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Hierarchical space-time models for fire ignition and percentage of land burned by wildfires

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Abstract Policy responses for local and global fire management as well as international green-gas inventories depend heavily on the proper understanding of the annual fire extend as well as its spatial variation across any given study area. Proper statistical models are important tools in quantifying these fire risks. We propose Bayesian methods to model jointly the probability of ignition and fire sizes in Australia and New Zeland. The data set on which we base our model and results consists of annual observations of several meteorological and topographical explanatory variables, together with the percentage of land burned over a grid with resolution of 1° across Austalia and New Zealand. Our model and conclusions bring improvements on the results reported by Russell-Smith et al. in Int J Wildland Fire, 16:361–377 (2007) based on a similar data set.

Keywords Wildfires · Bayesian hierarchical models · Spatial statistics

1 Introduction

A series of recent studies have modeled spatial patterns of fire incidence as a function of a set of environmental covariates, at geographical scales ranging from regional (Spessa et al. 2005) to continental (Russell-Smith et al. 2007; Archibald et al. 2009; Sá et al. under review) and global (Krawchuk et al. 2009). Spessa et al. (2005) and

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Russell-Smith et al. (2007) focused on Australia and relied on generalized linear models to quantify fire-environment relations. Archibald et al. (2009) and Sá et al. (under review) studied African fires and used, respectively, regression trees and geographically weighted regression. The global fire analysis by Krawchuk et al. (2009) was performed with generalized additive models. Spatial data analysis requires careful consideration of autocorrelation and heterogeneity issues. Spatial autocorrelation (or spatial dependency) is the tendency for objects located close together to be more similar than those farther apart (Legendre 1993). Heterogeneity (or spatial non-stationarity) is not a property of the data but of the modeled relationship, and describes the tendency for relationships to vary in space. Lack of consideration of these issues may lead to biased parameter estimates, errors in significance tests, and sub-optimal prediction (Anselin and Griffith 1988).

The Australian studies (Spessa et al. 2005; Russell-Smith et al. 2007) fail to address or discuss spatial statistics issues at all. Archibald et al. (2009) discuss spatial autocorrelation but do not address it explicitly, since no technique is available to incorporate its effects in regression tree models. Data sub-sampling inherent in their ensemble models, is expected to reduce spatial autocorrelation, but the authors admit to some degree of uncertainty in statistical tests, potentially leading to inclusion of spurious explanatory variables. Only the most important splits in regression trees were kept, in an attempt to minimize this problem. Krawchuk et al. (2009) also acknowledged the problem of spatial dependence and its potential effects on variable and model selection. They did not explicitly incorporate a spatial component in their models, but performed a geostatistical analysis of the data to determine a spatial sub-sampling pattern capable of attenuating the effects of spatial autocorrelation. Sá et al. (under review) analysed spatial non-stationarity of fire-environment relationships in sub-Saharan Africa, using geographically weighted regression (GWR). This technique, which explicitly incorporates spatial locations of the data, was used to explore the assumption of spatial stationarity of relationships between fire and a set of environmental covariates. However, as we will see later, it is questionable whether certain properties of spatial data can be adequately dealt with by the standard weighted least squares method, based on an independent Gaussian error structure.The main purpose of the present study is to introduce space-time hierarchical Bayesian modeling to pyrogeography, and to highlight its advantages relatively to modeling approaches used so far. In order to do this, we use spatial data for fire occurrence and area burned in Australia and New Zealand during the period 1998–2006, together with maps of climatic, vegetation, and anthropogenic covariates. This data set is defined on a continental grid of $1^\circ \times 1^\circ$ cells, totaling 750 grid points. Fire affected areas in each grid cell between 1998 and 2006, derived from satellite imagery, were used as the response variable, whereas dry season severity, precipitation over the wet season, type of landcover, among other variables observed in each grid cell, were used as a set of explanatory variables. No attempt was made to model the temporal and spatial variation in the explanatory variables. The principal objective of this study is to explore the relative importance of the explanatory variables under analysis and, as a consequence, assess the spatial variation of fire affected areas at a continental scale.

