this basis, a centralized decision model and a coordination decision model will be developed, analyzed and compared for the ASI in this section.

4.2.1 Centralized decision model

Under the centralized decision model, the detailed decision sequences are as follows: the IP will first decide the IMS effort s, the assembly system will first decide the retail price p, and then decide the stock factor z. The two-stage Stackelberg game model for the ASI under centralized decision can be formulated as:

$$\begin{cases} \max\limits_{s} \Pi_{IP}^{s}(s) \\ s.t.\ \max\limits_{p,z} \quad \Pi_{SC}^{s}(p,z,s) \end{cases} \tag{5}$$

Solving this two-stage Stackelberg game problem, we get the equilibrium IMS effort s_c, the equilibrium retail price p_c^s, the distribution function of the centralized equilibrium stock factor $F\left(z_c^s\right)$ and the equilibrium production quantity q_c^s. Furthermore, the equilibrium profits of the assembly system and IP can be calculated as Π_{SC}^{sc} and Π_{IP}^{sc} (see Table 2 for detailed analytical modeling results).

Obviously, only when the condition $\Pi_{SC}^{sc} \geq \Pi_{SC}^{c}$ holds, i.e., only when the incentive ratio indicator $\alpha \equiv \rho^{b-\theta-1} s_c^{\kappa} \geq 1$, (where $\rho = \frac{c+\lambda c_e+(1-\lambda)c_s+\sum_{i=1}^{n} c_i}{c+\lambda c_e+(1-\lambda)c_s+\sum_{i=1}^{n} c_i+t}$), would the assembly system have the incentive to introduce integrated management service (IMS). Furthermore, only when the condition $\Pi_{IP}^{sc} \geq 0$ holds, i.e., only when the incentive ratio indicator $\beta \equiv 2tg^{-1}\rho^{b-\theta} q_c s_c^{\kappa-2} \geq 1$, would the IP have the incentive to provide integrated management service (IMS).

4.2.2 Coordination decision model

Under this scenario, the detailed decision sequences are as follows: the IP first decides IMS effort s, and then, the suppliers simultaneously offer the assembler a revenue sharing contract in which suppliers charge a lower wholesale price w_i from the assembler; if the assembler accepts the contract, he will place an order of quantity q with the module suppliers, after the final product is assembled by modules or components, he will sell the final product through omnichannel at regular retail price p and decide the stock factor z when the selling season starts, and sell the leftover stock through omnichannel at salvage price ηp in the clearance season. Finally, the assembler will share a fraction $1-\delta$ of his net revenue with the suppliers (supplier i will get a fraction $\frac{c_i}{\sum_{i=1}^{n} c_i}(1-\delta)$ of the assembler's sharing revenue), where δ is the revenue keeping fraction of the assembler, and $0 \leq \delta \leq 1$. The revenue shared by the assembler with supplier i is as follows:

$$T_i^s = \frac{c_i}{\sum_{i=1}^{n} c_i}(1-\delta)\left\{ \begin{array}{r} pv(p,s)\cdot E[\min\{z,x\}] + \eta pv(p,s)\cdot E\left[(z-x)^+\right] \\ -\left[c+\lambda c_e + (1-\lambda)c_s + t\right]v(p,s)z \end{array} \right\}$$

Thus, the profit functions of the supplier i and assembler under revenue sharing contract are as follows:

$$\Pi_{S_i}^{sc}\left(w_i\right) = \Pi_{S_i}^{s}\left(w_i\right) + T_i^s$$

$$\Pi_A^{sc}(p,z,s) = \Pi_A^s(p,z,s) - \sum_{i=1}^{n} T_i^s$$

The two-stage Stackelberg game problem for the AS under the revenue sharing contract is as follows:

$$\begin{array}{l} \textit{Feasible domain of } \delta^* \textit{ is derived from solving } \Pi_A^{sc}(\delta) \geq \Pi_A^c \textit{ and } \Pi_{S_i}^{sc}(\delta) \geq \Pi_{S_i}^c \\ s.t. \left\{ \begin{array}{l} w_i^{sc}(\delta), \Pi_{S_i}^{sc}(\delta) \textit{ and } \Pi_A^{sc}(\delta) \textit{ are derived from solving the following problem} \\ s.t. \left\{ \begin{array}{l} p_r^s\left(w_1,\ldots,w_i,\ldots,w_n\right) = p_c^s,\ F\left(z_r^s\right) = F\left(z_c^s\right) \\ s.t. \left\{ \begin{array}{l} p_r^s\left(w_1,\ldots,w_i,\ldots,w_n\right), F\left(z_r^s\right) \textit{ are derived from solving } \max\limits_{p,z} \ \Pi_A^{sc}\left(p,z,s_c\right) \\ p_c^s, F\left(z_c^s\right),\ q_c^s \textit{ and } \Pi_{SC}^{sc} \textit{ are derived from solving } \max\limits_{p,z} \ \Pi_{SC}^{s}\left(p,z,s_c\right) \end{array} \right. \end{array} \right. \end{array} \right. \end{array} \tag{6}$$

Solving this two-stage Stackelberg game problem, we get the feasible domain of revenue keeping rate δ^*, the coordinated wholesale price w_i^{sc}, the equilibrium retail price p_c^s, the distribution function of the centralized equilibrium stock factor $F\left(z_c^s\right)$ and the equilibrium production quantity q_c^s. Furthermore, the coordinated profits of the supplier i, the assembler and the assembly system can be calculated as $\Pi_{S_i}^{sc}$, Π_A^{sc} and Π_{SC}^{sc}. (See Table 2 for the detailed analytical modeling results).

The analytical results of Sect. 4.2 are summarized in Table 2. The centralized strategy neglects the roles of the suppliers in making crucial pricing and production quantity decisions and therefore is inferior to the coordination strategy regarding the derived solutions. Thus, the centralized decision results are not shown in Table 2 and will be ruled out in coming discussions.

4.3 Comparisons and discussions of analytical results

Based on the analytical results derived above, the key findings are drawn and summarized as follows:

4.3.1 Findings from OMO mode

(1) When the suppliers make simultaneous (SI) decisions under the OMO mode, (i) the assembler's equilibrium profit is $\frac{(b-\theta)}{(b-\theta-1)}$ times of any supplier's profit. That

is, $\frac{\Pi_A^d}{\Pi_{S_i}^d} = \frac{b-\theta}{b-\theta-1}$, $i=1, 2, \ldots, n$. (ii) the supplier $(i+1)$'s profit equals the supplier i's profit. That is, $\frac{\Pi_{S_{i+1}}^d}{\Pi_{S_i}^d} = 1$, $i=1, 2, \ldots, n$. Thus, all suppliers gain the same profit even though their production costs may be different. (iii) the equilibrium profits of the assembly system and its members decrease as the number of the suppliers increases.

(2) When the suppliers make sequential (SE) decisions under the OMO mode, (i) the assembler's equilibrium profit is $\left[\frac{(b-\theta)}{(b-\theta-1)}\right]^{n-i+1}$ times of the supplier i's profit. That is, $\frac{\Pi_A^{d'}}{\Pi_{S_i}^{d'}} = \left(\frac{b-\theta}{b-\theta-1}\right)^{n-i+1}$, $i=1, 2, \ldots, n$. (ii) the supplier $i+1$ gains $\frac{(b-\theta)}{(b-\theta-1)}$ times of the supplier i's profit. That is, $\frac{\Pi_{S_{i+1}}^{d'}}{\Pi_{S_i}^{d'}} = \frac{b-\theta}{b-\theta-1}$, $i=1, 2, \ldots, n$. Thus, the supplier who moves later, gains more profits, i.e., there exists a last-mover advantage when the suppliers make sequential decisions. (iii) the equilibrium profits of the assembly system and its members decrease as the number of the suppliers increases.

(3) Under the OMO mode, only when $\phi^* \in \left[\underline{\phi}, \bar{\phi}\right]$, would the members of assembly system have the economic incentive to coordinate the supply chain and achieve Pareto improvement of operational performance. Hereinto,

$$\underline{\phi} = \max\left\{\left(\frac{b-\theta-n}{b-\theta}\right)^{b-\theta-1}, \left(\frac{b-\theta-1}{b-\theta}\right)^{n(b-\theta-1)}\right\},$$

$$\bar{\phi} = \min_{i\in N}\left\{1 - \frac{\sum_{i=1}^{n} c_i}{c_i}\max\left\{\frac{b-\theta-1}{b-\theta-n}\left(\frac{b-\theta-n}{b-\theta}\right)^{b-\theta}, \left(\frac{b-\theta-1}{b-\theta}\right)^{n(b-\theta)-i+1}\right\}\right\}$$

(4) Under the OMO mode, (i) when the decentralized strategy is taken, the retail price under SE decision is lower than that of SI decision, the ordering quantity of SE decision is larger than that of SI decision, and the profits of SE decision are higher than those of SI decision. In brief, the SE strategy can deliver better results than the SI strategy. (ii) the retail price of the coordination strategy is lower than that of SE strategy, the ordering quantity of the coordination strategy is larger than that of SE strategy, and the profits of coordination strategy are higher than those of SE strategy. In brief, the coordination strategy can deliver better results than the decentralized strategies.

4.3.2 Findings from OMI mode

(1) Under OMI mode, as the IMS effort-elasticity index of the expected demand increases, the IMS effort will increase, the ordering quantity of the final product and the profit of the assembly system will first decrease and then increase. Furthermore, as the IMS fee increases, the IMS effort will increase, the ordering quantity of the final product and the profit of the assembly system will increase; As the cost coefficient of IMS effort increases, the IMS effort will decrease, the

ordering quantity of the final product and the profit of the assembly system will decrease.

(2) Under OMI mode, a *bidimensional incentive region matrix* can be formulated to identify the incentive conditions of introducing and providing IMS and determine the cooperation region between AS and IP (see Table 3). This *bidimensional incentive region matrix* is composed of two key incentive ratio indicators α and β. When α and β are in different intervals, different situations can be derived and discussed:

(i) In quadrant I, $\alpha \geq 1$ and $0 < \beta < 1$, the AS would have the incentive to introduce IMS, while the IP would not have the incentive to provide IMS, i.e., the cooperation between the AS and IP failed.

