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Interpreting contracts: the purposive approach and non-comprehensive incentive contracts

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Abstract
Real world contracts often contain incentive clauses that fail to fully specify conditions triggering payments, giving rise to legal disputes. When complete contract generate Pareto efficient allocations the L&E literature advocates that courts should fill in the missing clauses. This logic does not directly extend to environments with moral hazard, where complete contracts result in constrained efficient allocations. Despite this inefficiency we find that when agency and marginal agency costs are congruent, the legal system can do no better than guide its courts to complete contracts according to the parties' intentions.

Keywords Asymmetric information · Balance of probabilities · Incomplete contracts · Judicial system · Courts

JEL Classification D82 · D86 · K40

1 Introduction

Real world contracts are usually designed to align incentives. That purpose is partially thwarted by contract incompleteness which may trigger opportunistic behavior and lead to conflicts. In a court of law resolving such conflicts will require an ex-post interpretation of contractual clauses. This juridical issue is embedded in a fundamental debate among legal scholars and practitioners concerning the interpretation of documents where some advocate a "literal approach" and others favor a "purposive interpretation". For contracting disputes, the law and economics literature

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usually supports the latter approach based on the intuition that completing contracts should generate a Pareto efficient allocation.[1] In this paper, we focus on a subclass of contracting problems where this rationale does not apply.

In our analysis, contract incompleteness arises from a *non-comprehensive* description of the conditions triggering payment in a moral hazard environment.[2] The presence of informational frictions implies that guiding courts to resolve payment disputes by completing contracts according to the parties' initial purpose would enforce Pareto inefficient allocations. Clearly, this invalidates the aforestated justification underlying the purposive approach. For this context, we find a condition which nevertheless ensures that courts can do no better than apply that approach.

To fix ideas, consider two recent UK court cases. In *Rainy Sky S.A. and others (Appellants) v Kookmin Bank (Respondent) [2011]*, the dispute centered around the repayment of a bond in a multi-party contract between buyers, a shipbuilder and a bank. The contract foresaw some installment payments by the buyers. In order to align ex-post incentives, these payments were guaranteed by a performance bond at the Kookmin Bank. At some point in time, the shipbuilder went bankrupt and the buyers demanded activation of the bond, which was denied by the bank.[3]

The second case *Commerzbank AG v Keen [2006,2007]* concerned the payment of discretionary bonuses. At some point in time, Mr. Keen was not awarded a bonus for a specific period during which he had worked on ground that he was no longer employed by the Commerzbank AG when the actual decisions were made. He challenged that decision in court. While the two problems differ in scope and in many other dimensions, there is nevertheless a key similarity; in both cases the legal difficulty resulted from the non-comprehensive specification of the conditions triggering payment. While the problem is obviously constitutive in the case of a *discretionary* bonus scheme, it was also present in *Rainy Sky v Kookmin Bank* where the Supreme Court noted that the contract's articles were not "intended to set out all the circumstances in which the refund guarantee should operate".

This kind of ambiguity concerning payment-related details of incentive clauses designed to resolve a moral hazard problem are not unique to the above cases. For instance, there are numerous other examples which involve non-comprehensive bonus schemes [e.g. *Takacs v Barclays Services Jersey Ltd [2006]* and *Ridgway v JP Morgan Chase Bank National Association [2007]* in the UK and recently *Bryan Burns v. RBS Securities, Inc.,* 151 Conn. App. 451 (2014)]. Moreover, there are many other types of contracts with ambiguities in the conditions triggering payments; standard rental agreements use vague terms like "normal wear and tear" to determine whether deposits are to be refunded; construction contracts often specify penalties for delays and/or inadequate performance without providing an exhaustive

[1] This point is related to the Posnerian Principle which is further discussed in the literature review section.

[2] We follow the terminology from Hart (1995) who defines *non- comprehensive* contracts as agreements which fail to "specify all parties' obligations in all future states of the world, to the fullest extent possible (i.e. to the extent that these obligations are observable and verifiable)".

[3] For further details, see https://www.supremecourt.uk/cases/uksc-2010-0127.html. All quotations below related to this case are taken from the official press release summarizing it.

list describing such events; "Best efforts" and "reasonable efforts" clauses in employment contracts do not explicitly define what these terms mean, promotion and dismissal causes are generally not fully spelled out, etc.[4]

The Law plays a fundamental role for contracts with this kind of ambiguity. First, at the bargaining stage because the negotiation leading to the contract occurs in its shadow where the parties take expected court behavior into consideration. And, second, because the parties' anticipation with respect to the outcome of a potential legal challenge impacts their respective behavior after the agreement has been signed. Naturally, this raises the question of the optimal procedure that should be applied to legal conflicts arising from a non-comprehensive clause in an incentive contract.

In *Rainy Sky v Kookmin Bank* the judges on the UK Supreme court followed a purposive approach.[5] They resolved the ambiguity by asking what "a reasonable person would have understood the parties to have meant". Answering that question guided the court in ruling for the buyers. The court's approach essentially reproduces the procedure to complete (incomplete) contracts according to the parties' intentions. In so doing, it implements the Posnerian principle which calls upon the legal process to mimic the market.[6] This logical chain is based on the notion that non-comprehensive incentive schemes may be regarded as a special kind of incomplete contracts. Therefore, applying the Posnerian principle for both cases may initially appear very natural. However, there is an important caveat with this approach which raises doubt as to whether the standard rationale for mimicking the market should apply to the current context. Specifically, it is well known from the literature that in moral-hazard environments profit maximizing incentive contracts do *not* implement the Pareto efficient allocation.

In order to analyze these questions from a welfare perspective, we introduce a stylized principal-agent framework characterized by moral hazard with respect to the agent's effort. To align the latter's incentives, contracts condition the agent's payments on the outcome of a proxy variable that is correlated with effort. We assume that due to transactions costs, it is impossible to comprehensively describe ex-ante the conditions triggering payment. As a result, the bonus scheme becomes discretionary. This feature raises the prospect of an ex-post opportunistic behavior with respect to the principal's obligation to pay the bonus. In this setup, the court system becomes an essential institution to guarantee that the principal honors its promise.[7] Moreover, the process leading to a court decision for or against an agent's claim matters both at the contracting stage and for the incentives generated by a given discretionary bonus scheme.

[4] See for example the Goetz and Scott (1981) discussion of ambiguities associated with the "best efforts" clause.

[5] Historically, UK courts favored a literal approach to contractual interpretation, but this has been progressively replaced by a purposive approach as in the current example.

[6] See Posner (2007).

[7] The economic literature examined numerous other disciplining devices such as repeated interactions, market reputation, property rights, authority and renegotiation [for further details, see e.g. the survey by Kornhauser and MacLeod (2010)]. In our analysis we ignore these possibilities in order to focus on the role of the judicial system.

Against this backdrop, we provide a positive evaluation of the preceding adjudication method on contract design and the resulting efficiency of the principal/agent relationship. We then use the model to conduct a normative analysis. Our main finding specifies a *congruency condition* which guarantees that completing contracts according to the purposive approach yields the constrained second-best allocation. Intuitively, that condition requires that a change in the set triggering incentive payments causes agency costs and marginal agency costs to shift in the same direction. When this condition fails to hold, we argue that courts should take a more comprehensive look at the circumstances and potentially consider additional societal aspects.[8] In this sense, our analysis provides a positive explanation to the difficulties that legal scholars and supreme courts have faced concerning the "literal" versus the "purposive" approaches to contract interpretation.