Statistical methods and models so far used in the existing literature can be improved in many directions: Percentage of land burned as well as the occurrence of ignition in each grid cell show high degree of spatial and temporal dependency for low resolution grid data, and this dependence probably is even stronger in data sets with higher resolution than the one we use. Although part of this dependence is explained through the explanatory variables, a significant portion of this dependence is due to latent, unaccounted spatially and temporarily varying variables. Therefore, standard Gaussian regression methods based on independent errors (Spessa et al. 2005; Russell-Smith et al. 2007; Krawchuk et al. 2009) may be inappropriate. These unaccounted dependence structures have particularly undesired and strong effects on the statistical tests carried on the regression parameters.

Data set on ignition and percentage of land burned within each grid cell typically contains large numbers of 0 values, that is, there are many grid points where no fire occurs during the year. Under these conditions, the usual arcsine transformation (e.g. Russell-Smith et al. 2007) is not particularly satisfactory if a substantial number of the proportions are equal to 0 or 1 or for values at the extreme ends of the possible range (near 0 and near 1).

Latent, unaccounted spatially and temporarily varying variables add significant bias to the residuals and result in the underestimation of the structural variation in the response variables. Therefore, inclusion of latent, spatially and temporally dependent random factors in the link functions and in the regression should reduce bias and improve the overall quality of the regression. They may also indicate future research directions in searching for other explanatory variables having direct effect on fire regimes.

Inference on the ignition probability and percentage of land burned should not be done separately, since these two events are interconnected. Hence, the likelihood should be specified jointly within a single model.

Finally, the use of Bayesian hierarchical models brings with it the usual benefits of Bayesian methodology, such as incorporating sampling variation of model parameters as well as prior expert knowledge into the model.

The structure of the paper is as follows: In Sect. 2, we explain the data set in detail. In Sect. 3, we introduce the model, whereas in Sect. 3.2, we report on the model fitting. In Sect. 3.3, we give the conclusions taken from the fitted model, and finally in Sect. 4, we comment on how our model improves on previous research and summarize the most important conclusions.

2 Data

All data surfaces used for analysis were compiled on a grid of $1^\circ \times 1^\circ$ cells covering Australian and New Zealand land mass, resulting in an equidistant lattice with 750 cells. This is a relatively coarse grid as compared to the grid used by Russell-Smith et al. (2007). Let $s_i = (latitude, longitute)$ be the coordinates of the i-th pixel centroid in this lattice, and $Y(i, t)$ denote the percentage of land burned in pixel i, during year $t = 1998, \ldots, 2006$. The data set under study is of the form

$$(Y(i,t), X_1(i,t), X_2(i,t), \ldots, X_7(i,t), \quad i = 1, \ldots, 750, \quad t = 1, \ldots, 9),$$

where we denote by $X_j(i, t)$, the j-th covariate observed at pixel i at time t, and $Y(i, t)$ is the dependent variable representing the estimated burned fraction over the year, taken from the Global Fire Emissions Database (GFED) (Giglio et al. 2006). Some of the covariates are static in time and they will be denoted by $X_j(i)$ The description of the covariates are as follows:

1. $X_1(i, t)$: Dry Season Severity, observed from the Tropical Rainfall Mapping Mission (TRMM) satellite data (native resolution 0.25°, aggregated to 1°), calculated as the values of 2T-Ppt, where T is mean monthly temperature, and Ppt is monthly precipitation, summed over the months with a positive value of this difference (Breckle 2002).
2. $X_2(i, t)$: Precipitation over the wet season (or growing season) observed from TRMM satellite data. Wet season months are those for which the difference defined above is negative.
3. $X_3(i)$: Human footprint (static), as defined in Sanderson et al. (2002).
4. $X_4(i)$: Tree landcover (static), varying from 0 (no cover) to 1 (full cover), based on Global Land Cover 2000 data (Bartholome and Belward 2005).
5. $X_5(i)$: Grass and Shrubs landcover (static), same as above.
6. $X_6(i)$: Agricultural Landcover (static), same as above.
7. $X_7(i, t)$: Maximum number of consecutive dry pentads (pentad = 5 days) over the corresponding year, computed from the Climate Predicition Center (CPC) Merged Analyses of Precipitation product (CMAP), described by Xie and Arkin (1997).

