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Clarifications of Certain Ambiguities and Failings of Poisson's Ratios in Linear Viscoelasticity

Harry H. Hilton

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Abstract Detailed new analytical investigations are presented describing the behavior of Class I, II and III viscoelastic Poisson's ratios (PR). Their previously demonstrated dependence on stress-time histories, which lead to the inability to consider them as universal viscoelastic material properties and the incapacity to produce a general elastic–viscoelastic correspondence principle (EVCP) based, is expanded. A new Class VI PR is analytically derived from the viscoelastic constitutive relations in the Fourier transform (FT) space to achieve the proper FT form of the elastic/viscoelastic correspondence principle, i.e., the elastic-viscoelastic analogy. However, even though this PR Class is a pure universal material property function, it still fails to provide a convenient and useful path to a correspondence principle due to its inopportune constitutive form in real time space vis-à-vis a thermodynamic model with equivalent attributes. Consequently, no general EVCP involving PRs can be formulated. The derived Class VI PRs are equivalent to the defined Class III PRs with 1-D loadings (stresses).

Keywords Elastic/viscoelastic correspondence principle · Integral-differential relations · Material characterization · Poisson's ratio · Viscoelasticity

Mathematics Subject Classification (2000) 46N20 · 47G10 · 47G20 · 47N20 · 65R20

1 Introduction

In linear elasticity, Poisson's ratio (PR) [1] is a well defined material property on par with moduli and compliances, which have all proven to be ideal vehicles for characterization of

This paper is dedicated in memory of my dear friend and cherished colleague Dr. Donald E. Carlson, Professor Emeritus of Theoretical and Applied Mechanics at UIUC, to honor his many fundamental research contributions in mechanics, and for his tireless devotion to his students and to the profession.

H.H. Hilton (✉)
Aerospace Engineering Department, College of Engineering and Private Sector Program Division, National Center for Supercomputing Applications, University of Illinois at Urbana-Champaign (UIUC), 104 S. Wright Street, 316 Talbot Lab., MC-236, Urbana, IL 61801-2935, USA
e-mail: h-hilton@illinois.edu

material behavior. These PRs have the further highly convenient attribute in that for each elastic material they are an absolute constant, possibly varying only with temperature. Unfortunately the same invariant properties and material behavior identification do not hold for viscoelastic PRs.

The fundamental difficulty with viscoelastic PRs, in contrast to elastic ones, is not in the scarcity of their definitions (at least 6 Classes) but in their utility or more precisely the lack thereof. These PRs are not universal material property identifiers due to their strong dependence on stress histories and their lack of exportability from specific to general loading conditions. Furthermore, their introduction into constitutive relations remains problematic and does not lead to EVCPs.

The fact that viscoelastic Poisson's ratios (PRs) are time dependent was conclusively demonstrated by experimental evidence as early as 1987 [2–9]. The impossibility of formulating an elastic-viscoelastic correspondence principle (analogy) (EVCP) [10, 11] based on PRs has been proven by different analytical formulations in [12–14]. Yet, publications continue to appear in respectable archival journals which miss the point by either assuming that viscoelastic PRs are constants or that the EVCP applies to PRs or by extending special PR concepts into general regions where they have been proven to be inapplicable or by using all of the above. References [15–18] are some representative recent examples.[1] The Class II PRs defined by $\nu_{12}^{II}(x,t) \stackrel{\text{def}}{=} \epsilon_{22}(x,t)/\epsilon_{11}^{0}(x)$ for time independent strains in the 1-D loaded direction are sensitive to a number of non-material contributions. However, PR values obtained from Class II experiments are of little import unless the data properly captures the initial stress and strain buildups in time [19–21]. The initial transient loading effects provide major contributions to material characterizations due to viscoelastic energy dissipation and memory properties [7–9, 19–21].

The Class III PR definition [12–14] as the ratio of two transforms of perpendicular strains has no physical denotation and is less than useful in its time-space inversion counterpart. Furthermore, it can be shown that PRs cannot be part of viscoelastic constitutive relations and that they cannot encompass the EVCP.

Hence, extreme caution must be exercised when applying any of these three viscoelastic PRs as they are not universal material property functions. Unlike relaxation moduli and creep compliances, PRs determined under one type of loading cannot necessarily be exported nor applied to other loading conditions. For the same material, with each distinct loading condition a different equally non-ubiquitous PR function prevails in identical time frames.

It must be further noted that in the real world, constant strains can be obtained only after some initial loading phase and time have passed. The significant impacts and influences of viscoelastic memory and energy dissipation on these start up patterns vis-à-vis proper material property determinations have been demonstrated analytically [19–21] and experimentally [7–9]. If these startup phenomena are included in any analysis designed to characterize material property, as indeed they should, no time-independent strains exist over the entire experimental time interval and hence any application of Class II PRs is spurious. Similar loading conditions apply to time-independent stresses, stress and strain rate, as they too in their own right require time delayed start up initiations and appropriate analytical descriptions.

Other important viscoelastic PR publications may be found in [24–27].

[1] A bibliography and evaluations of prior publications regarding specious uses of viscoelastic PRs may be found in [12–14].

In isotropic materials such as high polymers, bulk relaxation moduli exceed by several orders of magnitudes shear relaxation moduli and are not necessarily accompanied by Class I, II and III PRs approaching values of 0.5 or any other constant value. It has been amply demonstrated experimentally [2–9] and analytically [12–14] that Class I PRs (ν^I) are not limited to the elastic range of $-1 \leq \nu^E(x) \leq 0.5$ and may substantially exceed this range at either or both ends because of their stress and stress rate dependencies. The same can be said for the other PR classes.

It will be shown in this paper that viscoelastic PRs have two major shortcomings: (A) they are not universal material descriptors but rather are loading specific quantities that cannot be exported from one loading condition to another and (B) they do not lend themselves to a construction of an EVCP. In the final analysis, linear viscoelastic characterization in terms of either moduli or compliances or creep or relaxation functions removes all PR ambiguities and remains the protocol of choice.

