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Does counterfeiting benefit genuine manufacturer? The role of production costs

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Abstract A monopolist sells a luxury genuine product which can be illegally copied and sold by a competitive fringe of counterfeiters. Fines imposed on caught counterfeiters are pocketed by the genuine firm. We prove that if production costs are low, then the genuine manufacturer would lobbying for high penalties so that counterfeiters should be thrown out of the market. In this case, the presence of counterfeiters does not provide any benefit to the producer of the original product. Whenever the production cost is neither too high nor too low, the optimal fine guarantees a positive demand for the genuine product as well as for the fake; the genuine producer is better off than in a world without counterfeiters. If production costs are too high, the genuine firm has no more incentive to produce. Its remaining goal is to collect penalty money from counterfeiters. Again, the presence of counterfeiters provides a benefit to the genuine manufacturer. Finally, a comparison between full protection and null protection policies is performed.

Keywords Counterfeiting · Fines · Intellectual property rights · Social welfare

JEL Classification D21 · D23 · D42

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1 Introduction

According to Grossman and Shapiro (1988b), counterfeiting is defined as "illegally copying of a brand-name label or other distinguishing trademarks" without the brand owner's authorization. Counterfeiting is an illicit activity linked to intellectual property rights (IPR) infringement. It boasts a vast literature in economics and law. In most countries, counterfeiting is punishable by criminal law as well as civil laws, with penalties ranging from injunctions to damages to detention. Counterfeiting differs from patents and copyright infringements in regards to the ranking of qualities and market channels. The effective quality of products with a patent infringement may be higher than that of the legitimate product and consumers may even unknowingly purchase them in legal outlets. Counterfeit products, by contrast, are generally inferior to authentic goods. In addition, they are usually sold in illegal market channels.

There are two principal markets for trademark infringing products. In the first (the primary market), counterfeiters and pirates infiltrate distribution channels with products that are often substandard. Consumers unwittingly purchase these products, thinking that they are genuine. In fact, they have been deceived. Zhang and Zhang (2015) deal with the issue of restructuring marketing channels penetrated by deceptive counterfeiting by incorporating the wholesale price decisions, consumers' risk attitude towards counterfeits and consumer loyalty towards the reliable stores. Their main finding is that the brand name company should continue to sell, sometimes exclusively, through the general channel despite deceptive counterfeiting under various conditions. The secondary market involves consumers who, under certain conditions, are willing to purchase counterfeit products that they know are not genuine (non-deceptive counterfeiting).

Counterfeiting damages brand owners' reputations and lowers consumer confidence in the affected brands. It also causes missed sales opportunities and actual job losses by manufacturers and retailers. Counterfeiting also deprives national economies of customs duties and tax revenues. The infiltration of organized crime in counterfeiting activity may seriously threat the health and safety of consumers and the national security. Counterfeiting is an important issue to study. An OECD report (OECD 2007) indicates that international trade in counterfeit and pirated products could have been up to USD 200 billion in 2005. This total does not include domestically produced and consumed counterfeit and pirated products and the significant volume of pirated digital products being distributed via the Internet. If these items were added, the total magnitude of counterfeiting and piracy worldwide could be several hundred billion dollars more and this engenders strong and sustained action from governments, business and consumers.

Although counterfeiting hits every sector, it notably influences the luxury industries. Luxury goods are mainly consumed for their status conveying properties (Corneo and Jeanne 1997). In the luxury industries, price often enhances consumption utility, as the price tag of the luxury product can signal one's wealth and prestige (Veblen effect; see, for example, Leibenstein 1950; Bagwell and Bernheim 1996). Many consumers buy fake luxury items knowingly and at a price much lower than genuine price. The proliferation of unauthorized imitations of luxury goods can make the genuine items less desirable to their "snob" traditional consumers (brand dilution). Creative industries are deprived of a legitimate income leading to an under-innovation outcome (e.g., Arrow 1962; Grossman and Shapiro 1988b). Nevertheless, luxury products and their unauthorized imitations frequently coexist, sometimes in close geographic proximity (Barnett 2005; Castro et al. 2008). Barnett (2005) guessed that a flattery effect increasing the snob value of originals is created by counterfeiting.

Yao (2005a) presents a single-good market model in which genuine and counterfeit products are sold while the laws of intellectual property rights are enforced. The quality and price choices of a monopolist and the total social welfare in response to different IPR enforcements are investigated via a vertically differentiated model. A penalty in terms of monetary fines is imposed to counterfeiters due to their illegal activity. Fines are assumed to be pegged to the price of the genuine product. It is shown that if the degree to which the genuine product is imitated is low then the non-protection policy should be adopted. When the imitation rate is high then the counterfeit monitoring policy should be used, whereby a high imitation rate should be accompanied with a high monitoring rate.

The conventional wisdom is that counterfeiting affects branded goods negatively. However, a number of papers suggest that counterfeiting may actually benefit certain luxury brands. Romani et al. (2012) show how the market presence of luxury counterfeit items can increase consumers' willingness to pay for well-known original brands, but not for lesser-known ones. They address the underlying psychological mechanisms. Yao (2005b) focuses on the relationship between IPR enforcement and the monopoly price under a counterfeit monitoring regime. He shows that strictly enforcing IPR laws may cause a side effect in the luxury industries: the price of the luxury good may exceed that in the absence of counterfeit products. This is because in its model fines are claimed by the IPR holder. Consequently, the producer may obtain greater profits in the presence of counterfeiting than in its absence. Zhang et al. (2012) focus on how non-deceptive counterfeits affect the price, market share and profitability of brand name products. They also consider the strategies for brand name companies to fight counterfeiting.

Bekir et al. (2012) consider, too, a luxury monopolist and a competitive fringe of counterfeiters operating in a market with Veblen effects. Following Yao (2005a, b) the monopolist is allowed to pocket sanctions imposed to counterfeiters but without requiring sanctions pegged to the price of genuine items. Authors claim that even when the genuine producer can shape the 'rules of the game' (namely, the amount of sanctions imposed on counterfeiters), he will not necessarily seek to eliminate all counterfeiters. The policy implication is that proper IPR law and enforcement must consider how legitimate producers will adjust their prices in response to policy. Bekir et al. (2012) above claims are the results of what is written in Sect. 3 of their paper and in particular Lemma 2 and Figs. 3 and 4. Unfortunately, the proof of Lemma 2 contains a mistake while Figs. 3 and 4 are not correct due to miscalculations.

In this paper, according to Yao (2005a, b), and Bekir et al. (2012), a market where a monopolist sells a luxury genuine product protected by conventional IPR laws is considered. However, a competitive fringe of counterfeiters can illegally copy and sell the product without the permission of the monopolist. The original manufacturer can undertake some actions to deter counterfeiters from their activity, particularly through lobbying to improve the enforcement of IPR laws. We suppose that each counterfeiter incurs no production cost, except if it is caught and forced to pay a fine proportional to the quantity sold. As practised in some countries, we assume that the fines are pocketed by the genuine firm (Yao 2005b). Suppose that the (linear) production costs of the genuine firm are higher or equal to those of the counterfeiters.

Section 2 is dedicated to the model description ant to the computation of demand functions and profits.

In Sect. 3 production costs of the genuine items are assumed equal to those of fakes. Then the model reduces exactly to the Bekir et al. (2012) one. We correct their claim and prove that the genuine producer is never better off by the presence of counterfeiters.

Their results, however, may be recovered in the more general scenario of asymmetric production costs and this is proven in Sect. 4. We prove that if the production costs are not too low, then the genuine firm can take advantage of the presence of counterfeiting. This is not completely unexpected because production costs play an important role in the literature on counterfeiting. Far for provide a comprehensive review of the literature on the subject, here we just mention some relevant examples. In Yao (2005a), Mussa and Rosen (1978) and Grossman and Shapiro (1988a), the variable production cost of a genuine item depends on its quality; moreover the optimal quality (and, consequently, the optimal production cost) is endogenously computed, optimizing the utility of the firm. Zhang et al. (2012) prove that if the production cost of the genuine product is low, then authentic items and fakes coexist in the market; otherwise, the authentic product will be driven out of the market due to its high production cost. Zhang and Zhang (2015) show that when the production cost are above a given threshold, then the general distribution channel will definitely carry counterfeits. Tsai et al. (2012) establish a duopoly model, with horizontal and vertical differentiation simultaneously, to investigate how counterfeiting affects firms' market power and consumer's purchasing behavior. One of their findings tells that when the production cost of a genuine product increases, the consumers who originally purchased this genuine product may continue to purchase the genuine one, purchase the genuine of the other brand, or, quite interestingly, purchase the counterfeiting product of the other brand.

In Sect. 5 some policy implications that may result from our model are addressed. The social planner, being able to predict the optimal price that will be practised by the genuine firm, can choose the amount of the fine and the protection policy level in order to maximize the social welfare. A comparison between full protection and null protection policies is performed. If the social planner cares solely or mainly for consumer surplus, then it is more convenient to maintain a non-protection policy against a full protection one, regardless of the enforcement costs. By contrast, if sufficient attention is paid to the genuine firm profits, then the comparison result depends on the enforcement costs. Non-protection policy is desirable in the presence of high costs; otherwise the social planner is better off by performing full protection.

Section 6 concludes the paper and suggests some extensions.

2 The model

Let us consider a market where a monopolist sells a luxury genuine product protected by conventional IPR laws. However, a competitive fringe of counterfeiters can illegally copy and sell the product without the permission of the monopolist. The original manufacturer can undertake some actions to deter counterfeiters from their activity, particularly through lobbying to improve the enforcement of IPR laws. In particular, the genuine firm can seek strategically to shape the rules of the game and get a fine level to maximize its utility. Note that usually it less costly for public authorities to increase the fine level compared to the cost of increasing the probability of detection (Becker 1968). Consequently, it is intuitively easier for genuine firms to influence public decisions on the fine amount level.

The demand for luxury products comes from a continuum of consumers indexed by a parameter θ, which is uniformly distributed over the interval [0, 1]. The taste parameter θ can be interpreted as the valuation that the consumer gives to the good. This is consistent with the literature on models for vertically differentiated goods (see e.g., Mussa and Rosen 1978 for a more detailed discussion).

Genuine and counterfeited products are denoted, respectively, with subscripts g and c. Every individual demands either one unit of the good or nothing. As genuine and counterfeited products are imperfect substitutes and vertically differentiated, the valuation of a fake product is lower than the valuation of an original item. Consequently, the valuation of a fake compared to the valuation of a genuine item is discounted by the factor α, with $0 < \alpha < 1$. Each counterfeiter incurs no production cost, except if it is caught and forced to pay a fine proportional to the quantity sold. The per unit fine payable by a detected counterfeiter will be denoted by the non negative number f. As practiced in some countries, we assume that the fines are pocketed by the genuine firm (Yao 2005b). The utility of a consumer is given by:

$$\begin{cases} \theta - p_g & \text{if she buys the genuine product} \\ \alpha\theta - p_c & \text{if she buys the fake product} \\ 0 & \text{if she buys none} \end{cases}$$

where p_g and p_c are, respectively, the price of the genuine and fake products.

Assume that the genuine firm incurs linear production costs and let c be the unitary cost. If c is higher than one, then the genuine manufacturer can not stay on the market making positive profits. Therefore, in the following we suppose

$$0 \leq c \leq 1.$$

According to Bekir et al. (2012), the genuine firm's R&D costs are neglected. These costs are in fact relevant ex-ante to assess whether or not to enter the market. However they do not affect the choice of the price of the product nor the amount of the optimal fine. Furthermore, Bekir et al. (2012) assume that the luxury monopolist incurs a fixed cost k, corresponding to expenditures related to stronger enforcement of their intellectual property rights. Nevertheless, such costs are taken to be independent of both the amount of the penalty and the commitment to track down counterfeiters. As a result, however, they become irrelevant in the entire subsequent analysis. Thus, for simplicity, we set $k = 0$. The monopolist profit is expressed by

$$\pi_g = (p_g - c)D_g + \phi f D_c$$

where D_g and D_c are the demands addressed to the genuine manufacturer and to to the counterfeiters, respectively. The counterfeiters earn $p_c D_c$ whenever they are not caught, otherwise they incur in a loss of fD_c. Hence, the average counterfeiters profit is

$$\pi_c = ((1 - \phi)p_c - \phi f)D_c$$

where ϕ is the probability of being caught.

A world without counterfeiting serves as a benchmark with a world where counterfeiters co-exist with the luxury monopolist. If counterfeiters do not exist, the demand addressed to the monopolist is expressed by $D_g = \max(0, 1 - p_g)$. The profit of the luxury monopolist is given by $\pi_g = (p_g - c)D_g$. Its maximum $\hat{\pi}_g = (1 - c)^2/4$ is reached for $\hat{p}_g = (1 + c)/2$.

Counterfeiters are supposed all identical and they can enter and exit the market freely. Therefore, following Bekir et al. (2012), the net expected pay-off due to counterfeiting should be zero in equilibrium and the price of a fake is

$$p_c = \frac{\phi f}{1 - \phi}.$$

A consumer buys a genuine item if she receives a positive utility and if such a utility is greater than to that she would get by buying a fake. Similarly, a consumer buys a fake if she receives a positive utility and if such a utility is greater than to that she would get by buying a genuine item. The demands of the genuine and of the

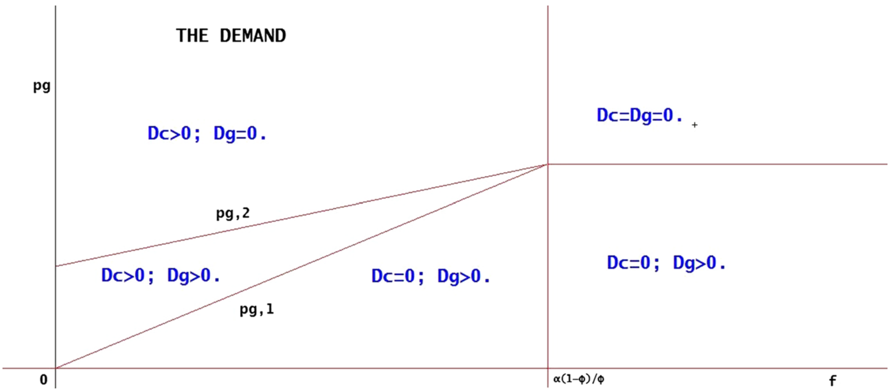

Fig. 1 The demand as a function of the fine f and of the price of genuine item p_g

counterfeited products depend on the fine f and on the price p_g of an original item. They are easily computed in the following Proposition and illustrated in Fig. 1.

Proposition 1 (The demand) *Let*

$$\tilde{f} = \frac{\alpha(1-\phi)}{\phi}; \quad p_{g1} = \frac{\phi f}{\alpha(1-\phi)}; \quad p_{g2} = 1 - \alpha + \frac{\phi f}{1-\phi}.$$

- *Let* $0 \le f < \tilde{f}$.
  - *If* $0 \le p_g \le p_{g1}$, *then* $D_c = 0$ *and* $D_g = 1 - p_g > 0$;
  - *if* $p_{g1} < p_g < p_{g2}$ *then* $D_c = \dfrac{\alpha p_g - \dfrac{\phi f}{1-\phi}}{\alpha(1-\alpha)} > 0$ *and* $D_g = \dfrac{1 - \alpha - p_g + \dfrac{\phi f}{1-\phi}}{1-\alpha} > 0$;
  - *if* $p_{g2} \le p_g$, *then* $D_c = 1 - \dfrac{\phi f}{\alpha(1-\phi)} > 0$ *and* $D_g = 0$.

- *Let* $\tilde{f} \le f$.
  - *If* $0 \le p_g < 1$, *then* $D_c = 0$ *and* $D_g = 1 - p_g > 0$;
  - *if* $1 \le p_g$, *then* $D_c = 0$ *and* $D_g = 0$.

Proof See Appendix.

We observe that when the amount of the fine overcomes the threshold $\tilde{f}$, counterfeiters are thrown out of the market and the genuine producer acts as a monopolist. Therefore in the rest of the paper we limit our analysis to the case

$$0 \le f \le \tilde{f}.$$

As an immediate consequence of Proposition 1, the profit π_g of the genuine producer is given as follows.