Other independent variables known to affect the fire incidence, such as spatial heterogeneity of land cover, wind speed and direction, topography, etc., were not considered, due to their perceived secondary role, at the spatial scale of the analysis. These unobserved covariates will be represented in the model by a latent, random effect with a specific spatio-temporal dependence structure. Some of the covariates in the list exhibit strong time and space variation, but at this stage, there will be no attempt to model these temporal and spatial structures in these independent variables; they will be simply included in our model as fixed explanatory variables.

2.1 Preliminary data analysis

The data set reveals some characteristics which need to be taken into account while modeling. The data contain an unusually high number of 0's, and non-zero observations are highly skewed to the left with a fairly heavy right tail. Data also show strong spatial and temporal dependence, which can not be ignored in the model. Therefore, regression methods, assuming uncorrelated error structures will give erroneous results.

Let $Y^+(i, t) = Y(i, t)|Y(i, t) > 0$ represent the non-zero observations. Due to the skewness of the data, either a proper model and the consequent generalized linear model has to be chosen or an adequate transformation has to be found so that the convenient Gaussian structure can be used in modeling the transformed data. Assuming (the crude) assumption that the observations are independent, the models and transformations given below were considered as part of a preliminary data analysis, to find the option which, in principle, would be more suitable.

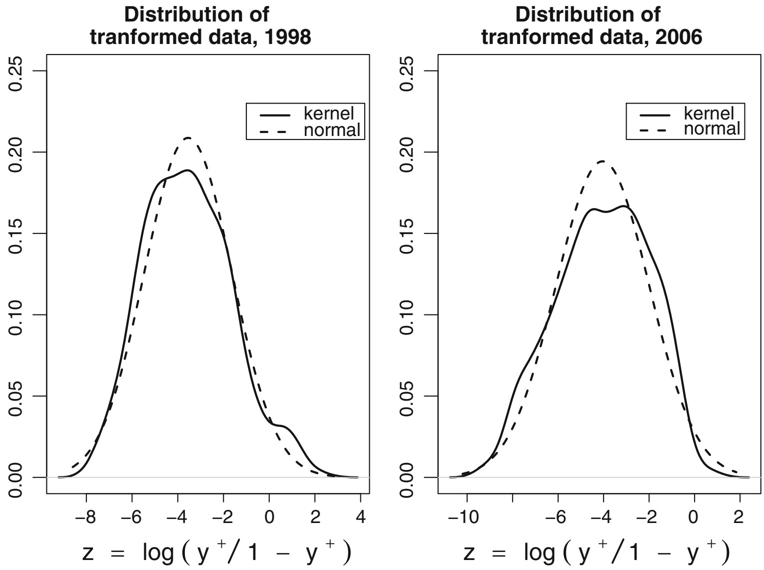

Fig. 1 Normal approximation for $\log\left(\frac{Y^{+}(i,t)}{1-Y^{+}(i,t)}\right)$; years 1998, 2006

$$\begin{aligned} &\text{Case (1)} \ : Y^{+}(i,t) \sim Beta(\theta_1, \theta_2), \\ &\text{Case (2)} \ : -\log(Y^{+}(i,t)) \sim Gamma(\alpha, \beta), \\ &\text{Case (3)} \ : \log\left(\frac{Y^{+}(i,t)}{1-Y^{+}(i,t)}\right) \sim Normal(\mu_1, \sigma_1^2), \\ &\text{Case (4)} \ : \text{Arcsine transformation } \arcsin(\sqrt{Y^{+}(i,t)}) \sim Normal(\mu_2, \sigma_2^2). \end{aligned}$$

The transformation considered in case (3) resulted in residuals which are more compatible with normal distribution, for the null model and a model with all the covariates, than the transformation in case (4).

In Fig. 1 we represent this approximation for the null model, for the years 1998 and 2006.

As expected, due to the large number of zeros and skewed distribution, the commonly used arcsin transformation was inadequate in obtaining Gaussian-like data, even when only the non-zero values are considered. (see Fig. 2).