2 Analysis

2.1 General Formulation for Class I PRs

In 1829, Poisson first introduced the ratio bearing his name in a monumental memoir, which also defines a number of other lasting fundamental elastic concepts [1]. The original legacy definition of PRs, designated as the Class I PR in [13] and [14],

$$\nu^I_{ji}(x,t) \overset{\text{def}}{=} -\underbrace{\frac{\epsilon_{\underline{jj}}(x,t)}{\epsilon_{\underline{ii}}(x,t)}}_{\text{universal}} = -\underbrace{\frac{\int_{-\infty}^{t} C_{\underline{jj}kl}(x,t-t')\,\sigma_{kl}(x,t')\,dt'}{\int_{-\infty}^{t} C_{\underline{ii}mn}(x,t-t')\,\sigma_{mn}(x,t')\,dt'}}_{\text{non-universal, dependent on stress history}} \equiv \nu_{ji}(x,t) \quad i \neq j \quad (1)$$

is a universal statement applying to all materials, when expressed *solely in terms of strains*.[2] However, in linear viscoelastic media the strains and these Class I PRs are implicitly dependent not only on material properties but also on time, stresses and their time histories "trapped" in the time integrals. Thus, when (1) is augmented by constitutive relations, a dependence on material properties clearly emerges and one sees that, in contrast to what occurs with homogeneous elastic materials [28], PRs are no longer unique material properties, but rather are functions of time and stress histories. It has been noted in [28] that, even in nonhomogeneous and functionally graded elasticity, PRs are stress dependent and not all-encompassing material property descriptors as indeed they are in homogeneous elastic media.

To illustrate the severe restrictions imposed on the PRs, consider in components relative to Cartesian basis with coordinates $x = \{x_i\} = \{x_1, x_2, x_3\}$ a 1-D stress state in an isothermal process in an isotropic linear viscoelastic medium with loading solely in the x_1-direction. Then,

$$\sigma_{11} \neq 0 \qquad \epsilon_{22} = \epsilon_{33} = -\nu^I_{21}\epsilon_{11} \qquad E^*_{\underline{mmmm}} = E^*_{\underline{nnnn}} \qquad E^*_{\underline{iijj}} = E^*_{\underline{jjii}} \quad i \neq j, \quad (2)$$

[2] The usual summation convention applies unless indices are underlined, in which case summation is suppressed.

in which case the

$$\sigma_{11}(x,t) = \underbrace{\int_{-\infty}^{t} E^*_{1111}(x,t-t')\,\epsilon_{11}(x,t')\,dt'}_{=\,\mathrm{INT}_1} + 2\underbrace{\int_{-\infty}^{t} E^*_{1122}(x,t-t')\,\epsilon_{22}(x,t')\,dt'}_{=\,\mathrm{INT}_2}$$

$$= \int_{-\infty}^{t} E^*_{1111}(x,t-t')\,\epsilon_{11}(x,t')dt'$$

$$- 2\underbrace{\int_{-\infty}^{t} E^*_{1122}(x,t-t')\,\nu^I_{21}(x,t')\,\epsilon_{11}(x,t')\,dt'}_{=\,\mathrm{INT}_3} \tag{3}$$

and

$$0 = \underbrace{\int_{-\infty}^{t} E^*_{2211}(x,t-t')\,\epsilon_{11}(x,t')\,dt'}_{=\,\mathrm{INT}_4}$$

$$- \underbrace{\int_{-\infty}^{t} E^*_{2222}(x,t-t')\,\nu^I_{21}(x,t')\,\epsilon_{11}(x,t')\,dt'}_{=\,\mathrm{INT}_5}$$

$$- \underbrace{\int_{-\infty}^{t} E^*_{2211}(x,t-t')\,\nu^I_{21}(x,t')\epsilon_{11}(x,t')\,dt'}_{=\,\mathrm{INT}_6}. \tag{4}$$

Alternately, for this 1-D loading isotropic case with $\sigma_{11} \neq 0$ and $\sigma_{22} = \sigma_{33} = 0$,

$$\epsilon_{11}(x,t) = \int_{-\infty}^{t} C_{1111}(x,t-t')\,\sigma_{11}(x,t')\,dt' = \int_{-\infty}^{t} \frac{\sigma_{11}(x,t')}{E(x,t-t')}\,dt'. \tag{5}$$

The Fourier transform (FT) of a function $u(x,t)$ is defined by

$$\mathcal{F}\{u(x,t)\} \;=\; \overline{\overline{u}}(x,\omega) \;=\; \int_{-\infty}^{\infty} u(x,t)\,\exp(-\imath\,\omega\,t)\,dt. \tag{6}$$

FTs and Laplace transforms (LTs) can, of course, be used interchangeably, provided they exist, However in the present analysis FTs are preferable to two-sided LTs since FTs avoid the necessity of prescribing specific ICs at $t = 0$. For $-\infty \leq t < 0$, all state variables are at rest. In the course of solving a problem, the lower limit of (6) is for all practical purposes replaced by zero, but then proper and unique ICs must be introduced at $t = 0$.

Moreover, applying FTs to (5) results in

$$\overline{\overline{C}}_{1111}(x,\omega) \;=\; \overline{\overline{C}}(x,\omega) \;=\; \frac{1}{\overline{\overline{E}}(x,\omega)} \;\neq\; \frac{1}{\overline{\overline{E}}_{1111}(x,\omega)} \tag{7}$$

with $E(x,t)$ is the relaxation modulus equivalent to the elastic Young's modulus E^0 and $C(x,t)$ is the corresponding compliance.