- If $0 \le p_g \le p_{g1}$, then $\pi_g = (p_g - c)(1 - p_g)$;
- if $p_{g1} < p_g < p_{g2}$ then $\pi_g = (p_g - c)\left(\dfrac{1 - \alpha - p_g + \dfrac{\phi f}{1-\phi}}{1-\alpha}\right) + \phi f \left(\dfrac{\alpha p_g - \dfrac{\phi f}{1-\phi}}{\alpha(1-\alpha)}\right)$;

- if $p_{g2} \leq p_g$, then $\pi_g = \phi f\left(1 - \frac{\phi f}{\alpha(1-\phi)}\right)$.

3 The Bekir et al. (2012) case: $c = 0$

In this section we consider the case $c = 0$ which corresponds to the one treated in Bekir et al. (2012). The following proposition gives the optimal choice of the genuine producer.

Proposition 2 (The optimal profit) *Let*

$$\hat{f} = \frac{\alpha(1-\alpha)(1-\phi)}{\phi[2(1-\alpha)+\alpha\phi]}; \quad p^*_{go} = \frac{(1-\phi)(1-\alpha)+\phi(2-\phi)f}{2(1-\phi)};$$

$$\pi^*_{go} = \frac{\alpha(1-\alpha)^2(1-\phi)^2 + 2\alpha(1-\alpha)(1-\phi)(2-\phi)\phi f + [\alpha(2-\phi)^2 - 4(1-\phi)]\phi^2 f^2}{4\alpha(1-\alpha)(1-\phi)^2};$$

$$\pi_{g1} = \frac{\phi f}{\alpha(1-\phi)}\left(1 - \frac{\phi f}{\alpha(1-\phi)}\right).$$

- *If* $0 \leq f < \hat{f}$ *then the optimal price chosen by the genuine producer is* $p_g = p^*_{go}$, *both demands* D_c *and* D_g *are positive and the ensuing profit is* $\pi_{g,max} = \pi^*_{go}$.
- *If* $\hat{f} \leq f \leq \tilde{f}/2$ *then the optimal price chosen by the genuine producer is* $p_g = p_{g1} < 1/2$. *The demand of the genuine item is positive while the demand of the fake is nil. The ensuing profit is* $\pi_{g,max} = \pi_{g1} < 1/4$.
- *If* $\tilde{f}/2 < f \leq \tilde{f}$ *then the optimal price chosen by the genuine producer is* $p_g = 1/2$. $D_g = 1/2, D_c = 0$ *and the ensuing profit is* $\pi_{g,max} = 1/4$.

Proof See Appendix.

If $0 \leq f < \hat{f}$, then the optimal profit of the genuine firm is obtained allowing a positive demand for both genuine product and fake. Whenever $f \geq \hat{f}$, the optimal profit of the genuine firm is obtained in such a way the demand for counterfeit products is nil, counterfeiting is deterred and the monopolist does not pocket any revenue from fines. In the following proposition, the fine giving the maximum profit to the genuine producer is computed.

Proposition 3 (The best fine) *The optimal profit* $\pi_{g,max}$ *is strictly increasing with respect to the fine* f *as* $0 \leq f \leq \tilde{f}/2$; *it is constant as* $\tilde{f}/2 < f \leq \tilde{f}$. *The highest value* 1 / 4 *of* $\pi_{g,max}$ *is obtained for any* $\tilde{f}/2 < f \leq \tilde{f}$.

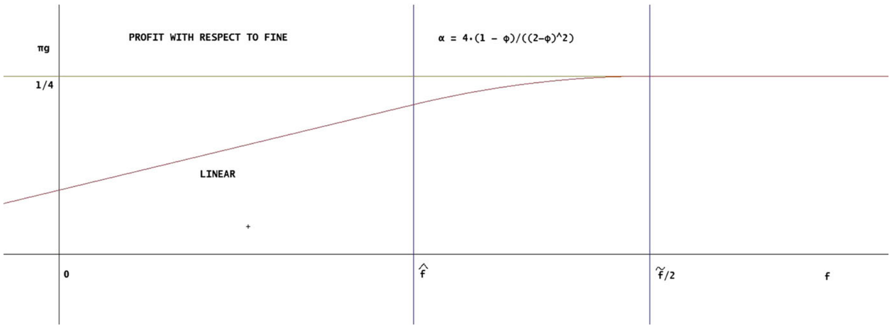

Fig. 2 The optimal profit as a function of the fine *f*. The linear case $\alpha = \alpha^*$

Proof See Appendix.

The behaviour of the optimal profit $\pi_{g,max}$ with respect to the fine *f* is depicted in Figs. 2, 3 and 4. As $0 \leq f \leq \hat{f}$, $\pi_{g,max}$ is a concave function if $\alpha < \alpha^*$, a convex function if $\alpha > \alpha^*$, a linear function if $\alpha = \alpha^*$, where

$$\alpha^* = \frac{4(1-\phi)}{(2-\phi)^2}.$$

Moreover $\pi_{g,max}$ is a continuous[1] non decreasing function.

Note that Fig. 3 corresponds to Fig. 3, p. 657 of Bekir's et al. paper while Fig. 4 corresponds to Fig. 4, p. 658. As you can see, they report a value of π^*_{go} in $f = \hat{f}$ greater than 1/4 and this is not true.[2] We also observe that when $\hat{f} < f < \tilde{f}/2$, then the optimal choice of the genuine producer is the price p_{g1} and the demand of the fake is zero. But this does not correspond to a world without counterfeiters. In fact the optimal price p_{g1} is lower than $\hat{p}_g = 1/2$ and the profit π_{g1} is inferior to $\hat{\pi}_g = 1/4$. For such values of the penalty, an increase in the price of genuine items leads to a positive demand of the fake. In other words, a world without counterfeiters at all is different from a world where there is a threat of counterfeiting. The genuine producer would gain the same profit as in a world without counterfeiters only if the fine exceeds the threshold $\tilde{f}/2$.

In the conclusion of their paper, Bekir et al. (2012) write "Unlike the conventional wisdom, that suggests that the monopolist will prefer completely deterring counterfeiters, our results state that even when the genuine producer can strongly shape the 'rules of the game' (namely, the penalties imposed on counterfeiters), he will not entirely eliminate counterfeiting. When fines are slightly lower than the deterrence threshold, then the monopolist pockets the fines and the demand addressed to the counterfeiters is enough to generate the positive collateral

[1] It is $\pi^*_{go}(f = \hat{f}) = \dfrac{(1-\alpha)(1-\alpha+\alpha\phi)}{(2-2\alpha+\alpha\phi))^2} = \pi_{g1}(f = \hat{f})$ and $\pi_{g1}(f = \tilde{f}/2) = \dfrac{1}{4} = \hat{\pi}_g$.

[2] It is $\pi^*_{go}(\hat{f}) = \dfrac{(1-\alpha)(1-\alpha+\alpha\phi)}{(2-2\alpha+\alpha\phi)^2} < \dfrac{1}{4}$.

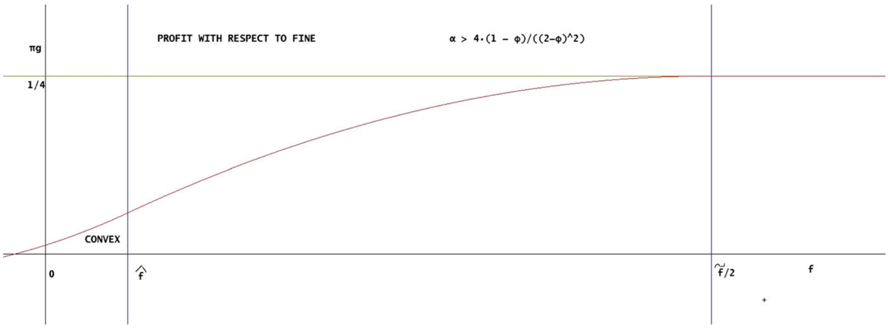

Fig. 3 The optimal profit as a function of the fine f. The convex case $\alpha > \alpha^*$

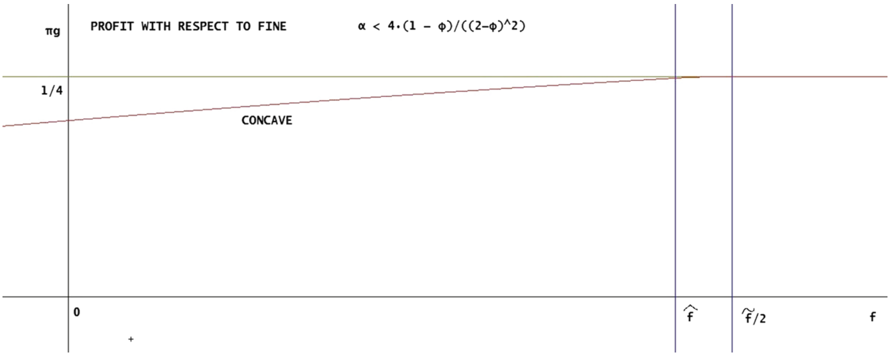

Fig. 4 The optimal profit as a function of the fine f. The concave case $\alpha < \alpha^*$

effects. In sum, the coexistence of genuine and counterfeit items associated with a certain level of counterfeiters monitoring can make the genuine producer better off".

As a consequence of what has been shown above, this interesting conclusion (which is the main contribution of their paper) is not true. Their results, however, may be recovered in the more general scenario of asymmetric production costs which will be considered in the next section.

4 Asymmetric production cost

In this section, the production cost for a genuine item is assumed to be $c \geq 0$ while the cost for an imitation remains equal to zero. Let[3]

[3] Note that if $c = 0$, then $p_g^* = p_{go}^*$ and $\pi_g^* = \pi_{go}^*$.

$$f_1=\frac{\alpha(1-\phi)[1-\alpha+c]}{\phi[2(1-\alpha)+\alpha\phi]};\ f_2=\frac{\alpha(1-\phi)(1+c)}{2\phi}.$$
$$f_3=\frac{\alpha(1-\phi)\left[\sqrt{1-\alpha}(\phi-(2-\phi)c)+c\phi-(2-\phi)(1-\alpha)\right]}{\phi\left[\alpha(2-\phi)^2-4(1-\phi)\right]}.$$
$$f_4=\frac{(1-\phi)[c-(1-\alpha)]}{\phi^2};\quad f_5=\frac{\sqrt{\alpha(1-\phi)}\sqrt{\alpha(1-\phi)-(1-c)^2}+\alpha(1-\phi)}{2\phi}.$$
$$f_6=\frac{\alpha(1-\phi)[c\phi-(2-\phi)(1-\alpha)]}{\phi\left[\alpha(2-\phi)^2-4(1-\phi)\right]};\ \tilde{f}=\frac{\alpha(1-\phi)}{\phi}.$$
$$c_1=\frac{\phi}{2-\phi};\ c_2=1-\alpha;\ c_3=1-\alpha+\frac{\alpha\phi}{2};\ c_4=1-\sqrt{\alpha(1-\phi)}.$$
$$c_5=\frac{2\alpha\phi(1-\phi)\sqrt{1-\alpha}-\alpha(\phi^2-6\phi+4)+4(1-\phi)}{\alpha\phi^2+4(1-\phi)}.$$
$$p_g^*=\frac{(1-\phi)(1-\alpha+c)+\phi(2-\phi)f}{2(1-\phi)}.$$
$$p_{g1}=\frac{\phi f}{\alpha(1-\phi)};\ p_{g2}=1-\alpha+\frac{\phi f}{1-\phi};\ \hat{p}_g=\frac{1+c}{2}.$$
$$\pi_g^*=\frac{c^2\alpha(1-\phi)^2-2c\alpha(1-\phi)\left[f\phi^2+(1-\alpha)(1-\phi)\right]+2\alpha(1-\alpha)(1-\phi)(2-\phi)\phi f+[\alpha(2-\phi)^2-4(1-\phi)]\phi^2f^2}{4\alpha(1-\alpha)(1-\phi)^2}.$$
$$\pi_{g1}=\left(\frac{\phi f}{\alpha(1-\phi)}-c\right)\left(1-\frac{\phi f}{\alpha(1-\phi)}\right);\quad \hat{\pi}_g=\frac{(1-c)^2}{4};\quad \pi_{g,2}=\frac{f\phi[\alpha(1-\phi)-f\phi]}{\alpha(1-\phi)}.$$
$$\tilde{\alpha}=\frac{2(1-\phi)}{2-\phi};\ \alpha^*=\frac{4(1-\phi)}{(2-\phi)^2}.$$

Let $\pi_{g,max}$ be the highest achievable profit by the genuine producer. It depends on the unitary production cost c and on the unitary fine f.

Proposition 4 (The optimal profit) *We distinguish two cases:*

1. $0<\alpha\le\tilde{\alpha}$.
   - *Let* $0\le c<c_1$. *The optimal price chosen by the genuine producer is* $p_g=p_g^*$ *if* $0\le f<f_1, p_g=p_{g1}<\hat{p}_g$ *if* $f_1\le f<f_2$, $p_g=\hat{p}_g$ *if* $f_2\le f\le\tilde{f}$.
   - *Let* $c_1\le c<c_2$. *The optimal price chosen by the genuine producer is* $p_g=p_g^*$ *if* $0\le f\le f_3, p_g=\hat{p}_g$ *if* $f_3\le f\le\tilde{f}$.
   - *Let* $c_2\le c<c_5$. *The optimal price chosen by the genuine producer is any* $p_g\ge p_{g2}$ *if* $0\le f<f_4, p_g=p_g^*$ *if* $f_4\le f<f_3, p_g=\hat{p}_g$ *if* $f_3\le f\le\tilde{f}$.
   - *Let* $c_5\le c\le 1$. *The optimal price chosen by the genuine producer is any* $p_g\ge p_{g2}$ *if* $0\le f\le f_5, p_g=\hat{p}_g$ *if* $f_5\le f\le\tilde{f}$.

2. $\tilde{\alpha}<\alpha<1$.
   - *Let* $0\le c<c_2$. *The optimal price chosen by the genuine producer is* $p_g=p_g^*$ *if* $0\le f<f_1, p_g=p_{g1}<\hat{p}_g$ *if* $f_1\le f<f_2$, $p_g=\hat{p}_g$ *if* $f_2\le f\le\tilde{f}$.
   - *Let* $c_2\le c<c_1$. *The optimal price chosen by the genuine producer is any* $p_g\ge p_{g2}$ *if* $0\le f<f_4, p_g=p_g^*$ *if* $f_4\le f<f_1, p_g=p_{g1}<\hat{p}_g$ *if* $f_1\le f<f_2$, $p_g=$

$\hat{p}_g$ if $f_2 \le f \le \tilde{f}$.

- *Let $c_1 \le c < c_5$. The optimal price chosen by the genuine producer is any $p_g \ge p_{g2}$ if $0 \le f < f_4, p_g = p_g^*$ if $f_4 \le f < f_3, p_g = \hat{p}_g$ if $f_3 \le f \le \tilde{f}$.*
- *Let $c_5 \le c \le 1$. The optimal price chosen by the genuine producer is any $p_g \ge p_{g2}$ if $0 \le f < f_5, p_g = \hat{p}_g$ if $f_5 \le f \le \tilde{f}$.*

Proof See Appendix.

Depending on f and c, the highest profit is achieved by the genuine firm choosing one of the following prices $p_g = p_g^*, p_g = p_{g1}, p_g = \hat{p}_g$, any $p_g \ge p_{g2}$.

If the production costs of the genuine firm and the amount of the fines are quite low, then the most profitable choice for of the company is the price $p_g = p_g^*$. In this case both demands D_c and D_g are positive and the ensuing profit is $\pi_{g,max} = \pi_g^*$. Legal and illegal products coexist in the market.

Low production costs and intermediate levels of fines lead the firm to sell at the price $p_g = p_{g1} < \hat{p}_g$ and get the profit $\pi_{g,max} = \pi_{g1}$. In this case, consumers buy exclusively the original items and nobody buys a fake. Nevertheless the market does not reflect exactly "a world without counterfeiters" and the firm obtains profits that are lower than the ones given in a true monopoly scenario. An increase in the price of genuine items leads shortly to a positive demand of the fake.

As the production costs of the genuine firm increase while the amount of the fines remains low enough, the most profitable choice for of the company becomes any price $p_g \ge p_{g2}$. In this case, only the demand of fakes is positive while nobody buys a genuine item. Since production costs are higher enough, the firm may have no incentives to produce a genuine item and its only goal is to rake in the penalty money from counterfeiters. If so, then no any genuine item appears in the market. Since no genuine product is available for copying, then counterfeiting activities should disappear in the market within a short time.[4]

High penalties induce the firm to practice the monopoly price $\hat{p}_g$, regardless of costs. Consumers buy exclusively the original items and the firm obtains exactly the profits attainable in a "a world without counterfeiters".