Although the models given in cases (1) and (2) are good alternatives, the inferences on the corresponding generalized linear models are harder as compared to the Gaussian model (3), without extra benefit attached. The dependence structure among the covariates and their combined effect on the dependent variable is somewhat complicated and interactions among the covariates are also suspected to exist. It is not computationally viable to run full factorial model with all levels of interactions. Therefore upon a preliminary data analysis combined with wildfire expert opinion, a reduced factorial model which included all the covariates and some second order interactions was initially analyzed.

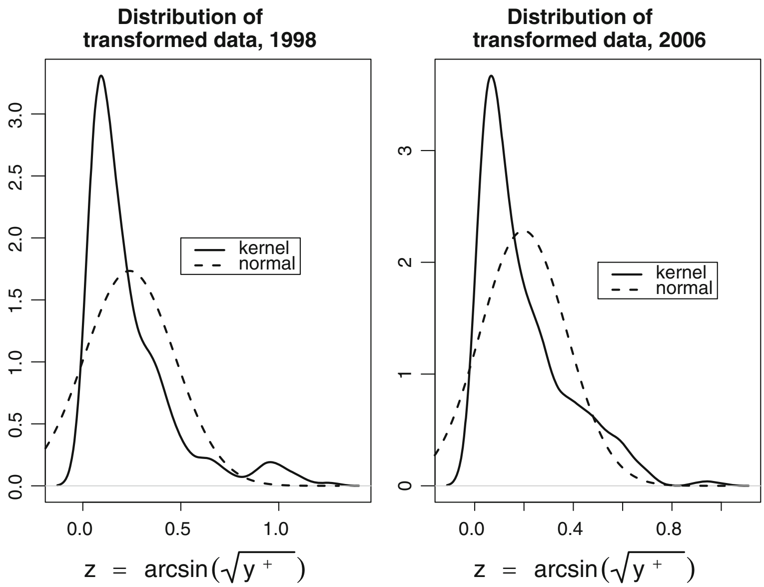

Fig. 2 Normal approximation for arcsin $\left(\sqrt{Y^{+}(i,t)}\right)$; years 1998, 2006

3 Statistical methodology and models

Our objective is to quantify the relative importance of independent variables as determinants of the fire incidence, as well as explaining the space-time variation in the response variable. One possible way of handling such data and take adequate conclusions is the use of geographically weighted regression analysis (GWR) (Fotheringham et al. 2002). The main objective of GWR is to carry out the usual multiple regression analysis using the well known weighted least squares method, where the weights are chosen taking into consideration the distances between the spatial coordinates over which the observations are taken. However, certain aspects of data, which were reported in the previous section, are not compatible with the standard weighted least squares method based on independent Gaussian error structure. Therefore, we suggest an alternative hierarchical model to take into account the characteristics of this data set.

3.1 Joint areal model for ignition probability and percentage of area burned

The geographical area under study is partitioned into a finite number or areal units, which in our case are the fixed grid cells of size $1^\circ \times 1^\circ$. These cells are areal units with well defined boundaries. Hence, we consider the cell positions s_i as their respective centroids and take the data as areal data. Models for areal data are easier to deal with, since the hidden spatio-temporal random effect is a discrete space-time process which can be assumed to be a nearest neighbor Markov process (for example, conditionally autoregressive process (CAR), or Gaussian Intrinsic autoregressive process (ICAR)), and can be simulated without the curse of dimension. See Banerjee et al. (2004). As an alternative, it is possible to assume that the data are a realization of a continuous parameter spatial process and model the data using a point referenced model (Banerjee et al. 2004). In fact, in more general terms, the data can be modeled as a marked spatial point process, where the points are the centroids of individual fire perimeters and the marks are the percentage of burned area. However, this modeling strategy will result in computational difficulties arising from numerical inversion of matrices of size 750×750, which is not very feasible, although there are ways of avoiding it (Banerjee et al. 2008).

We define Bernoulli variables

$$R(i,t) = \begin{cases} 1, & \text{with probability } p_1(i,t) = 1 - p_0(i,t); \\ 0, & \text{with probability } p_0(i,t), \end{cases} \tag{1}$$

where, $p_1(i,t)$ represents the probability that a fire is ignited at the pixel i with centroid at s_i, during year t.