Use of FTs transforms (3) and (4) into

$$\overline{\overline{\sigma}}_{11}(x,\omega) = \overline{\overline{E^*}}_{1111}(x,\omega)\,\overline{\overline{\epsilon}}_{11}(x,\omega) + 2\,\overline{\overline{E^*}}_{1122}(x,\omega)\,\overline{\overline{\epsilon}}_{22}(x,\omega)$$

$$= \overline{\overline{E^*}}_{1111}(x,\omega)\,\overline{\overline{\epsilon_{11}}}(x,\omega) \;-\; 2\,\overline{\overline{E^*}}_{2211}(x,\omega)\,\overline{\overline{\nu^I_{21}\,\epsilon_{11}}}(x,\omega) \tag{8}$$

and

$$\begin{aligned} 0 &= \overline{\overline{E^*}}_{2211}(x,\omega)\,\overline{\overline{\epsilon}}_{11}(x,\omega) \;-\; \overline{\overline{E^*}}_{2222}(x,\omega)\,\overline{\overline{\epsilon}}_{22}(x,\omega) \;-\; \overline{\overline{E^*}}_{2211}(x,\omega)\,\overline{\overline{\epsilon}}_{22}(x,\omega) \\ &= \overline{\overline{E^*}}_{2211}(x,\omega)\,\overline{\overline{\epsilon}}_{11}(x,\omega) \;-\; \overline{\overline{E^*}}_{2222}(x,\omega)\,\overline{\overline{\nu^I_{21}\,\epsilon_{11}}}(x,\omega) \\ &\quad - \overline{\overline{E^*}}_{2211}(x,\omega)\,\overline{\overline{\nu^I_{21}\,\epsilon_{11}}}(x,\omega), \end{aligned} \tag{9}$$

where

$$\underbrace{\overline{\overline{\nu^I_{21}\,\epsilon_{11}}}(x,\omega)}_{\substack{\text{not the FT of a}\\ \text{convolution integral}}} = \mathcal{F}\{\nu^I_{21}(x,t)\,\epsilon_{11}(x,t)\} = \underbrace{\int_{-\infty}^{\infty} \nu^I_{21}(x,t)\,\epsilon_{11}(x,t)\,\exp(-\imath\,\omega\,t)\,dt}_{\substack{\text{not a convolution integral, but}\\ \text{part of the constitutive relations}}}$$

$$\neq \overline{\overline{\nu^I_{21}}}(x,\omega)\,\overline{\overline{\epsilon}}_{11}(x,\omega) = \underbrace{\int_{-\infty}^{t} \nu^I_{21}(x,t-t')\,\epsilon_{11}(x,t')\,dt'.}_{\substack{= \text{FT is a convolution integral, but}\\ \text{not part of constitutive relations}}} \tag{10}$$

Consequently, no EVCP can emerge from the combinations displayed in (8) to (10).

2.2 Special Class II PRs

The Class II PR is the special and highly restricted case of Class I with one time independent strain component and a 1-D stress field $\sigma_{11}(x,t) \neq 0$ [12–14], namely

$$\epsilon_{11}(x,t) = \epsilon^0_{11}(x)\;H(t) \;\equiv\; \epsilon^0_{11}(x) \quad \text{for } t \geq 0 \tag{11}$$

and

$$\nu^{II}_{j1}(x,t) \;\stackrel{\text{def}}{=}\; -\frac{\epsilon_{jj}(x,t)}{\epsilon^0_{11}(x)} \quad \text{and} \quad \overline{\overline{\nu^{II}_{j1}}}(x,t) \;=\; -\frac{\overline{\overline{\epsilon}}_{jj}(x,t)}{\epsilon^0_{11}(x)} \quad \text{with } j \neq 1. \tag{12}$$

For Class II PRs and *for this special case only*, (8) and (9) reduce to

$$\overline{\overline{\sigma}}_{11}(a,\omega) = \big[\overline{\overline{E}}_{1111}(x,\omega) \;-\; 2\,\overline{\overline{E}}_{1122}(x,\omega)\,\overline{\overline{\nu^{II}_{21}}}(x,\omega)\big]\epsilon^0_{11}(x) \quad \text{Class II PRs only,} \tag{13}$$

$$\big(\overline{\overline{E}}_{1122}(x,\omega) - \big[\overline{\overline{E}}_{1111}(x,\omega) + \overline{\overline{E}}_{1122}(x,\omega)\big]\overline{\overline{\nu^{II}_{21}}}(x,\omega)\big)\epsilon^0_{11}(x) = 0 \quad \text{Class II PRs only,} \tag{14}$$

which yield

$$\overline{\overline{\nu^{II}_{21}}}(x,\omega) \;=\; \frac{\overline{\overline{E}}_{1122}(x,\omega)}{\overline{\overline{E}}_{1111}(x,\omega) + \overline{\overline{E}}_{1122}(x,\omega)} \quad \text{Class II PRs only.} \tag{15}$$

Finally, using the first of each of (8) and (9) and eliminating $\overline{\overline{\epsilon}}_{22}$, one obtains for the viscoelastic Young's modulus the universal relation[3]

$$\overline{\overline{E}}(x,\omega) \;=\; \frac{\overline{\overline{E}}^{2}_{1111}(x,\omega) + \overline{\overline{E}}_{1111}(x,\omega)\,\overline{\overline{E}}_{1122}(x,\omega) - 2\,\overline{\overline{E}}^{2}_{1122}(x,\omega)}{\overline{\overline{E}}_{1111}(x,\omega) + \overline{\overline{E}}_{1122}(x,\omega)}, \tag{16}$$

where the expression (16) does not depend on PRs. Similar expressions independent of ν can be derived for the complex shear $\overline{\overline{G}}$ and bulk $\overline{\overline{K}}$ moduli. See [7–9]. These expressions are obtained by recasting the constitutive relations of (8) into the form

$$\begin{aligned}
\sigma_{11}(x,t) &= \int_{-\infty}^{t} \frac{1}{3}\left[4\,G^{*}(x,t-t') + K^{*}(x,t-t')\right]\epsilon_{11}(x,t')\,dt' \\
&\quad + \int_{-\infty}^{t} \frac{1}{3}\left[2\,K^{*}(x,t-t') - 4\,G^{*}(x,t-t')\right]\epsilon_{22}(x,t')\,dt' \\
&= \int_{-\infty}^{t} \underbrace{\frac{1}{3}\left[4\,G(x,t-t') + K(x,t-t')\right]}_{=\,E_{1111}(x,t-t')} \frac{\partial \epsilon_{11}(x,t')}{\partial t'}\,dt' \\
&\quad + \int_{-\infty}^{t} \underbrace{\frac{1}{3}\left[2\,K(x,t-t') - 4\,G(x,t-t')\right]}_{=\,2\,E_{1122}(x,t-t')} \frac{\partial \epsilon_{22}(x,t')}{\partial t'}\,dt'.
\end{aligned} \tag{17}$$