The following proposition gives, for any $c \ge 0$, a description of $\pi_{g,max}$ as a function of f. Moreover, the value f_{opt} of the fine f which guarantees the highest profit π_{opt} to the genuine firm is computed.

Proposition 5 (The best fine) *We distinguish two cases:*

1. $0 < \alpha \le \alpha^*$.

   - *If $0 \le c < c_1$, then the highest profit attainable by the genuine producer is guaranteed by any $f \ge f_{opt} = f_2$. The profit is $\pi_{opt} = \hat{\pi}_g$. It is $D_c = 0, D_g > 0$.*

[4] The author explicitly acknowledges one of the anonymous referees for this highlight and argues that this issue could be more appropriately discussed in the framework of a dynamic model.

- *If* $c_1 \leq c < c_3$, *then the highest profit attainable by the genuine producer is guaranteed by* $f = f_6$. *The profit is* $\pi_{opt} = \pi_g^*(f = f_6) > \hat{\pi}_g$. *It is* $D_c > 0, D_g > 0$.
- *If* $c_3 \leq c \leq 1$, *then the highest profit attainable by the genuine producer is guaranteed by* $f = \tilde{f}/2$. *The profit is* $\pi_{opt} = \pi_{g,2}\left(f = \tilde{f}/2\right) > \hat{\pi}_g$. *It is* $D_c > 0, D_g = 0$.

2. $\alpha^* < \alpha < 1$.

- *If* $0 \leq c < c_4$, *then the highest profit attainable by the genuine producer is guaranteed by any* $f \geq f_{opt} = f_2$. *The profit is* $\pi_{opt} = \hat{\pi}_g$. *It is* $D_c = 0, D_g > 0$.
- *If* $c_4 \leq c \leq 1$, *then the highest profit attainable by the genuine producer is guaranteed by* $f = \tilde{f}/2$. *The profit is* $\pi_{opt} = \pi_{g,2}\left(f = \tilde{f}/2\right) > \hat{\pi}_g$. *It is* $D_c > 0, D_g = 0$.

Proof See Appendix.

If production costs are low, then the genuine manufacturer would lobbying for high penalties. Counterfeiters should be thrown out of the market and the original producer could sell at the optimum price achievable in a world without counterfeiters and get the same profit. In this case, (which includes the one investigated in Bekir et al. 2012), the presence of counterfeiters does not provide any benefit to the producer of the original product.

Whenever the production cost is neither too high nor too low, the optimal fine $f = f_6$ guarantees a positive demand for the genuine product as well as for the fake. In this case the maximum profit π_{opt} is higher than the one obtainable in a world without counterfeiters. The presence of counterfeiters provides a benefit to the producer of the original products.

If production costs are too high, the genuine firm has no more incentive to produce. Its remaining goal is to collect penalty money from counterfeiters. The optimum fine would be $f = \tilde{f}/2$. The demand for the original items is nil and consumers buy only fakes. The maximum profit π_{opt} is higher than the one obtainable in a world without counterfeiters and it is obtained only cashing in fines. Again, the presence of counterfeiters provides a benefit to the producer of the original products. In this case the genuine item disappears and the market is full of fake products. As noted above, since no genuine product is available for copying, counterfeiting activities should disappear in the market in a short time.

To sum up, if the production costs are not too low, the genuine firm can take advantage of the presence of counterfeiting. It is thus recovered the Bekir's et al. (2012) claim.

At the end of this section, we explicitly highlight that if the production costs of the genuine firm are the same as those of counterfeiters (i.e., both equal to zero), then the general result in Sect. 4 return to that of Sect. 3. In fact, if $c = 0$, Proposition 4 states: " The optimal price chosen by the genuine producer is $p_g = p_g^*$

if $0 \le f < f_1, p_g = p_{g1} < \hat{p}_g$ if $f_1 \le f < f_2$, $p_g = \hat{p}_g$ if $f_2 \le f \le \tilde{f}$". Since as $c = 0$, it is $f_1 = \hat{f}, f_2 = \tilde{f}/2$, we conclude that Proposition 4 reduces exactly to Proposition 2 in the previous section. Similarly Proposition 5 returns to Proposition 3.

5 Social planning

In this section some policy implications that may result from the model above are mentioned. In Yao (2005a), social welfare is assumed to be the unweighted sum of consumer and producer surplus minus the IPR enforcement costs. It seems reasonable to guess that the government or the legislature may have a different view of the company's revenue compared to consumer welfare. Therefore it seems more appropriate to take the social welfare W as the weighted sum of consumer and producer surplus minus the IPR enforcement costs.

$$W = qC_{surp} + (1 - q)\pi_g - M\phi$$

where C_{surp} is the consumer surplus and the IPR enforcement costs are assumed to be proportional to the probability of catching counterfeiters. $q \in [0, 1]$ is the weight assigned by the firm to the consumers surplus against the weight $1 - q$ assigned to the firm profits. When both the demands of the genuine item and the fake are positive, the consumer plus C_{surp} is given by the sum of two contributions: the surplus of consumers buying the genuine product (C_{surpg}) and the one of consumers who buy the counterfeit product (C_{surpc}).

$$C_{surp} = C_{surpc} + C_{surpg}$$

where

$$C_{surpc} = \int_{\hat{\theta}_c}^{\hat{\theta}_g} (\alpha\theta - p_c) d\theta; \quad C_{surpg} = \int_{\hat{\theta}_g}^{1} (\theta - p_g) d\theta. \tag{1}$$

and

$$\hat{\theta}_c = \frac{p_c}{\alpha}, \quad \hat{\theta}_g = \frac{p_g - p_c}{1 - \alpha}. \tag{2}$$

Substituting (2) in (1), we obtain

$$C_{surpc} = \frac{(\alpha p_g - p_c)^2}{2\alpha(1 - \alpha)^2}; \quad C_{surpg} = \frac{(pg - p_c - 1 + \alpha)\left(p_c + (1 - 2\alpha)p_g - 1 + \alpha\right)}{2(1 - \alpha)^2}.$$

When the demand of the genuine item is positive while the one of the fake is nil, then

$$C_{surpc} = 0; \quad C_{surpg} = \int_{p_g}^{1} (\theta - p_g) d\theta = \frac{(1 - p_g)^2}{2}. \tag{3}$$

When the demand of the fake is positive while the one of the genuine item is nil, then

$$C_{surpc} = \int_{\hat{\theta}_c}^{1} (\alpha\theta - p_c)d\theta = \frac{(p_c - \alpha)^2}{2\alpha}; \quad C_{surpg} = 0.$$

The social planner, being able to predict the optimal price of the genuine firm as described in Proposition 4, adopts the amount of the fine f and the protection policy level ϕ which maximize the social welfare.

Three IPR-enforcement policies associated with ϕ are provided as follows:

1. The policy with $\phi = 1$ is called the full-protection policy.
2. The policy with $\phi = 0$ is called the non-protection policy due to no monitoring actions against counterfeiting. Under this policy no enforcement costs are incurred.
3. The policy with $0 < \phi < 1$ is called the counterfeit monitoring policy.

A complete and detailed analysis of the social planner optimization problem is greatly complicated due to the number of parameters involved; it should be possible the subject of a subsequent paper. Nevertheless some interesting considerations will be made here by comparing full protection policy with non-protection policy.

5.1 Full protection policy

Towards understanding the impact of counterfeiting on total social welfare, it is instructive to consider the case $\phi = 1$ (the full-protection policy) when the fakes are excluded from the market completely. The optimal price chosen by the genuine firm is $p_g^* = 1 + c/2$ leading to the firm profit $\pi_g^* = (1 - c)^2/4$. From (3), the consumer surplus is given by $C_{surpg} = (1 - c)^2/8$ so that the social welfare is

$$W = q\frac{(1 - c)^2}{8} + (1 - q)\frac{(1 - c)^2}{4} - M \qquad (4)$$

5.2 Non-protection policy

The policy with $\phi = 0$ is called the non-protection policy due to no monitoring actions against counterfeiting. Since counterfeiters operate in a perfectly competitive market, each counterfeiter earns zero economic profits in equilibrium. Since there are no fines, the counterfeiters incur zero cost and price zero their items, that is $p_c = 0$. The demand of the genuine items and of the fake are given by Proposition 1 as follows:

- if $0 \le p_g < 1 - \alpha$ then $D_c = \dfrac{p_g}{1 - \alpha} > 0$ and $D_g = \dfrac{1 - \alpha - p_g}{1 - \alpha} > 0$;
- if $1 - \alpha \le p_g$, then $D_c = 1$ and $D_g = 0$.

We distinguish two cases:

1. Let $0 \le c < 1-\alpha$. Proposition 4 gives the optimal price of the genuine item $p_g^* = (1-\alpha+c)/2$ and the corresponding optimal profit $\pi_g^* = (1-\alpha-c)^2/(4(1-\alpha))$. Both the demands of genuine and counterfeited items are positive and the consumer surplus are given by

$$C_{surpc} = \frac{\alpha(1-\alpha+c)^2}{8(1-\alpha)^2};\ C_{surpg} = \frac{(1-\alpha-c)[1+\alpha-2\alpha^2+c(2\alpha-1)]}{8(1-\alpha)^2}.$$

The total consumer surplus is

$$C_{surp} = \frac{c^2-2(1-\alpha)c+(1-\alpha)(1+3\alpha)}{8(1-\alpha)}$$

and the social welfare is

$$W = q\frac{c^2-2(1-\alpha)c+(1-\alpha)(1+3\alpha)}{8(1-\alpha)} + (1-q)\frac{(1-\alpha-c)^2}{4(1-\alpha)} \qquad (5)$$

2. Let $1-\alpha \le c \le 1$.
In this case the demand for genuine items is nil while the market is covered by fakes, that is $D_g = 0, D_c = 1$. The social welfare coincides with the consumer surplus. Precisely, it is

$$W = q\frac{\alpha}{2} \qquad (6)$$

Comparing (5) and (6) with (4) we obtain the following Proposition.

Proposition 6 *Let*

$$M^* = \begin{cases} \dfrac{\alpha((1-\alpha)(2-5q)-(2-q)c^2)}{8(1-\alpha)} & \text{if} \quad 0 \le c < 1-\alpha; \\ \dfrac{(2-q)c^2-2(2-q)c+2-q-4\alpha q}{8} & \text{if} \quad 1-\alpha < c \le 1. \end{cases}$$

$$q^* = \begin{cases} \dfrac{2(1-\alpha-c^2)}{5(1-\alpha)-c^2} & \text{if} \quad 0 \le c < 1-\alpha; \\ \dfrac{2(1-c)^2}{(1-c)^2+4\alpha} & \text{if} \quad 1-\alpha < c \le 1. \end{cases}$$

1. *Let $0 \le q < q^*$. The social planner is better off by full protection if the IPR enforcement costs are low enough, that is if $0 \le M < M^*$. Otherwise no protection policy is preferred.*
2. *Let $q^* \le q \le 1$. The social planner prefers no-protection policy with respect to full protection no matter the IPR enforcement costs are.*

Proof See Appendix.

Note that comparing the null with the full protection policy, the amount of the fine does not play any role. This is not surprising. If the null protection policy is implemented, then counterfeiters are never caught and then they don't pay any penalty. If, by contrast, the full protection policy is enforced, then counterfeiters do not enter the market and, again, no fine is paid. The situation is different whenever a generic monitoring policy is accomplished. In such cases, the social welfare depends both on the intensity of the monitoring policy and the amount of the sanctions imposed.

Proposition 6 says that if the social planner cares solely or mainly for consumer surplus, then it is more convenient to maintain a non-protection policy against a full protection one, regardless of the enforcement costs. By contrast, if sufficient attention is paid to the genuine firm profits by the social planner, then the comparison result depends on the enforcement costs. Non-protection policy is desirable in the presence of high costs; otherwise the social planner is better off by performing full protection.

6 Conclusion

In the conclusion of their paper, Bekir et al. (2012) write "... the coexistence of genuine and counterfeit items associated with a certain level of counterfeiters monitoring can make the genuine producer better off". Unfortunately, that paper contains two mistakes: one mistake is a miscalculation; the other comes from confusing a world without counterfeiting at all with a world threatened by counterfeiters. Overcoming such errors, we show that the genuine producer would gain at most the same profit as in a world without counterfeiters. This happens whenever the fine exceeds some threshold.

Things change if asymmetric production costs are introduced. If the production costs of the genuine firm and the amount of the fines are both low enough, then the most profitable choice for the company allows both genuine and fake demands. Legal and illegal products coexist in the market. Low production costs and intermediate levels of fine lead the firm to sell at a price such that consumers buy exclusively the original items and nobody buys a fake. Nevertheless the market does not reflect exactly "a world without counterfeiters" and the firm obtains profits that are lower than the ones given in a monopoly scenario. An increase in the price of genuine items leads shortly to a positive demand of the fake. As the production costs of the genuine firm increase while the amount of the fines remains low, the company best choice is such that only the demand of fakes is positive while nobody buys a genuine item. But, since no genuine product is available for copying, then counterfeiting activities should disappear in the market within a short time. This apparent paradox should be better explained in the framework of a dynamic model. High penalties induce the firm to practice the monopoly price, regardless of costs. Consumers buy exclusively the original items and the firm obtains exactly the profits attainable in a "a world without counterfeiters".

When the genuine producer can strongly shape the ‘rules of the game’ (namely, the penalties imposed on counterfeiters), he would prefer high penalties whenever production costs are low. Counterfeiters should be thrown out of the market and the original producer could sell at the optimum price achievable in a world without counterfeiters and get the same profit. In this case the presence of counterfeiters does not provide any benefit to the producer of the original product. Whenever the production cost is neither too high nor too low, the optimal fine guarantees a positive demand for the genuine product as well as for the fake. In this case the maximum profit is higher than the one obtainable in a world without counterfeiters. The presence of counterfeiters provides a benefit to the producer of the original products. If production costs are too high, the genuine firm has no more incentive to produce. Its remaining goal is to collect penalty money from counterfeiters. The optimum fine gives no demand for the original items, consumers buy only fakes. The maximum profit is again higher than the one obtainable in a world without counterfeiters and it is obtained only cashing in fines. Also in this case the presence of counterfeiters provides a benefit to the producer of the original products. To sum up, if the production costs are not too low, the genuine firm can take advantage of the presence of counterfeiting. It is thus recovered the Bekir’s et al. (2012) claim. In the final part of the paper, some policy implications are addressed.

Defining the social welfare as the weighted sum of consumer and producer surplus minus the IPR enforcement costs, we compare the null protection with the full protection policy. If the social planner cares solely or mainly for consumer surplus, then it is more convenient to maintain a non-protection policy against a full protection one, regardless of the enforcement costs. By contrast, if sufficient attention is paid to the genuine firm profits by the social planner, then the comparison result depends on the enforcement costs. Non-protection policy is desirable in the presence of high costs; otherwise the social planner is better off by performing full protection.

The genuine firm is able to lobby the authorities successfully to fight counterfeiting. Nevertheless, in this paper the genuine firm does not burden any costs including lobbying and law enforcement costs. However, the enforcement of the IPR laws needs enforcement costs. In their paper Bekir et al. (2012) appear to consider such costs. But they were assumed to be independent of both the amount of the penalty and the commitment to track down counterfeiters. In this way, however, these costs are irrelevant. On the other hand the amount of sanctions is considered exogenous with respect to the choices of the genuine firm and we only calculate the amount of the penalty that produces the highest profits for the company. It would be very interesting to study the joint contribution of the company (through lobbying) and the public authorities in the determination of the amount of penalties and/or the level of protection policy. A possible framework would be the bargaining theory. However this issue could be addressed in a subsequent paper.

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Appendix

Proof of Proposition 1 The consumer indexed by $\theta = p_{g1} = \frac{p_c}{\alpha}$ has utility zero buying a fake. The consumer indexed by $\theta = p_g$ has utility zero buying a genuine item. The consumer indexed by $\hat{\theta}_g = \frac{p_g - p_c}{1-\alpha}$ is indifferent between buying the counterfeit product or the original one. Remember that $p_c = \frac{\phi f}{1-\phi}$.

Note that it is: (1) $p_{g1} < p_g < \frac{p_g - p_c}{1-\alpha}$ or (2) $\frac{p_g - p_c}{1-\alpha} < p_g < p_{g1}$.

Let us consider the case (1).