Let $\mathbf{V}(i,t) = (V_1(i,t), V_2(i,t))$ be a latent bivariate spatio-temporal process. Here, we assume that V_1 is the latent process representing the unobserved explanatory variables having influence on the ignition process, whereas V_2 is the latent process representing the unobserved explanatory variables having influence on the fire sizes.

We assume that these latent processes have simple additive space-time structures, namely we assume that

$$V_l(i,t) = V_l(i) + \delta_l(t), \quad l = 1, 2 \tag{2}$$

where, $V_1(i)$ and $V_2(i)$ are dependent spatial processes (we define the nature of this dependence later in this section), whereas $\delta_1(t)$ and $\delta_2(t)$ are temporal processes, independent of each other. This model does not take into consideration space-time interaction, and is relatively easy to implement. Another possibility, for further studies, is to model each $V_l(i,t)$, $l = 1, 2$ as a dynamic CAR model

$$V_l(i,t) = \rho_l V_l(i,t-1) + \eta_l(i,t), \quad l = 1, 2 \tag{3}$$

where $\rho_l \in (0,1)$, $\eta_l(i,t)$ are iid random variables each having the distribution $N(0, \zeta^2)$ and $V_l(i,1)$ are Markov Random Field with a CAR prior.

Let

$$Z(i,t) = \begin{cases} \log\left(\frac{Y(i,t)}{1-Y(i,t)}\right), & \text{if } 0 < Y(i,t) < 1, \\ 0, & \text{if } Y(i,t) = 0. \end{cases} \tag{4}$$

and denote by $\mathbf{Z}$, $\mathbf{X}$, $\mathbf{R}$ and $\mathbf{V}$

$$
\begin{aligned}
&(Z(i,t), \quad i = 1, 2, \ldots, 750, \quad t = 1, 2, \ldots, 9),\\
&(\mathbf{X}(i,t), \quad i = 1, 2, \ldots, 750, \quad t = 1, 2, \ldots, 9),\\
&(R(i,t), \quad i = 1, 2, \ldots, 750, \quad t = 1, 2, \ldots, 9),\\
&(\mathbf{V}(i,t), \quad i = 1, 2, \ldots, 750, \quad t = 1, 2, \ldots, 9),
\end{aligned}
$$

respectively. Let $\boldsymbol{\Theta}$ be all the (random) model parameters to be defined later.

We observe $\mathbf{Z}$ and $\mathbf{R}$ and for the conditional likelihood

$$f(\mathbf{Z}, \mathbf{R}|\mathbf{X}, \mathbf{V}, \boldsymbol{\Theta}) = f(\mathbf{Z}|\mathbf{R}, \mathbf{X}, \mathbf{V}, \boldsymbol{\Theta}) f(\mathbf{R}|\mathbf{X}, \mathbf{V}, \boldsymbol{\Theta})$$

we have the model

$$f(\mathbf{Z}, \mathbf{R}|\mathbf{V}, \mathbf{X}, \boldsymbol{\Theta}) = \prod_{t=1}^{9} \prod_{i=1}^{750} f\left(z(i,t), |R(i,t), \mathbf{X}(i,t), \mathbf{V}(i,t), \boldsymbol{\Theta}\right) f\left(R(i,t)|\mathbf{X}(i,t), \mathbf{V}(i,t), \boldsymbol{\Theta}\right). \tag{5}$$

Assuming, as stated in the previous section, that for $0 < Y(i,t) < 1$, $Z(i,t)$ is normally distributed, we have

$$f(z(i,t)|\mathbf{R}, \mathbf{V}, \mathbf{X}, \boldsymbol{\Theta}) = \left[\phi(\mu(i,t), \sigma^2)\right]^{R(i,t)}, \tag{6}$$

where $\phi(\mu, \sigma^2)$ denotes the normal density function, and the conditional distribution of $R(i,t)$ is Bernoulli with $P[R(i,t) = 1] = p_1(i,t)$.