For polymers and other organic materials, but not metals, the bulk relaxation moduli are generally several orders of magnitude larger than the shear moduli and changes in volume are significantly slower to develop than changes in shape. As a consequence, for such materials,

$$K(x,t) \gg G(x,t) \quad \text{and} \quad \tau_K \gg \tau_G \quad \text{or} \quad \mathcal{O}[\tau_K] > \mathcal{O}[\tau_G] \tag{18a}$$

$$\text{whereas } E_{1111}(x,t) \approx E_{1122}(x,t) \quad \text{and} \quad \tau_{1111} \approx \tau_{1122}, \tag{18b}$$

where $\mathcal{O}$ means "order of magnitude." Consequently, while both forms are equally valid, the constitutive relations (3) are preferable to and more manageable than (17).

For isotropic viscoelastic media, these expressions also yield the following relation for multi-dimensional stress fields:

$$K(x,t) = \frac{3}{2}\left[\underline{E_{iiii}}(x,t) \;+\; 2\,\underline{E_{iijj}}(x,t)\right] \quad \text{and} \quad G(x,t) = \frac{1}{2}\left[\underline{E_{iiii}}(x,t) \;-\; \underline{E_{iijj}}(x,t)\right], \quad \text{with } i \neq j. \tag{19}$$

Caveat The results displayed in this section, except for (16) to (19), are only valid for the limited case of Class II PRs where the strains in the loaded direction are time-independent.

Additionally, a most important phenomenon which must be considered in any viscoelastic material property characterization analysis and experiment is the undeniable fact that

[3] Relaxation modulus and creep compliance relations not involving PRs are termed "universal." Viscoelastic PRs, on the other hand, since they are dependent on stress histories are non-universal material properties with time functions specific to each loading cycle.

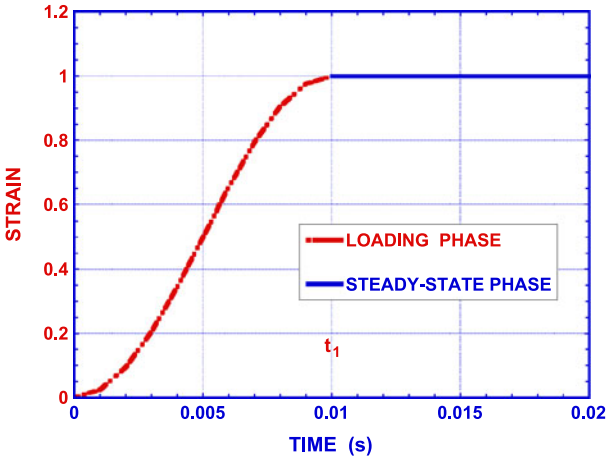

Fig. 1 Transient and steady-state strain

physically time-independent strains or stresses, or their first-order time derivatives for $t \geq 0$, do not suddenly appear out of nowhere as has been demonstrated in [7–9] and [19–21]. (See Fig. 1 and the Appendix.) This negates the physically possible existence of Class II PRs since the one time-independent strain so vital to this definition does not exist over the entire time span of the experiments, i.e., $t \geq 0$. Such time-independent strains or stresses, or the lack thereof, are of particularly significance when the needed integral transforms are evaluated.

Unlike elastic materials with their algebraic constitutive relations, viscoelastic media have memory, dissipate energy and have time-integral stress–strain relations with moduli that degrade in time. Consequently, loading sequences and time are of primary importance and cannot be ignored or marginalized.

It is readily seen that Class II PRs are a subset of Class III viscoelastic PRs.

2.3 Special Class III PRs

Class III PRs are defined in terms of Fourier (FT) or Laplace (LT) transforms as [12–14]

$$\overline{\overline{\nu_{ji}^{III}}}(x,\omega) \stackrel{\text{def}}{=} -\frac{\overline{\overline{\epsilon}}_{\underline{jj}}(x,\omega)}{\overline{\overline{\epsilon}}_{\underline{ii}}(x,\omega)} \neq \overline{\overline{\nu_{ji}^{I}}}(x,\omega) = -\int_{-\infty}^{\infty} \frac{\epsilon_{\underline{jj}}(x,t')}{\epsilon_{\underline{ii}}(x,t')} \exp(-\imath\,\omega\, t')\, dt' \quad i \neq j. \tag{20}$$

Note that for the 1-D loading case, $\sigma_{11}(x,t) \neq 0$, the Class III transform PR is stress independent[4]

$$\overline{\overline{\nu_{j1}^{III}}}(x,\omega) = -\frac{\overline{\overline{C}}_{\underline{jj}11}(x,\omega)}{\overline{\overline{C}}_{1111}(x,\omega)} \quad \text{for } j \neq 1 \text{ and } \sigma_{11}(x,t) \neq 0 \text{ all other } \sigma_{\underline{jj}} = 0. \tag{21}$$

[4] In a later section, this will be identified as a Class VI PR ($\nu_{ji}^{VI}(x,t)$ with $j \neq i$).