(ii) In quadrant II, $0 < \alpha < 1$ and $0 < \beta < 1$, the AS would not have the incentive to introduce IMS, and the IP would not have the incentive to provide IMS, i.e., the cooperation between the AS and IP failed.

(iii) In quadrant III, $0 < \alpha < 1$ and $\beta \geq 1$, the AS would not have the incentive to introduce IMS, while the IP would have the incentive to provide IMS, i.e., the cooperation between the AS and IP failed.

(iv) In quadrant IV, $\alpha \geq 1$ and $\beta \geq 1$, the AS would have the incentive to introduce IMS, and the IP would have the incentive to provide IMS, i.e., the cooperation between the AS and IP can be achieved.

Apparently, only when key incentive ratio indicators $\alpha \geq 1$ and $\beta \geq 1$, would the AS and IP have the incentive to introduce and provide integrated management service (IMS).

(3) Under OMI mode, only when $\delta^* \in \left[\underline{\delta}, \bar{\delta}\right]$, would the members of assembly system have the economic incentive to coordinate the supply chain and achieve Pareto improvement of operational performance. Hereinto, $\underline{\delta} = \frac{\phi^* \Pi_{SC}^{c}}{\Pi_{SC}^{sc}}$, $\bar{\delta} = \frac{\Pi_{SC}^{sc} - (1-\phi^*)\Pi_{SC}^{c}}{\Pi_{SC}^{sc}}$.

Table 3 Bidimensional Incentive Region Matrix

		IP	
		$0 < \beta < 1$	$\beta \geq 1$
AS	$\alpha \geq 1$	Quadrant I. *AS introduces IMS, and IP does not provide IMS.* Cooperation failed.	Quadrant IV. *AS introduces IMS, and IP provides IMS.* Cooperation achieved.
	$0 < \alpha < 1$	Quadrant II. *AS does not introduce IMS, and IP does not provide IMS.* Cooperation failed.	Quadrant III. *AS does not introduce IMS, and IP provides IMS.* Cooperation failed.

4.3.3 Findings from OMI mode versus OMO mode

(1) OMI mode versus OMO mode, (i) the optimal stock factor under OMI mode equals that under OMO mode; (ii) the optimal retail price of the final product under OMI mode is ρ^{-1} times of that under OMO mode; (iii)the optimal ordering quantity of the final product under OMI mode is $\rho\alpha$ times of that under OMO mode; (iv) the optimal profit of the supply chain under OMI mode is α times of that under OMO mode. That is, $\frac{z_c^s}{z_c} = 1$, $\frac{p_c^s}{p_c} = \frac{1}{\rho}$, $\frac{q_c^s}{q_c} = \rho\alpha$, $\frac{\Pi_{SC}^{sc}}{\Pi_{SC}^{c}} = \alpha$. Hereinto, $\rho = \frac{c+\lambda c_e+(1-\lambda)c_s+\sum_{i=1}^{n} c_i}{c+\lambda c_e+(1-\lambda)c_s+\sum_{i=1}^{n} c_i+t}$, $\alpha \equiv \rho^{b-\theta-1} s_c^{\kappa}$.

(2) Under the OMO mode and OMI mode, as the price-elasticity index of the expected demand decreases, the ordering quantity of the final product will increase, and the profit of the assembly system will also increase. Furthermore, as the mutual fusion coefficient between channels increases, the ordering quantity of the final product and the profit of the assembly system will increase. Besides, as the module costs, the assembly cost, the operational cost of the online- or offline- channel decrease, the optimal retail price will decrease, and the optimal ordering quantity and the optimal profits of the assembly system and its members will increase.

(3) Be it under OMO mode or OMI mode, the revenue sharing contract mechanism can effectively coordinate the members of the AS to make the best pricing and quantity decisions that create the best profits for all members. Besides, the supplier who incurs more costs gains more coordinated profits in the coordination decision of assembly system.

The numerical and sensitivity analyses conducted in the next section validate and reveal the key analytical findings of this section with a real example, thus providing a more powerful explanation for the theoretical findings and comparisons drawn in this section.

5 Numerical and sensitivity analyses

An electronic product is selected from the Chinese market for the purpose of numerical and sensitivity analyses [36]. The setting of parameters and their values are listed in Table 4. They will serve as inputs to the analytical models developed in Sect. 4.1, 4.2 and 4.3.

The AS is composed of four key module suppliers and one assembler with direct omnichannel, i.e., $n = 4$, and $i = 1, 2, 3, 4$. The unit costs of the modules are represented by c_1, c_2, c_3, and c_4 valued at 149, 53, 80 and 60 USD/unit respectively. The unit assembly cost of the final product c is 50 USD/unit. The operational cost of the online channel c_e is 26 USD/unit and the operational cost of the offline channel c_s is 39 USD/unit. The IMS fee t is 1 USD/unit. The cost coefficient of IMS effort g is $1E+6$. The salvage discount price factor η is set at 50%. The market demand share of the online channel λ is set at 0.6. The maximum

Table 4 Parameters setting

	Parameters	Value
c	Assembly cost (USD/unit)	50
c_1	1st module cost (USD/unit)	149
c_2	2nd module cost (USD/unit)	53
c_3	3rd module cost (USD/unit)	80
c_4	4th module cost (USD/unit)	60
c_e	Operational cost of the online channel (USD/unit)	26
c_s	Operational cost of offline channel (USD/unit)	39
t	Integrated management service (IMS) fee (USD/unit)	1
g	Cost coefficient of IMS effort	1E+6
a	Maximum possible demand	5E+17
b	Price-elasticity index of the expected demand	5.0
θ	Mutual fusion coefficient between channels	0.5
κ	IMS effort-elasticity index of the expected demand	0.5
η	Salvage discount price factor	50%
λ	Market demand share of the online channel	0.6
·	Revenue keeping rate under OMO mode	0.5
δ	Revenue keeping rate under OMI mode	0.5
μ	Mean value of random factor	100
σ	Standard deviation of random factor	10

possible demand a is set at 5E+17. The price-elasticity index of expected demand b is set at 5.0. The mutual fusion coefficient between channels θ is set at 0.5. The IMS effort-elasticity index of the expected demand κ is set at 0.5. The revenue keeping rate under OMO mode ϕ is 0.5. The revenue keeping rate under OMI mode δ is 0.5. The random factor x obeys normal distribution, i.e. $x \sim N(\mu, \sigma^2)$, and $\mu = 100$, $\sigma = 10$. A is set at 0.1 and B is set at 1000.

5.1 Numerical analysis

The numerical analysis results of all models are shown in Table 5. The findings are summarized and discussed below:

Simultaneous (SI) versus Sequential (SE) versus Coordination (CO) Decisions under OMO Mode For all model types under OMO mode, it is found that: (1) the stock factor: $SI = SE = CO$; (2) the retail price: $CO < SE < SI$; (3) the ordering quantity: $CO > SE > SI$; and (4) the profits of the AS and its members: $CO > SE > SI$. Obviously, coordination (CO) decision outperforms the other decisions regarding key indicators of operational performance.

OMI Mode versus OMO Mode Since coordination decision performs best under OMO mode, we will just compare coordination decision under OMI mode with that under OMO mode. Comparing the coordination decision under *OMI* mode with that under *OMO* mode in Table 5, it is found that: (1) based on the

Table 5 Numerical Analysis Results under OMO Mode and OMI Mode

Scenarios	OMO Mode			OMI Mode
Outcomes	Decentralized strategy		Coordination strategy (CO)	Coordination strategy (COI)
	Simultaneous (SI)	Sequential (SE)		
s_*	NA	NA	NA	2.33
z_*	99	99	99	99
p_*	6418	1949	713	714
q_*	365	78,036	7192,731	10,899,326
w_1^*	1238	305	75	75
w_2^*	1142	253	27	27
w_3^*	1169	337	40	40
w_4^*	1149	391	30	30
$\Pi_{S_1}^*$	511,558	33,168,154	559,388,948	849,213,194
$\Pi_{S_2}^*$	397,879	12,137,896	243,710,390	369,978,848
$\Pi_{S_3}^*$	397,879	15,605,867	86,688,931	131,603,214
$\Pi_{S_4}^*$	397,879	20,064,686	130,851,216	198,646,361
Π_A^*	397,879	25,797,453	98,138,412	148,984,771
Π_{SC}^*	2,103,073	106,774,057	1,118,777,897	1,698,426,388
Π_{IP}^*	NA	NA	NA	8,174,494
ϕ^*, δ^*	NA	NA	$\phi^* \in [0.03, 0.87]; set at 0.5$	$\delta^* \in [0.33, 0.67]; set at 0.5$
α, β	NA	NA	NA	$\alpha = 1.52 > 1, \beta = 4.00 > 1$

bidimensional incentive region matrix derived above, the dual incentive ratio indicators $\alpha = 1.52 \geq 1$ and $\beta = 4.00 \geq 1$, i.e., the dual incentive ratio indicators are in the quadrant $\{\alpha \geq 1 \text{ and } \beta \geq 1\}$ of *bidimensional incentive region matrix*, thus, AS and IP would have the incentive to introduce and provide IMS, and the cooperation between AS and IP can be achieved under OMI mode; (2) the stock factor under OMI mode equals that under OMO mode. (3) the retail price under OMI mode is higher than that under OMO mode; (4) the ordering quantity under OMI mode is larger than that under OMO mode; (5) the profits of the AS and its members under OMI mode are higher than those under OMO mode.

Demand growth effect versus cost increasing effect Under OMI mode, the introduction of IMS can enhance the value added for consumers, and lead to a growth in demand. The profit growth effect brought about by this demand growth can be called the demand growth effect. Meanwhile, this demand growth is at the expense of IMS fee/cost payment. This IMS fee/cost payment effect caused by this demand growth can be called the cost increasing effect. On this basis, we can calculate that the demand growth effect is 590,547,817 and the cost increasing effect is 10,899,326. The demand growth effect is stronger than the cost increasing effect. Therefore, it is worth introducing IMS for the AS to improve the operational performance.

Coordination mechanism based on revenue sharing contract Be it under OMO mode or OMI mode, a revenue sharing contract can effectively coordinate the AS and achieve better operational performances for its members.