The remainder of the paper is structured as follows. The next section provides a short literature review. Section 3 describes the moral hazard framework. In Sect. 4, we analyze the benchmark environment solving for the profit maximizing comprehensive contract. Section 5 introduces non-comprehensive contracts and studies in two subsections the Completing Contract procedure (in the sequel, the CC procedure), and a generalization. That subsection also analyzes the benevolent regulator's decision (aiming at efficiently guiding the courts) and derives the main results. Finally, Sect. 6 summarizes the paper and offers some concluding remarks. Proofs are relegated to an "Appendix".

2 Literature review

In a broad context our paper is associated with fundamental developments in the legal profession. Historically, in most legal systems courts favored a literal approach to contractual interpretation.[9] This has been progressively replaced by a purposive approach over the course of the last century.[10,11] In the UK, it is now settled that the interpretation of contracts should follow a purposive methodology.[12] Nevertheless, the matter is not yet fully resolved as evidenced by the recent book of the former Israeli Supreme Court president Barak (2005).

[8] A similar informal argument may be found in Barak (2005). We return to this issue in the conclusion and discussion section at the end of the paper.

[9] For instance, as stated by Lord Cozens-Hardy (Master of the Rolls, 1907–1918) "it is the duty of the court... to construe the document according to the ordinary grammatical meaning of the words used therein".

[10] Quoting Baron Hoffmann (Lord of Appeal in Ordinary, 1995–2009) "the meaning which a document ... would convey to a reasonable man is not the same thing as the meaning of its words. The meaning of words is a matter of dictionaries and grammars; the meaning of the document is what the parties using those words against the relevant background would reasonably have been understood to mean".

[11] The literature sometime distinguishes between the purposive and the contextualist approaches (e.g. Goetz and Scott 1981). For our purpose, both are equivalent.

[12] The judgement *Investors Compensation Scheme Ltd. v West Bromwich Building Society [1997] UKHL 28* provides a frequently cited precedent in English contract law which directs courts to apply the purposive interpretation.

Within law and economics, our analysis is related to the debate around the Posnerian Principle. That Principle typically applies to situations in which parties are prevented (for cost reasons) from writing a complete contract which would have generated a Pareto efficient allocation. In case of a legal challenge, the Principle suggests that the litigation process should interpret contracts on the basis of the parties' intentions thereby "mimicking the market".[13] In the current context that logic does not directly apply because the parties would implement inefficient outcomes even under comprehensive contracting. Therefore, interpreting contracts according to the parties' intentions no longer leads to the implementation of a first-best allocation. Some scholars have recommended a more "objective" approach that focuses on the public interest taking into account market failures and limited rationality [see for instance Zamir (1997)]. It turns out that in our analysis, both approaches lead to the same conclusion implementing a constrained efficient allocation provided the aforementioned congruency property is satisfied. The tension between these approaches remains unresolved when that condition fails to hold.

Our analysis is also related to the research on the optimal breach of contracts. Analogous to our model, the key element in that literature is that contracts do not include all the relevant contingencies. However, in that context the missing clauses are those which would justify a breach. As is well known, for instance from Shavell (1984), these contractual omissions can be successfully counteracted by remedies for breach and renegotiations. Unlike this literature where the occurrence of a breach, becomes known, in our context the asymmetric information structure implies that neither the principal nor the court ever observe the agent's actions.

More closely related to our work, Shavell (2006) analyzes situations where contracting parties choose not to include certain contingencies—for cost reasons—knowing that courts will interpret the contract. He shows that "interpretation of contracts is in the interests of contracting parties" and that it "is superior to enforcement of contracts as written". From the same perspective, Anderlini et al. (2011) analyze how the court's intervention could improve welfare in the case of incomplete contracts. Related to our conceptual framework, Hadfield (1994) discusses cases in which contracts are "necessarily incomplete" by which she means that they "unavoidably fail to include terms that the parties would both prefer to include". However, unlike our analysis, Hadfield focuses on court limitations and resulting implications in adjudication.

There is a somewhat related literature which studies standard of proofs in courts [e.g. Demougin and Fluet (2006) and the literature therein]. In that literature, courts are required to decide on a claim in light of imperfect information. The judicial system guides courts, in particular, through definitions of standards of proofs typically designed to maximize the appropriate social goal (depending on the context). Close

[13] The following quotes from Posner (2007, p. 250) summarize what we interpret as the Posnerian Principle: "In settings where the cost of allocating resources by voluntary transactions is prohibitively highy—where, in other words, market transactions are infeasible—the common law prices behavior in such a way as to mimic the market... [T]he common law establishes property rights, regulates their exchange, and protects them against unreasonable interference—all to the end of facilitating the operation of the free market, and where the free market is unworkable of simulating its results."

to our analysis, Fluet (2003) uses a buyer-seller model where contracts are non-comprehensive with respect to quality. In contrast to our conclusion, he finds that in his setup the Balance of Probabilities standard maximizes total surplus. We address this difference in the discussion section at the end of the paper.

Within the economics discipline, our analysis relates to the vast literature on incomplete contracts where incompleteness has been invoked to analyze governance structures [for a recent reference see Aghion and Holden (2011)]; the property rights literature (e.g. Grossman and Hart 1986; Hart and Moore 1990, 1999); authority and delegation (e.g. Aghion and Tirole 1997; Baker et al. 1999). Moreover, contract incompleteness has been studied in the context of the design of incentive contracts [e.g. Tirole (1999) and Tirole (2009) which emphasize the role of renegotiations in static settings, and Levin (2003) in the context of relational environments]. Unlike this literature, it is the role of the legal system and its courts as a disciplining device which is at the core of our analysis.

3 The model

In this section, we describe a standard non repeated moral hazard problem between a risk-neutral principal (e.g. a firm) and a risk-neutral agent (e.g. an employee, a subcontractor or another firm). The principal owns a production technology requiring an agent's input referred to hereafter as effort. We denote the level of effort by $a \geq 0$ and the value it generates to the principal by $v(a)$. The function $v(\cdot)$ satisfies standard regularity requirements whereby $v(0) = 0$, $v'(\cdot) > 0$ and $v''(\cdot) < 0$. Undertaking effort generates utility costs to the agent. We denote the monetary equivalent of these costs by $c(a)$ satisfying the regularity conditions $c(0) = c'(0) = 0$, for all $a > 0$, we have $c'(a) > 0$ and $c''(a) > 0$.

As is standard in the principal-agent literature, the principal holds all of the bargaining power and makes a take-it-or-leave-it contract offer to the agent.[14] We suppose that the agent has an outside option of zero and that he is financially constrained requiring wage payments to be non-negative in all states of the world.

Moral hazard results because neither effort, nor the cost it generates or the value it produces are verifiable. Instead, the parties have access to a monitoring technology which functions as follows. After the agent has produced effort a, monitoring generates with an exogenous probability $m \in (0, 1)$ a proxy variable x and no information with probability $(1 - m)$. It is common knowledge that the realization of x is drawn from a distribution $G(x;a)$ over the support $[0, \bar{x}]$. The case of no information is represented hereafter by the outcome Ø.

[14] Models with more evenly distributed bargaining power have been explored in the literature; for instance, Nash bargaining and Rubinstein bargaining (e.g. Pitchford 1998; Demougin and Helm 2006). In our analysis, changing the allocation of bargaining power would not affect the findings.