State equations are the mean function $\mu(i,t)$ and the ignition probability $p_1(i,t)$, through which the spatial dependence will be introduced. We assume the following structures for $\mu(i,t)$ and for $p_1(i,t)$.

$$\mu(i,t) = \beta_0 + \mathbf{X}(i,t)^T \boldsymbol{\beta} + V_1(i) + \delta_1(t) \tag{7}$$

$$\log \frac{p_1(i,t)}{1 - p_1(i,t)} = \eta_0 + \mathbf{X}(i,t)^T \boldsymbol{\eta} + V_2(i) + \delta_2(t), \tag{8}$$

where $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_k)$, $\boldsymbol{\eta} = (\eta_1, \ldots, \eta_k)$ are regression parameters.

We assume that $\mathbf{V}(i) = (V_1(i), V_2(i))$ is a coregionalization model (Banerjee et al. 2004; Gelfand et al. 2004) having the simple intrinsic specification

$$
\begin{aligned}
V_1(i) &= \nu_1 W_0(i) + \nu_2 W_1(i),\\
V_2(i) &= W_1(i),
\end{aligned}
$$

and $W_0(i)$ and $W_1(i)$ are independent ICAR models given by

$$W_0(i)|w_0(j), \quad j \neq i \sim N\left(\sum_j b_{ij} w_0(j), \frac{1}{\tau_0}\right), \tag{9}$$

$$W_1(i)|w_1(j), \quad j \neq i \sim N\left(\sum_j c_{ij} w_1(j), \frac{1}{\tau_1}\right). \tag{10}$$

with τ_0 and τ_1 being precision parameters. Here, the parameters ν_1 and ν_2 control the degree of dependence between the two latent processes.

We assume that the temporal processes $\delta_1(t)$ and $\delta_2(t)$ are independent of each other, each having the identical structure of a random walk process of order 1. This corresponds to having a set of temporally correlated random effects $\delta_l(t), l = 1, 2, t = 1, \ldots, N$ (where $N = 9$ is the number of equally-spaced time points). In the case of a random walk of order 1 we may write (Thomas et al. 2004)

$$\begin{aligned}
\delta_l(t)|\delta_l(t^*) &\sim N\left(\delta_l(t+1), \frac{1}{\phi}\right) \quad \text{for } t = 1 \\
&\sim N\left(\frac{\delta_l(t-1) + \delta_l(t+1)}{2}, \frac{2}{\phi}\right) \quad \text{for } t = 2, \ldots, N-1 \\
&\sim N\left(\delta_l(t-1), \frac{1}{\phi}\right) \quad \text{for } t = N, \quad l = 1, 2
\end{aligned}$$

where $\delta_l(t^*)$ denotes all elements of δ_l except $\delta_l(t)$. This can be implemented in WinBUGS as a ICAR model (Thomas et al. 2004). Note that, $\delta_1(t)$ and $\delta_2(t)$ are temporal processes acting globally over the area under study.

The model parameters and hyperparameters are given by

$$\tau = \frac{1}{\sigma^2}, \quad \boldsymbol{\beta}^* = (\beta_0, \beta_1, \ldots, \beta_k), \quad \boldsymbol{\eta}^* = (\eta_0, \eta_1, \ldots \eta_k), \ \nu_1, \ \nu_2,$$

and the the precision parameters of the latent processes involved. Since sound *prior* information for the parameters of the model is not available, we assume diffuse independent priors for the parameters. Specifically, we assume

1. $\boldsymbol{\beta}^*$ independent $N(0, 1000)$, $\boldsymbol{\eta}^*$ independent $N(0, 1000)$.
2. $\nu_1 \sim N(1, 10)$, $\nu_2 \sim N_0(1, 10)$
3. $Gamma(0.01, 0.01)$ for precison parameters.

3.2 Analysis of the hierarchical model

The software WinBUGS (Lunn et al. 2000) was used to analyze the proposed Bayesian hierarchical model.