Summary Across the game-theoretical decision models, we have noticed several phenomena. First, the worst strategy is the *SI* strategy which causes an extremely low demand for the assembled product due to its exceptionally high retail price. Second, the *SE* strategy provides much better solutions and higher profits than the *SI* strategy. Third, the coordinated strategy outperforms the decentralized strategy regarding the profits of AS.

In sum, through the above pairwise model comparisons, the coordination strategy with integrated management service (IMS) is identified as the best operational strategy for the AS. This finding provides profound practical implications for IMS adopting decision and operational strategy selection for the AS, which helps enhance quick response ability and improve operational performance.

5.2 Sensitivity analysis

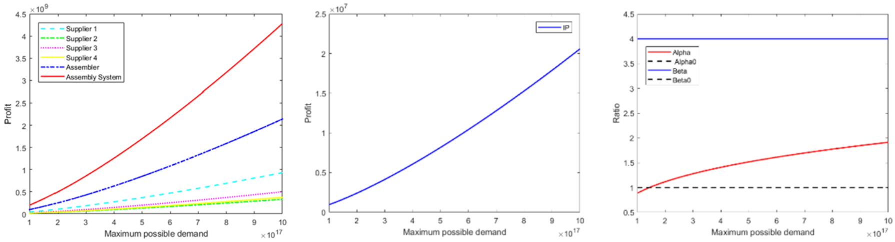
Since, from the analysis in Sect. 5.1, OMI mode is found to be the most attractive business mode for omnichannel practitioners, the sensitivity analysis will focus on how the changes of seven key parameters of the models under OMI mode impact the profits of the members in the AS. Seven key parameters are: *maximum possible demand* (a), *online channel demand share* (λ), *price-elasticity index of the expected demand* (b), *mutual fusion coefficient between channels* (θ), *IMS effort-elasticity index of the expected demand* (κ), *cost coefficient of IMS effort* (g), and *IMS fee* (t). The increment scale and range of change of each parameter in the sensitivity analysis are listed in Table 6.

5.2.1 Maximum possible demand (*a*)

The sensitivity analysis results of the maximum possible demand (a) are shown in Fig. 2. There is a positive relationship between profits and a. The profits of suppliers, assembler, AS and IP increase as the maximum possible demand (a) increases.

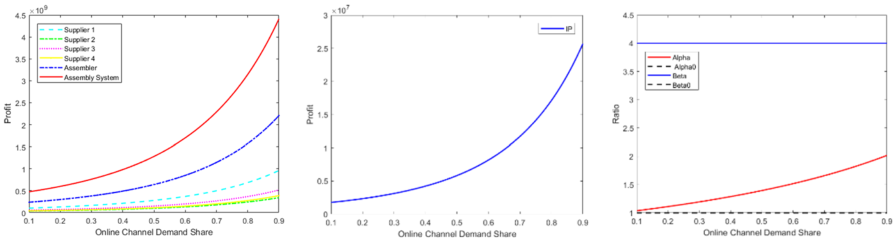
Table 6 Ranges of key parameters for sensitivity analysis

	Parameters	Original value	± Increment	Range
a	Maximum possible demand	5E+17	1E+16	[1E+17, 1E+18]
λ	Online channel demand share	0.6	0.01	[0.1, 0.9]
b	Price-elasticity index of the expected demand	5.0	0.01	[5.0, 6.0]
θ	Mutual fusion coefficient between channels	0.5	0.01	[0.1, 0.9]
κ	IMS effort-elasticity index of the expected demand	0.5	0.01	[0.1, 0.9]
g	Cost coefficient of IMS effort	1E+6	1E+4	[1E+5, 1E+6]
t	IMS fee	1.0	0.1	[1.0, 10.0]



(a) Profits of AS (b) Profits of IP (c) Incentive Ratio Indicators

Fig. 2 Impact of *maximum possible demand* (a) Change on Profits and Incentive Ratio Indicators



(a) Profits of AS (b) Profits of IP (c) Incentive Ratio Indicators

Fig. 3 Impact of *online channel demand share* (λ) Change on Profits and Incentive Ratio Indicators

The AS' s incentive ratio indicator of adopting IMS (α) increases as the maximum possible demand (a) increases. Especially, when the maximum possible demand (a) is less than about 1.5E+17, $\alpha<1$, i.e., the AS does not have the incentive to adopt IMS; on the contrary, when the maximum possible demand (a) is more than about 1.5E+17, $\alpha>1$, i.e., the AS has the incentive to adopt IMS.

The IP' s incentive ratio indicator of providing IMS (β) remains unchanged as the maximum possible demand (a) increases. Furthermore, The IP' s incentive ratio indicator of providing IMS $\beta>1$, i.e., the IP always has the incentive to provide IMS.

The finding may imply that, when the market scale reaches a certain degree, the AS and IP would have the incentive to adopt and provide IMS.

5.2.2 Online channel demand share (λ)

The sensitivity analysis results of the online channel demand share (λ) are shown in Fig. 3. It is clear there is a positive exponential relationship between profits and λ. The profits of suppliers, assembler, AS and IP exponentially increase as the online channel demand share (λ) increases.

The AS' s incentive ratio indicator of adopting IMS (α) increases as the online channel demand share (λ) increases. Furthermore, the AS' s incentive ratio indicator of adopting IMS $\alpha>1$, i.e., the AS always has the incentive to adopt IMS.

The IP' s incentive ratio indicator of providing IMS (β) remains unchanged as the online channel demand share (λ) increases. Furthermore, The IP' s incentive ratio indicator of providing IMS $\beta > 1$, i.e., the IP always has the incentive to provide IMS.

The finding may imply that, regardless of how much online channel demand share (λ) is, the AS and IP always have the incentive to adopt and provide IMS.

5.2.3 Price-elasticity index of the expected demand (b)

The sensitivity analysis results of the price-elasticity index of the expected demand (b) are shown in Fig. 4. It is clear there is a reverse exponential relationship between profits and b. The profits of suppliers, assembler, AS and IP exponentially decrease as the price-elasticity index of the expected demand (b) increases.

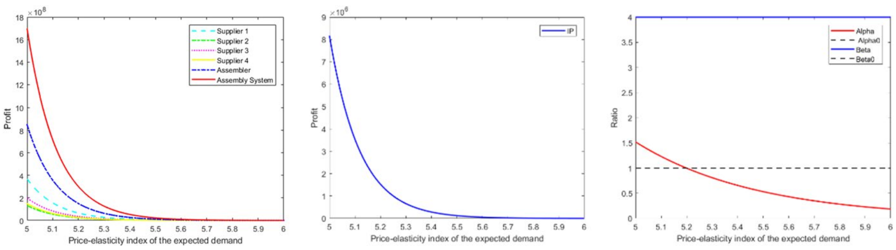
The AS' s incentive ratio indicator of adopting IMS (α) decreases as the price-elasticity index of the expected demand (b) increases. Especially, when the price-elasticity index of the expected demand (b) is less than about 5.2, $\alpha > 1$, i.e., the AS has the incentive to adopt IMS; on the contrary, when the price-elasticity index of the expected demand (b) is more than about 5.2, $\alpha < 1$, i.e., the AS does not have the incentive to adopt IMS.

The IP's incentive ratio indicator of providing IMS (β) remains unchanged as the price-elasticity index of the expected demand (b) increases. Furthermore, The IP' s incentive ratio indicator of providing IMS $\beta > 1$, i.e., the IP always has the incentive to provide IMS.

The finding may imply that, when the price-elasticity index of the expected demand is lower than a certain value, the AS and IP would have the incentive to adopt and provide IMS.

5.2.4 Mutual fusion coefficient between channels (θ)

The sensitivity analysis results of the mutual fusion coefficient between channels (θ) are shown in Fig. 5. It is clear there is a positive exponential relationship between profits and θ. The profits of suppliers, assembler, AS and IP exponentially increase as the mutual fusion coefficient between channels (θ) increases.



(a) Profits of AS (b) Profits of IP (c) Incentive Ratio Indicators

Fig. 4 Impact of *Price Elasticity Index of the Expected Demand* (b) Change on Profits and Incentive Ratio Indicators

The AS’ s incentive ratio indicator of adopting IMS (α) increases as the mutual fusion coefficient between channels (θ) increases. Especially, when the mutual fusion coefficient between channels (θ) is less than about 0.3, $\alpha < 1$, i.e., the AS does not have the incentive to adopt IMS; on the contrary, when the mutual fusion coefficient between channels (θ) is more than about 0.3, $\alpha > 1$, i.e., the AS has the incentive to adopt IMS.

The IP’ s incentive ratio indicator of providing IMS (β) remains unchanged as the mutual fusion coefficient between channels (θ) increases. Furthermore, The IP’ s incentive ratio indicator of providing IMS $\beta > 1$, i.e., the IP always has the incentive to provide IMS.

The finding may imply that, when the mutual fusion coefficient between channels (θ) is higher than a certain value, the AS and IP would have the incentive to adopt and provide IMS.

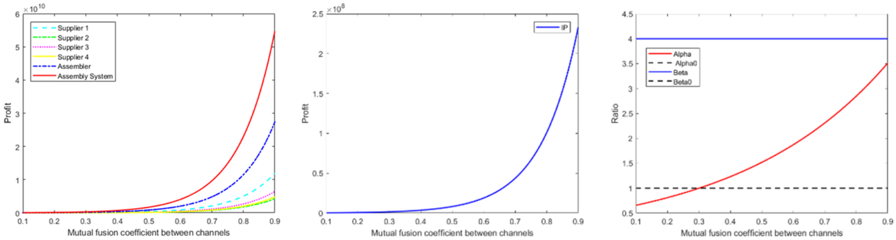
5.2.5 IMS effort-elasticity index of the expected demand (κ)

The sensitivity analysis results of the IMS effort-elasticity index of the expected demand (κ) are shown in Fig. 6. It is clear there is a positive exponential relationship between profits and κ. The profits of suppliers, assembler, AS and IP exponentially increase as the IMS effort-elasticity index of the expected demand (κ) increases.

The AS’s incentive ratio indicator of adopting IMS (α) increases as the IMS effort-elasticity index of the expected demand (κ) increases. Furthermore, the AS’ s incentive ratio indicator of adopting IMS $\alpha > 1$, i.e., the AS always has the incentive to adopt IMS.