However for multi-axial loadings, this is not the case since the normal stresses do not factor out of the ratio of the time integrals

$$\overline{\overline{\nu_{ji}^{III}}}(x,\omega) = -\frac{\int_{-\infty}^{\infty}\int_{-\infty}^{t}\sum_{k=1}^{3} C^{*}_{\underline{jj}kk}(x,t-t')\,\sigma_{kk}(x,t')\,dt'\,\exp(-\imath\,\omega t)\,dt}{\int_{-\infty}^{\infty}\int_{-\infty}^{t}\sum_{k=1}^{3} C^{*}_{\underline{ii}kk}(x,t-t')\,\sigma_{kk}(x,t')\,dt'\,\exp(-\imath\,\omega t)dt}$$

$$= -\frac{\sum_{k=1}^{3}\overline{\overline{C}}_{\underline{jj}kk}(x,\omega)\,\overline{\overline{\sigma}}_{kk}(x,\omega)}{\sum_{k=1}^{3}\overline{\overline{C}}_{\underline{ii}kk}(x,\omega)\,\overline{\overline{\sigma}}_{kk}(x,\omega)} \quad i \neq j. \tag{22}$$

This is because the normal stresses entering the sums in the numerator and denominator of (22) cannot generally be cancelled out. Furthermore, inverting (20) leads to the not so useful relations

$$\epsilon_{\underline{jj}}(x,t) \;=\; -\int_{-\infty}^{t} \nu_{j\underline{i}}^{III}(x,t-t')\,\epsilon_{\underline{ii}}(x,t')\,dt' \quad i \neq j \tag{23}$$

in real time space.

2.4 Classes IV and V PRs

These somewhat artificially defined PRs are given by [12–14]

$$\nu_{jt}^{\mathbf{IV}}(x,t) \stackrel{\text{def}}{=} -\frac{\log[1+\epsilon_{\underline{jj}}(x,t)]}{\log[1+\epsilon_{\underline{ii}}(x,t)]} \tag{24}$$

and

$$\frac{\partial \nu_{ji}^{\mathbf{V}}(x,t)}{\partial t} \stackrel{\text{def}}{=} -\frac{\frac{\partial \epsilon_{\underline{jj}}(x,t)}{\partial t}}{\frac{\partial \epsilon_{\underline{ii}}(x,t)}{\partial t}}. \tag{25}$$

Neither Class IV nor V PRs provide any help in formulating an EVCP.

Remark In linear viscoelasticity, universal material properties are those which are independent of stress and/or strains. Typical examples are relaxation moduli and functions, and creep compliances and functions. However, viscoelastic Class I through V PRs fail to meet this prescription due to their stress-time history dependence and, therefore, cannot be shared among different loading conditions.

2.5 A Few Additional Remarks Regarding EVCP and the New Class VI PR[5]

In this section the following question is asked and answered: "What form of viscoelastic PRs is required to properly state the constitutive relations in terms of PRs so that an EVCP can be realized?"

Since the correspondence principle has been proven inapplicable when PRs are involved [12–14] the usual elastic relations for shear and bulk moduli have no viscoelastic counterparts in terms of PRs, i.e.,

[5] See also (3) to (10).

$$\begin{aligned}
&\overline{\overline{G}} \neq \frac{\overline{\overline{E}}}{2(1+\overline{\overline{\nu^I}})}, \qquad \overline{\overline{K}} \neq \frac{\overline{\overline{E}}}{1-2\,\overline{\overline{\nu^I}}},\\
&\overline{\overline{\sigma}}_{ij}(x,\omega) \neq \overline{\overline{\sigma}}^{e}_{ij}\Big[x,\omega,\overline{\overline{E}}_{ijkl}(x,\omega),\overline{\overline{\nu^I}}(x,\omega),\underbrace{\overline{\overline{X}}_i(x,\omega),\overline{\overline{U}}_i(x,\omega),}_{\text{boundary conditions}}\underbrace{\overline{\overline{\alpha T}}(x,\omega)}_{\text{thermal strains}}\Big], \quad \text{etc..}
\end{aligned} \tag{26}$$

However, without PRs the expressions

$$\begin{aligned}
&\overline{\overline{E}} = \frac{3\,\overline{\overline{G}}}{1+\overline{\overline{G}}/\overline{\overline{K}}},\\
&\overline{\overline{\sigma}}_{ij}(x,\omega) = \overline{\overline{\sigma}}^{e}_{ij}\Big[x,\omega,\overline{\overline{E}}_{ijkl}(x,\omega),\underbrace{\overline{\overline{X}}_i(x,\omega),\overline{\overline{U}}_i(x,\omega),}_{\text{boundary conditions}}\underbrace{\overline{\overline{\alpha T}}(x,\omega)}_{\substack{\text{thermal}\\ \text{strains}}}\Big], \quad \text{etc.},
\end{aligned} \tag{27}$$

and

$$\overline{\overline{\epsilon}}_{ij}(x,\omega) = \overline{\overline{\epsilon}}^{e}_{ij}\Big[x,\omega,\overline{\overline{C}}_{ijkl}(x,\omega),\underbrace{\overline{\overline{X}}_i(x,\omega),\overline{\overline{U}}_i(x,\omega),}_{\text{boundary conditions}}\underbrace{\overline{\overline{\alpha T}}(x,\omega)}_{\substack{\text{thermal}\\ \text{strains}}}\Big], \quad \text{etc.}, \tag{28}$$

are universally valid, as are the expressions (16) to (19).

Difficulties with generating a proper EVCP stem from the inherent assemblage of terms in the elastic constitutive relations when PRs are involved. In terms of moduli/compliances the 3-D Hooke's law and the corresponding linear viscoelastic constitutive relations are both derivable from first principles [22] and for elastic media have the form

$$\epsilon^E_{ij}(x,t) = C^0_{\underline{ijkl}}(x)\Big[\sigma^E_{kl}(x,t) + \delta_{\underline{ij}}\,\alpha\,T(x,t)\Big]. \tag{29}$$

However, when the isotropic Hooke's law is assembled with elastic PRs, in an almost ad hoc manner one direction at a time to yield [23]

$$\epsilon^E_{jj}(x,t) = \underbrace{\frac{1}{E^0(x)}}_{=\,C^0(x)}\Big[\sigma^E_{\underline{jj}}(x,t) - \nu^0_{21}(x)\Big(\sigma^E_{\underline{ii}}(x,t) + \sigma^E_{\underline{kk}}(x,t)\Big) + \alpha\,T(x,t)\Big] \quad i\neq j\neq k. \tag{30}$$