Table 1 Deviance information criterion

Model	DIC
(i)	13399.98
(ii)	12806.78
(iii)	10958.46

In the initial model, all the standardized covariates , the relevant interactions as well as the random effects, enter linearly in the respective link functions of the model, so that

$$\mu(i,t) = \beta_0 + \mathbf{X}(i,t)^T\boldsymbol{\beta} + V_1(i) + \delta_1(t) \tag{11}$$

$$\log\frac{p_1(i,t)}{1-p_1(i,t)} = \eta_0 + \mathbf{X}(i,t)^T\boldsymbol{\eta} + V_2(i) + \delta_2(t). \tag{12}$$

Selection of variables was done using the Deviance Information Criterion (DIC) coupled with information about the marginal posterior distribution of regression parameters. A variable (or interaction) is included in the model if the posterior distribution of the corresponding coefficient is away from 0 (Ntzoufras 2009). This was identified by computing, for each regression parameter, the probability of the smallest HPD interval containing zero and keeping the corresponding variables for which this probability was large. After each variable was taken out of the model, DIC was computed. The model with the smallest DIC value was chosen as the final model. Table 1 shows the values of DIC for (i) the model with all the covariates but without interaction terms, (ii) the model with all the covariates and interaction terms of order two between climatological variables and type of land cover, and (iii) the final model, as described next.

The final model has the following structure For $\mu(i,t)$ and for $\log\frac{p_1(i,t)}{1-p_1(i,t)}$:

$$\begin{aligned}
\mu(i,t) &= \beta_0 + \beta_1 X_1(i,t) + \beta_2 X_2(i,t) + \beta_3 X_4(i)\\
&\quad + \beta_4 X_5(i) + \beta_5 X_7(i,t) + \beta_6 X_1(i,t)X_4(i)\\
&\quad + \beta_7 X_2(i,t)X_5(i) + \beta_8 X_1(i,t)X_5(i) + \beta_9 X_1(i,t)X_6(i)\\
&\quad + V_1(i) + \delta_1(t),\\
\log\frac{p_1(i,t)}{1-p_1(i,t)} &= \eta_0 + \eta_1 X_1(i,t) + \eta_2 X_2(i,t) + \eta_3 X_4(i,t)\\
&\quad + \eta_4 X_5(i) + \eta_5 X_6(i) + \eta_6 X_7(i,t)\\
&\quad + \eta_7 X_2(i,t)X_4(i) + \eta_8 X_1(i,t)X_4(i) + \eta_9 X_2(i,t)X_5(i)\\
&\quad + V_2(i) + \delta_2(t).
\end{aligned}$$

The corresponding summary statistics for the posterior distribution of the parameters are given in Tables 2 and 3.

Table 2 Summary statistics for the marginal posterior distributions for the parameters of the linear model for $\mu(i, t)$

Var. name	Parameter	Mean	SD	MCerror	2.5%	Median	97.5%
Intercept	β_0	−4.24	0.04	0.00	−4.33	−4.24	−4.17
X_1	β_1	0.20	0.06	0.00	0.09	0.201	0.31
X_2	β_2	0.27	0.04	0.00	0.19	0.27	0.36
X_4	β_3	0.42	0.088	0.00	0.24	0.41	0.59
X_5	β_4	0.12	0.09	0.00	−0.06	0.12	0.30
X_7	β_5	0.12	0.03	0.00	0.06	0.12	0.19
X_1X_4	β_6	0.52	0.08	0.00	0.37	0.52	0.66
X_2X_5	β_7	0.40	0.04	0.00	0.27	0.35	0.42
X_1X_5	β_8	0.34	0.10	0.00	0.17	0.35	0.52
X_1X_6	β_9	0.34	0.07	0.00	0.19	0.34	0.48

Table 3 Summary statistics for the marginal posterior distributions for the parameters of log $\frac{p_1(i,t)}{1-p_1(i,t)}$

Var. names	Parameters	Mean	SD	MCerror	2.5%	Median	97.5%
Intercept	η_0	2.24	0.12	0.01	2.02	2.23	2.47
X_1	η_1	−0.21	0.12	0.01	−0.45	−0.21	0.02
X_2	η_2	0.49	0.12	0.01	0.25	0.49	0.73
X_4	η_3	1.58	0.24	0.02	1.10	1.58	2.04
X_5	η_4	1.11	0.23	0.02	0.65	1.11	1.55
X_6	η_5	0.80	0.15	0.01	0.51	0.80	1.09
X_7	η_6	0.17	0.068	0.00	0.04	0.17	0.31
X_2X_4	η_7	0.19	0.19	0.01	−0.09	0.19	0.45
X_2X_4	η_8	0.25	0.11	0.01	0.03	0.25	0.48
X_2X_5	η_9	0.78	0.18	0.01	0.44	0.78	1.13