If $\frac{p_g - p_c}{1-\alpha} < 1$ (that is $p_g < p_{g2}$) then consumer indexed between $\theta = \hat{\theta}_g$ and $\theta = 1$ buy the original item so that $D_g = 1 - \hat{\theta}_g = \frac{1 - \alpha - p_g + \frac{\phi f}{1-\phi}}{1-\alpha}$. Consumers indexed between $\theta = p_{g1}$ and $\theta = \hat{\theta}_g$ buy the fake and then $D_c = \hat{\theta}_g - p_{g1} = \frac{\alpha p_g - \frac{\phi f}{1-\phi}}{\alpha(1-\alpha)}$.

If $1 \leq \frac{p_g - p_c}{1-\alpha}$ (that is $p_g \geq p_{g2}$) and $p_{g1} < 1$ then consumers indexed between $\theta = p_{g1}$ and $\theta = 1$ buy the fake so that $D_c = 1 - p_{g1} = 1 - \frac{\phi f}{\alpha(1-\phi)}$ and nobody buys the original product.

If $p_{g1} \geq 1$ then nothing is bought, that is $D_c = D_g = 0$.

Let us consider the case (2).

If $p_g < 1$ then consumer indexed between $\theta = p_g$ and $\theta = 1$ buy the original item so that $D_g = 1 - p_g$ and nobody buys the fake.

If $1 \leq p_g$ then nothing is bought, that is $D_c = D_g = 0$. □

Proof of Proposition 2 It is $\hat{f} < \frac{\tilde{f}}{2}$. Note that π_g is constant with respect to p_g when $p_g \geq p_{g2}$.

- Let $0 \leq f < \hat{f}$.

  If $0 \leq p_g \leq p_{g1}$ then $\pi_g = p_g(1 - p_g)$ and it is increasing (being $p_g < \frac{1}{2}$).

  If $p_{g1} < p_g < p_{g2}$ then

$$\pi_g = p_g \left(\frac{1 - \alpha - p_g + \frac{\phi f}{1-\phi}}{1-\alpha} \right) + \phi f \left(\frac{\alpha p_g - \frac{\phi f}{1-\phi}}{\alpha(1-\alpha)} \right).$$

The graph of π_g is a concave parabola. Its derivative with respect to p_g is

$$\frac{d\pi_g}{dp_g} = \frac{f\phi(2-\phi) + (1-\phi)(2p_g - (1-\alpha))}{(1-\phi)(1-\alpha)}.$$

It is

$$\frac{d\pi_g(p_{g1})}{dp_g} = \frac{\alpha(1-\alpha)(1-\phi) - f\phi(2-2\alpha+\alpha\phi)}{\alpha(1-\phi)(1-\alpha)}.$$

$\frac{d\pi_g(p_{g1})}{dp_g} > 0$ iff $f < \hat{f}$. It is

$$\frac{d\pi_g(p_{g2})}{dp_g} = -\frac{(1-\alpha)(1-\phi) + f\phi^2}{(1-\phi)(1-\alpha)} < 0.$$

It follows that π_g is increasing for $p_{g1} \le p_g \le p_g^*$ and it is decreasing for $p_g^* \le p_g \le p_{g2}$. We conclude that when $0 \le f < \hat{f}$, π_g reaches it maximum at $p_g = p_g^*$.

- Let $\hat{f} \le f \le \frac{\tilde{f}}{2}$.

  If $0 \le p_g \le p_{g1}$ then $\pi_g = p_g(1-p_g)$ and it is increasing (being $p_g < \frac{1}{2}$).
  If $p_{g1} < p_g < p_{g2}$ then the graph of π_g is a concave parabola and it is

  $$\frac{d\pi_g(p_{g1})}{dp_g} < 0; \quad \frac{d\pi_g(p_{g2})}{dp_g} < 0$$

  It follows that π_g is decreasing for $p_{g1} \le p_g \le p_{g2}$. We conclude that when $\hat{f} \le f \le \frac{\tilde{f}}{2}$, π_g reaches it maximum at $p_g = p_{g1}$.

- Let $\frac{\tilde{f}}{2} < f \le \tilde{f}$.

  If $0 \le p_g \le p_{g1}$ then $\pi_g = p_g(1-p_g)$ and (being $p_{g1} > \frac{1}{2}$) it is increasing with respect to p_g for $0 \le p_g \le \frac{1}{2}$, decreasing with respect to p_g for $\frac{1}{2} \le p_g \le p_{g1}$.
  If $p_{g1} < p_g < p_{g2}$ then the graph of π_g is a concave parabola and it is

  $$\frac{d\pi_g(p_{g1})}{dp_g} < 0; \quad \frac{d\pi_g(p_{g2})}{dp_g} < 0.$$

  It follows that π_g is decreasing for $p_{g1} \le p_g \le p_{g2}$. We conclude that when $\hat{f} \le f \le \frac{\tilde{f}}{2}$, π_g reaches its maximum at $p_g = \frac{1}{2}$. □

Proof of Proposition 3 The graph of π_g^* with respect to the fine f is a concave parabola if $\alpha < \frac{4(1-\phi)}{(2-\phi)^2} = \alpha^*$, a convex parabola if $\alpha > \alpha^*$. π_g^* is a linear function with respect to f in the particular case $\alpha = \alpha^*$.

- Let $\alpha = \alpha^*$.
In this case, it is

$$\pi_g^* = \frac{\phi(2(2-\phi)^3 f + \phi(1-\phi))}{4(1-\phi)(2-\phi)^2}$$
$$\frac{d\pi_g^*}{df} = \frac{\phi(2-\phi)}{2(1-\phi)} > 0$$

It is

$$\pi_{g1} = \frac{\phi f(2-\phi)^2(4(1-\phi)^2 - \phi f(2-\phi)^2)}{16(1-\phi)^4}$$
$$\frac{d\pi_{g1}}{df} = \frac{\phi(2-\phi)^2(2(1-\phi)^2 - \phi f(2-\phi)^2)}{8(1-\phi)^4}$$

It is $\frac{d\pi_{g1}}{df} > 0$ if $f < \frac{2(1-\phi)^2}{\phi(2-\phi)^2} = \frac{\tilde{f}}{2}$.
Hence if $\alpha = \alpha^*$ the optimal profit $\pi_{g,max}$ is increasing with respect to f and its maximum value $\frac{1}{4}$ is obtained for any $f \geq \frac{\tilde{f}}{2} = \frac{2(1-\phi)^2}{\phi(2-\phi)^2}$.

- Let $\alpha \neq \alpha^*$.
The graph of π_g^* with respect to the fine f is a parabola.
It is

$$\frac{d\pi_g^*}{df} = \frac{\phi\Big(\alpha(1-\alpha)(1-\phi)(2-\phi) + \Big(\alpha(2-\phi)^2 - 4(1-\phi)\Big)\phi f\Big)}{2\alpha(1-\alpha)(1-\phi)^2}$$
$$\frac{d\pi_g^*}{df}(0) = \frac{\phi(2-\phi)}{2(1-\phi)} > 0$$
$$\frac{d\pi_g^*}{df}(\hat{f}) = \frac{\phi^2}{(1-\phi)(2-2\alpha+\alpha\phi)} > 0$$

Hence π_g^* is increasing with respect to f for $f \in [0, \hat{f}]$.

$$\pi_{g1} = \frac{\phi f}{\alpha(1-\phi)}\left(1 - \frac{\phi f}{\alpha(1-\phi)}\right) < \frac{1}{4}.$$
$$\frac{d\pi_{g1}}{df} = \frac{\phi(\alpha(1-\phi) - 2\phi f)}{\alpha^2(1-\phi)^2}$$

It is $\frac{d\pi_{g1}}{df} > 0$ if $f < \frac{\tilde{f}}{2}$.
Hence if $\alpha \neq \alpha^*$ the optimal profit $\pi_{g,max}$ is increasing with respect to f and its maximum value $\frac{1}{4}$ is obtained for any $f \geq \frac{\tilde{f}}{2}$. □

In order to prove Proposition 4, we need the following preliminary Lemmas.

Lemma 1 *Let* $0 \leq p_g \leq p_{g1}$. *Then* $\pi_g = (p_g - c)(1 - p_g)$. *Consequently*

- *If* $0 \leq f \leq f_2$ *then* π_g *is increasing with respect to* $p_g \in [0, p_{g1}]$ *and reaches its maximum* π_{g1} *at* $p_g = p_{g1}$.
- *If* $f_2 \leq f \leq \tilde{f}$ *then* π_g *is increasing with respect to* $p_g \in [0, \hat{p}_g]$ *and decreasing in* $[\hat{p}_g, p_{g1}].\pi_g$ *reaches its maximum* $\hat{\pi}_g$ *at* $p_g - \hat{p}_g$.

Proof It follows from the first order conditions.

Lemma 2 *Let* $p_{g1} \leq p_g \leq p_{g2}$. *Then*

$$\pi_g = (p_g - c)\left(\frac{1 - \alpha - p_g + \dfrac{\phi f}{1 - \phi}}{1 - \alpha}\right) + \phi f\left(\frac{\alpha p_g - \dfrac{\phi f}{1 - \phi}}{\alpha(1 - \alpha)}\right).$$

Consequently

- $\dfrac{\partial \pi_g}{\partial p_g} = 0$ *if* $p_g = p_g^*$.
- $\dfrac{\partial \pi_g}{\partial p_g}(p_g = p_{g1}) > 0$ *iff* $f < f_1$.
- $\dfrac{\partial \pi_g}{\partial p_g}(p_g = p_{g2}) > 0$ *iff* $f < f_4$.

Proof It is trivial.

Lemma 3 *Let*

$$\omega_1 = \frac{2 - 2\alpha + \alpha\phi}{2 - \alpha\phi}; \quad \omega_2 = 1 - \alpha + \alpha\phi.$$

- *If* $0 < \alpha < \tilde{\alpha}$ *then* $0 < c1 < c2 < \omega_1 < \omega_2 < 1$;
- *if* $\tilde{\alpha} < \alpha < 1$ *then* $0 < c2 < c1 < \omega_1 < \omega_2 < 1$.

Proof It is trivial.

Lemma 4

- *Let* $0 < \alpha < \tilde{\alpha}$.
  - *If* $0 < c < c_1$ *then* $f_4 < 0 < f_1 < f_2 < \tilde{f}$;
  - *if* $c_1 < c < c_2$ *then* $f_4 < 0 < f_2 < f_1 < \tilde{f}$;

- *if* $c_2 < c < \omega_1$ *then* $0 < f_4 < f_2 < f_1 < \tilde{f}$;
- *if* $\omega_1 < c < \omega_2$ *then* $0 < f_2 < f_4 < f_1 < \tilde{f}$;
- *if* $\omega_2 < c \le 1$ *then* $0 < f_2 < \tilde{f} < f_1 < f_4$;

- *Let* $\tilde{\alpha} < \alpha < 1$.
  - *If* $0 < c < c_2$ *then* $f_4 < 0 < f_1 < f_2 < \tilde{f}$;
  - *if* $c_2 < c < c_1$ *then* $0 < f_4 < f_1 < f_2 < \tilde{f}$;
  - *if* $c_1 < c < \omega_1$ *then* $0 < f_4 < f_2 < f_1 < \tilde{f}$;
  - *if* $\omega_1 < c < \omega_2$ *then* $0 < f_2 < f_4 < f_1 < \tilde{f}$;
  - *if* $\omega_2 < c \le 1$ *then* $0 < f_2 < \tilde{f} < f_1 < f_4$;

Proof It is trivial.

Lemma 5 *Note that the profit of the genuine producer is constant with respect to* p_g *for* $p_g \ge p_{g2}$*; consequently, in order to find the optimal price of the genuine item, it is sufficient to examine the behaviour of* π_g *with respect to* p_g *for* $0 \le p_g \le p_{g2}$.

1. $0 < \alpha < \tilde{\alpha}$.
   - *Let* $0 \le c < c_1$.
     Then
     - *If* $0 \le f \le f_1$*, then* π_g *is increasing with respect to* p_g *for* $0 \le p_g \le p_{g1}, \pi_g$ *is increasing with respect to* p_g *for* $p_{g1} \le p_g \le p_g^*$ *and decreasing for* $p_g^* \le p_g \le p_{g2}$. *We conclude that* π_g *reaches its maximum at* $p_g = p_g^*$.
     - *If* $f_1 \le f \le f_2$ *then* π_g *is increasing with respect to* p_g *for* $0 \le p_g \le p_{g1}, \pi_g$ *is decreasing for* $p_{g1} \le p_g \le p_{g2}$. *We conclude that* π_g *reaches its maximum at* $p_g = p_{g1}$.
     - *If* $f_2 \le f < \tilde{f}$ *then then* π_g *is increasing with respect to* p_g *for* $0 \le p_g \le \hat{p}_g$ *and decreasing for* $\hat{p}_g \le p_g \le p_{g1}, \pi_g$ *is decreasing for* $p_{g1} \le p_g \le p_{g2}$. *We conclude that* π_g *reaches its maximum at* $p_g = \hat{p}_g$.

   - *Let* $c_1 \le c < c_2$.
     - *If* $0 \le f \le f_2$*, then* π_g *is increasing with respect to* p_g *for* $0 \le p_g \le p_{g1}, \pi_g$ *is increasing with respect to* p_g *for* $p_{g1} \le p_g \le p_g^*$ *and decreasing for* $p_g^* \le p_g \le p_{g2}$. *We conclude that when* $0 \le f \le f_3, \pi_g$ *reaches its maximum at* $p_g = p_g^*$.
     - *If* $f_2 \le f \le f_1$*, then* π_g *is increasing with respect to* p_g *for* $0 \le p_g \le \hat{p}_g$ *and decreasing for* $\hat{p}_g \le p_g \le p_{g1}, \pi_g$ *is increasing with respect to* p_g *for* $p_{g1} \le p_g \le p_g^*$ *and decreasing for* $p_g^* \le p_g \le p_{g2}$. *Therefore in this case* π_g

reaches its maximum at $p_g = \hat{p}_g$ or at $p_g = p_g^*$ and it is necessary to compare π_g^* and $\hat{\pi}_g$.