Assumption 1 The distribution function $G(x;a)$ satisfies;

1. $g(x;a) > 0$ over the support $[0, \overline{x}]$;
2. the strict Monotone Likelihood Ratio Property (hereafter MLRP) and the Convexity of the Density Function Condition (hereafter CDFC).[15]

If monitoring is successful, the principal can make the realization of x verifiable at no additional cost. In the alternative, all other parties (i.e. the agent and potentially a third party called upon to enforce the contract as, for instance, a judge) obtain no information. In that case, from the standpoint of these other parties the result is indistinguishable from the outcome $\emptyset$.[16] The principal's ability to avoid reporting a realization of x that it deems disfavorable by claiming that no information was realized, introduces an additional moral hazard issue on the part of the principal.

Before concluding this section, some remarks is in order with respect to the specification $m < 1$. In order to obtain optimal contracts that are simple, we assumed risk-neutrality and imposed MLRP for the distribution of the proxy variable. Intuitively, MLRP implies that larger realizations of x are more indicative of high effort whereas risk-neutrality ensures that the optimal incentive scheme is a bonus contract. Therefore, in the absence of both moral hazard on the part of the principal ($m = 1$) and some financial constraints (either on the agent or on the principal), the optimal contract selects the most informative realization of the proxy, hence, the largest possible value $x = \overline{x}$.[17] Since this implies that agent almost never receives the bonus, ensuring participation requires that the bonus converges towards infinity. Of course, from a practical perspective this result is clearly meaningless; infinite payments do not exist, and if they were to exist, it would be unreasonable to assume risk-neutrality of the agent as is well known from the St. Petersburg paradox.

The literature has explored numerous avenues to escape this nonsensical result, for instance limited liability (see e.g. Kim 1997) or double moral hazard (see e.g. Bental et al. 2012). In the current model, we used a simplified version of the latter idea. Intuitively, the presence of double moral hazard limits the size of the bonus to ensure that the principal's temptation to cheat (by claiming that he did not observe the proxy) does not become excessively large.

4 Comprehensive contracts

In this section, we consider the benchmark scenario where parties can write comprehensive contracts. The analysis serves two purposes which are presented separately. In the first subsection, we show that the optimal comprehensive agreement which

[15] MLRP implies a number of useful properties, such as first-order stochastic dominance. It is an important condition in the literature on contracting under informational asymmetries. CDFC was introduced by Rogerson (1985) as a sufficient condition for the first-order approach (i.e. the principal can substitute the agent's first-order condition with respect to effort for the true incentive compatibility constraint).

[16] For instance, suppose x is the output of an electronic device and $(1 - m)$ the probability that it breaks down. If x has been realized, the principal may claim that the machine failed. However, if the principal communicates the result of monitoring, x is non-manipulable.

[17] See for instance the discussions in Innes (1990) and Kim (1995).

implements a given effort level $a > 0$ is a bonus contract. Subsequently, we will use this result to justify restricting non-comprehensive contracts to discretionary bonus schemes. The second subsection derives the principal's cost of implementing effort and solves the principal's overall decision problem for the benchmark environment.

4.1 Bonus contracts

In this subsection, we derive the principal's cost minimizing comprehensive contract that implements an exogenously set effort level a. We follow the standard approach in the Principal-Agent literature including the effort to be implemented within the contract and requiring that the agreement be *incentive feasible*.[18] Hence, a contract is a triplet $\mathcal{C} = \left\{a, w_{\emptyset},\ w(x)\right\}$ which demands of the agent to undertake the non-verifiable effort a conditioning his remuneration on the outcome $\emptyset$ or on $x \in [0, \bar{x}]$.

In our environment, the incentive feasibility conditions require that the contract satisfies the agent's financial restriction for any possible outcome $\emptyset$ or $x \in [0, \bar{x}]$. Moreover, it must induce the agent to accept the contract. In addition, in the ensuing interaction between the agent and the principal, the former must be motivated to undertake the agreed upon effort and the latter to never conceal the outcome of monitoring. The last requirement imposes $w_{\emptyset} \geq w(x)$ for all the possible realizations of x.

Altogether, the incentive feasible contract, $\mathcal{C} = \left\{a, w_{\emptyset},\ w(x)\right\}$, which minimizes the principal's cost is the solution to the optimization problem:

$$\min_{w_{\emptyset},\ w(x)} \ w_{\emptyset}(1-m) + m\int_0^{\bar{x}} w(x)g(x;a)dx \quad \text{(I)}$$

$$a = \arg\max_{\hat{a}} \ w_{\emptyset}(1-m) + m\int_0^{\bar{x}} w(x)g(x;\hat{a})dx - c(\hat{a}) \quad \text{(IC)}$$

$$w_{\emptyset}(1-m) + m\int_0^{\bar{x}} w(x)g(x;a)dx - c(a) \geq 0 \quad \text{(PC)}$$

$$w(x) \geq 0 \quad \text{(AFC)}$$

$$w_{\emptyset} \geq w(x) \quad \text{(PFC)}$$

where the (IC), (PC) conditions and the agent's Financial Constraint (AFC) ensure respectively that the agent undertakes the effort a, that he accepts the contract and that payments are feasible. Moreover, the principal's Feasibility Constraint (PFC) guarantees that the principal never finds it beneficial to conceal the realization of x. In the "Appendix", we prove the following result.

[18] See e.g. Laffont and Martimort (2002).

Proposition 1 *For every* $a > 0$, *the solution to problem (I) is a bonus contract* $\mathcal{B}^a = \{a, B^a, x^a\}$ *defined by a bonus* $B^a > 0$ *and a critical proxy* $x^a \in (0, \bar{x})$ *such that*

$$w_\emptyset = B^a \text{ and } w(x) = \begin{cases} 0 & \text{if } x < x^a \\ B^a & \text{if } x \geq x^a \end{cases} . \tag{1}$$

The intuition underlying the proposition is the following. As mentioned above, with risk-neutral participants and a financially constrained agent, it is well known from the existing literature that $w(x)$ only takes two values and that the agent extracts rent.[19] Moreover, due to MLRP the critical value x^a partitions the support $[0, \bar{x}]$ into a low and a high payment interval. Due to the constraint on the agent's reward, the lower payment is set at 0 while at the payment relevant set, $x \geq x^a$, it is B^a. Finally, from (PFC) $w_\emptyset$ is set at B^a.

In the sequel, we will analyze cases leading to non-comprehensive contracting. For that purpose it is useful to consider an incentive feasible bonus contract $\mathcal{B} = \{a, B, z\}$ where z is some exogenously given critical value (instead of the optimal critical value x^a).[20] The agent's incentive compatibility requirement for implementing a becomes[21]:

$$a = \arg\max_{\hat{a}} \; [1 - mG(z;\hat{a})]B - c(\hat{a}) \tag{IC'}$$

Taking the first-order condition of (IC') yields the bonus required in order to incentivize the desired effort level, i.e.[22]:

$$B = \frac{c'(a)}{-mG_a(z;a)} > 0 . \tag{2}$$

The sign follows from strict MLRP (which guarantees strict First-Order Stochastic Dominance and, thus, implies that $G_a(z;a) < 0$ for all $0 < z < \bar{x}$). We use the solution for B to define the principal's *agency cost*, $C^P(a, z)$, associated with incentivizing effort a for an exogenously given z:

$$C^P(a, z) = \frac{1 - mG(z;a)}{-mG_a(z;a)} c'(a) . \tag{3}$$

For environment with risk-neutral parties these agency costs consist of the agent's effort costs and the latter's rent. In the "Appendix", we verify that the current moral hazard setup mimics standard results summarized by the following proposition.

[19] For further details see Demougin and Fluet (1998) and the literature cited therein.

[20] As a convention, we use z for critical vaues that have not been optimally chosen by the principal. Otherwise we use x.