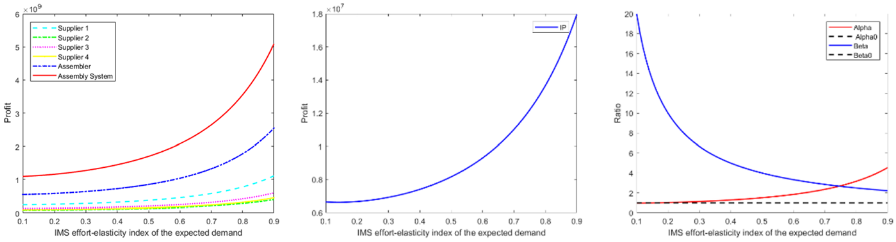
The IP’s incentive ratio indicator of providing IMS (β) decreases as the IMS effort-elasticity index of the expected demand (κ) increases. Furthermore, The IP’s incentive ratio indicator of providing IMS $\beta > 1$, i.e., the IP always has the incentive to provide IMS.

The finding may imply that, regardless of how much IMS effort-elasticity index of the expected demand (κ) is, the AS and IP always have the incentive to adopt and provide IMS.



(a) Profits of AS (b) Profits of IP (c) Incentive Ratio Indicators

Fig. 5 Impact of *Mutual fusion Coefficient* (θ) Change on Profits and Incentive Ratio Indicators



(a) Profits of AS (b) Profits of IP (c) Incentive Ratio Indicators

Fig. 6 Impact of *IMS effort-elasticity index of the expected demand* (κ) Change on Profits and Incentive Ratio Indicators

5.2.6 Cost coefficient of IMS effort (g)

The sensitivity analysis results of the cost coefficient of IMS effort (g) are shown in Fig. 4. It is clear there is a reverse exponential relationship between profits and g. The profits of suppliers, assembler, AS and IP exponentially decrease as the cost coefficient of IMS effort (g) increases.

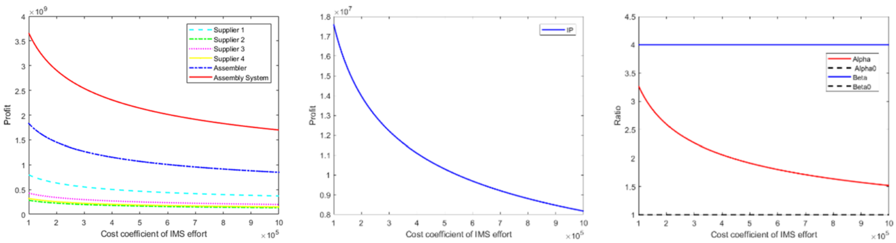
The AS' s incentive ratio indicator of adopting IMS (α) decreases as the cost coefficient of IMS effort (g) increases. Furthermore, the AS' s incentive ratio indicator of adopting IMS $\alpha > 1$, i.e., the AS always has the incentive to adopt IMS.

The IP' s incentive ratio indicator of providing IMS (β) remains unchanged as the cost coefficient of IMS effort (g) increases. Furthermore, The IP' s incentive ratio indicator of providing IMS $\beta > 1$, i.e., the IP always has the incentive to provide IMS.

The finding may imply that, regardless of how much cost coefficient of IMS effort (g) is, the AS and IP would have the incentive to adopt and provide IMS (Fig. 7).

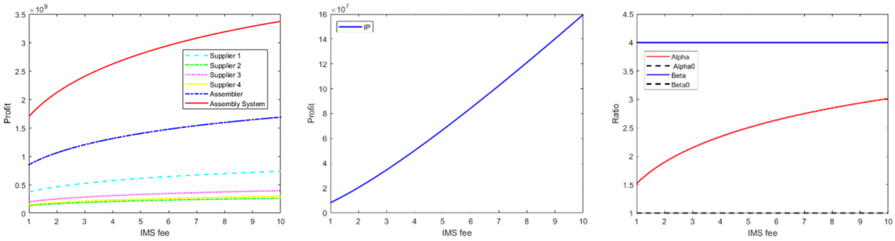
5.2.7 IMS fee (t)

The sensitivity analysis results of the IMS fee (t) are shown in Fig. 8. It is clear that there is a positive exponential relationship between profits and t. The profits of suppliers, assembler, AS and IP exponentially increase as the IMS fee (t) increases.



(a) Profits of AS (b) Profits of IP (c) Incentive Ratio Indicators

Fig. 7 Impact of *cost coefficient of IMS effort* (g) Change on Profits and Incentive Ratio Indicators



(a) Profits of AS (b) Profits of IP (c) Incentive Ratio Indicators

Fig. 8 Impact of *IMS fee* (t) Change on Profits and Incentive Ratio Indicators

The AS' s incentive ratio indicator of adopting IMS (α) increases as the IMS fee (t) increases. Furthermore, the AS' s incentive ratio indicator of adopting IMS $\alpha > 1$, i.e., the AS always has the incentive to adopt IMS.

The IP' s incentive ratio indicator of providing IMS (β) remains unchanged as the IMS fee (t) increases. Furthermore, The IP' s incentive ratio indicator of providing IMS $\beta > 1$, i.e., the IP always has the incentive to provide IMS.

The finding may imply that, regardless of how much the IMS fee (t) is, the AS and IP would have the incentive to adopt and provide IMS.

To summarize, the sensitivity analysis of seven key parameters on OMI mode provides valuable implications for both theoretical and practical understanding of the research questions.

6 Managerial insights

Despite its tremendous global growth over the last few years, ecommerce sales still represent only 8.3% of total retail sales in the U.S. [29]. Even though more and more consumers are used to buying things like books, shoes and electronics online, the majority of spending still takes place in brick and mortar outlets. In fact, apart from Amazon, all the top ten retailers in the U.S. are old-school, brick-and-mortar stores (Thau 2017). Most shoppers conduct online research before making big purchases. Being able to channel even a small percentage of these customers straight from their online research to offline stores would represent a massive potential for brick-and-mortar stores. This potential has driven major assemblers with direct retailing channel, either e-commerce or traditional retailing channel, to jump into an omnichannel commerce wagon. This study and its key theoretical findings provide new and useful theoretical and practical insights into the assembler with direct omnichannel.

6.1 Theoretical insights

Based on the foregoing discussions, the following theoretical insights can be derived and summarized as follows:

6.1.1 Operational strategies under the OMO mode

First, when the decentralized strategy is taken, sequential (SE) strategy outperforms simultaneous (SI) strategy regarding the operational performance for the AS. This is a typical 'late-mover advantage'. Second, the coordination strategy based on the revenue sharing contract (RSC) outperforms SE strategy regarding the operational performance for the AS. Therefore, the RSC based coordination strategy is the best operational strategy to increase operational performance for the AS, and the SE strategy would be the second-best choice for the AS, if the coordination strategy is ruled out due to non-economic reasons.

6.1.2 Incentives of IMS adoption and operational strategies under the OMI mode

First, only when the incentive ratio indicator $\alpha \geq 1$ would the AS have the incentive to introduce IMS, and only when the incentive ratio indicator $\beta \geq 1$ would the IMS provider (IP) have the incentive to provide IMS. In other words, only when the dual incentive ratio indicators are in the quadrant $\{\alpha \geq 1$ and $\beta \geq 1\}$ of *bidimensional incentive region matrix* would AS have the incentive to introduce IMS and IP have the incentive to provide IMS, and thus, the cooperation between AS and IP can be achieved and the operational performance of AS and IP can be improved. Second, with the help of IMS effort, the ability of collaborative operations and quick response for the AS can be effectively improved, the value added for customers can be effectively strengthened, and thus the growth in demand can be effectively reached. Besides, it should be noted that this demand growth is at the expense of IMS fee/cost payment. When the demand growth effect is stronger than the cost increasing effect (in the numerical analysis part, demand growth effect 590,547,817 > cost increasing effect 10,899,326), the operational performance of the AS can be effectively improved. Third, a coordination strategy based on revenue sharing contract (RSC) can effectively improve the operational performance of the ASI. Finally, reducing the module costs, the assembly cost, the operational costs of online- or offline- channel, and the cost of IMS effort can effectively improve the operational performance of the ASI. Furthermore, a higher maximum possible demand, a higher online channel demand share, a lower price-elasticity index of the expected demand, a higher mutual fusion coefficient between channels, a higher IMS effort-elasticity index of the expected demand, and a higher IMS fee, are conducive to the improvement of the operational performance of the ASI. Therefore, reducing operational costs and IMS costs, attracting more demand to the low-cost channel, assembling and selling a lower price elasticity product, enhancing the communication and integration between the ecommerce and physical channels, strengthening mutual fusion effects of omnichannel, introducing, adopting and providing IMS, and setting a relatively higher IMS fee, would be good marketing and operational strategies for assembly system with IMS in omnichannel business scenario.

6.2 Practical insights

From the operational management practical perspective, the practical insights can be derived and summarized as follows:

In management practice, the AS is generally operated under a decentralized decision mode with suppliers' simultaneous or sequential decision. Based on the modeling and numerical analyses and discussions, the operational performance of the AS under the decentralized decision mode is not Pareto optimal, and still has space for improvement. Due to the different parts quality, process technology, cost control and delivery ability of different suppliers, the AS under decentralized decision often faces the risk of supply interruption, and thus resulting in non-cooperative operation and slow response of the AS and forming a "neck stuck" phenomenon in the AS. This is harmful to the healthy and sustainable operations of the AS.

To address these issues, the assembler with direct omnichannel and multiple suppliers usually try to implement collaborative operations through contractual mechanism. From the theoretical perspective, the AS may achieve coordinative operations and realize Pareto improvement of the operational performance for all the stakeholders. Nevertheless, from the practical perspective, it is still quite challenging to guarantee effective implementation of the contract. Due to the complicated structure and multiple participants in the AS, the assembler has to deal with the complicated collaborative business relationships with multiple suppliers, resulting in non-cooperative operations and slow-response ability in the AS. Meanwhile, owing to the lack of effective demand information sharing and collaborative planning, forecasting and replenishment mechanism across the AS, suppliers are often unable to guarantee the perfect matching supply of parts and components for the assembly process of the assembler, which may lead to assembly disruption.