- *If $f_1 \le f \le \tilde{f}$ then π_g is increasing with respect to p_g for $0 \le p_g \le \hat{p}_g$ and decreasing for $\hat{p}_g \le p_g \le p_{g1}, \pi_g$ is decreasing with respect to p_g for $p_{g1} \le p_g \le p_{g2}$. We conclude that π_g reaches its maximum at $p_g = \hat{p}_g$.*

- *Let $c_2 \le c < \omega_1$.*
  - *If $0 \le f \le f_4$, then π_g is increasing with respect to p_g for $0 \le p_g \le p_{g1}, \pi_g$ is increasing with respect to p_g for $p_{g1} \le p_g \le p_{g2}$. We conclude that π_g reaches its maximum at any $p_g \ge p_{g2}$.*
  - *If $f_4 \le f \le f_2$, then π_g is increasing with respect to p_g for $0 \le p_g \le p_{g1}, \pi_g$ is increasing with respect to p_g for $p_{g1} \le p_g \le p_g^*$ and decreasing for $p_g^* \le p_g \le p_{g2}$. We conclude that π_g reaches its maximum at $p_g = p_g^*$.*
  - *If $f_2 \le f \le f_1$, then π_g is increasing with respect to p_g for $0 \le p_g \le \hat{p}_g$ and decreasing for $\hat{p}_g \le p_g \le p_{g1}, \pi_g$ is increasing with respect to p_g for $p_{g1} \le p_g \le p_g^*$ and decreasing for $p_g^* \le p_g \le p_{g2}$. Therefore in this case π_g reaches its maximum at $p_g = \hat{p}_g$ or at $p_g = p_g^*$ and it is necessary to compare π_g^* and $\hat{\pi}_g$.*
  - *If $f_1 \le f \le \tilde{f}$ then π_g is increasing with respect to p_g for $0 \le p_g \le \hat{p}_g$ and decreasing for $\hat{p}_g \le p_g \le p_{g1}, \pi_g$ is decreasing with respect to p_g for $p_{g1} \le p_g \le p_{g2}$. We conclude that π_g reaches its maximum at $p_g = \hat{p}_g$.*

- *Let $\omega_1 \le c < \omega_2$.*
  - *If $0 \le f \le f_2$, then π_g is increasing with respect to p_g for $0 \le p_g \le p_{g1}, \pi_g$ is increasing with respect to p_g for $p_{g1} \le p_g \le p_{g2}$. We conclude that when $0 \le f \le f_1, \pi_g$ reaches its maximum at any $p_g \ge p_{g2}$.*
  - *If $f_2 \le f \le f_4$, then π_g is increasing with respect to p_g for $0 \le p_g \le \hat{p}_g$ and decreasing for $\hat{p}_g \le p_g \le p_{g1}, \pi_g$ is increasing with respect to p_g for $p_{g1} \le p_g \le p_{g2}$. Therefore in this case π_g reaches its maximum at $p_g = \hat{p}_g$ or at any $p_g \ge p_{g2}$ and it is necessary to compare $\pi_{g,2}$ and $\hat{\pi}_g$.*
  - *If $f_4 \le f \le f_1$, then π_g is increasing with respect to p_g for $0 \le p_g \le \hat{p}_g$ and decreasing for $\hat{p}_g \le p_g \le p_{g1}, \pi_g$ is increasing with respect to p_g for $p_{g1} \le p_g \le p_g^*$ and decreasing for $p_g^* \le p_g \le p_{g2}$. Therefore in this case π_g reaches its maximum at $p_g = \hat{p}_g$ or at $p_g = p_g^*$ and it is necessary to compare π_g^* and $\hat{\pi}_g$.*
  - *If $f_1 \le f \le \tilde{f}$ then π_g is increasing with respect to p_g for $0 \le p_g \le \hat{p}_g$ and decreasing for $\hat{p}_g \le p_g \le p_{g1}, \pi_g$ is decreasing with respect to p_g for $p_{g1} \le p_g \le p_{g2}$. We conclude that π_g reaches its maximum at $p_g = \hat{p}_g$.*

- *Let* $\omega_2 \leq c \leq 1$.
  - *If* $0 \leq f \leq f_2$, *then* π_g *is increasing with respect to* p_g *for* $0 \leq p_g \leq p_{g1}$, π_g *is increasing with respect to* p_g *for* $p_{g1} \leq p_g \leq p_{g2}$. *We conclude that* π_g *reaches its maximum at any* $p_g \geq p_{g2}$.
  - *If* $f_2 \leq f \leq \tilde{f}$, *then then* π_g *is increasing with respect to* p_g *for* $0 \leq p_g \leq \hat{p}_g$ *and decreasing for* $\hat{p}_g \leq p_g \leq p_{g1}$, π_g *is increasing with respect to* p_g *for* $p_{g1} \leq p_g \leq p_{g2}$. *Therefore in this case* π_g *reaches its maximum at* $p_g = \hat{p}_g$ *or at any* $p_g \geq p_{g2}$ *and it is necessary to compare* $\pi_{g,2}$ *and* $\hat{\pi}_g$.

2. $\tilde{\alpha} < \alpha < 1$.

- *Let* $0 \leq c < c_2$.
  - *If* $0 \leq f \leq f_1$, *then* π_g *is increasing with respect to* p_g *for* $0 \leq p_g \leq p_{g1}$, π_g *is increasing with respect to* p_g *for* $p_{g1} \leq p_g \leq p_g^*$ *and decreasing for* $p_g^* \leq p_g \leq p_{g2}$. *We conclude that* π_g *reaches its maximum at* $p_g = p_g^*$.
  - *If* $f_1 \leq f \leq f_2$ *then* π_g *is increasing with respect to* p_g *for* $0 \leq p_g \leq p_{g1}$, π_g *is decreasing for* $p_{g1} \leq p_g \leq p_{g2}$. *We conclude that* π_g *reaches its maximum at* $p_g = p_{g1}$.
  - *If* $f_2 \leq f < \tilde{f}$ *then then* π_g *is increasing with respect to* p_g *for* $0 \leq p_g \leq \hat{p}_g$ *and decreasing for* $\hat{p}_g \leq p_g \leq p_{g1}$, π_g *is decreasing for* $p_{g1} \leq p_g \leq p_{g2}$. *We conclude that* π_g *reaches its maximum at* $p_g = \hat{p}_g$.

- *Let* $c_2 \leq c < c_1$.
  - *If* $0 \leq f \leq f_4$, *then* π_g *is increasing with respect to* p_g *for* $0 \leq p_g \leq p_{g1}$, π_g *is increasing with respect to* p_g *for* $p_{g1} \leq p_g \leq p_{g2}$. *We conclude that* π_g *reaches its maximum at any* $p_g \geq p_{g2}$.
  - *If* $f_4 \leq f \leq f_1$, *then* π_g *is increasing with respect to* p_g *for* $0 \leq p_g \leq p_{g1}$, π_g *is increasing with respect to* p_g *for* $p_{g1} \leq p_g \leq p_g^*$ *and decreasing for* $p_g^* \leq p_g \leq p_{g2}$. *We conclude that* π_g *reaches its maximum at* $p_g = p_g^*$.
  - *If* $f_1 \leq f \leq f_2$ *then* π_g *is increasing with respect to* p_g *for* $0 \leq p_g \leq p_{g1}$, π_g *is decreasing for* $p_{g1} \leq p_g \leq p_{g2}$. *We conclude that* π_g *reaches its maximum at* $p_g = p_{g1}$.
  - *If* $f_2 \leq f < \tilde{f}$ *then then* π_g *is increasing with respect to* p_g *for* $0 \leq p_g \leq \hat{p}_g$ *and decreasing for* $\hat{p}_g \leq p_g \leq p_{g1}$, π_g *is decreasing for* $p_{g1} \leq p_g \leq p_{g2}$. *We conclude that* π_g *reaches its maximum at* $p_g = \hat{p}_g$.

- *Let* $c_1 \leq c < \omega_1$.

- *If* $0 \le f \le f_4$, *then* π_g *is increasing with respect to* p_g *for* $0 \le p_g \le p_{g1}$, π_g *is increasing with respect to* p_g *for* $p_{g1} \le p_g \le p_{g2}$. *We conclude that* π_g *reaches its maximum at any* $p_g \ge p_{g2}$.
- *If* $f_4 \le f \le f_2$, *then* π_g *is increasing with respect to* p_g *for* $0 \le p_g \le p_{g1}$, π_g *is increasing with respect to* p_g *for* $p_{g1} \le p_g \le p_g^*$ *and decreasing for* $p_g^* \le p_g \le p_{g2}$. *We conclude that* π_g *reaches its maximum at* $p_g = p_g^*$.
- *If* $f_2 \le f \le f_1$, *then* π_g *is increasing with respect to* p_g *for* $0 \le p_g \le \hat{p}_g$ *and decreasing for* $\hat{p}_g \le p_g \le p_{g1}$, π_g *is increasing with respect to* p_g *for* $p_{g1} \le p_g \le p_g^*$ *and decreasing for* $p_g^* \le p_g \le p_{g2}$. *Therefore in this case* π_g *reaches its maximum at* $p_g = \hat{p}_g$ *or at* $p_g = p_g^*$ *and it is necessary to compare* π_g^* *and* $\hat{\pi}_g$.
- *If* $f_1 \le f \le \tilde{f}$ *then* π_g *is increasing with respect to* p_g *for* $0 \le p_g \le \hat{p}_g$ *and decreasing for* $\hat{p}_g \le p_g \le p_{g1}$, π_g *is decreasing with respect to* p_g *for* $p_{g1} \le p_g \le p_{g2}$. *We conclude that* π_g *reaches its maximum at* $p_g = \hat{p}_g$.

- *Let* $\omega_1 \le c < \omega_2$.

  - *If* $0 \le f \le f_2$, *then* π_g *is increasing with respect to* p_g *for* $0 \le p_g \le p_{g1}$, π_g *is increasing with respect to* p_g *for* $p_{g1} \le p_g \le p_{g2}$. *We conclude that when* $0 \le f \le f_1$, π_g *reaches its maximum at any* $p_g \ge p_{g2}$.
  - *If* $f_2 \le f \le f_4$, *then* π_g *is increasing with respect to* p_g *for* $0 \le p_g \le \hat{p}_g$ *and decreasing for* $\hat{p}_g \le p_g \le p_{g1}$, π_g *is increasing with respect to* p_g *for* $p_{g1} \le p_g \le p_{g2}$. *Therefore in this case* π_g *reaches its maximum at* $p_g = \hat{p}_g$ *or at any* $p_g \ge p_{g2}$ *and it is necessary to compare* $\pi_{g,2}$ *and* $\hat{\pi}_g$.
  - *If* $f_4 \le f \le f_1$, *then* π_g *is increasing with respect to* p_g *for* $0 \le p_g \le \hat{p}_g$ *and decreasing for* $\hat{p}_g \le p_g \le p_{g1}$, π_g *is increasing with respect to* p_g *for* $p_{g1} \le p_g \le p_g^*$ *and decreasing for* $p_g^* \le p_g \le p_{g2}$. *Therefore in this case* π_g *reaches its maximum at* $p_g = \hat{p}_g$ *or at* $p_g = p_g^*$ *and it is necessary to compare* π_g^* *and* $\hat{\pi}_g$.
  - *If* $f_1 \le f \le \tilde{f}$ *then* π_g *is increasing with respect to* p_g *for* $0 \le p_g \le \hat{p}_g$ *and decreasing for* $\hat{p}_g \le p_g \le p_{g1}$, π_g *is decreasing with respect to* p_g *for* $p_{g1} \le p_g \le p_{g2}$. *We conclude that* π_g *reaches its maximum at* $p_g = \hat{p}_g$.

- *Let* $\omega_2 \le c < 1$.

  - *If* $0 \le f \le f_2$, *then* π_g *is increasing with respect to* p_g *for* $0 \le p_g \le p_{g1}$, π_g *is increasing with respect to* p_g *for* $p_{g1} \le p_g \le p_{g2}$. *We conclude that* π_g *reaches its maximum at any* $p_g \ge p_{g2}$.
  - *If* $f_2 \le f \le \tilde{f}$, *then then* π_g *is increasing with respect to* p_g *for* $0 \le p_g \le \hat{p}_g$ *and decreasing for* $\hat{p}_g \le p_g \le p_{g1}$, π_g *is increasing with respect to* p_g *for* $p_{g1} \le p_g \le p_{g2}$. *Therefore in this case* π_g *reaches its maximum at* $p_g = \hat{p}_g$ *or at any* $p_g \ge p_{g2}$ *and it is necessary to compare* $\pi_{g,2}$ *and* $\hat{\pi}_g$.

Proof It is a consequence of Lemmas 1, 2, 3 and 4.

Lemma 6 *Let*

$$\lambda_1 = \frac{\alpha(1-\phi) - \sqrt{\alpha(1-\phi)}\sqrt{\alpha(1-\phi)-(1-c)^2}}{2\phi}.$$

$$\lambda_2 = \frac{\alpha(1-\phi) + \sqrt{\alpha(1-\phi)}\sqrt{\alpha(1-\phi)-(1-c)^2}}{2\phi}.$$

- *If* $0 < c < c_4$ *then* $\pi_{g,2} < \hat{\pi}_g$ *for any value of* f;
- *if* $c_4 < c$ *then* $\pi_{g,2} > \hat{\pi}_g$ *for* $\lambda_1 < f < \lambda_2$ *while* $\pi_{g,2} < \hat{\pi}_g$ *for* $f < \lambda_1$ *or* $f > \lambda_2$.

Proof Note that $\pi_{g,2} > \hat{\pi}_g$ if

$$4\phi^2 f^2 - 4\alpha\phi(1-\phi)f + \alpha(1-\phi)(1-c)^2 < 0.$$

From Lemma 5 we need to compare $\hat{\pi}_g$ and $\pi_{g,2}$ in the cases

- $\omega_1 < c < \omega_2$ and $f_2 < f < f_4$;
- $\omega_2 < c < 1$ and $f_2 < f < \tilde{f}$.

Lemma 7

1. $c_4 < \omega_1$ *for any* $\alpha, \phi \in [0, 1]$.
2. $\lambda_1 < f_2$ *for any* $0 < \alpha < 1, 0 < \phi < 1, c > 0$.
3. $\lambda_2 > f_2$ *for any* $0 < \alpha < 1, 0 < \phi < 1, c > \omega_1$.
4. $\lambda_2 > f_4$ *if* $\omega_1 < c < c_5, \lambda_2 < f_4$ *if* $c_5 < c < 1$.
5. $\lambda_2 < \tilde{f}$ *for any* $\alpha, \phi \in]0, 1[$.

Proof

1. $c_4 < \omega_1$ if $4\alpha(1-\phi) - (2-\alpha\phi)^2 < 0$, that is $-4(1-\alpha) - \alpha^2\phi^2 < 0$.
2. $\lambda_1 < f_2$ if $\sqrt{1-\phi}\sqrt{\alpha(1-\phi)-(1-c)^2} + c\sqrt{\alpha}(1-\phi) > 0$.
3. $\lambda_2 > f_2$ if

$$\sqrt{1-\phi}\sqrt{\alpha(1-\phi)-(1-c)^2} > c\sqrt{\alpha}(1-\phi)$$

or

$$[1+\alpha(1-\phi)]c^2 - 2c + [1-\alpha+\alpha\phi] < 0$$

that is if

$$\frac{1-\alpha+\alpha\phi}{1+\alpha+\alpha\phi} < c < 1.$$

Since $\frac{1-\alpha+\alpha\phi}{1+\alpha+\alpha\phi} < \omega_1$ for any $\alpha, \phi \in]0,1[$ we conclude that $\lambda_2 > f_2$ for any $c > \omega_1$.

4. $\lambda_2 < f_4$ if

$$\sqrt{\alpha}\phi\sqrt{1-\phi}\sqrt{\alpha(1-\phi)-(1-c)^2} < (1-\phi)[2c+2\alpha-\alpha\phi-2]$$

that is

$$\begin{cases} c > c_3 \\ \alpha\phi^2(1-\phi)\left[\alpha(1-\phi)-(1-c)^2\right] < (1-\phi)^2[2c+2\alpha-\alpha\phi-2]^2 \end{cases}$$

or

$$\begin{cases} c > c_3 \\ c < y_1 \quad or \quad c > y_2 \end{cases}$$

where

$$y_1 = \frac{-2\alpha\phi(1-\phi)\sqrt{1-\alpha} - \alpha(\phi^2-6\phi+4) + 4(1-\phi)}{\alpha\phi^2 + 4(1-\phi)}.$$

$$y_2 = \frac{2\alpha\phi(1-\phi)\sqrt{1-\alpha} - \alpha(\phi^2-6\phi+4) + 4(1-\phi)}{\alpha\phi^2 + 4(1-\phi)};$$

It is $y_1 < \omega_1 < y_2 < \omega_2$ and $c_3 < \omega_1$ for any $\alpha, \phi \in]0,1][$. Observing that $c_5 = y_2$ completes the proof.

5. $\lambda_2 < \tilde{f}$ if $\sqrt{1-\phi}\sqrt{\alpha(1-\phi)-(1-c)^2} < \sqrt{\alpha}(1-\phi)$ or $\alpha(1-\phi)-(1-c)^2 < \alpha(1-\phi)$ or $-(1-c)^2 < 0$.

Lemma 8

1. *Let* $\omega_1 < c < c_5$. *Then* $\pi_{g,2} > \hat{\pi}_g$ *for any* $f \in]f_2, f_4[$.
2. *Let* $c_5 < c < \omega_2$. *Then*
   - $\pi_{g,2} > \hat{\pi}_g$ *if* $f \in]f_2, f_5[$;
   - $\pi_{g,2} < \hat{\pi}_g$ *if* $f \in]f_5, f_4[$;
3. *Let* $\omega_2 < c < 1$. *Then*

- $\pi_{g,2} > \hat{\pi}_g$ *if* $f \in]f_2, f_5[$;
- $\pi_{g,2} < \hat{\pi}_g$ *if* $f \in]f_5, \tilde{f}[$.

Proof It is a consequence of Lemmas 6 and 7. Note that $f_5 = \lambda_2$.

Lemma 9 *Let* $c > c_1$ *and*

$$z_1 = \frac{\alpha(1-\phi)\{c\phi - (1-\alpha)(2-\phi) - \sqrt{1-\alpha}[2c - c\phi - \phi]\}}{\phi\left[\alpha(2-\phi)^2 - 4(1-\phi)\right]};$$

$$z_2 = \frac{\alpha(1-\phi)\{c\phi - (1-\alpha)(2-\phi) + \sqrt{1-\alpha}[2c - c\phi - \phi]\}}{\phi\left[\alpha(2-\phi)^2 - 4(1-\phi)\right]}.$$

- *If* $0 < \alpha < \alpha^*$ *then* $z_2 < z_1$, $\pi_g^* > \hat{\pi}_g$ *for* $z_2 < f < z_1$ *while* $\pi_g^* < \hat{\pi}_g$ *for* $z < z_2$ *or* $z > z_1$;
- *if* $\alpha^* < \alpha < 1$ *then* $z_1 < z_2$, $\pi_g^* < \hat{\pi}_g$ *for* $z_1 < f < z_2$ *while* $\pi_g^* > \hat{\pi}_g$ *for* $z < z_1$ *or* $z > z_2$.