[21] For any contract $\mathcal{B} = \{a, B, z\}$ satisfying (1), we have:

$$(1 - m)B + m \int_z^{\bar{x}} Bg(x;a)dx = [1 - mG(z\,;a)]B .$$

[22] Hence at the critical value $z = x^a$, we obtain $B = B^a$.

Proposition 2 *Under comprehensive contracting, for any* (a, z) *with* $a > 0$ *the agent's rent is positive and increasing in effort.*

As is well known from the literature, the positive rent drives a wedge between the principal's cost and societal costs. Moreover, in our setup this wedge becomes larger as effort is increasing.

4.2 Profit maximization

In this subsection, we solve for the profit maximizing comprehensive incentive feasible contract, $\mathcal{B}^C = \{a^C, B^C, x^C\}$.[23] That contract solves:

$$\max_{a,z} \; v(a) - C^P(a, z). \tag{II}$$

where B^C follows from (2) evaluated at (a^C, x^C). Hence, the profit maximizing agent's effort a^C and the associated critical proxy x^C must satisfy the following system of first-order conditions:

$$\begin{cases} v'(a) - C_a^P(a, z) = 0 \\ -C_z^P(a, z) = 0 \end{cases} \tag{4}$$

This system provides two insights. The first equation implies that, as usual in moral hazard environments, the principal implements a *constrained efficient* solution which induces too little effort relative to the First-Best outcome ($a^C < a^{FB}$). Intuitively, the principal perceives the agent's rent as a cost and therefore equates the marginal benefit of effort to the sum of the social marginal costs and the agent's marginal rent. Given that the marginal rent is strictly positive, the principal finds it beneficial to implement too little effort.

The second insight is related to the aforementioned role of the parameter m in our model. The second first-order condition in (4) is equivalent to requiring that the following equality holds at the point (a^C, x^C)[24]:

$$\frac{g_a\left(x^C;a^C\right)}{g\left(x^C;a^C\right)} = \left(\int_{x^C}^{\bar{x}} \frac{g_a(x;a^C)}{g(x;a^C)} \frac{g(x;a^C)}{1 - G(z;a^C)} dx\right) \cdot \left[\frac{1 - G(x^C;a^C)}{m^{-1} - G(x^C;a^C)}\right]. \tag{5}$$

Clearly, at the limit $m = 1$, the square bracket in (5) is 1 for any effort level. Hence, for that case finding x^C requires equating the likelihood ratio on the LHS to its

[23] For parsimony of notation, we write $\mathcal{B}^C = \mathcal{B}^{a^C}$, $B^C = B^{a^C}$ and $x^C = x^{a^C}$.

[24] Satisfying the second equation in (4),

$$C_z^P(a^C, x^C) = \left[\frac{\partial}{\partial z}\left(\frac{1 - mG(a^C, x^C)}{-mG_a(a^C, x^C)}\right)\right] c'(a^C),$$

requires that the numerator of the derivative in the square bracket, $mg(a^C, x^C)G_a(a^C, x^C) + \left(1 - mG(a^C, x^C)\right)g_a(a^C, x^C)$, is set at 0, which can be rewritten as (5).

conditional expected value. Given that the likelihood ratio is strictly increasing by the MLRP assumption, for all a^C, equality can only occur at $x^C = \overline{x}$.[25]

As m decreases, the conditional expected value of the likelihood ratio is multiplied by a factor that is smaller than 1. Accordingly, x^C becomes smaller. To gain an intuition, consider again the case $m = 1$ with $x^C = \overline{x}$.[26] This requires a very large bonus since it is almost never paid. Suppose now that m is slightly decreased. Assuming the principal does not adjust the scheme, the principal would now pay the very large bonus with a strictly positive probability which clearly cannot be cost minimizing. Hence, the principal reduces the bonus which, in turn, forces an increase in the likelihood of payment in order to maintain incentives aligned.

5 Non-comprehensive incentive contracts

In the foregoing section, we solved for the comprehensive contract, $\mathcal{B}^C = \left\{a^C, B^C, x^C\right\}$. An essential part of that contract is the ability to provide a description of all the states triggering the bonus payment. In our context, it requires the capability to verbalize x^C and the ability to compare the realized value of x with x^C. In that respect, the simplicity of the model may be misleading. For instance, the assumption that the proxy is a real number gives the impression that verbalizing x and x^C should be feasible.

However, x could be a sufficient statistic which summarizes the realization of a large multidimensional random vector with elements that could be quantitative, qualitative, objective and/or subjective and therefore not verbalizable. In fact, requiring a fully comprehensive contract that verbalizes ex-ante the payment relevant set, may force the contract to restrict the metric it uses. To take an example familiar to academics, consider the issue of granting tenure to a young colleague. One could write a comprehensive promotion rule that counts publications according to some well defined metric. However, this is rarely done because it would ignore additional information which is relevant to the decision, for instance, personality traits, collegiality etc. which are themselves impossible to exhaustively describe. Once these considerations are allowed to be present, it is not only their measurability that is a problem, but also their comparability to a critical value.[27]

Beyond these theoretical reflections, it is a *matter-of-fact* that many real life contracts include some non-comprehensive clauses as demonstrated by the examples cited in the introduction. Indeed, as noted by the UK Supreme Court in *Rainy Sky v Kookmin Bank* contracts' articles are not necessarily intended to set out all the

[25] Of course, from the discussion at the end of Sect. 3, the result is not surprising since, with $m = 1$, there is no moral hazard issue on the part of the principal.

[26] This feature is common to many moral hazard environment with risk-neutral parties provided the principal is not financially constrained (see e.g. Innes 1990). For our purpose, introducing such a restriction is not useful because it does not generate a tension between private and social considerations.

[27] For instance, what is the meaning of being a "good citizen" of an academic unit? Or in the context of the examples presented in the introduction, what is a comprehensive measure for "normal wear and tear" or "best effort"?

circumstances in which incentive payment operates. To embed these real-world facts within the current framework, we introduce two assumptions.

Assumption 2 Except for trivial cases, critical likelihood ratios cannot be specified in an implementable manner at the contracting stage.

Assumption 3 After effort a has been undertaken and x has been realized likelihood ratios can be computed.

Both assumptions mimic the literature on incomplete contracts initiated by the seminal papers Grossman and Hart (1986) and Hart and Moore (1990). That literature aims at analyzing realistic environments where contracts cannot specify the parties' actions under all possible contingencies. To do so, it explicitly rules out the ability to condition actions on events that are otherwise foreseeable.[28] However, it also stipulates that issues which could not be contracted upon ex-ante, become clear once the state of the world has been realized.[29]

In a similar vein to that literature, Assumption 2 captures the idea that the dimensionality and complexity of the informational space, which generates the sufficient statistic x, prevent it from being described at the contracting stage in a way that is ex-post implementable, either directly or indirectly. However, it does not cover trivial cases where, for instance, the firm promises to either always or never pay. Assumption 3 then supposes that ex-post the information set becomes sufficiently simple to enable the computation of likelihood ratios. To fix ideas, reconsider the above tenure example. Typically, at the promotion stage, the information provided in the dossier of an assistant professor becomes quite concrete, facilitating a comparison of the candidate's performance to some notional standards of the deciding committee.

In terms of the current model, Assumption 2 rules out directly conditioning bonus payments on a critical x or indirectly by describing the payment relevant set. Accordingly, we restrict the ensuing analysis to discretionary bonus contracts, $\mathcal{D} = \{a, B\}$, which only specify the bonus, $B \geq 0$, and a mutually agreed upon understanding of the agent's expected level of effort.[30] Absent a description of the payment-relevant set, this introduces the potential for opportunistic behavior on the part of the principal.