Against this background, the professional third-party, IMS providers (IPs), came into being. For example, there are many IPs in the electronics industry, such as Arrow Electronics, Avnet, WPG Holdings, Future Electronics, etc. These IMS providers (IPs) can provide various comprehensive IMS solutions for the AS to enhance their ability of quick response, boost their collaborative operations, and improve their operational performance. Specifically, they can provide a unified logistics and information flow management platform for the AS, enhance the demand information sharing between the assembler and multiple suppliers and help multiple suppliers collaboratively carry out matching collection and distribution of the corresponding parts and components to the assembler in accordance with the assembly requirements of product. With the help of IPs, the ability of collaborative operations and quick response can be effectively enhanced, the mutual fusion effect between online- and offline- channels can be effectively strengthened, the value added for customers can be effectively raised, the demand can be effectively increased and the operational performance can be effectively improved in the AS.

Now, research questions aforementioned can be answered. First, it is worth introducing IMS for the AS to enhance their ability of quick response, boost their collaborative operations, and improve their operational performance. Second, a *bidimensional incentive region matrix* can be applied to identify the incentive regions for AS and IP. Only when the dual incentive ratio indicators are in the quadrant $\{\alpha \geq 1$

and $\beta \geq 1$} of *bidimensional incentive region matrix* would AS have the incentive to introduce IMS and IP have the incentive to provide IMS, and thus, the cooperation between AS and IP can be achieved and the operational performance of AS and IP can be improved. Third, coordination strategy based on revenue sharing contract (RSC) can effectively improve the operational performance of the ASI. Therefore, introducing and providing IMS will not only improve the operational performance for the AS, but also contribute to the development of integrated management service (IMS) industry and cultivation of new economic growth points from the perspective of macro economy.

In order to boost the healthy and sustainable development of IMS industry, effective governance policies and scientific industry standards should be established and improved, and corresponding fiscal and tax policy support should be established and implemented according to the practical situation. Besides, advanced and applicable IMS technologies and corresponding comprehensive solutions can be spread and applied to assembly systems with direct omnichannel. Furthermore, IMS providers with professional technology and strong financing ability can be cultivated, supported and expanded, assembly systems with direct omnichannel can be encouraged to introduce IMS, and the competition order of IMS market and development environment of IMS industry should be regulated and optimized.

In brief, when certain conditions are met, introducing and providing IMS is not only beneficial to improving the operational performance of AS, but also conducive to developing integrated management service (IMS) industry and cultivating new economic growth points. A coordination strategy based on revenue sharing contract (RSC) can effectively improve the operational performance of AS and IP.

7 Conclusion

In the context of the development of the omnichannel retailing mode, introducing and providing integrated management service (IMS) and corresponding comprehensive solutions have important theoretical value and practical significance for the assembly system with direct omnichannel (AS) to optimize their operational performance, further develop IMS industry, and realize new economic growth points. The incentive conditions/regions of adopting IMS and the operational strategies/performance for the AS are important issues calling for urgent solution. To tackle these issues, a centralized, two decentralized and a coordination decision models for the AS are developed and analyzed, and a centralized and a coordination decision models for the ASI are further developed, analyzed and compared. Based on an electronic product case, the corresponding numerical and sensitivity analyses are conducted. On this basis, the analytical and numerical results are compared and validated to derive managerial insights. The research results indicate that: (1) under OMO mode, the coordination strategy based on the revenue sharing contract (RSC) is the best operational strategy to improve operational performance for the AS. (2) under OMI mode, only when the dual incentive ratio indicators are in the quadrant {$\alpha \geq 1$ and $\beta \geq 1$} of *bidimensional incentive region matrix* would AS and IP have the incentive to introduce and provide

IMS. Introducing and providing IMS can effectively improve the operational performance of the AS. Furthermore, the RSC-based coordination strategy can effectively improve the operational performance of the ASI. (3) under OMI mode, reducing operational costs and IMS costs, attracting more demand to the low-cost channel, assembling and selling a lower price elasticity product, enhancing the communication and integration between the ecommerce and physical channels, strengthening mutual fusion effects of omnichannel, introducing, adopting and providing IMS, and setting a relatively higher IMS fee, can effectively improve the operational performance for the ASI.

In terms of theoretical contribution, the existing literatures seldom cover the incentive conditions and cooperation regions regarding the introduction and adoption of the integrated management service (IMS) in the AS and the corresponding operational strategies/decisions/performance for the ASI. We address the literature gaps by cross-fertilizing the areas of mechanism design, operational strategies and management theory. This paper proposes a novel and useful approach toward incentive conditions, cooperation regions and operational strategies for the ASI from the perspective of game-theoretical modeling and comparative analysis. With regard to practical contribution, we shed new light on studies of the incentives of IMS adoption in AS. This paper provides a framework for understanding when the IMS would be introduced and provided in the AS and how to achieve coordinative operations across the AS. The modeling and numerical results can be effectively used to help AS and IP make appropriate operational decisions/strategies and optimize their operational performance.

Due to the shortage of relevant literature, limited research fund and difficulty in collecting empirical data, this study focuses mainly on the theoretical exploration of the value of IMS for the assembly system in an omnichannel business mode. Even though insightful findings are discovered, there are still many important research issues worthy of further exploration in the future. First, the assembly system may be extended to a three-echelon supply chain composed of multiple complementary module suppliers, an assembler and omnichannel retailer in future research. Second, different market power and related decision structure may be taken into account in the ASI. Third, other types of coordination contracts can also be considered in the ASI. Finally, fairness concern or overconfidence of decision-makers in the assembly system can also be included in future studies.

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Compliance with ethical standards

Conflict of interest On behalf of all authors, the corresponding author states that there is no conflict of interest.

Appendix: Proofs for analytical results of game-theoretical decision models

Based on Sect. 3 modeling notations and assumptions, this section conducts an extensive game-theoretical modeling of the equilibrium and coordination conditions for the assembly system with direct omnichannel and IMS. In the models to follow, note that the superscript or subscript c represents centralized decision and coordination decision under omnichannel mode (i.e., OMO mode); the superscript or subscript d: decentralized decision with suppliers' simultaneous action under OMO mode; the superscript or subscript d': decentralized decision with suppliers' sequential action under OMO mode; the superscript or subscript sc: centralized decision and coordination decision under omnichannel mode with IMS (i.e., OMI mode).

Appendix 1: Proofs for analytical results of Sect. 4.1 game-theoretical decision models under OMO mode

Under OMO mode, the assembly system with direct omnichannel does not introduce IMS, i.e., $t = 0$, $g = 0$, $s = 1$ and $\kappa = 0$. A centralized decision model, two decentralized decision models and a coordination decision model will be developed, analyzed and compared for the assembly system with direct omnichannel in this section.

Centralized decision model

When the assembly system under OMO mode takes a centralized strategy, the optimal profit function of the system can be formulated as follows:

$$\begin{aligned}\max_{p,z} \quad \Pi_{SC}(p,z) = py(p)\cdot E[\min\{z,x\}] + \eta py(p)\cdot E\left[(z-x)^+\right] \\ -\left[c + \lambda c_e + (1-\lambda)c_s + \sum\nolimits_{i=1}^{n} c_i\right]y(p)z\end{aligned} \tag{7}$$

When the distribution of random variable x satisfies the *IGFR* condition, the first order conditions $\left(p_c, z_c\right)$ determine a unique solution to the above optimization problem. Solving the first-order condition of the optimization profit function with respect to (w.r.t.) the stock factor z and the retail price p, we can get the optimal retail price and the distribution function of the centralized optimal stock factor as follows:

$$p_c = \frac{b-\theta}{b-\theta-1}\frac{\left[c + \lambda c_e + (1-\lambda)c_s + \sum_{i=1}^{n} c_i\right]z_c}{z_c - (1-\eta)\Lambda\left(z_c\right)} \tag{8}$$

$$F\left(z_c\right) = \frac{1}{(1-\eta)(b-\theta)} + \frac{(b-\theta-1)\Lambda\left(z_c\right)}{(b-\theta)z_c} \tag{9}$$

where $\Lambda\left(z_c\right) = \int_A^{z_c} \left(z_c - x\right)f(x)dx$.

Then, we can have the centralized optimal production quantity as follows:

$$q_c = y(p_c)z_c \tag{10}$$

Substituting the optimal stock factor z_c and the optimal retail price p_c into the profit function of the assembly system under OMO mode, we can obtain the optimal profit of the assembly system as follows:

$$\Pi_{SC}^{c} = \frac{1}{b-\theta-1}\left[c + \lambda c_e + \left(1-\lambda_0\right)c_s + \sum\nolimits_{i=1}^{n} c_i\right]q_c \tag{11}$$

Decentralized decision model

Stackelberg game model (suppliers' simultaneous decisions) Under this scenario, the detailed decision sequences are as follows: all the module suppliers decide their wholesale price w_i simultaneously, and then, the assembler with direct omnichannel decides the retail price p and stock factor z. The two-stage Stackelberg game model for the decentralized assembly system with direct omnichannel can be formulated as:

$$\begin{cases} \begin{cases} \max\limits_{w_1} \; \Pi_{S_1}\left(w_1, q_d\left(w_1, \dots, w_i, \dots, w_n\right)\right) \\ \qquad\qquad\qquad\qquad \vdots \\ \max\limits_{w_i} \; \Pi_{S_i}\left(w_i, q_d\left(w_1, \dots, w_i, \dots, w_n\right)\right) \\ \qquad\qquad\qquad\qquad \vdots \\ \max\limits_{w_n} \; \Pi_{S_n}\left(w_n, q_d\left(w_1, \dots, w_i, \dots, w_n\right)\right) \end{cases} \\ s.t. \begin{cases} p_d\left(w_1, \dots, w_i, \dots, w_n\right), F\left(z_d\right), q_d\left(w_1, \dots, w_i, \dots, w_n\right) \\ \textit{are derived from solving the following problem} \\ \max\limits_{p,z} \; \Pi_A(p,z) \end{cases} \end{cases}$$

(1) Assembler's Decision

When the assembler takes a decentralized strategy, the optimal profit function can be formulated as follows:

$$\begin{aligned} \max_{p,z} \; \Pi_A(p,z) = & py(p) \cdot E[\min\{z,x\}] + \eta py(p) \cdot E\left[(z-x)^+\right] \\ & - \left[c + \lambda c_e + (1-\lambda)c_s + \sum\nolimits_{i=1}^{n} w_i\right]y(p)z \end{aligned} \tag{12}$$

Solving the first-order condition of the optimization problem with respect to the stock factor z and the retail price p, we can get the reaction function of optimal retail price w.r.t. the wholesale price and the distribution function of the optimal stock factor as follows:

$$p_d\left(w_1,\dots,w_i,\dots,w_n\right)=\frac{c+\lambda c_e+(1-\lambda)c_s+\sum_{i=1}^{n}w_i}{c+\lambda c_e+(1-\lambda)c_s+\sum_{i=1}^{n}c_i}p_c \tag{13}$$

$$F\left(z_d\right)=F\left(z_c\right) \tag{14}$$

where $\Lambda\left(z_d\right)=\int_A^{z_d}\left(z_d-x\right)f(x)dx$.