Specifically, the pitfalls in producing a coherent EVCP arise from the triple-product elastic combinations and their FTs:[6]

$$\frac{\nu^0_{21}(x)\sigma^E_{\underline{ii}}(x,t)}{E^0(x)} = C^0(x)\nu^0_{21}(x)\sigma^E_{\underline{ii}}(x,t) \quad\Longrightarrow\quad C^0(x)\nu^0_{21}(x)\overline{\overline{\sigma^E_{\underline{ii}}}}(x,\omega). \tag{31}$$

Such expression require a viscoelastic triple FT product counterpart $\overline{\overline{C}}(x,\omega)\,\overline{\overline{\nu^D_{21}}}(x,\omega)\times\overline{\overline{\sigma}}_{\underline{ii}}(x,\omega)$, which can only be generated from real time combinations determined by its FT inversions

$$\mathcal{F}^{-1}\Big\{\overbrace{\underbrace{\overline{\overline{C}}(x,\omega)\,\overline{\overline{\nu^D_{21}}}(x,\omega)}_{=\,-\,\overline{\overline{C}}_{1122}(x,\omega)}\,\overline{\overline{\sigma}}_{\underline{ii}}(x,\omega)}^{\substack{=\text{ FT combination needed to}\\ \text{produce an EVCP}}}\Big\}$$

[6] This is but one of the two triple products in the constitutive relations. The other one involves $\sigma_{\underline{kk}}$ and has an analogous formulation.

$$= \begin{cases} \int_{-\infty}^{t} C(x,t-t') \left\{ \int_{-\infty}^{t'} \nu_{21}^{D}(x,t'-s)\ \sigma_{\underline{ii}}(x,s)\ ds \right\} dt' & \text{or} \\ \int_{-\infty}^{t} \underbrace{\left\{ \int_{-\infty}^{t'} C(x,t'-s)\ \nu_{21}^{D}(x,s)\ ds \right\}}_{=-\,C_{1122}(x,t')} \sigma_{\underline{ii}}(x,t-t')\ dt' & \text{or} \\ -\int_{-\infty}^{t} C_{1122}(x,t-t')\ \sigma_{\underline{ii}}(x,t')\ dt' & \text{always true.} \end{cases} \tag{32}$$

The above three expressions indicate how the constitutive relations need to be formulated in the real time space so that the desired FT triple products can be realized.

It then follows from this inverse heuristic approach that

$$-\underbrace{\overline{\overline{C}}(x,\omega)}_{=\overline{\overline{C}}_{1111}(x,\omega)}\ \overline{\overline{\nu_{21}^{D}}}(x,\omega)\ \overline{\overline{\sigma}}_{\underline{ii}}(x,\omega) = \overline{\overline{C}}_{1122}(x,\omega)\ \overline{\overline{\sigma}}_{\underline{ii}}(x,\omega), \tag{33}$$

with the definition of a new Class VI viscoelastic PR:

$$\overline{\overline{\nu_{21}^{D}}}(x,\omega) \equiv \overline{\overline{\nu_{21}^{VI}}}(x,\omega) \overset{\text{def}}{=} -\frac{\overline{\overline{C}}_{1122}(x,\omega)}{\overline{\overline{C}}(x,\omega)} = -\overline{\overline{C}}_{1122}(x,\omega)\ \overline{\overline{E}}(x,\omega) \tag{34}$$

and

$$\nu_{21}^{VI}(x,t) = -\int_{-\infty}^{t} C_{1122}(x,t-t')\ E(x,t')\ dt' = -\int_{-\infty}^{t} E(x,t-t')\ C_{1122}(x,t')\ dt'. \tag{35}$$

At least three more rhetorical questions remain: (A) What, if any, is the physical meaning of the Class VI PRs, ν_{21}^{VI}? (B) What is their relation to the strains? (C) What is the physical meaning, if any, of the first two integrals of (32)? However, due to their appearance as triple products in the above constitutive relations, they cannot provide formulations for EVCP. Even though the Class VI PRs are pure universal material property descriptors, they are less useful than the compliances that enter them.

There is a certain irony accompanying the creation of the Class VI PR. The intent is to produce a viscoelastic look-alike to the structure of the elastic constitutive relations containing a mixture of moduli, stresses and the original Class I elastic PR [1]. What emerges as a Class VI PR is a function independent of strains and their history and one that is only dependent on certain moduli and compliances. Of course, in the elastic case the same path following (31) leads to $\sigma_{11} \neq 0$ and all other $\sigma_{ij} = 0$:[7]

$$\text{elastic} \quad \Longrightarrow \quad -\nu_{21}^{0VI}(x) = -\frac{C_{1122}^{0}(x)}{C^{0}(x)} = \underbrace{-\frac{C_{1122}^{0}(x)\ \sigma_{11}^{0}(x,t)}{C^{0}(x)\ \sigma_{11}^{0}(x,t)} = -\frac{\epsilon_{22}^{0}(x,t)}{\epsilon_{11}^{0}(x,t)}}_{\text{1-D loading only}} = \nu_{21}^{0}(x). \tag{36}$$

For a viscoelastic medium, ν_{21}^{VI} can be similarly interpreted as

$$\text{viscoelastic} \quad \Longrightarrow \quad \overline{\overline{\nu_{21}^{VI}}}(x,\omega) = -\frac{\overline{\overline{C}}_{1122}(x,\omega)}{\overline{\overline{C}}(x,\omega)}$$

[7] For non-auxetic elastic materials, $C_{1122}^{0} < 0$ and $C_{1122}(x,t) < 0$ while $E(x,t) > G(x,t) > 0$. For auxetic materials the reverse inequalities hold and the elastic PR's are in the range $-1 \leq \nu_{21}^{0} < 0$.