In the property rights context, the equivalent to Assumption 3 allows the parties to renegotiate ex-post thereby reducing the negative impact of opportunistic

[28] See Grossman and Hart (1986), page 696 which states that "(A) basic assumption of the model is that the production decisions ... are sufficiently complex that they cannot be specified completely in an initial contract between the firms."

[29] Grossman and Hart (1986, p. 696), state that although production decision "is ex ante noncontractible, we suppose that, once the state of the world is determined, the (small number of) relevant aspects of the production allocation become clear and the parties can negotiate or recontract over these (costlessly)".

[30] Strictly speaking, this class of bonus schemes is only a subset of all possible non-comprehensive contracts. However, we find in the sequel that under the appropriate institutional setup, there exists a simple condition which ensures that this restriction is without loss of generality.

behavior. In that context, the ex-ante allocation of property rights becomes an important matter because of its impact on the outcome of bargaining at the renegotiation stage. In our framework, renegotiations are clearly not useful because after the agent has undertaken effort, the parties' interests are diametrically opposed; the principal always wants to avoid paying the bonus while the agent always wants to be paid. In such a context, enabling a contractual relationship requires the existence of an outside actor to curb opportunism. In the sequel, we delegate that role to courts embedded in a judicial system. Courts are modelled as perfect agents of society,[31] which exactly follow the guidance provided by the legal system through non-formal standards, criteria, procedural rules, directives for the interpretation and evaluation of information and arguments presented in the course of a trial, etc.[32]

Accordingly, suppose the judicial system has defined a set of rules and procedures guiding courts on how to adjudicate a payment dispute involving a contract of the form $D = \{a, B\}$. At a court hearing, the agent's effort has already been undertaken and the information summarized by x has been realized—i.e., by Assumption 3 the court can calculate likelihood ratios. The court then applies the specified procedure to decide upon the validity of the claim. Note, however, that neither the court nor the legal system need to verbalize the payment relevant set. Nevertheless, at the contracting stage, the parties are assumed to rationally anticipate the procedure's impact on the resulting payment set.

5.1 Completing contracts

In this subsection, we specify an operationalization of the purposive approach where courts are directed to complete non-comprehensive contracts by applying the precept that "the meaning of the document is what the parties using those words against the relevant background would reasonably have been understood to mean".[33]

From the foregoing section, a discretionary bonus contract, $\mathcal{D} = \{a, B\}$, may be thought of as a "normal" bonus contract, $\mathcal{B} = \{a, B, z\}$, where the critical z which defines the payment relevant set has been omitted. With this in mind, a non-formal guidance for implementing the purposive approach could be to instruct courts to select *the options they think the parties would have picked had they been able to specify the payment relevant set and bargain over it beforehand.*[34]

In our context, a court facing a request to adjudicate a payment dispute involving a contract $\mathcal{D} = \{a, B\}$ would, first, use a to compute a critical value, $z^{CC}(a)$, according to the second equation in (4). Second, it compares the realization of x to the imputed value $z^{CC}(a)$ and find in favor of the agent if and only if $x \geq z^{CC}(a)$. Under

[31] In particular, we assume that there no agency issues between a court and society. Nonetheless, the court must be given the proper guidance in order to implement society's objective.

[32] That role can also be performed by private institutions via arbitration or mediation settings. However, these institutions also function in the shadow of the Law.

[33] From Baron Hoffmann's quote in footnote 10.

[34] This is a paraphrase of a well known reference by Easterbrook (1983) which states that when "completing contracts, courts ordinarily select the options they think the parties would have picked had they thought of the subsequently surfacing problems and been able to bargain about them beforehand at no cost".

this procedure, the formation of beliefs is straightforward since the court implements the same payment-relevant set which the parties would have themselves agreed upon under comprehensive contracting had they wanted to implement a.[35]

Proposition 3 *Guiding courts to complete contracts implements* $z = x^C$, $B = B^C$ *and* $a = a^C$. *Specifically, the parties agree to the non-comprehensive contract* $\mathcal{D}^C = \left\{a^C, B^C\right\}$ *and courts use the critical value* $z^{CC}(a^C) = x^C$.

Proof In order to determine the profit maximizing effort level under the completing contract procedure, the principal solves:

$$\max_{a} \; v(a) - C^P(a, z^{CC}(a)) \tag{6}$$

Hence the first-order condition yields:

$$v'(a) - C^P_a(a, z^{CC}(a)) - C^P_z(a, z^{CC}(a)) \cdot z^{CC}_a(a) = 0 \tag{7}$$

However, from the definition of $z^{CC}(a)$, we have $C^P_z(a, z^{CC}(a)) = 0$ which together with (4) verifies the claim.

□

This result simply states that by completing contracts according to the parties' intentions, courts allow the agency-partners to achieve the allocation they would have obtained under comprehensive contracting.[36] In that respect, the CC procedure implements the Posnerian Principle which calls upon the legal process to mimic the market.[37]

5.2 Completing contracts and welfare

In the previous subsection, we started with the CC procedure and found that it implemented the same allocation as under complete contracting. In the absence of market failures, it is obviously welfare maximizing. However, in the current context this is not clear since in a moral hazard environment comprehensive contracting induces an inefficient allocation.

A priori, one could envisage many other procedures to address a payment dispute arising from the contract $\mathcal{D} = \{a, B\}$ and evaluate their welfare implication. For

[35] This observation on the formation of beliefs bears some relevance on a debate among legal scholars as to whether the purposive approach implies an increase of the parties' uncertainty. Our finding supports the opposite view; see also Barak (2005) for a similar conclusion.

[36] It is in this sense that restricting the analysis to discretionary bonus schemes is without loss of generality (see footnote 30).

[37] See Posner (2007).

instance, courts could be guided to decide by attempting to determine whether the agent actually performed the contracted effort.[38] In Common Law countries, this would require the court to rule on a Balance of Probabilities (BoP).[39] In our context, a natural interpretation of BoP would be that upon observing evidence x a court rules in favor of payment if and only if it cannot find a smaller effort level which appears more likely. Formally, this defines the payment relevant set

$$\mathcal{S}(a) = \left\{ x \in [0, \overline{x}] \mid \forall a' \leq a,\ g(x, a') \leq g(x, a) \right\} . \tag{8}$$

In the "Appendix", we prove that the resulting payment relevant set is an interval of the form $\mathcal{S}(a) = [z^{BoP}(a), \overline{x}]$ so that courts would enforce paying B whenever it observes $x \geq z^{BoP}(a)$.

From a normative perspective, one would like to search for the optimal guidance among *all* possible procedures. In order to operationalize that search over court guidance, we assume the existence of a benevolent regulator who provides court guidance with the aim of maximizing social welfare—measured by the expected sum of surpluses generated by the contract between the principal and the agent.[40]

Technically, there is no obvious way to directly integrate the verbalization restriction into the regulator's optimization problem. In order to circumvent this difficulty, we proceed in two steps. First, we *temporarily imagine* that, contrary to Assumption 2, the regulator can verbalize a procedure which defines critical likelihood ratios and solve for the optimal procedure. In a second step, we find a condition under which the fictitious regulator cannot improve upon the allocation obtained under the CC procedure.

The fictitious regulator is thought to have the same information as the principal, and not as the agent, since he does not know the latter's private information. The timing of this hypothetical regulator-principal-agent game is as follows; (1) the regulator verbalizes a procedure which implements a standard z^R; (2) the principal makes a take-it-or-leave-it offer $\mathcal{D} = \{a, B\}$; (3) the agent decides whether to participate; (4) conditional on participation the agent undertakes effort; (5) x is realized and the principal decides whether to pay B; (6) the agent decides whether or not to challenge the principal's decision. In the latter case, payments are made according to the principal's decision. In case of a challenge, the regulator adjudicates in accordance with the standard z^R.