Proof Note that $\pi_g^* < \hat{\pi}_g$ if

$$\phi^2\left[\alpha(2-\phi)^2 - 4(1-\phi)\right]f^2 - 2\alpha\phi(1-\phi)[c\phi - (1-\alpha)(2-\phi)]f + \alpha^2(1-\phi)^2\left[c^2 - (1-\alpha)\right] < 0.$$

Solving the equation

$$\phi^2\left[\alpha(2-\phi)^2 - 4(1-\phi)\right]f^2 - 2\alpha\phi(1-\phi)[c\phi - (1-\alpha)(2-\phi)]f + \alpha^2(1-\phi)^2\left[c^2 - (1-\alpha)\right] = 0$$

with respect to c we obtain the solutions z_1 and z_2. It is $z1 < z_2$ if $\alpha > \alpha^*$ and $z1 > z_2$ if $\alpha < \alpha^*$.

From Lemma 5 we need to compare $\hat{\pi}_g$ and π_g^* in the cases

- $c_1 < c < \omega_1$ and $f_2 < f < f_1$;
- $\omega_1 < c < \omega_2$ and $f_4 < f < f_1$.

Lemma 10 Let $c > c_1$.

1. *Let*

$$\sigma_1 = \frac{2(1-\phi)(1-\alpha)[2 - 2\alpha + \alpha\phi] - \alpha\phi^2\sqrt{1-\alpha}}{4(1-\phi)(1-\alpha) - \alpha\phi(2-\phi)\sqrt{1-\alpha}}.$$

It is $z_1 < f_4$ *for any* $0 < \alpha < 1, 0 < \phi < 1, c > \sigma_1$.

2. $f_2 < z_1$ *for any* $0 < \alpha < 1, 0 < \phi < 1, c > c_1$.
3. $z_1 < f_1$ *for any* $0 < \alpha < 1, 0 < \phi < 1, c > c_1$.
4. *Let*

$$\sigma_2 = \frac{\alpha\phi^2\sqrt{1-\alpha} + 2(1-\alpha)(1-\phi)[\alpha\phi - 2\alpha + 2]}{\alpha\phi\sqrt{1-\alpha}(2-\phi) + 4(1-\phi)(1-\alpha)}.$$

It is $z_2 < f_4$ *if*

- $0 < \alpha < \alpha^*, 0 < \phi < 1, c > \sigma_2$;
- $\alpha^* < \alpha < 1, 0 < \phi < 1, c < \sigma_2$.

5. *It is* $z_2 < f_2$ *if* $\alpha < \alpha^*$.
6. *It is* $z_2 < f_1$ *if* $\alpha < \alpha^*$.

Proof

1. Let

$$\rho_1 = \frac{4(1-\alpha)(1-\phi) - \alpha\phi(2-\phi)\sqrt{1-\alpha}}{\alpha(2-\phi)^2 - 4(1-\phi)}$$

It is $z_1 < f_4$ if ($\rho_1 < 0$ and $c > \sigma_1$) or ($\rho_1 > 0$ and $c < \sigma_1$).
Since it is $\rho_1 < 0$ for any $\alpha, \phi \in]0, 1[$, it follows that $z_1 < f_4$ if $c > \sigma_1$.

2. $f_2 < z_1$ if

$$\frac{\left[2\sqrt{1-\alpha} + 2\alpha - \alpha\phi - 2\right][c\phi - 2c + \phi]}{\alpha(2-\phi)^2 - 4(1-\phi)} > 0$$

which is satisfied for any $0 < \alpha < 1, 0 < \phi < 1, c > c_1$.

3. $z_1 < f_1$ if

$$\frac{[c\phi - 2c + \phi]\left[\sqrt{1-\alpha}(2 - 2\alpha + \alpha\phi) - 2(1-\alpha)\right]}{\alpha(2-\phi)^2 - 4(1-\phi)} < 0$$

which is satisfied for any $0 < \alpha < 1, 0 < \phi < 1, c > c_1$.

4. Let

$$\rho_2 = \frac{-4(1-\alpha)(1-\phi) - \alpha\phi(2-\phi)\sqrt{1-\alpha}}{\alpha(2-\phi)^2 - 4(1-\phi)}$$

It is $z_2 < f_4$ if ($\rho_2 < 0$, and $c < \sigma_2$) or ($\rho_2 > 0$ and $c > \sigma_2$).
Since it is $\rho_2 < 0$ if $\alpha > \alpha^*$, it follows that $z_2 < f_4$ if $c < \sigma_2$ and $\alpha > \alpha^*$ or if $c > \sigma_2$ and $\alpha < \alpha^*$.

5. It is trivial.
6. It is trivial.

Lemma 11 *It is*

$$\sigma_2 < \omega_1 < \sigma_2 < \omega_1$$

for any $0 < \alpha < 1, 0 < \phi < 1$.

Proof

1. $\sigma_2 < \omega_1$ if

$$-\frac{2(1-\alpha)^{3/2} + (1-\alpha)[2 - 2\alpha + \alpha\phi]}{\alpha\phi\sqrt{1-\alpha}(2-\phi) + 4(1-\phi)(1-\alpha)} < 0.$$

2. $\omega_1 < \sigma_1$ if

$$\frac{2\alpha\phi(1-\phi)(1-\alpha)\left[2 - 2\alpha + \alpha\phi - 2\sqrt{1-\alpha}\right]}{(2-\alpha\phi)\left[\alpha\phi\sqrt{1-\alpha}(2-\phi) - 4(1-\alpha)(1-\phi)\right]} > 0.$$

   Note that the numerator and the denominator of the above fraction are both positive if $\alpha > \alpha^*$ and both negative if $\alpha < \alpha^*$.

3. $\sigma_1 < \omega_2$ if

$$\frac{-\left\{\alpha\phi(1-\phi)\left[\sqrt{1-\alpha}(2 - 2\alpha + \alpha\phi) - 2(1-\alpha\right]\right\}}{4(1-\alpha)(1-\phi) - \alpha\phi\sqrt{1-\alpha}(2-\phi)} > 0.$$

   Note that the numerator and the denominator of the above fraction are both positive if $\alpha < \alpha^*$ and both negative if $\alpha > \alpha^*$.

Lemma 12

1. *Let* $c_1 < c < \omega_1$. *Then*
   - $\pi_g^* > \hat{\pi}_g$ *if* $f \in]f_2, z_1[$;
   - $\pi_g^* < \hat{\pi}_g$ *if* $f \in]z_1, f_1[$;

2. *Let* $\omega_1 < c < \sigma_1$. *Then*
   - $\pi_g^* > \hat{\pi}_g$ *if* $f \in]f_4, z_1[$;
   - $\pi_g^* < \hat{\pi}_g$ *if* $f \in]z_1, f_1[$;

3. *Let* $\sigma_1 < c < \omega_2$. *Then* $\pi_g^* < \hat{\pi}_g$ *if* $f \in]f_4, f_1[$.

Proof It is a consequence of Lemmas 9, 10 and 11.

Now we are ready to prove Proposition 4.

Proof of Proposition 4 Since $f_3 = z_1$ and since it can be shown that $\sigma_1 = c_5$, from Lemma 5, 8 and 12 we obtain the following results.

Note that the profit of the genuine producer is constant with respect to p_g for $p_g \geq p_{g2}$; consequently, in order to find the optimal price of the genuine item, it is sufficient to examine the behaviour of π_g with respect to p_g for $0 \leq p_g \leq p_{g2}$.

1. $0 < \alpha < \tilde{\alpha}$.
   - Let $0 \leq c < c_1$.
     Then
     - If $0 \leq f \leq f_1$, then π_g is increasing with respect to p_g for $0 \leq p_g \leq p_{g1}$, π_g is increasing with respect to p_g for $p_{g1} \leq p_g \leq p_g^*$ and decreasing for $p_g^* \leq p_g \leq p_{g2}$. We conclude that π_g reaches its maximum at $p_g = p_g^*$.
     - If $f_1 \leq f \leq f_2$ then π_g is increasing with respect to p_g for $0 \leq p_g \leq p_{g1}$, π_g is decreasing for $p_{g1} \leq p_g \leq p_{g2}$. We conclude that π_g reaches its maximum at $p_g = p_{g1}$.
     - If $f_2 \leq f < \tilde{f}$ then then π_g is increasing with respect to p_g for $0 \leq p_g \leq \hat{p}_g$ and decreasing for $\hat{p}_g \leq p_g \leq p_{g1}$, π_g is decreasing for $p_{g1} \leq p_g \leq p_{g2}$. We conclude that π_g reaches its maximum at $p_g = \hat{p}_g$.

   - Let $c_1 \leq c < c_2$.
     - If $0 \leq f \leq f_2$, then π_g is increasing with respect to p_g for $0 \leq p_g \leq p_{g1}$, π_g is increasing with respect to p_g for $p_{g1} \leq p_g \leq p_g^*$ and decreasing for $p_g^* \leq p_g \leq p_{g2}$. We conclude that when $0 \leq f \leq f_3$, π_g reaches its maximum at $p_g = p_g^*$.
     - If $f_2 \leq f \leq f_3$, then π_g is increasing with respect to p_g for $0 \leq p_g \leq \hat{p}_g$ and decreasing for $\hat{p}_g \leq p_g \leq p_{g1}$, π_g is increasing with respect to p_g for $p_{g1} \leq p_g \leq p_g^*$ and decreasing for $p_g^* \leq p_g \leq p_{g2}$. Therefore in this case π_g reaches its maximum at $p_g = \hat{p}_g$ or at $p_g = p_g^*$. Comparing π_g^* and $\hat{\pi}_g$ we conclude that π_g reaches its maximum at $p_g = p_g^*$.
     - If $f_3 \leq f \leq f_1$, then π_g is increasing with respect to p_g for $0 \leq p_g \leq \hat{p}_g$ and decreasing for $\hat{p}_g \leq p_g \leq p_{g1}$, π_g is increasing with respect to p_g for $p_{g1} \leq p_g \leq p_g^*$ and decreasing for $p_g^* \leq p_g \leq p_{g2}$. Therefore in this case π_g reaches its maximum at $p_g = \hat{p}_g$ or at $p_g = p_g^*$. Comparing π_g^* and $\hat{\pi}_g$ we conclude that π_g reaches its maximum at $p_g = \hat{p}_g$.
     - If $f_1 \leq f \leq \tilde{f}$ then π_g is increasing with respect to p_g for $0 \leq p_g \leq \hat{p}_g$ and decreasing for $\hat{p}_g \leq p_g \leq p_{g1}$, π_g is decreasing with respect to p_g for $p_{g1} \leq p_g \leq p_{g2}$. We conclude that π_g reaches its maximum at $p_g = \hat{p}_g$.

- Let $c_2 \leq c < \omega_1$.
    - If $0 \leq f \leq f_4$, then π_g is increasing with respect to p_g for $0 \leq p_g \leq p_{g1}$, π_g is increasing with respect to p_g for $p_{g1} \leq p_g \leq p_{g2}$. We conclude that π_g reaches its maximum at any $p_g \geq p_{g2}$.
    - If $f_4 \leq f \leq f_2$, then π_g is increasing with respect to p_g for $0 \leq p_g \leq p_{g1}$, π_g is increasing with respect to p_g for $p_{g1} \leq p_g \leq p_g^*$ and decreasing for $p_g^* \leq p_g \leq p_{g2}$. We conclude that π_g reaches its maximum at $p_g = p_g^*$.
    - If $f_2 \leq f \leq f_3$, then π_g is increasing with respect to p_g for $0 \leq p_g \leq \hat{p}_g$ and decreasing for $\hat{p}_g \leq p_g \leq p_{g1}$, π_g is increasing with respect to p_g for $p_{g1} \leq p_g \leq p_g^*$ and decreasing for $p_g^* \leq p_g \leq p_{g2}$. Therefore in this case π_g reaches its maximum at $p_g = \hat{p}_g$ or at $p_g = p_g^*$. Comparing π_g^* and $\hat{\pi}_g$ we conclude that π_g reaches its maximum at $p_g = p_g^*$.
    - If $f_3 \leq f \leq f_1$, then π_g is increasing with respect to p_g for $0 \leq p_g \leq \hat{p}_g$ and decreasing for $\hat{p}_g \leq p_g \leq p_{g1}$, π_g is increasing with respect to p_g for $p_{g1} \leq p_g \leq p_g^*$ and decreasing for $p_g^* \leq p_g \leq p_{g2}$. Therefore in this case π_g reaches its maximum at $p_g = \hat{p}_g$ or at $p_g = p_g^*$. Comparing π_g^* and $\hat{\pi}_g$ we conclude that π_g reaches its maximum at $p_g = \hat{p}_g$.
    - If $f_1 \leq f \leq \tilde{f}$ then π_g is increasing with respect to p_g for $0 \leq p_g \leq \hat{p}_g$ and decreasing for $\hat{p}_g \leq p_g \leq p_{g1}$, π_g is decreasing with respect to p_g for $p_{g1} \leq p_g \leq p_{g2}$. We conclude that π_g reaches its maximum at $p_g = \hat{p}_g$.

- Let $\omega_1 \leq c < c_5$.
    - If $0 \leq f \leq f_2$, then π_g is increasing with respect to p_g for $0 \leq p_g \leq p_{g1}$, π_g is increasing with respect to p_g for $p_{g1} \leq p_g \leq p_{g2}$. We conclude that when $0 \leq f \leq f_1$, π_g reaches its maximum at any $p_g \geq p_{g2}$.
    - If $f_2 \leq f \leq f_4$, then π_g is increasing with respect to p_g for $0 \leq p_g \leq \hat{p}_g$ and decreasing for $\hat{p}_g \leq p_g \leq p_{g1}$, π_g is increasing with respect to p_g for $p_{g1} \leq p_g \leq p_{g2}$. Therefore in this case π_g reaches its maximum at $p_g = \hat{p}_g$ or at any $p_g \geq p_{g2}$. Comparing $\hat{\pi}_g$ and $\pi_{g,2}$ we conclude that π_g reaches its maximum at any $p_g \geq p_{g2}$.
    - If $f_4 \leq f \leq f_3$, then π_g is increasing with respect to p_g for $0 \leq p_g \leq \hat{p}_g$ and decreasing for $\hat{p}_g \leq p_g \leq p_{g1}$, π_g is increasing with respect to p_g for $p_{g1} \leq p_g \leq p_g^*$ and decreasing for $p_g^* \leq p_g \leq p_{g2}$. Therefore in this case π_g reaches its maximum at $p_g = \hat{p}_g$ or at $p_g = p_g^*$. Comparing π_g^* and $\hat{\pi}_g$ we conclude that π_g reaches its maximum at $p_g = p_g^*$.
    - If $f_3 \leq f \leq f_1$, then π_g is increasing with respect to p_g for $0 \leq p_g \leq \hat{p}_g$ and decreasing for $\hat{p}_g \leq p_g \leq p_{g1}$, π_g is increasing with respect to p_g for $p_{g1} \leq p_g \leq p_g^*$ and decreasing for $p_g^* \leq p_g \leq p_{g2}$. Therefore in this case π_g

reaches its maximum at $p_g = \hat{p}_g$ or at $p_g = p_g^*$. Comparing π_g^* and $\hat{\pi}_g$ we conclude that π_g reaches its maximum at $p_g = \hat{p}_g$.

- If $f_1 \le f \le \tilde{f}$ then π_g is increasing with respect to p_g for $0 \le p_g \le \hat{p}_g$ and decreasing for $\hat{p}_g \le p_g \le p_{g1}, \pi_g$ is decreasing with respect to p_g for $p_{g1} \le p_g \le p_{g2}$. We conclude that π_g reaches its maximum at $p_g = \hat{p}_g$.