$$= \underbrace{-\frac{\overline{\overline{C}}_{1122}(x,\omega)\,\overline{\overline{\sigma}}_{11}(x,\omega)}{\overline{\overline{C}}(x,\omega)\,\overline{\overline{\sigma}}_{11}(x,\omega)} = -\frac{\overline{\overline{\epsilon}}_{22}(x,\omega)}{\overline{\overline{\epsilon}}_{11}(x,\omega)} = \overbrace{\overline{\overline{\nu_{21}^{III}}}(x,\omega)}^{\text{special 1-D case}}}_{\text{1-D loading only}}, \quad (37)$$

with the real time convolution integral property

$$\epsilon_{22}(x,t) = -\int_{-\infty}^{t} \nu_{21}^{VI}(x,t-t')\,\epsilon_{11}(x,t')\,dt' \quad \text{with } \sigma_{11} \neq 0 \text{ and } \sigma_{\underline{ij}} = 0 \text{ for j} \neq 1. \quad (38)$$

It should be noted that the Class VI PR is a special case of the general Class III PR with $\sigma_{11} \neq 0$ and $\sigma_{22} = \sigma_{33} = 0$. Neither Class III nor Class VI PRs have a physical definition or have values could be measured experimentally in the time domain. While it is certainly possible to measure the strains $\epsilon_{ij}(x,t)$ in real time, the time functions associated with this PR class are complicated and the closed form evaluation of the integral in (38) is generally difficult. The second integral of (32) clearly represents the proper form of the triple FT product involving PRs in the time domain. The character of the time function defining $\nu_{21}^{VI}(x,t)$ can be deduced from the inversion of the FTs, (35), since the Prony series for C_{1122} and C are known expressions determined from generalized Kelvin or Maxwell models. However, since (35) requires the a priori determination of the compliances, one can justifiably inquire why bother with ν_{21}^{VI} when the characterization is complete with the knowledge of either set of the C_{ijkl} or E_{ijkl}. Here, it is worth emphasizing that although detailed knowledge of these PRs still does not produce a useful EVCP, and either the moduli or the compliances do so handily.

In the viscoelastic constitutive relations, there appears to be no clear protocol based on thermodynamically sound principles, to derive directly, in real time space, either of the first two triple integrals displayed in (32). Only through mimicking Hooke's law in the FT space can (32) be so postulated in an inverse fashion. In a larger sense then, the introduction of PRs into viscoelastic stress-strain relations can be interpreted as an artificial act of faith not based on first principles, i.e., with PRs related to thermodynamic derivatives.

3 Discussion and Conclusions

The definitions and properties of the six classes of PRs and relaxation moduli are summarized in Table 1.

The general inability to produce an EVCP in terms of PRs is one reason for interdicting PR use in viscoelastic characterizations and analyses. For instance, elastic Timoshenko beam shear coefficients (TSC) are heavily dependent on elastic Class I PRs (and cross-section geometry), hence necessitating a new PR-independent viscoelastic analysis without recourse to elastic TSC results [29]. The viscoelastic counterparts to the elastic Timoshenko shear coefficients become non-general shear functions of time, specific to cross-section, material properties and loading histories.

Consequently, general linear viscoelastic constitutive relation characterizations should preferably be in terms of moduli, compliances or creep and relaxation functions but not PRs. Experimental determinations of viscoelastic PRs, therefore, are of extremely limited importance and unneeded since universal material properties such as moduli cannot be derived from PRs. Efforts should instead be directed to determining relaxation moduli or creep compliances, or to constructing creep or relaxation functions. Indeed, the 3-D isotropic viscoelastic characterization in terms of relaxation moduli and/or creep compliances has been

Table 1 Summary of properties of viscoelastic poisson's ratios and moduli

Class	Equation $i \neq j$	Physical meaning	How Obtained	EVCP	Appears in Time Space Constitutive Relations	Universal Material Property
I	$\nu^{I}_{ji}(x,t) \stackrel{\text{def}}{=} -\dfrac{\epsilon_{\underline{jj}}(x,t)}{\epsilon_{\underline{ii}}(x,t)}$	Yes	Defined	No	Yes	No
II	$\nu^{II}_{j1}(x,t) \stackrel{\text{def}}{=} -\dfrac{\epsilon_{\underline{jj}}(x,t)}{\epsilon^{0}_{11}(x)}$	Yes	Defined[a]	Yes[a]	Yes	No
III	$\overline{\overline{\nu}}^{III}_{ji}(x,\omega) \stackrel{\text{def}}{=} -\dfrac{\overline{\overline{\epsilon}}_{\underline{jj}}(x,\omega)}{\overline{\overline{\epsilon}}_{\underline{ii}}(x,\omega)}$	No	Defined	No	No	No
IV	$\nu^{\mathbf{IV}}_{jt}(x,t) \stackrel{\text{def}}{=} -\dfrac{\log[1+\epsilon_{\underline{jj}}(x,t)]}{\log[1+\epsilon_{\underline{ii}}(x,t)]}$	No	Defined	No	No	No
V	$\dfrac{\partial \nu^{\mathbf{V}}_{ji}(x,t)}{\partial t} \stackrel{\text{def}}{=} -\dfrac{\dfrac{\partial \epsilon_{\underline{jj}}(x,t)}{\partial t}}{\dfrac{\partial \epsilon_{\underline{ii}}(x,t)}{\partial t}}$	Yes	Defined	No	No	No
VI	$\nu^{VI}_{ji}(x,t) = -\dfrac{C_{\underline{jjii}}(x,t)}{C(x,t)}$	Yes	Derived	Yes	Yes	Yes
N/A	$E_{mnkl}(x,t), C_{mnkl}(x,t)$	Yes	Derived	Yes	Yes	Yes

[a]Of limited usage

successfully achieved [7–9, 30] through experiments based on 1-D loading and 2-D photogrammetric strain measurements. Alternately, simultaneous extension and torsion experiments result in the determination of isotropic viscoelastic Young's, bulk and shear moduli [31, 32].