Then, we have the reaction function of the decentralized optimal production quantity w.r.t. the wholesale price as follows:

$$q_d\left(w_1,\dots,w_i,\dots,w_n\right)=\left[\frac{c+\lambda c_e+(1-\lambda)c_s+\sum_{i=1}^{n}c_i}{c+\lambda c_e+(1-\lambda)c_s+\sum_{i=1}^{n}w_i}\right]^{b-\theta}q_c \tag{15}$$

(2) Suppliers' Simultaneous Decisions

For the case when the module suppliers make a simultaneous decision, substituting the reaction function of the optimal production quantity w.r.t. the wholesale price $q_d\left(w_1,\dots,w_i,\dots,w_n\right)$ into the module supplier i's profit function, we can obtain the optimal profit function for the module supplier i as follows:

$$\max_{w_i}\ \Pi_{S_i}\left(w_i\right)=\left(w_i-c_i\right)q_d\left(w_1,\dots,w_i,\dots,w_n\right),\quad i=1,2,\dots,n \tag{16}$$

Solving the first-order condition of the module supplier i's profit function with respect to the wholesale price w_i, and deriving the reaction function of the supplier i's wholesale price w_i w.r.t. the other suppliers' wholesale price $\left\{w_1,\dots,w_{i-1},w_{i+1},\dots,w_n\right\}$, when condition $b-\theta>n$ holds, we can obtain the unique Nash-equilibrium wholesale price w_i^d. of the ith supplier as follows:

$$w_i^d=\frac{1}{b-\theta-n}\left[c+\lambda c_e+(1-\lambda)c_s+\sum_{i=1}^{n}c_i\right]+c_i,\quad i=1,2,\dots,n \tag{17}$$

Plugging the supplier i's equilibrium wholesale price w_i^d into the reaction function of the optimal retail price w.r.t. the wholesale price $p_d\left(w_1,\dots,w_i,\dots,w_n\right)$, and the reaction function of the optimal production quantity w.r.t. the wholesale price $q_d\left(w_1,\dots,w_i,\dots,w_n\right)$, then we can get the equilibrium retail price, the equilibrium stock factor and the equilibrium ordering quantity as follows:

$$p_d=\frac{b-\theta}{b-\theta-n}p_c \tag{18}$$

$$F\left(z_d\right)=F\left(z_c\right) \tag{19}$$

$$q_d=\left(\frac{b-\theta-n}{b-\theta}\right)^{b-\theta}q_c \tag{20}$$

Plugging the equilibrium stock factor, the equiliium retail price, and the equilibrium ordering quantity into the profit functions, we can obtain the equilibrium profits of the module supplier i, the assembler and the assembly system as follows:

$$\Pi_{S_i}^{d}=\frac{b-\theta-1}{b-\theta}\left(\frac{b-\theta-n}{b-\theta}\right)^{b-\theta-1}\Pi_{SC}^{c},\quad i=1,2,\ldots,n \tag{21}$$

$$\Pi_{A}^{d}=\left(\frac{b-\theta-n}{b-\theta}\right)^{b-\theta-1}\Pi_{SC}^{c} \tag{22}$$

$$\Pi_{SC}^{d}=\left[(n+1)-\frac{n}{b-\theta}\right]\left(\frac{b-\theta-n}{b-\theta}\right)^{b-\theta-1}\Pi_{SC}^{c} \tag{23}$$

Stackelberg game model (suppliers' sequential decisions) Under this scenario, the detailed decision sequences are as follows: all the module suppliers first make their wholesale price w_i sequentially: the module supplier 1 first decides his wholesale price w_1, and then, the module supplier 2 decides his wholesale price w_2 based on supplier 1's decision, …, and finally, the module supplier n decides his wholesale price w_n based on decisions of suppliers 1, 2,…,n-1; and then, the assembler with direct omnichannel decides the retail price p and stock factor z. The $(n+1)$-stage Stackelberg game model for the decentralized assembly system with direct omnichannel can be formulated as:

$$\begin{cases}\begin{cases}\max\limits_{w_1}\quad \Pi_{S_1}\left(w_1,q_{d'}\left(w_1,\ldots,w_i,\ldots,w_n\right)\right)\\ s.t.\ \max\limits_{w_2}\quad \Pi_{S_2}\left(w_2|w_1,q_{d'}\left(w_1,\ldots,w_i,\ldots,w_n\right)\right)\\ \vdots\\ s.t.\ \max\limits_{w_i}\quad \Pi_{S_i}\left(w_i|w_1,w_2,\ldots,w_{i-1},q_{d'}\left(w_1,\ldots,w_i,\ldots,w_n\right)\right)\\ \vdots\\ s.t.\ \max\limits_{w_n}\quad \Pi_{S_n}\left(w_n|w_1,w_2,\ldots,w_{n-1},q_{d'}\left(w_1,\ldots,w_i,\ldots,w_n\right)\right)\end{cases}\\ s.t.\begin{cases}p_{d'}\left(w_1,\ldots,w_i,\ldots,w_n\right),F\left(z_{d'}\right),q_{d'}\left(w_1,\ldots,w_i,\ldots,w_n\right)\\ \textit{are derived from solving the following problem}\\ \max\limits_{p,z}\quad \Pi_A(p,z)\end{cases}\end{cases}$$

(1) Assembler's Decision

Likewise, when the assembler takes a decentralized strategy, the optimal profit function can be formulated as follows:

$$\max_{p,z} \quad \Pi_A(p,z) = py(p) \cdot E[\min\{z,x\}] + \eta py(p) \cdot E\left[(z-x)^+\right] - \left[c + \lambda c_e + (1-\lambda)c_s + \sum_{i=1}^{n} w_i\right] y(p)z$$

Solving the first-order condition of the optimization problem with respect to the stock factor z and the retail price p, we can get the reaction function of optimal retail price w.r.t. the wholesale price and the distribution function of the optimal stock factor as follows:

$$p_{d'}\left(w_1,\dots,w_i,\dots,w_n\right) = \frac{c + \lambda c_e + (1-\lambda)c_s + \sum_{i=1}^{n} w_i}{c + \lambda c_e + (1-\lambda)c_s + \sum_{i=1}^{n} c_i} p_c$$

$$F\left(z_{d'}\right) = F\left(z_c\right)$$

where $\Lambda\left(z_{d'}\right) = \int_A^{z_{d'}} \left(z_{d'} - x\right) f(x)dx$

Then, we have the reaction function of the decentralized optimal production quantity w.r.t. the wholesale price as follows:

$$q_{d'}\left(w_1,\dots,w_i,\dots,w_n\right) = \left[\frac{c + \lambda c_e + (1-\lambda)c_s + \sum_{i=1}^{n} c_i}{c + \lambda c_e + (1-\lambda)c_s + \sum_{i=1}^{n} w_i}\right]^{b-\theta} q_c$$

(2) Suppliers' Sequential Decisions

When the module suppliers make sequential decisions, substituting the reaction function of the optimal production quantity w.r.t. the wholesale price $q_{d'}\left(w_i\right)$ into the module supplier i's profit function, assuming the wholesale price of the 1st, 2nd, …, (n-1)th module supplier $w_1, w_2, \dots, w_{n-1}$ is given, then we can obtain the optimal profit function for the module supplier n as follows:

$$\max_{w_n} \quad \Pi_{S_n}\left(w_n | w_1, w_2, \dots, w_{n-1}\right) = \left(w_n - c_n\right) q_{d'}\left(w_1,\dots,w_i,\dots,w_n\right) \tag{24}$$

Solving the first-order condition of the module supplier n's profit function with respect to the wholesale price w_n, deriving the reaction function of the supplier n's wholesale price w_n w.r.t. the other suppliers' wholesale price $\left\{w_1,\dots,w_{n-1}\right\}$, plugging $w_n^{d'}\left(w_1,\dots,w_{n-1}\right)$ into the supplier (n-1)'s profit function, and solving the first-order condition of the module supplier (n-1)'s profit function with respect to the wholesale price w_{n-1}, we can get the reaction function of the supplier (n-1)'s wholesale price w_{n-1} w.r.t. the other suppliers' wholesale price $\left\{w_1,\dots,w_{n-2}\right\}$ as $w_{n-1}^{d'}\left(w_1,\dots,w_{n-2}\right)$; likewise, we can get $w_{n-2}^{d'}\left(w_1,\dots,w_{n-3}\right)$,…,$w_2^{d'}\left(w_1\right)$,$w_1^{d'}$, then we can obtain $w_2^{d'},\dots,w_n^{d'}$ via backward induction. Hence, for $b-\theta > 1$, the unique Nash-equilibrium wholesale price $w_i^{d'}$ of the ith supplier is as follows:

$$w_i^{d'} = \frac{(b-\theta)^{i-1}}{(b-\theta-1)^i}\left[c + \lambda c_e + (1-\lambda)c_s + \sum_{i=1}^{n} c_i\right] + c_i, \quad i = 1,2,\dots,n \tag{25}$$