In this hypothetical setting, a rational regulator would anticipate the effect of the standard on effort. Specifically, given a standard z the regulator would expect the effort level[41]

$$a^*(z) = \max_a \ v(a) - C^P(a, z) . \tag{III}$$

[38] In the current context, this could be interpreted as an attempt to operationalize the literal interpretation of the discretionary bonus scheme.

[39] See, for example, a verdict by the Court of Appeals of Utah, *Mark Technologies Corp. v. Utah Resources Intern., Inc., 147 p.3d 509* (2006) which involved the enforcement of a "best efforts" clause.

[40] Note that since the informational rents cancel out, the benevolent regulator will want to implement the efficient solution despite the moral hazard context.

[41] Technically, $a^*(z)$ is implicitly defined by the first equation in (4).

From the foregoing, we know that for all $z \in (0, \bar{x})$ the principal finds it advantageous to design a contract which induces too little effort relative to the fist-best solution.[42] Accordingly, in order to maximize welfare a benevolent regulator should select a procedure with an associated standard z^R that induces the largest possible effort level $a^*(z^R)$. Hence, in our hypothetical environment the regulator would select the standard to minimize the principal's marginal costs of inducing effort. Formally, this requires defining the critical likelihood ratio z^R by $C^P_{az}(a^*(z), z) = 0$ which generates the allocation associated with effort $a^*(z^R)$.

Next, we compare this outcome with the allocation obtained under comprehensive contracting or equivalently the CC procedure. From the foregoing, we know that the principal selects x^C to minimize his costs of implementing $a^*(z^C)$. Technically, x^C is implicitly defined by $C^P_z(a^*(z), z) = 0$ (see the equation system 4).

Clearly, these two solutions coincide whenever

$$\text{sign}\left(C^P_z(a, z)\right) = \text{sign}\left(C^P_{az}(a, z)\right) \tag{9}$$

holds. Intuitively, (9) ensures that for all effort levels, the functions $C^P(a, z)$ and $C^P_a(a, z)$ attain a minimum in z at the same point. Economically, (9) means that the principal's agency costs and marginal agency costs are *congruent*.[43] Next result summarizes this finding.

Proposition 4 *Assuming condition* (9) *is satisfied, even if a benevolent regulator was able to verbalize a procedure which defines a critical likelihood ratio, he would not be able to implement an allocation that improves upon the constrained efficient allocation.*

In other words, assuming the condition (9) holds and principals use discretionary bonus schemes, even a benevolent regulator who *could verbalize any* critical likelihood ratio associated with *any* principal-agent problem, could *not* improve upon the allocation resulting from applying the CC procedure. The result trivially extends to the more realistic case where the legal system does not possess an all-encompassing ability to verbalize procedures. Next result summarizes the findings.

Corollary 5 *Assuming that contractual parties, courts and the legal system all satisfy Assumptions* 2 *and* 3, *and condition* (9) *is satisfied, then the CC procedure implements the best possible allocation.*

While Corollary 5 should be very useful to guide courts, it is not formulated in terms of the model's primitives which raises the question whether (9) is ever

[42] See the discussion just below the system (4).

[43] An analogous congruency property has been introduced in the literature dealing with moral-hazard adverse-selection environments (e.g. McAfee and McMillan (1987) analyzing competition for agency contracts, and McAfee and McMillan (1991) and Vander Veen (1995) deriving optimal contracts for teams).

satisfied, and if so under what conditions. At the formal level, we provide two examples based on the primitives verifying that while there exist cases where (9) holds, it does not always hold. For this purpose, we first provide a technical *sufficient condition* on the elasticity of the distribution function $G(z;a)$ guaranteeing that (9) holds. Using the respective first-order conditions (24) and (25) for the principal's and regulator's problems, we prove in the "Appendix" the following result:

Proposition 6 *Suppose the distribution* $G(z;a)$ *satisfies*

$$\frac{aG_{aa}(z;a)}{G_a(z;a)} = h(a) \text{ for all } z \in \left(0, \bar{x}\right) \tag{10}$$

then by selecting the CC procedure the regulator implements the constrained efficient allocation.

Notice, however, that since (10) is sufficient, but not necessary for (9) to hold, there may be many additional environments that produce congruent agency and marginal agency costs.[44] By Corollary 5, for all such environments the CC procedure would also attain the constrained efficient allocation.

In the remainder of the subsection we turn to the aforementioned examples. Technically, the sufficient condition (10) demands that the elasticity of $G_a(z;a)$ with respect to effort be independent of z. To gain further insight on that requirement and keeping in mind that $G(\cdot;\cdot)$ is restricted to satisfy MLRP and CDFC, we draw on a theory paper by LiCalzi and Spaeter (2003) which studies two families of distributions complying with the latter properties. We now show that (10) holds for all members of one family and is never satisfied for the distributions belonging to the other.

Members of the first family of distributions introduced by LiCalzi and Spaeter (2003) take the generic form

$$G^1(x;a) = x + \beta(x)\gamma(a) \quad \text{with } x \in [0, 1] \tag{11}$$

where the functions $\beta(\cdot)$ and $\gamma(\cdot)$ must satisfy appropriate conditions reproduced in the "Appendix".[45] For that family of distributions (10) is satisfied since

$$\frac{aG^1_{aa}(z;a)}{G^1_a(z;a)} = \frac{a\gamma''(a)}{\gamma'(a)} . \tag{12}$$

The second family of distributions is generically characterized by

[44] For instance, Bental et al. (2014) uses a moral hazard adverse selection model where the agent's effort only takes two possible values. In that environment (9) holds without additional restrictions on the distribution of the proxy used to align incentives.

[45] Observe that $G^1(x, a)$ is closely related to the formulation in Hart and Holmstrom (1987) where

$$G(x, a) = \gamma(a)F(x) + (1 - \gamma(a))H(x) .$$

$$G^2(x;a) = \delta(x)e^{\beta(x)\gamma(a)} \quad \text{with } x \in [0,1] \tag{13}$$

where the functions $\delta(\cdot)$, $\beta(\cdot)$ and $\gamma(\cdot)$ must satisfy requirements also listed in the "Appendix". In particular, $\beta(\cdot)$ cannot be constant and in our context the function $\gamma(\cdot)$ must be strictly increasing. For this case, (10) is clearly not satisfied because

$$\frac{aG_{aa}(z;a)}{G_a(z;a)} = \frac{a\gamma''(a)}{\gamma'(a)} + a\beta(z)\gamma'(a)\,, \tag{14}$$

so that $z^R \neq x^C$.[46]

6 Discussion and concluding remarks

There exists an extensive economic literature analyzing interactions subjected to informational asymmetries. Typically, models of such interactions involve the design of incentive schemes which are assumed to be comprehensive. However, real world contracts often contain incentive clauses that do not fully specify the conditions triggering their implementation. The resulting contractual ambiguities create the potential for opportunistic behavior. In that context, the legal system becomes one of the mechanisms which are used to discipline the contracting parties.

In order to study the relationship between non-comprehensive contracting and the legal environment, we introduced a stylized moral hazard model between a principal and an agent. In the analysis, we invoke an assumption which rules out fully comprehensive contracting. Subject to that restriction, incentive schemes take the form of discretionary bonus agreements giving rise to potential payment-related disputes.