- The first five classes of viscoelastic PRs are time-dependent functions of stresses and stress histories. Hence, they vary with loading conditions and are not unique universal material properties. Furthermore, they are nonlinear functions of strains even though their corresponding relaxation moduli remain linear, i.e., independent of stresses and strains and their time histories.
- In linear viscoelastic media, relaxation moduli and creep compliances, unlike PRs, are unique time-dependent universal material property functions that are independent of loading conditions.
- Elastic and viscoelastic moduli and compliances can be directly related to thermodynamic derivatives [22], while PRs are defined as strain ratios.
- The needed scalar product combination $\overline{\overline{\nu_{j\underline{i}}}}(x,\omega)\overline{\overline{\epsilon_{\underline{ii}}}}(x,\omega)$ with $i \neq j$ cannot be realized in the viscoelastic constitutive relations and, therefore, no general EVCP based on PRs can be formulated.
- The ability to experimentally determine PRs is not in question. Each experimental protocol, however, produces some non-universal PR Class I to V result (a stress history dependent special PR time function) that cannot be incorporated into general linear viscoelastic constitutive relations.

- Class II PRs are highly specialized entities depending on the presence of a time-independent strain component over the entire time range. Loading transients and viscoelastic material memory effects (see Fig. 1) render such entities unattainable.
- Sensitivity of modulus/compliance modeling to starting transients has been conclusively demonstrated analytically and experimentally in [7–9] and [19–21]. In these references, it has been shown that neglect of such start ups in favor of steady-state conditions leads to erroneous moduli/characterization functions. These serious and large errors occur even in the absence of any PR considerations.
- Any interrelations between integral transforms of Class II PRs and relaxation moduli and/or creep compliances are limited to constant strain conditions for *the entire time interval* $t \geq 0$. If physically ever-present starting transients are properly included (see Appendix) then Class II PRs are not universal material property identities and, therefore, they cannot be generalized to any and all loading conditions. Nor do they lead to EVCP expressions.
- Any appeal to a Class III PR [13, 14] for an EVCP is fallacious since one cannot make the non-physical PRs $\overline{\overline{\nu_{ji}^{III}}}(x, \omega)$ appear in any shape or form in the constitutive relations and satisfy the triple product requirements. Consequently, Class III PRs also cannot produce an EVCP.
- Due to their intrinsic definitions, neither of the Class IV nor V PRs are capable of producing an EVCP.
- The new Class VI PRs have their genesis in a definition starting in the FT space, based on a form of the constitutive relations that can be interpreted as supporting an EVCP. However, they are devoid of physical meaning and the stress-strain relations do not transform into a form in time space that can be derived independently from thermodynamic first principles. They represent a group of material property time functions independent of stress and strain histories. Nevertheless, their utility is far less than the relaxation moduli and creep compliances that define them.
- The arbitrary introduction of any form of PRs that depends on stresses and their time history changes linear viscoelastic constitutive relations into nonlinear ones and, therefore, constitutes a false and improper transformation.
- While PRs continue to serve elasticity extremely well, they are nevertheless an anachronism in viscoelasticity due to (A) their inability to capture an EVCP and (B) their inability to universally characterize material properties.
- *It has been repeatedly demonstrated that the inherent dependencies of viscoelastic PRs on stresses and loading histories render them non-universal non-descriptors of material properties incapable of leading to EVCPs. Hence, their precarious use in characterization and/or stress analysis is counterproductive and counter-indicated. Their use in any form or in any one of its six categories should, therefore, be abandoned as, in doing so, one generally encounters spurious results and conclusions.*
- *In the final analysis, the use of the first five viscoelastic PR classes leads to incorrect conclusions and when applied produces PR functions that are not material pure descriptors. The universal formulation of viscoelastic constitutive relations solely in terms of moduli, compliances or relaxation and creep functions without any PR involvement should remain the operational bona fide path of first and only choice.*

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Appendix: Starting Transients and Class II PRs

In Refs. [7–9] and [19–21], the important and significant influences of initial loading or strain transients in relation to viscoelastic media characterization have been demonstrated analytically and experimentally. When determining Class II PRs[8] consider an example of a steady-state constant 1-D strain $\epsilon_{11}(t) = \epsilon_{11}^0 H(t - t_1)$ that develops according to specified time-loading histories. For an applied constant strain ϵ_{11}^0, suitable transient and steady-state loading functions are (Fig. 1)

$$\epsilon_{11}(t) = \begin{cases} \underbrace{\frac{\epsilon_{11}^0}{2}(1 - \cos\frac{\pi t}{t_1})}_{\text{a sample loading strain function}} & 0 \le t \le t_1 \lesseqgtr t_0 \\ \underbrace{\epsilon_{11}^0 H(t - t_1)}_{\text{constant strain loading}} & t \ge t_1 \end{cases} \tag{39}$$

with initial and final conditions in the loading interval $0 \le t \le t_1$:

$$\epsilon_{11}(x, 0) = \frac{\partial \epsilon_{11}(x, 0)}{\partial t} = \frac{\partial \epsilon_{11}(x, t_1)}{\partial t} = 0. \tag{40}$$

Similar patterns hold for constant strain/stress rates and for time invariant stresses and, of course, other functions can be considered provided they meet the initial and final transition conditions of (40).

The time t_1 is defined by $\epsilon_{11}(t_1) = \epsilon_{11}^0$ as the time when the strain reaches its steady state value and is obtained from experimental observations. The time t_0 is defined by

$$E(t) \approx E^0 \quad t_{int} \le t \le t_0 \ll t_R. \tag{41}$$

Essentially, t_0 is the largest time before the modulus begins to relax and t_R is the time when the fully relaxed modulus value is first reached. The time t_{int} is equal to either $-\infty$ or 0 depending on how the lower limit of integration enters the constitutive relations. (See also discussion following (6).)

Remark Consequently, Class II PRs based on $\epsilon_{11}(t) = H(t)\,\epsilon_{11}^0$ for $0 \le t \le \infty$ are mathematically but not physically realizable. Additionally, since PRs are stress and stress-time history dependent, the Class II PRs are non-exportable to other loading conditions. Even if the loading sequence is taken into account, the eventually stabilized ϵ_{11}^0 will take on distinct values depending on specific loading histories.

To avoid intruding into the relaxed part of the viscoelastic modulus $E(t)$ and, thus, simplify characterization analyses, it is best to complete the loading phase during the time interval $t_1 < t_0$, where the "elastic" modulus E_0 is of physical relevance.

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