Plugging the supplier i's equilibrium wholesale price $w_i^{d'}$ into the reaction function of the optimal retail price w.r.t. the wholesale price $p_{d'}\left(w_1, \ldots, w_i, \ldots, w_n\right)$, and the reaction function of the optimal production quantity w.r.t. the wholesale price $q_{d'}\left(w_1, \ldots, w_i, \ldots, w_n\right)$, then we can get the equilibrium retail price, the equilibrium stock factor and the equilibrium ordering quantity as follows:

$$p_{d'} = \left(\frac{b-\theta}{b-\theta-1}\right)^n p_c \tag{26}$$

$$F\left(z_{d'}\right) = F\left(z_c\right) \tag{27}$$

$$q_{d'} = \left(\frac{b-\theta-1}{b-\theta}\right)^{n(b-\theta)} q_c \tag{28}$$

Plugging the equilibrium stock factor, the equilibrium retail price, and the equilibrium ordering quantity into the profit functions, we can obtain the equilibrium profits of the supplier i, the assembler and the assembly system as follows:

$$\Pi_{S_i}^{d'} = \left(\frac{b-\theta-1}{b-\theta}\right)^{n(b-\theta)-i+1} \Pi_{SC}^c, \quad i = 1, 2, \ldots, n \tag{29}$$

$$\Pi_A^{d'} = \left(\frac{b-\theta-1}{b-\theta}\right)^{n(b-\theta-1)} \Pi_{SC}^c \tag{30}$$

$$\Pi_{SC}^{d'} = \left[(b-\theta)\left(\frac{b-\theta-1}{b-\theta}\right)^{n(b-\theta-1)} - (b-\theta-1)\left(\frac{b-\theta-1}{b-\theta}\right)^{n(b-\theta)}\right] \Pi_{SC}^c \tag{31}$$

Coordination decision model

Under this scenario, the detailed decision sequences are as follows: the suppliers simultaneously offer the assembler a revenue sharing contract in which suppliers charge a lower wholesale price w_i from the assembler; if the assembler accepts the contract, he will place an order with quantity q to the module suppliers, after the final product is assembled by modules or components, he will sell the final product through omnichannel at regular retail price p and decide the stock factor z when the selling season starts, and sell the leftover stock through omnichannel at salvage price ηp in the clearance season. Finally, the assembler will share a fraction $(1-\phi)$ of his net revenue to the suppliers (supplier i will get a fraction $\frac{c_i}{\sum_{i=1}^n c_i}(1-\phi)$ of the assembler's sharing revenue), where ϕ is the revenue keeping fraction of the assembler, and $0 \le \phi \le 1$. The revenue shared by the assembler to supplier i is as follows:

$$T_i = \frac{c_i}{\sum_{i=1}^{n} c_i}(1-\phi)\left\{\begin{array}{l} py(p)\cdot E[\min\{z,x\}] + \eta py(p)\cdot E\left[(z-x)^+\right] \\ -\left[c+\lambda c_e + (1-\lambda)c_s\right]y(p)z \end{array}\right\}.$$

Thus, the profit functions of the supplier i and assembler under revenue sharing contract are as follows:

$$\Pi_{S_i}^c\left(w_i\right) = \Pi_{S_i}\left(w_i\right) + T_i$$

$$\Pi_A^c(p,z) = \Pi_A(p,z) - \sum\nolimits_{i=1}^{n} T_i$$

The optimal problem for the assembly system with direct omnichannel under the revenue sharing contract is as follows:

$$\textit{Feasible domain of } \phi^* \textit{ is derived from solving } \Pi_A^c(\phi) \geq \max\left\{\Pi_A^d, \Pi_A^{d'}\right\} \textit{ and } \Pi_{S_i}^c(\phi) \geq \max\left\{\Pi_{S_i}^d, \Pi_{S_i}^{d'}\right\}$$

$$s.t.\begin{cases} w_i^c(\phi), \Pi_{S_i}^c(\phi) \textit{ and } \Pi_A^c(\phi) \textit{ are derived from solving the following problem} \\ s.t.\begin{cases} p_r\left(w_1,\ldots,w_i,\ldots,w_n\right) = p_c,\ F\left(z_r\right) = F\left(z_c\right) \\ s.t.\begin{cases} p_r\left(w_1,\ldots,w_i,\ldots,w_n\right), F\left(z_r\right) \textit{ are derived from solving } \max\limits_{p,z}\ \Pi_A^c(p,z) \\ p_c, F\left(z_c\right),\ q_c \textit{ and } \Pi_{SC}^c \textit{ are derived from solving } \max\limits_{p,z}\ \Pi_{SC}(p,z) \end{cases} \end{cases} \end{cases}$$

The optimal problem for the assembler with omnichannel under the revenue sharing contract is as follows:

$$\begin{aligned} \max_{p,z}\quad \Pi_A^c(p,z) = {} & \phi py(p)\cdot E[\min\{z,x\}] + \phi\eta py(p)\cdot E\left[(z-x)^+\right] \\ & -\left[\phi c + \phi\lambda c_e + \phi(1-\lambda)c_s + \sum\nolimits_{i=1}^{n} w_i\right]y(p)z \end{aligned} \tag{32}$$

Solving the first-order condition of the optimization profit function with respect to the stock factor z and the retail price p, we can get the reaction function of optimal retail price w.r.t. the wholesale price, the distribution function of the optimal stock factor and the reaction function of optimal ordering quantity w.r.t. the wholesale price as follows:

$$p_r\left(w_1,\ldots,w_i,\ldots,w_n\right) = \frac{\phi c + \phi\lambda c_e + \phi(1-\lambda)c_s + \sum_{i=1}^{n} w_i}{\phi c + \phi\lambda c_e + \phi(1-\lambda)c_s + \phi\sum_{i=1}^{n} c_i}p_c \tag{33}$$

$$F\left(z_r\right) = F\left(z_c\right) \tag{34}$$

$$q_r\left(w_1,\ldots,w_i,\ldots,w_n\right) = \left[\frac{\phi c + \phi\lambda c_e + \phi(1-\lambda)c_s + \phi\sum_{i=1}^{n} c_i}{\phi c + \phi\lambda c_e + \phi(1-\lambda)c_s + \sum_{i=1}^{n} w_i}\right]^b q_c \tag{35}$$

where $\Lambda\left(z_r\right) = \int_A^{z_r}\left(z_r - x\right)f(x)dx$.

To coordinate the supply chain, the following conditions need to be satisfied: $p_r\left(w_1,\dots,w_i,\dots,w_n\right)=p_c$, and $F\left(z_r\right)=F\left(z_c\right)$. Then we have the reaction function of coordinated wholesale prices w.r.t. the revenue keeping rate: $w_i^c(\phi)=\phi c_i$.

Plugging the optimal stock factor, the optimal retail price, the optimal ordering quantity and the coordinated wholesale prices into the profit functions, we can obtain the reaction functions of coordinated profits of the module supplier i and the assembler w.r.t. the revenue keeping rate as follows:

$$\Pi_{S_i}^c(\phi)=\frac{c_i}{\sum_{i=1}^n c_i}(1-\phi)\Pi_{SC}^c,\quad i=1,2,\dots,n$$

$$\Pi_A^c(\phi)=\phi\Pi_{SC}^c$$

Obviously, only when the following two conditions hold: $\Pi_A^c(\phi)\ge\max\left\{\Pi_A^d,\Pi_A^{d'}\right\}$, $\Pi_{S_i}^c(\phi)\ge\max\left\{\Pi_{S_i}^d,\Pi_{S_i}^{d'}\right\}$, the members of assembly system under OMO mode would have the economic incentive to coordinate the omnichannel and achieve Pareto improvement of operational performance. Thus, the reasonable interval of revenue keeping rate can be derived as follows: $\phi^*\in\left[\underline{\phi},\bar{\phi}\right]$. Hereinto,

$$\underline{\phi}=\max\left\{\left(\frac{b-\theta-n}{b-\theta}\right)^{b-\theta-1},\left(\frac{b-\theta-1}{b-\theta}\right)^{n(b-\theta-1)}\right\},$$

$$\bar{\phi}=\min_{i\in N}\left\{1-\frac{\sum_{i=1}^n c_i}{c_i}\max\left\{\frac{b-\theta-1}{b-\theta-n}\left(\frac{b-\theta-n}{b-\theta}\right)^{b-\theta},\left(\frac{b-\theta-1}{b-\theta}\right)^{n(b-\theta)-i+1}\right\}\right\}.$$

On this basis, the coordinated wholesale prices, the coordinated profits of the module supplier i and the assembler can be obtained as follows:

$$w_i^c=\phi^*c_i,\quad i=1,2,\dots,n \tag{36}$$

$$\Pi_{S_i}^c=\frac{c_i}{\sum_{i=1}^n c_i}(1-\phi^*)\Pi_{SC}^c,\quad i=1,2,\dots,n \tag{37}$$

$$\Pi_A^c=\phi^*\Pi_{SC}^c \tag{38}$$

The analytical results of Sect. 4.1 are summarized in Table 2. The centralized strategy neglects the roles of the suppliers in making crucial pricing and production quantity decisions and therefore is inferior to the coordination strategy regarding the derived solutions. Thus, the centralized decision results are not shown in Table 2 and will be ruled out in the coming discussions.

Appendix B: Proofs for analytical results of Sect. 4.2 game-theoretical decision models under OMI mode

Under OMI mode, the assembly system with direct omnichannel introduces IMS, i.e., $t > 0$, $g > 0$, $s > 1$ and $\kappa \in (0,1)$. With the help of integrated management services (IMS), the assembly system with direct omnichannel can achieve effective coordinative management. Thus, the decentralized decision scenario does not exist under OMI mode and will not be considered in this section. On this basis, a centralized decision model and a coordination decision model will be developed, analyzed and compared for the assembly system with direct omnichannel and IMS in this section.

Centralized decision model

Under the centralized decision model, the detailed decision sequences are as follows: the IP will first decide the IMS effort s, the assembly system will first decide the retail price p, and then decide the stock factor z. The two-stage Stackelberg game model for the assembly system with direct omnichannel and IMS under centralized decision can be formulated as:

$$\begin{cases} \max\limits_{s} \Pi_{IP}^{s}(s) \\ s.t.\ \max\limits_{p,z} \quad \Pi_{SC}^{s}(p,z,s) \end{cases}$$

When the assembly system under OMI mode takes a centralized strategy, the optimal profit function of the system can be formulated as follows:

$$\begin{aligned} \max_{p,z} \quad \Pi_{SC}^{s}(p,z) = pv(p,s)\cdot E[\min\{z,x\}] + \eta pv(p,s)\cdot E\left[(z-x)^{+}\right] \\ -\left[c+\lambda c_e + (1-\lambda)c_s + \sum\nolimits_{i=1}^{n} c_i + t\right] v(p,s)z \end{aligned} \tag{39}$$

Likewise, solving the first-order condition of the optimization profit function with respect to (w.r.t.) the stock factor z and the retail price p, we can get the distribution function of the centralized optimal stock factor and the optimal retail price as follows:

$$F\left(z_c^s\right) = F\left(z_c\right) \tag{40}$$

$$p_c^s = \frac{1}{\rho} p_c \tag{41}$$

where $\rho = \frac{c+\lambda_0 c_e + (1-\lambda_0)c_s + \sum_{i=1}^{n} c_i}{c+\lambda_0 c_e + (1-\lambda_0)c_s + \sum_{i=1}^{n} c_i + t}$.