Drawing on the Law and Economics literature on incomplete contracts, we analyze a procedure which requires courts to complete non-comprehensive contracts according to the parties' initial intentions. We find a congruency condition which guarantees that, despite the tension between the principal's and social objectives, the CC procedure yields the constrained second-best allocation. Whether this condition holds in any specific case is an empirical matter. Specifically, in a given conflict the court will have to ascertain whether or not by applying more stringent conditions triggering payment the principal's cost of inducing effort and the marginal cost thereof respond in a conformal way. If the answer to that question is positive, the court should complete the contract according to the parties' intentions thereby implementing the constrained efficient allocation.

Our analysis does not provide any guidance as to what courts should do when congruency fails. An attempt to address this challenging issue can be found in a recent book by the former president of the Israeli supreme court Aharon Barak.[47]

[46] Suppose that in (13) for all x and a, we have $\delta(x) = 1$, $\gamma(a) = a$ and $\beta(x) = \ln F(x)$ where $F(\cdot)$ is a cdf over $x \in [0,1]$. In that case, we obtain $G(x;a) = [F(x)]^a$ which is the well known example from the Rogerson (1985) paper. For that case (14) holds so that in Rogerson's example (10) does not hold.

[47] Barak is well known for his advocacy of "active courts". Very much in the spirit of our paper, in the "Apropim" verdict (CA4628/93 State of Israel vs. Apropim [1995]), Barak first inferred the parties'

When discussing cases in which there is a discrepancy between private and social objectives (in our context when congruency fails), Barak (2005, p. 338) advocates that judges “may privilege objective purpose” and use “pragmatism to choose the best solution within the parameters (subjective and objective)” in order to “aspire to the purpose that best achieves justice”. In terms of a formal model, the “subjective purpose” could be interpreted as the parties’ intentions, the “objective purpose” as social welfare and “justice” in the Posnerian way as “maximizing welfare”. Based on this interpretation, Barak’s suggestion would imply that when congruency fails courts should not solely focus on the parties’ intentions and take broader societal considerations into account.

Our analysis suggests that neither case where congruency is satisfied or where it is not satisfied is vacuous. Formally, we introduced a sufficient condition that guarantees congruency. In our context, that sufficient condition relates to an elasticity requirement on the probability density function. We then provided two simple examples; in the one case the elasticity condition and, therefore, the congruency requirement were satisfied, while in the other example congruency fails.

We briefly discussed another verbalizable procedure typically used in Common Law countries. That procedure would guide courts to apply a Balance of Probabilities in order to evaluate whether the agent has produced the effort contracted upon. However, as implied by the potential efficiency of the CC procedure, the BoP method will, in general, not deliver the constrained efficient allocation. This may initially appear surprising given the numerous efficiency properties of BoP found in the Law and Economics literature, such as minimizing errors (see Brook 1982) and maximizing incentives (Demougin and Fluet 2006).

In that respect, the paper by Fluet (2003) helps shed some light as to why BoP does not deliver a constrained efficient allocation in our model. The environment analyzed by Fluet (2003) is very similar to ours; it is characterized by non-comprehensive contracting in a moral hazard buyer/seller framework. The contract between the buyer and the seller involves an up-front payment. Moreover, in case of a dispute with respect to the quality delivered, a court may enforce a penalty if it finds in favor of the buyer. In that environment, the penalty plays the same role as the bonus in our setup. However, in contrast to our model the penalty is assumed to be determined by the court according to “expectation damages” and, hence *out of the realm of negotiations by the parties*. It is this feature which explains the difference between our conclusion and the finding in Fluet (2003).[48] In our analysis, the court decides on the payment of a bonus which was *negotiated by the parties* at the contracting stage. This structure introduces a feedback loop between the anticipated court action (based upon some specified procedure) and the contract negotiated by the parties. This feedback loop is the feature which undermines the efficiency of the BoP procedure in our framework.

Footnote 47 (continued)
intentions in order to complete the contract and then decided upon the validity of a claim to implement a disputed incentive clause.

48 We thank Claude Fluet for pointing the above difference out.

While we used a specific model to exemplify the role of the congruency condition, its relevance extends to more general environments. Indeed, the core question is whether agency costs and marginal agency costs react in the same way to changes of the payment relevant set. Whenever this is the case, completing contracts should yield the constrained second-best allocation. Intuitively, the critical value triggering payment which minimizes the principal's cost of inducing effort would also minimize the marginal cost thereof, and hence generate the highest possible effort level under the informational constraints.

In a broader context our analysis may be associated with fundamental developments in the legal profession. Historically, in most legal systems courts favored a literal approach to contractual interpretation. This has been progressively replaced by a purposive approach over the course of the last century. However, in many jurisdictions the debate is still relevant as legal doctrines are replete with countervailing arguments in favor and against logical (textual) interpretation on the one hand, and purposive or teleological (policy-oriented or contextual) interpretation on the other. While the legal arguments refer mainly to legal consistency, judicial function or judicial activism, our point of view is that of incentives and economic efficiency. Instructing courts to complete contracts is in the spirit of the purposive approach. However, in our framework that procedure implements the constrained efficient allocation only if the congruency condition holds. This finding provides an economic rationale for the continued debate among legal scholars: how should courts be guided when the congruency condition fails to hold? Would a more grammatical approach generate economically desirable outcomes? This question is left for future research.

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Appendix

Proof of Proposition 1 Observe that due to the curvature assumption on $c(\cdot)$ and the CDFC requirement, the agent's objective function is concave. Hence, the (IC) condition can be substituted for the first-order condition of the agent's optimization problem. Initially ignoring the (PC) requirement and converting (I) into a maximization yields the Lagrangian:

$$\mathcal{L} = -w_{\emptyset}(1-m) - m\int_0^{\bar{x}} w(x)g(x;a)dx + \lambda\left(m\int_0^{\bar{x}} w(x)g_a(x;a)dx - c'(a)\right) + \int_0^{\bar{x}} \xi(x)w(x)dx + \int_0^{\bar{x}} \zeta(x)\big(w_{\emptyset} - w(x)\big)dx \quad (15)$$

Taking the derivative with respect to $w_{\emptyset}$ and $w(x)$, we obtain:

$$\begin{cases} -(1-m) + \int_0^{\bar{x}} \zeta(x)w_{\emptyset}dx = 0 \\ -mg(x;a) + \lambda m g_a(x;a) + \xi(x) - \zeta(x) = 0 \end{cases} \quad (16)$$

which together with the complementary slackness conditions $\xi(x)w(x) = 0$, $\zeta(x)\big(w_{\emptyset} - w(x)\big) = 0$, the non-negativity requirements $\xi(x)$, $\zeta(x) \geq 0$ as well as the constraints (AFC) and (PFC) for all $x \in [0;1]$ implicitly define the solution.

Claim 1 *For any* $a > 0$ *we have* $\lambda \neq 0$.

Proof Suppose to the contrary. Then the second equation in (16) implies that for all $x \in [0;1]$

$$mg(x;a) + \zeta(x) = \xi(x) > 0 \quad (17)$$

since $m > 0$ and$g(x;a) > 0$ over the support. Accordingly, by complementary slackness $w(x) = 0$ over the support which violates the first-order condition of the agent's (IC) for any $a > 0$. □

Claim 2 *There can exist at most one point in* [0, 1] with $\zeta(x) = \xi(x) = 0$.

Proof Suppose $\zeta(x) = \xi(x) = 0$, then the second equation in (16) implies

$$-m + \lambda m \frac{g_a(x;a)}{g(x;a)} = 0 \quad (18)$$

verifying the claim by strict MLRP and $\lambda \neq 0$, $m > 0$. □

Since this potential one point is of measure zero, it is irrelevant to the optimization and will henceforth be ignored (together with the possibility $\zeta(x) = \xi(x) = 0$).