- Let $c_5 \le c < \omega_2$.
  - If $0 \le f \le f_2$, then π_g is increasing with respect to p_g for $0 \le p_g \le p_{g1}, \pi_g$ is increasing with respect to p_g for $p_{g1} \le p_g \le p_{g2}$. We conclude that when $0 \le f \le f_1, \pi_g$ reaches its maximum at any $p_g \ge p_{g2}$.
  - If $f_2 \le f \le f_5$, then π_g is increasing with respect to p_g for $0 \le p_g \le \hat{p}_g$ and decreasing for $\hat{p}_g \le p_g \le p_{g1}, \pi_g$ is increasing with respect to p_g for $p_{g1} \le p_g \le p_{g2}$. Therefore in this case π_g reaches its maximum at $p_g = \hat{p}_g$ or at any $p_g \ge p_{g2}$. Comparing $\hat{\pi}_g$ and $\pi_{g,2}$ we conclude that π_g reaches its maximum at any $p_g \ge p_{g2}$.
  - If $f_5 \le f \le f_4$, then π_g is increasing with respect to p_g for $0 \le p_g \le \hat{p}_g$ and decreasing for $\hat{p}_g \le p_g \le p_{g1}, \pi_g$ is increasing with respect to p_g for $p_{g1} \le p_g \le p_{g2}$. Therefore in this case π_g reaches its maximum at $p_g = \hat{p}_g$ or at any $p_g \ge p_{g2}$. Comparing π_g^* and $\hat{\pi}_g$ we conclude that π_g reaches its maximum at $p_g = \hat{p}_g$.
  - If $f_4 \le f \le f_3$, then π_g is increasing with respect to p_g for $0 \le p_g \le \hat{p}_g$ and decreasing for $\hat{p}_g \le p_g \le p_{g1}, \pi_g$ is increasing with respect to p_g for $p_{g1} \le p_g \le p_g^*$ and decreasing for $p_g^* \le p_g \le p_{g2}$. Therefore in this case π_g reaches its maximum at $p_g = \hat{p}_g$ or at $p_g = p_g^*$. Comparing π_g^* and $\hat{\pi}_g$ we conclude that π_g reaches its maximum at $p_g = \hat{p}_g$.
  - If $f_3 \le f \le f_1$, then π_g is increasing with respect to p_g for $0 \le p_g \le \hat{p}_g$ and decreasing for $\hat{p}_g \le p_g \le p_{g1}, \pi_g$ is increasing with respect to p_g for $p_{g1} \le p_g \le p_g^*$ and decreasing for $p_g^* \le p_g \le p_{g2}$. Therefore in this case π_g reaches its maximum at $p_g = \hat{p}_g$ or at $p_g = p_g^*$. Comparing π_g^* and $\hat{\pi}_g$ we conclude that π_g reaches its maximum at $p_g = \hat{p}_g$.
  - If $f_1 \le f \le \tilde{f}$ then π_g is increasing with respect to p_g for $0 \le p_g \le \hat{p}_g$ and decreasing for $\hat{p}_g \le p_g \le p_{g1}, \pi_g$ is decreasing with respect to p_g for $p_{g1} \le p_g \le p_{g2}$. We conclude that π_g reaches its maximum at $p_g = \hat{p}_g$.

- Let $\omega_2 \le c < 1$.
  - If $0 \le f \le f_2$, then π_g is increasing with respect to p_g for $0 \le p_g \le p_{g1}, \pi_g$ is increasing with respect to p_g for $p_{g1} \le p_g \le p_{g2}$. We conclude that π_g reaches its maximum at any $p_g \ge p_{g2}$.
  - If $f_2 \le f \le f_5$, then then π_g is increasing with respect to p_g for $0 \le p_g \le \hat{p}_g$ and decreasing for $\hat{p}_g \le p_g \le p_{g1}, \pi_g$ is increasing with respect to p_g for

$p_{g1} \le p_g \le p_{g2}$. Therefore in this case π_g reaches its maximum at $p_g = \hat{p}_g$ or at any $p_g \ge p_{g2}$. Comparing $\hat{\pi}_g$ and $\pi_{g,2}$ we conclude that π_g reaches its maximum at any $p_g \ge p_{g2}$.

- If $f_5 \le f \le \tilde{f}$, then $\pi_g =$ is increasing with respect to p_g for $0 \le p_g \le \hat{p}_g$ and decreasing for $\hat{p}_g \le p_g \le p_{g1}$, π_g is increasing with respect to p_g for $p_{g1} \le p_g \le p_{g1}$. Therefore in this case π_g reaches its maximum at $p_g = \hat{p}_g$ or at any $p_g \ge p_{g2}$. Comparing $\hat{\pi}_g$ and $\pi_{g,2}$ we conclude that π_g reaches its maximum at at $p_g = \hat{p}_g$.

- $\tilde{\alpha} < \alpha < 1$.
  - Let $0 \le c < c_2$.
    - If $0 \le f \le f_1$, then π_g is increasing with respect to p_g for $0 \le p_g \le p_{g1}$, π_g is increasing with respect to p_g for $p_{g1} \le p_g \le p_g^*$ and decreasing for $p_g^* \le p_g \le p_{g2}$. We conclude that π_g reaches its maximum at $p_g = p_g^*$.
    - If $f_1 \le f \le f_2$ then π_g is increasing with respect to p_g for $0 \le p_g \le p_{g1}$, π_g is decreasing for $p_{g1} \le p_g \le p_{g2}$. We conclude that π_g reaches its maximum at $p_g = p_{g1}$.
    - If $f_2 \le f < \tilde{f}$ then then π_g is increasing with respect to p_g for $0 \le p_g \le \hat{p}_g$ and decreasing for $\hat{p}_g \le p_g \le p_{g1}$, π_g is decreasing for $p_{g1} \le p_g \le p_{g2}$. We conclude that π_g reaches its maximum at $p_g = \hat{p}_g$.

  - Let $c_2 \le c < c_1$.
    - If $0 \le f \le f_4$, then π_g is increasing with respect to p_g for $0 \le p_g \le p_{g1}$, π_g is increasing with respect to p_g for $p_{g1} \le p_g \le p_{g2}$. We conclude that π_g reaches its maximum at any $p_g \ge p_{g2}$.
    - If $f_4 \le f \le f_1$, then π_g is increasing with respect to p_g for $0 \le p_g \le p_{g1}$, π_g is increasing with respect to p_g for $p_{g1} \le p_g \le p_g^*$ and decreasing for $p_g^* \le p_g \le p_{g2}$. We conclude that π_g reaches its maximum at $p_g = p_g^*$.
    - If $f_1 \le f \le f_2$ then π_g is increasing with respect to p_g for $0 \le p_g \le p_{g1}$, π_g is decreasing for $p_{g1} \le p_g \le p_{g2}$. We conclude that π_g reaches its maximum at $p_g = p_{g1}$.
    - If $f_2 \le f < \tilde{f}$ then then π_g is increasing with respect to p_g for $0 \le p_g \le \hat{p}_g$ and decreasing for $\hat{p}_g \le p_g \le p_{g1}$, π_g is decreasing for $p_{g1} \le p_g \le p_{g2}$. We conclude that π_g reaches its maximum at $p_g = \hat{p}_g$.
  - Let $c_1 \le c < \omega_1$.
    - If $0 \le f \le f_4$, then π_g is increasing with respect to p_g for $0 \le p_g \le p_{g1}$, π_g is increasing with respect to p_g for $p_{g1} \le p_g \le p_{g2}$. We conclude that π_g reaches its maximum at any $p_g \ge p_{g2}$.

- If $f_4 \leq f \leq f_2$, then π_g is increasing with respect to p_g for $0 \leq p_g \leq p_{g1}, \pi_g$ is increasing with respect to p_g for $p_{g1} \leq p_g \leq p_g^*$ and decreasing for $p_g^* \leq p_g \leq p_{g2}$. We conclude that π_g reaches its maximum at $p_g = p_g^*$.
- If $f_2 \leq f \leq f_3$, then π_g is increasing with respect to p_g for $0 \leq p_g \leq \hat{p}_g$ and decreasing for $\hat{p}_g \leq p_g \leq p_{g1}, \pi_g$ is increasing with respect to p_g for $p_{g1} \leq p_g \leq p_g^*$ and decreasing for $p_g^* \leq p_g \leq p_{g2}$. Therefore in this case π_g reaches its maximum at $p_g = \hat{p}_g$ or at $p_g = p_g^*$. Comparing π_g^* and $\hat{\pi}_g$ we conclude that π_g reaches its maximum at $p_g = p_g^*$.
- If $f_3 \leq f \leq f_1$, then π_g is increasing with respect to p_g for $0 \leq p_g \leq \hat{p}_g$ and decreasing for $\hat{p}_g \leq p_g \leq p_{g1}, \pi_g$ is increasing with respect to p_g for $p_{g1} \leq p_g \leq p_g^*$ and decreasing for $p_g^* \leq p_g \leq p_{g2}$. Therefore in this case π_g reaches its maximum at $p_g = \hat{p}_g$ or at $p_g = p_g^*$. Comparing π_g^* and $\hat{\pi}_g$ we conclude that π_g reaches its maximum at $p_g = \hat{p}_g$.
- If $f_1 \leq f \leq \tilde{f}$ then π_g is increasing with respect to p_g for $0 \leq p_g \leq \hat{p}_g$ and decreasing for $\hat{p}_g \leq p_g \leq p_{g1}, \pi_g$ is decreasing with respect to p_g for $p_{g1} \leq p_g \leq p_{g2}$. We conclude that π_g reaches its maximum at $p_g = \hat{p}_g$.

- Let $\omega_1 \leq c < c_5$.
  - If $0 \leq f \leq f_2$, then π_g is increasing with respect to p_g for $0 \leq p_g \leq p_{g1}, \pi_g$ is increasing with respect to p_g for $p_{g1} \leq p_g \leq p_{g2}$. We conclude that when $0 \leq f \leq f_1, \pi_g$ reaches its maximum at any $p_g \geq p_{g2}$.
  - If $f_2 \leq f \leq f_4$, then π_g is increasing with respect to p_g for $0 \leq p_g \leq \hat{p}_g$ and decreasing for $\hat{p}_g \leq p_g \leq p_{g1}, \pi_g$ is increasing with respect to p_g for $p_{g1} \leq p_g \leq p_{g2}$. Therefore in this case π_g reaches its maximum at $p_g = \hat{p}_g$ or at any $p_g \geq p_{g2}$. Comparing $\hat{\pi}_g$ and $\pi_{g,2}$ we conclude that π_g reaches its maximum at any $p_g \geq p_{g2}$.
  - If $f_4 \leq f \leq f_3$, then π_g is increasing with respect to p_g for $0 \leq p_g \leq \hat{p}_g$ and decreasing for $\hat{p}_g \leq p_g \leq p_{g1}, \pi_g$ is increasing with respect to p_g for $p_{g1} \leq p_g \leq p_g^*$ and decreasing for $p_g^* \leq p_g \leq p_{g2}$. Therefore in this case π_g reaches its maximum at $p_g = \hat{p}_g$ or at $p_g = p_g^*$. Comparing π_g^* and $\hat{\pi}_g$ we conclude that π_g reaches its maximum at $p_g = p_g^*$.
  - If $f_3 \leq f \leq f_1$, then π_g is increasing with respect to p_g for $0 \leq p_g \leq \hat{p}_g$ and decreasing for $\hat{p}_g \leq p_g \leq p_{g1}, \pi_g$ is increasing with respect to p_g for $p_{g1} \leq p_g \leq p_g^*$ and decreasing for $p_g^* \leq p_g \leq p_{g2}$. Therefore in this case π_g reaches its maximum at $p_g = \hat{p}_g$ or at $p_g = p_g^*$. Comparing π_g^* and $\hat{\pi}_g$ we conclude that π_g reaches its maximum at $p_g = \hat{p}_g$.

  - If $f_1 \le f \le \tilde{f}$ then π_g is increasing with respect to p_g for $0 \le p_g \le \hat{p}_g$ and decreasing for $\hat{p}_g \le p_g \le p_{g1}, \pi_g$ is decreasing with respect to p_g for $p_{g1} \le p_g \le p_{g2}$. We conclude that π_g reaches its maximum at $p_g = \hat{p}_g$.

- Let $c_5 \le c < \omega_2$.
  - If $0 \le f \le f_2$, then π_g is increasing with respect to p_g for $0 \le p_g \le p_{g1}, \pi_g$ is increasing with respect to p_g for $p_{g1} \le p_g \le p_{g2}$. We conclude that when $0 \le f \le f_1, \pi_g$ reaches its maximum at any $p_g \ge p_{g2}$.
  - If $f_2 \le f \le f_5$, then π_g is increasing with respect to p_g for $0 \le p_g \le \hat{p}_g$ and decreasing for $\hat{p}_g \le p_g \le p_{g1}, \pi_g$ is increasing with respect to p_g for $p_{g1} \le p_g \le p_{g2}$. Therefore in this case π_g reaches its maximum at $p_g = \hat{p}_g$ or at any $p_g \ge p_{g2}$. Comparing $\hat{\pi}_g$ and $\pi_{g,2}$ we conclude that π_g reaches its maximum at any $p_g \ge p_{g2}$.
  - If $f_5 \le f \le f_4$, then π_g is increasing with respect to p_g for $0 \le p_g \le \hat{p}_g$ and decreasing for $\hat{p}_g \le p_g \le p_{g1}, \pi_g$ is increasing with respect to p_g for $p_{g1} \le p_g \le p_{g2}$. Therefore in this case π_g reaches its maximum at $p_g = \hat{p}_g$ or at any $p_g \ge p_{g2}$. Comparing π_g^* and $\hat{\pi}_g$ we conclude that π_g reaches its maximum at $p_g = \hat{p}_g$.
  - If $f_4 \le f \le f_1$, then π_g is increasing with respect to p_g for $0 \le p_g \le \hat{p}_g$ and decreasing for $\hat{p}_g \le p_g \le p_{g1}, \pi_g$ is increasing with respect to p_g for $p_{g1} \le p_g \le p_g^*$ and decreasing for $p_g^* \le p_g \le p_{g2}$. Therefore in this case π_g reaches its maximum at $p_g = \hat{p}_g$ or at $p_g = p_g^*$. Comparing π_g^* and $\hat{\pi}_g$ we conclude that π_g reaches its maximum at $p_g = \hat{p}_g$.
  - If $f_1 \le f \le \tilde{f}$ then π_g is increasing with respect to p_g for $0 \le p_g \le \hat{p}_g$ and decreasing for $\hat{p}_g \le p_g \le p_{g1}, \pi_g$ is decreasing with respect to p_g for $p_{g1} \le p_g \le p_{g2}$. We conclude that π_g reaches its maximum at $p_g = \hat{p}_g$.

- Let $\omega_2 \le c < 1$.
  - If $0 \le f \le f_2$, then π_g is increasing with respect to p_g for $0 \le p_g \le p_{g1}, \pi_g$ is increasing with respect to p_g for $p_{g1} \le p_g \le p_{g2}$. We conclude that π_g reaches its maximum at any $p_g \ge p_{g2}$.
  - If $f_2 \le f \le f_5$, then then π_g is increasing with respect to p_g for $0 \le p_g \le \hat{p}_g$ and decreasing for $\hat{p}_g \le p_g \le p_{g1}, \pi_g$ is increasing with respect to p_g for $p_{g1} \le p_g \le p_{g2}$. Therefore in this case π_g reaches its maximum at $p_g = \hat{p}_g$ or at any $p_g \ge p_{g2}$. Comparing $\hat{\pi}_g$ and $\pi_{g,2}$ we conclude that π_g reaches its maximum at any $p_g \ge p_{g2}$.

- If $f_5 \le f \le \tilde{f}$, then $\pi_g =$ is increasing with respect to p_g for $0 \le p_g \le \hat{p}_g$ and decreasing for $\hat{p}_g \le p_g \le p_{g1}, \pi_g$ is increasing with respect to p_g for $p_{g1} \le p_g \le p_{g1}$. Therefore in this case π_g reaches its maximum at $p_g = \hat{p}_g$ or at any $p_g \ge p_{g2}$. Comparing $\hat{\pi}_g$ and $\pi_{g,2}$ we conclude that π_g reaches its maximum at at $p_g = \hat{p}_g$.

In order to prove Proposition 5, we need the following preliminary Lemmas.

Lemma 13 *It is*

1.

$$\frac{\partial \pi_g^*}{\partial f} > 0 \Longleftrightarrow \begin{cases} f < f_6 & and \quad 0 < \alpha < \alpha^*; \\ f > f_6 & and \quad \alpha^* < \alpha < 1. \end{cases}$$

2.

$$\frac{\partial \pi_{g,2}}{\partial f} > 0 \Longleftrightarrow f < \frac{\tilde{f}}{2}.$$

3.

$$\frac{\partial \pi_{g1}}{\partial f} > 0 \Longleftrightarrow f < f_2.$$

Proof It is trivial.