Then, we can obtain the reaction function of optimal production quantity w.r.t. IMS effort as follows:

$$q_c^s(s) = \rho^{b-\theta} s^{\kappa} q_c \tag{42}$$

Substituting $q_c^s(s)$, z_c and p_c^s into the profit function of IP under OMI mode, we can obtain the optimal problem for IP as follows:

$$\max_s \Pi_{IP}^s(s) = t\rho^{b-\theta} s^{\kappa} q_c - \frac{1}{2} g s^2 \tag{43}$$

When the condition $\kappa(\kappa-1)t\rho^{b-\theta} q_c s^{\kappa-2} < g$ holds, we can obtain the equilibrium IMS effort as follows:

$$s_c = \left(\frac{g}{\kappa t \rho^{b-\theta} q_c} \right)^{\frac{1}{\kappa-2}} \tag{44}$$

Plugging s_c into $q_c^s(s)$, we can obtain the equilibrium order quantity and profit of assembly supply chain as follows:

$$q_c^s = \rho^{b-\theta} s_c^{\kappa} q_c \tag{45}$$

Substituting s_c,z_c,p_c^s and q_c^s into the profit functions of assembly supply chain and IP under OMI mode, we can obtain the equilibrium profit of the assembly supply chain and IP as follows:

$$\Pi_{SC}^{sc} = \rho^{b-\theta-1} s_c^{\kappa} \Pi_{SC}^{c} \tag{46}$$

$$\Pi_{IP}^{sc} = t q_c^s - \frac{1}{2} g s_c^2 \tag{47}$$

Obviously, only when the condition $\Pi_{SC}^{sc} \geq \Pi_{SC}^{c}$ holds, i.e., only when the incentive ratio indicator $\alpha \equiv \rho^{b-\theta-1} s_c^{\kappa} \geq 1$, the assembly supply chain would have incentive to introduce integrated management service (IMS). Furthermore, only when the condition $\Pi_{IP}^{sc} \geq 0$ holds, i.e., only when the incentive ratio indicator $\beta \equiv 2tg^{-1}\rho^{b-\theta} q_c s_c^{\kappa-2} \geq 1$, the IP would have incentive to provide integrated management service (IMS).

Coordination decision model

Under this scenario, the detailed decision sequences are as follows: the IP first decides IMS effort s, and then, the suppliers simultaneously offer the assembler a revenue sharing contract in which suppliers charge a lower wholesale price w_i from the assembler; if the assembler accepts the contract, he will place an order with quantity q to the module suppliers, after the final product is assembled by modules or components, he will sell the final product through omnichannel at regular retail price p and decide the stock factor z when the selling season starts, and sell the left-over stock through omnichannel at salvage price ηp in the clearance season. Finally, the assembler will share a fraction $1-\delta$ of his net revenue to the suppliers (supplier i will get a fraction $\frac{c_i}{\sum_{i=1}^{n} c_i}(1-\delta)$ of the assembler's sharing revenue), where δ is the revenue keeping fraction of the assembler, and $0 \le \delta \le 1$. The revenue shared by the assembler to supplier i is as follows:

$$T_i^s = \frac{c_i}{\sum_{i=1}^{n} c_i}(1-\delta)\left\{ \begin{array}{r} pv(p,s)\cdot E[\min\{z,x\}] + \eta pv(p,s)\cdot E\left[(z-x)^+\right] \\ -\left[c+\lambda c_e+(1-\lambda)c_s+t\right]v(p,s)z \end{array} \right\}$$

Thus, the profit functions of the supplier i and assembler under revenue sharing contract are as follows:

$$\Pi_{S_i}^{sc}\left(w_i\right) = \Pi_{S_i}^{s}\left(w_i\right) + T_i^s$$

$$\Pi_A^{sc}(p,z,s) = \Pi_A^{s}(p,z,s) - \sum_{i=1}^{n} T_i^s$$

The two-stage Stackelberg game problem for the assembly system with direct omnichannel under the revenue sharing contract is as follows:

$$\begin{array}{r} \textit{Feasible domain of } \delta^* \textit{ is derived from solving } \Pi_A^{sc}(\delta) \ge \Pi_A^c \textit{ and } \Pi_{S_i}^{sc}(\delta) \ge \Pi_{S_i}^c \\ s.t.\left\{ \begin{array}{r} w_i^{sc}(\delta), \Pi_{S_i}^{sc}(\delta) \textit{ and } \Pi_A^{sc}(\delta) \textit{ are derived from solving the following problem} \\ s.t.\left\{ \begin{array}{r} p_r^s\left(w_1,\dots,w_i,\dots,w_n\right) = p_c^s,\ F\left(z_r^s\right) = F\left(z_c^s\right) \\ s.t.\left\{ \begin{array}{r} p_r^s\left(w_1,\dots,w_i,\dots,w_n\right), F\left(z_r^s\right) \textit{ are derived from solving } \max\limits_{p,z}\ \Pi_A^{sc}\left(p,z,s_c\right) \\ p_c^s, F\left(z_c^s\right),\ q_c^s \textit{ and } \Pi_{SC}^{sc} \textit{ are derived from solving } \max\limits_{p,z}\ \Pi_{SC}^{s}\left(p,z,s_c\right) \end{array} \right. \end{array} \right. \end{array} \right. \end{array}$$

The optimal problem for the assembler under the revenue sharing contract is as follows:

$$\begin{aligned} \max_{p,z}\quad \Pi_A^{sc}(p,z) = {} & \delta pv(p,s)\cdot E[\min\{z,x\}] + \delta\eta pv(p,s)\cdot E\left[(z-x)^+\right] \\ & -\left[\delta c+\delta\lambda c_e+\delta(1-\lambda)c_s+\delta t+\sum_{i=1}^{n} w_i\right]v(p,s)z \end{aligned} \tag{48}$$

Likewise, solving the first-order condition of the optimization profit function with respect to the stock factor z and the retail price p, we can get the reaction function of optimal retail price w.r.t. the wholesale price, the distribution function of the optimal stock factor and the reaction function of optimal ordering quantity w.r.t. the wholesale price as follows:

$$p_r^s\left(w_1,\dots,w_i,\dots,w_n\right) = \frac{\delta c+\delta\lambda c_e+\delta(1-\lambda)c_s+\delta t+\sum_{i=1}^{n} w_i}{\delta c+\delta\lambda c_e+\delta(1-\lambda)c_s+\delta t+\delta\sum_{i=1}^{n} c_i}p_c^s \tag{49}$$

$$F\left(z_r^s\right) = F\left(z_c^s\right) \tag{50}$$

$$q_r^s\left(w_1,\dots,w_i,\dots,w_n\right)=\left[\frac{\delta c+\delta\lambda c_e+\delta(1-\lambda)c_s+\delta t+\delta\sum_{i=1}^{n}c_i}{\delta c+\delta\lambda c_e+\delta(1-\lambda)c_s+\delta t+\sum_{i=1}^{n}w_i}\right]^b q_c^s \quad (51)$$

Where $\Lambda\left(z_r^s\right)=\int_A^{z_r^s}\left(z_r^s-x\right)f(x)dx$

To coordinate the omnichannel in the assembly system, the following conditions need to be satisfied: $p_r^s\left(w_1,\dots,w_i,\dots,w_n\right)=p_c^s$, and $F\left(z_r^s\right)=F\left(z_c^s\right)$. Then we have the reaction function of coordinated wholesale prices w.r.t. the revenue keeping rate: $w_i^{sc}(\delta)=\delta c_i$.

Plugging the optimal stock factor, the optimal retail price, the optimal ordering quantity and the coordinated wholesale prices into the profit functions, we can obtain the reaction functions of coordinated profits of the module supplier *i* and the assembler w.r.t. the revenue keeping rate as follows:

$$\Pi_{S_i}^{sc}(\delta)=\frac{c_i}{\sum_{i=1}^{n}c_i}(1-\delta)\Pi_{SC}^{sc},\quad i=1,2,\dots,n$$

$$\Pi_A^{sc}(\delta)=\delta\Pi_{SC}^{sc}$$

Only when the following two conditions hold: $\Pi_A^{sc}(\delta)\geq\Pi_A^c$, $\Pi_{S_i}^{sc}(\delta)\geq\Pi_{S_i}^c$, the members of assembly system under OMI mode would have the economic incentive to coordinate the omnichannel and achieve Pareto improvement of operational performance. Thus, the reasonable interval of revenue keeping rate can be derived as follows: $\delta^*\in\left[\underline{\delta},\bar{\delta}\right]$. Hereinto, $\underline{\delta}=\frac{\phi^*\Pi_{SC}^c}{\Pi_{SC}^{sc}}$, $\bar{\delta}=\frac{\Pi_{SC}^{sc}-(1-\phi^*)\Pi_{SC}^c}{\Pi_{SC}^{sc}}$.

On this basis, the coordinated wholesale prices, the coordinated profits of the module supplier *i* and the assembler can be obtained as follows:

$$w_i^{sc}=\delta^*c_i,\quad i=1,2,\dots,n \quad (52)$$

$$\Pi_{S_i}^{sc}=\frac{c_i}{\sum_{i=1}^{n}c_i}(1-\delta^*)\Pi_{SC}^{sc},\quad i=1,2,\dots,n \quad (53)$$

$$\Pi_A^{sc}=\delta^*\Pi_{SC}^{sc} \quad (54)$$

The analytical results of Sect. 4.2 are summarized in Table 2. The centralized strategy neglects the roles of the suppliers in making crucial pricing and production quantity decisions and therefore is inferior to the coordination strategy regarding the derived solutions. Thus, the centralized decision results are not shown in Table 2 and will be ruled out in the coming discussions [1–3, 31].

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