Claim 3 *There is no x in the support for which* $\zeta(x)$, $\xi(x) > 0$.

Proof Suppose to the contrary that at $x_1 \in (0, \bar{x})$ we have $\zeta(x_1)$, $\xi(x_1) > 0$. Then, by complementary slackness $w(x_1) = w_{\emptyset} - w(x_1) = 0$. As a result, (PFC) implies that for all x, we have $0 \leq w(x) \leq w_{\emptyset} = 0$. Hence $w(x) = 0$ for all $x \in (0, \bar{x})$ which violates the first-order condition of the agent's (IC) for any $a > 0$. □

Taking $x \in (0, \bar{x})$, there are two possible cases remaining; either $\zeta(x) = 0,\ \xi(x) > 0$ or $\zeta(x) > 0,\ \xi(x) = 0$. Clearly each of these cases must occur over a subset of the support with positive measure. Suppose to the contrary that $\xi(x) > 0$ almost everywhere (a.e. hereafter). This would imply $w(x) = 0$ a.e. Similarly $\zeta(x) > 0$ a.e. would yield $w(x) = w_{\emptyset}$ a.e. In either situation, setting positive incentives is not possible, leading to a contradiction.

To conclude the proof, observe that λ must be positive. Suppose it is negative, then the contract would pay $w_{\emptyset} > 0$ for small realization of x and 0 for large realization of x whereby the critical x^c would solve $-1 + \lambda \frac{g_a(x;a)}{g(x;a)} = 0$. But then setting incentives is not feasible.

Altogether, we now know that the contract which solves the simplified principal's problem where (PC) has been ignored is a bonus scheme paying B if $x \geq z$ for some critical value which partitions the support and pays 0 otherwise. Accordingly, we can rewrite the simplified problem as:

$$C(a) = \min_{B,z} [1 - mG(z;a)]B \quad \text{s.t.} \ - mG_a(z;a)B - c'(a) = 0 \tag{IV}$$

Substituting B, we define $C^P(a;z) = \frac{1-mG(z;a)}{-mG_a(z;a)} c'(a)$.[49] Observe that $C^P(0, z) = 0$ by $c'(0) = 0$. Moreover, we have

$$\frac{\partial C^P}{\partial a}(a, z) = c'(a) + \left[\frac{1 - mG(z;a)}{-mG_a(z;a)} \right] \left(c''(a) - \frac{G_{aa}(z;a)}{G_a(z;a)} c'(a) \right). \tag{19}$$

Strict MLRP, CDFC and the convexity of $c(\cdot)$ imply $\frac{\partial C^P}{\partial a}(a, z) > c'(a)$ for $a > 0$. Finally, note that by the envelope theorem, $C'(a) = \frac{\partial C^P}{\partial a}(a, x^c)$. Hence, the simplified problem also satisfies the (PC) requirement implying that its solution is identical to that of problem (I) thereby verifying the claim of Proposition 1. □

Proof of Proposition 2 The agent's rent is given by the difference

$$R(a, z) = C^P(a, z) - c(a). \tag{20}$$

From the proof of the foregoing proposition, we have $R(0, z) = 0$, $\frac{\partial R}{\partial a}(a, z) > 0$ and $R(a, z) > 0$ for all $a > 0$, thus, verifying the claim of Proposition 2. □

Lemma 7 *The set $\mathcal{S}(a)$ defined by (8) is an interval endogenously defined by a critical value $z^{BoP}(a)$, i.e. $\mathcal{S}(a) = [z^{BoP}(a), \bar{x}]$.*

Proof of Lemma 7 Suppose $x \in \mathcal{S}(a)$ i.e.

[49] Note that this is the same definition as (3) in the text.

$$\forall a' \leq a, \quad \frac{g(x;a)}{g(x;a')} \geq 1. \tag{21}$$

Moreover, MLRP states

$$\frac{d}{dx}\left[\frac{g(x;a)}{g(x;a')}\right] > 0\,. \tag{22}$$

Hence for any $y > x$ we have:

$$\frac{g(y;a)}{g(y;a')} > \frac{g(x;a)}{g(x;a')} \tag{23}$$

so that $\mathcal{S}(a)$ is an interval verifying the Lemma 7. □

Proof of Proposition 6 First, observe that the equation system (4) which defines (a^C, x^C) under comprehensive contracting can be rewritten as:

$$\begin{cases} v'(a) - c'(a) - \left[\frac{1 - mG(z;a)}{-mG_a(z;a)}\right]\left(c''(a) - \frac{G_{aa}(z;a)}{G_a(z;a)}c'(a)\right) = 0 \\ -\frac{\partial}{\partial z}\left[\frac{1 - mG(z;a)}{-mG_a(z;a)}\right]c'(a) = 0 \end{cases} \tag{24}$$

In the case where the regulator sets z, the pair $\left(a^R, z^R\right)$ which maximizes the regulator's objective is implicitly defined as the solution to the 2×2 system

$$\begin{cases} v'(a) - c'(a) - \left[\frac{1 - mG(z;a)}{-mG_a(z;a)}\right]\left(c''(a) - \frac{G_{aa}(z;a)}{G_a(z;a)}c'(a)\right) = 0 \\ \frac{\partial}{\partial z}\left(\left[\frac{1 - mG(z;a)}{-mG_a(z;a)}\right]\left(c''(a) - \frac{G_{aa}(z;a)}{G_a(z;a)}c'(a)\right)\right) = 0 \end{cases} \tag{25}$$

where the second equation follows by applying the implicit function theorem with respect to z on the first-order condition of (III). Intuitively, the regulator selects z^R to minimize the principal's marginal costs. The requirement (10) ensures that the systems (24) and (25) yield the same solution verifying the claim of Proposition 6. □

The LiCalzi and Spaeter distributions. For the sake of completeness, we briefly reproduce from LiCalzi and Spaeter (2003) the conditions characterizing the two distribution families satisfying MLRP and CDFC used in the Sect. 5.2. For the first family described by the generic form (11), the functions $\beta(\cdot)$ and $\gamma(\cdot)$ must satisfy:

1. $\beta(x)$ is a positive and concave function on the support $x \in [0, 1]$ such that $\lim_{x\downarrow 0} \beta(x) = \lim_{x\uparrow 1} \beta(x) = 0$ and $|\beta'(x)| \leq 1$ for all $x \in (0, 1)$;
2. $\gamma(a)$ is a decreasing and convex function for all $a \geq 0$ such that $|\gamma(a)| < 1$.

Moreover, for the second family given by (13), the functions $\delta(\cdot)$, $\beta(\cdot)$ and $\gamma(\cdot)$ are required to satisfy:

1. $\beta(x)$ is a non-constant, negative, increasing, and convex function on the support $x \in [0, 1]$ such that $\lim_{x\uparrow 1} \beta(x) = 0$;
2. $\gamma(a)$ is a strictly positive, increasing, and concave function for all $a \geq 0$;
3. $\delta(x)$ is a positive, strictly increasing, and concave function on the support $x \in [0, 1]$ such that $\lim_{x\downarrow 0} \delta(x) = 0$ and $\lim_{x\uparrow 1} \delta(x) = 1$.

Note that these are not the only distribution families for which MLRP and CDFC hold. For instance, the family of distributions $G(x;a) = [F(x)]^{\gamma(a)}$ where $F(\cdot)$ is a CDF defined over $x \in [0, 1]$ and $\gamma(\cdot)$ a strictly increasing and concave function also has the desired properties. Moreover, in line with the observation of footnote 46 that class does not satisfy (10).

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