Lemma 14 *Let*

$$\omega_3 = \frac{(1-\alpha)(2-\phi)}{\phi}.$$

It is

1. $c_2 < c_3 < c_5$ *for any* $\alpha, \phi \in]0,1[$;
2. $c_1 < c_3 \Longleftrightarrow \alpha < \alpha^*$;
3. $c_2 < \omega_3$ *for any* $\alpha, \phi \in]0,1[$;
4. $c_1 < \omega_3 \Longleftrightarrow \alpha < \alpha^*$;
5. $c_4 < c_3 \Longleftrightarrow \alpha < \alpha^*$;
6. $c_1 < c_4 \Longleftrightarrow \alpha < \alpha^*$.

Proof It is trivial.

Lemma 15 *It is* $c_4 < c_5$ *for any* $\alpha, \phi \in]0,1[$.

Proof $c_5 - c_4 > 0 \Longleftrightarrow$

$$2\sqrt{\alpha}(1-\phi)(2-\phi) < 2\sqrt{\alpha}\phi\sqrt{1-\alpha}(1-\phi) + \sqrt{1-\phi}\left[\alpha\phi^2 + 4(1-\phi)\right]$$

or

$$2\sqrt{\alpha}\sqrt{1-\phi}\left[2-\phi-\phi\sqrt{1-\alpha}\right]<\alpha\phi^2+4(1-\phi).$$

Squaring both sides of the above inequality, after some algebraic manipulations we obtain

$$8\alpha\phi(1-\phi)(2-\phi)\sqrt{1-\alpha}+\alpha^2\phi^2\left[\phi^2+4(1-\phi)\right]+16(1-\phi)^2(1-\alpha)>0$$

which is true for any $\alpha, \phi \in]0, 1[$.

Lemma 16 *It is*

1.

$$f_6>0 \iff \begin{cases} c<\omega_3 & \text{and} \quad 0<\alpha<\alpha^*; \\ c>\omega_3 & \text{and} \quad \alpha^*<\alpha<1. \end{cases}$$

2.

$$f_6<f_1 \iff \begin{cases} c>c_1 & \text{and} \quad 0<\alpha<\alpha^*; \\ c<c_1 & \text{and} \quad \alpha^*<\alpha<1. \end{cases}$$

3.

$$f_6<f_3 \iff \begin{cases} c>c_1 & \text{and} \quad 0<\alpha<\alpha^*; \\ c<c_1 & \text{and} \quad \alpha^*<\alpha<1. \end{cases}$$

4.

$$f_6<f_4 \iff \begin{cases} c>c_3 & \text{and} \quad 0<\alpha<\alpha^*; \\ c<c_3 & \text{and} \quad \alpha^*<\alpha<1. \end{cases}$$

5.

$$\frac{\tilde{f}}{2}<f_4 \iff c>c_3.$$

6.

$$\frac{\tilde{f}}{2}<f_5 \iff c>c_4.$$

Proof It is trivial.

Now we are ready to prove Proposition 5.

Proof of Proposition 5 From Lemmas 13, 14, 15 and 16 and from Proposition 4 we obtain the following results.

1. $0 < \alpha < \tilde{\alpha}$.

- Let $0 \leq c < c_1$.
Then

$$\pi_{g,max}(f) = \begin{cases} \pi_g^*(f) & if \quad 0 < f < f_1; \quad increasing; \\ \pi_{g1}(f) & if \quad f_1 < f < f_2; \quad increasing; \\ \hat{\pi}_g(f) & if \quad f_2 < f; \quad constant. \end{cases}$$

Therefore the highest profit for the genuine producer is guaranteed by any $f \geq f_{opt} = f_2$. The ensuing profit is $\pi_{opt} = \hat{\pi}_g$.

- Let $c_1 \leq c < c_2$.
Then

$$\pi_{g,max}(f) = \begin{cases} \pi_g^*(f) & if \quad 0 < f < f_3; \quad increasing\ in\ [0, f_6]\ and\ decreasing\ in [f_6, f_3]; \\ \hat{\pi}_g(f) & if \quad f_3 < f; \quad constant. \end{cases}$$

Therefore the highest profit for the genuine producer is guaranteed by $f = f_6$. The ensuing profit is $\pi_{opt} = \pi_g^*(f = f_6)$.

- Let $c_2 \leq c < c_3$.
Then

$$\pi_{g,max}(f) = \begin{cases} \pi_{g,2}(f) & if \quad 0 < f < f_4; \quad increasing; \\ \pi_g^*(f) & if \quad f_4 < f < f_3; \quad increasing\ in\ [f_4, f_6]\ and\ decreasing\ in [f_6, f_3]; \\ \hat{\pi}_g(f) & if \quad f_3 < f; \quad constant. \end{cases}$$

Therefore the highest profit for the genuine producer is guaranteed by $f = f_6$. The ensuing profit is $\pi_{opt} = \pi_g^*(f = f_6)$.

- Let $c_3 \leq c < c_5$.
Then

$$\pi_{g,max}(f) = \begin{cases} \pi_{g,2}(f) & if \quad 0 < f < f_4; \quad increasing\ in\ \left[0, \frac{\tilde{f}}{2}\right]\ and\ decreasing\ in \left[\frac{\tilde{f}}{2}, f_4\right]; \\ \pi_g^*(f) & if \quad f_4 < f < f_3; \quad decreasing; \\ \hat{\pi}_g(f) & if \quad f_3 < f; \quad constant. \end{cases}$$

Therefore the highest profit for the genuine producer is guaranteed by $f = \frac{\tilde{f}}{2}$.
The ensuing profit is $\pi_{opt} = \pi_{g,2}\left(f = \frac{\tilde{f}}{2}\right)$.

- Let $c_5 \leq c \leq 1$.
Then

$$\pi_{g,max}(f) = \begin{cases} \pi_{g,2}(f) & if \quad 0 < f < f_5; \quad increasing\ in\ \left[0, \frac{\tilde{f}}{2}\right]\ and\ decreasing\ in \left[\frac{\tilde{f}}{2}, f_5\right]; \\ \hat{\pi}_g(f) & if \quad f_5 < f; \quad constant. \end{cases}$$

Therefore the highest profit for the genuine producer is guaranteed by $f = \frac{\tilde{f}}{2}$.

The ensuing profit is $\pi_{opt} = \pi_{g,2}\left(f = \frac{\tilde{f}}{2}\right)$.

2. $\tilde{\alpha} < \alpha < \alpha^*$.

- Let $0 \le c < c_2$.
  Then

$$\pi_{g,max}(f) = \begin{cases} \pi_g^*(f) & if \quad 0 < f < f_1; \quad increasing; \\ \pi_{g1}(f) & if \quad f_1 < f < f_2; \quad increasing; \\ \hat{\pi}_g(f) & if \quad f_2 < f; \quad constant. \end{cases}$$

  Therefore the highest profit for the genuine producer is guaranteed by any $f \ge f_{opt} = f_2$. The ensuing profit is $\pi_{opt} = \hat{\pi}_g$.
- Let $c_2 \le c < c_1$.
  Then

$$\pi_{g,max}(f) = \begin{cases} \pi_{g,2}(f) & if \quad 0 < f < f_4; \quad increasing; \\ \pi_g^*(f) & if \quad f_4 < f < f_1; \quad increasing; \\ \pi_{g1}(f) & if \quad f_1 < f < f_2; \quad increasing; \\ \hat{\pi}_g(f) & if \quad f_2 < f; \quad constant. \end{cases}$$

  Therefore the highest profit for the genuine producer is guaranteed by any $f \ge f_{opt} = f_2$. The ensuing profit is $\pi_{opt} = \hat{\pi}_g$.
- Let $c_1 \le c < c_3$.
  Then

$$\pi_{g,max}(f) = \begin{cases} \pi_{g,2}(f) & if \quad 0 < f < f_4; \quad increasing; \\ \pi_g^*(f) & if \quad f_4 < f < f_3; \quad increasing\ in\ [f_4, f_6]\ and\ decreasing\ in [f_6, f_3]; \\ \hat{\pi}_g(f) & if \quad f_3 < f; \quad constant. \end{cases}$$

  Therefore the highest profit for the genuine producer is guaranteed by $f = f_6$. The ensuing profit is $\pi_{opt} = \pi_g^*(f = f_6)$.
- Let $c_3 \le c < c_5$.
  Then

$$\pi_{g,max}(f) = \begin{cases} \pi_{g,2}(f) & if \quad 0 < f < f_4; \quad increasing\ in \left[0, \frac{\tilde{f}}{2}\right]\ and\ decreasing\ in \left[\frac{\tilde{f}}{2}, f_4\right]; \\ \pi_g^*(f) & if \quad f_4 < f < f_3; \quad decreasing; \\ \hat{\pi}_g(f) & if \quad f_3 < f; \quad constant. \end{cases}$$

  Therefore the highest profit for the genuine producer is guaranteed by $f = \frac{\tilde{f}}{2}$.

  The ensuing profit is $\pi_{opt} = \pi_{g,2}\left(f = \frac{\tilde{f}}{2}\right)$.

- Let $c_5 \leq c \leq 1$.
  Then

$$\pi_{g,max}(f) = \begin{cases} \pi_{g,2}(f) & if \quad 0 < f < f_5; \quad increasing\ in \left[0, \frac{\tilde{f}}{2}\right] and\ decreasing\ in \left[\frac{\tilde{f}}{2}, f_5\right]; \\ \hat{\pi}_g(f) & if \quad f_5 < f; \quad constant. \end{cases}$$

  Therefore the highest profit for the genuine producer is guaranteed by $f = \frac{\tilde{f}}{2}$. The ensuing profit is $\pi_{opt} = \pi_{g,2}\left(f = \frac{\tilde{f}}{2}\right)$.

3. $\alpha^* < \alpha < 1$.

- Let $0 \leq c < c_2$.
  Then

$$\pi_{g,max}(f) = \begin{cases} \pi^*_g(f) & if \quad 0 < f < f_1; \quad decreasing\ in\ [0, f_6]\ and\ increasing\ in [f_6, f_1];; \\ \pi_{g1}(f) & if \quad f_1 < f < f_2; \quad increasing; \\ \hat{\pi}_g(f) & if \quad f_2 < f; \quad constant. \end{cases}$$

  Therefore the highest profit for the genuine producer is guaranteed by any $f \geq f_{opt} = f_2$. The ensuing profit is $\pi_{opt} = \hat{\pi}_g$.
- Let $c_2 \leq c < c_3$.
  Then

$$\pi_{g,max}(f) = \begin{cases} \pi_{g,2}(f) & if \quad 0 < f < f_4; \quad increasing; \\ \pi^*_g(f) & if \quad f_4 < f < f_1; \quad increasing; \\ \pi_{g1}(f) & if \quad f_1 < f < f_2; \quad increasing; \\ \hat{\pi}_g(f) & if \quad f_2 < f; \quad constant. \end{cases}$$

  Therefore the highest profit for the genuine producer is guaranteed by any $f \geq f_{opt} = f_2$. The ensuing profit is $\pi_{opt} = \hat{\pi}_g$.
- Let $c_3 \leq c < c_4$.
  Then

$$\pi_{g,max}(f) = \begin{cases} \pi_{g,2}(f) & if \quad 0 < f < f_4; \quad increasing\ in \left[0, \frac{\tilde{f}}{2}\right] and\ decreasing\ in \left[\frac{\tilde{f}}{2}, f_4\right]; \\ \pi^*_g(f) & if \quad f_4 < f < f_1; \quad decreasing\ in\ [f_4, f_6]\ and\ increasing\ in [f_6, f_1]; \\ \pi_{g1}(f) & if \quad f_1 < f < f_2; \quad increasing; \\ \hat{\pi}_g(f) & if \quad f_2 < f; \quad constant. \end{cases}$$

  Comparing $\pi_{g,2}\left(f = \frac{\tilde{f}}{2}\right)$ and $\hat{\pi}_g$ we obtain that the highest profit for the genuine producer is guaranteed by any $f \geq f_{opt} = f_2$. The ensuing profit is $\pi_{opt} = \hat{\pi}_g$.

- Let $c_4 \leq c < c_1$.
  Then

$$\pi_{g,max}(f) = \begin{cases} \pi_{g,2}(f) & \text{if} \quad 0 < f < f_4; \quad \textit{increasing in } \left[0, \frac{\tilde{f}}{2}\right] \textit{ and decreasing in} \left[\frac{\tilde{f}}{2}, f_4\right]; \\ \pi_g^*(f) & \text{if} \quad f_4 < f < f_1; \quad \textit{decreasing in } [f_4, f_6] \textit{ and increasing in} [f_6, f_1]; \\ \pi_{g1}(f) & \text{if} \quad f_1 < f < f_2; \quad \textit{increasing}; \\ \hat{\pi}_g(f) & \text{if} \quad f_2 < f; \quad \textit{constant}. \end{cases}$$

  Comparing $\pi_{g,2}\left(f = \frac{\tilde{f}}{2}\right)$ and $\hat{\pi}_g$ we obtain that the highest profit for the genuine producer is guaranteed by $f = \frac{\tilde{f}}{2}$. The ensuing profit is $\pi_{g,2}\left(f = \frac{\tilde{f}}{2}\right)$.

- Let $c_1 \leq c < c_5$.
  Then

$$\pi_{g,max}(f) = \begin{cases} \pi_{g,2}(f) & \text{if} \quad 0 < f < f_4; \quad \textit{increasing in } \left[0, \frac{\tilde{f}}{2}\right] \textit{ and decreasing in} \left[\frac{\tilde{f}}{2}, f_4\right]; \\ \pi_g^*(f) & \text{if} \quad f_4 < f < f_3; \quad \textit{decreasing in} [f_4, f_3]; \\ \hat{\pi}_g(f) & \text{if} \quad f_3 < f; \quad \textit{constant}. \end{cases}$$

  Therefore the highest profit for the genuine producer is guaranteed by $f = \frac{\tilde{f}}{2}$. The ensuing profit is $\pi_{g,2}\left(f = \frac{\tilde{f}}{2}\right)$.

- Let $c_5 \leq c \leq 1$.
  Then

$$\pi_{g,max}(f) = \begin{cases} \pi_{g,2}(f) & \text{if} \quad 0 < f < f_5; \quad \textit{increasing in } [0, \frac{\tilde{f}}{2}] \textit{ and decreasing in} [\frac{\tilde{f}}{2}, f_5]; \\ \hat{\pi}_g(f) & \text{if} \quad f_5 < f; \quad \textit{constant}. \end{cases}$$

  Therefore the highest profit for the genuine producer is guaranteed by $f = \frac{\tilde{f}}{2}$. The ensuing profit is $\pi_{opt} = \pi_{g,2}\left(f = \frac{\tilde{f}}{2}\right)$.

Proof of Proposition 6

- Let $0 \leq c < 1 - \alpha$.
  From (4) and (5) we have that $W(\phi = 0) > W(\phi = 1)$ iff $M > M^*$, where

$$M^* = \frac{\alpha((1-\alpha)(2-5q) - (2-q)c^2)}{8(1-\alpha)}$$

It is $\frac{\partial M^*}{\partial q} = \frac{\alpha(c^2 - 5(1-\alpha))}{8(1-\alpha)} < 0$ for any $c \in [0, 1-\alpha]$. Therefore M^* is a decreasing function of q. It is $M^* > 0$ iff $q < q^*$ where

$$q^* = \frac{2(1-\alpha-c^2)}{5(1-\alpha) - c^2}$$

It is $0 < q^* < 1$ for any $c \in [0, 1-\alpha[$.

- Let $1-\alpha \leq c \leq 1$.
  From (4) and (6) we have that $W(\phi = 0) > W(\phi = 1)$ iff $M > M^*$, where

$$M^* = \frac{(2-q)c^2 - 2(2-q)c + 2 - q - 4\alpha q}{8}$$

It is $\frac{\partial M^*}{\partial q} = -\frac{4\alpha + (1-c)^2}{8} < 0$ for any $c \in [1-\alpha, 1]$. Therefore M^* is a decreasing function of q. It is $M^* > 0$ iff $q < q^*$ where

$$q^* = \frac{2(1-c)^2}{(1-c)^2 + 4\alpha}$$

It is $0 < q^* < 1$ for any $c \in [1-\alpha, 1]$.

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