3.3 Results

The fitted model reveals that the consecutive number of dry pentads over the year (X_7), interaction between dry season severity (X_1) and tree land cover (X_4), precipitation over the wet season with a grass and shrub landcover (X_5) influence both the fire ignition and the percentage off land burned. On the other hand, the interaction between dy season severity (X_1) and grass and shrub land cover (X_5) only influences the percentage of land burned. Ignition is also influenced by the interaction between precipitation over the wet season (X_2) and tree land cover (X_4)

These findings show that the model captured some known key fire-environment relationships: fire occurrence and extent is favored by strong seasonal precipitation, that is, there is a marked difference between a wet season, when abundant water promotes

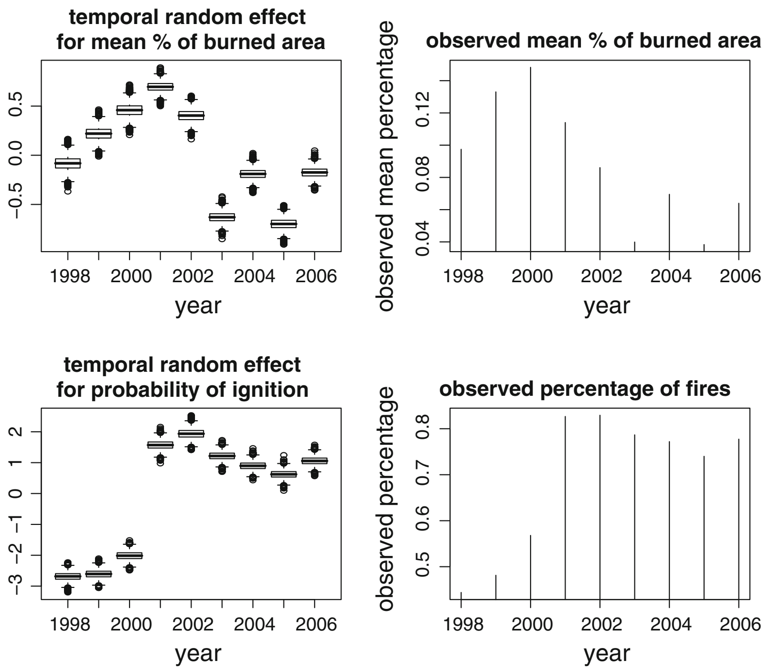

Fig. 3 Box-plot for the temporal random effects $\delta_1(t)$ and $\delta_2(t)$

plant growth, and a dry season, when plant fuels cure and become highly flammable. Interactions observed between climatic and vegetation variables are also very interpretable. Indeed, in tree-dominated ecosystems, woody fuels are abundant since they accumulate over several years, and meteorological conditions are the limiting factor for fire occurrence and extent. Under severe dry seasons, extensive burning tends to occur. In grass-dominated ecosystems, most fuel biomass is renewed annually and depends of rainfall availability during the previous wet season. Here, fuel availability, controlled by antecedent precipitation is the key determinant over fire occurrence and extent.

As it is displayed in Fig. 3, the temporal random effect captures the temporal pattern of the observed mean percentage of the land burned. This is also true for the observed percentage of fires and the corresponding random effect. This is a good evidence that the latent random effects are capturing some part of the temporal variation in the data.

Spatial effects were introduced through the processes V_1 and V_2. Although, in the initial model, the relation between these processes was taken as $V_1(i) = \nu_1 W_0(i) + \nu_2 W_1(i)$, $V_2(i) = W_1(i)$, with $W_0(i)$ and $W_1(i)$ independent ICAR models, the simulated values of the posterior distribution for the parameter ν_1 converged to zero, meaning that this parameter was not significantly different from zero. Hence, in the final model the spatial effects were modeled as $V_1(i) = \nu_2 W_1(i)$, $V_2(i) = W_1(i)$, with $W_1(i)|w_1(j), j \neq i \sim N\left(\sum_j c_{ij} w_1(i), 1/\tau_1\right)$, implying that a single latent process

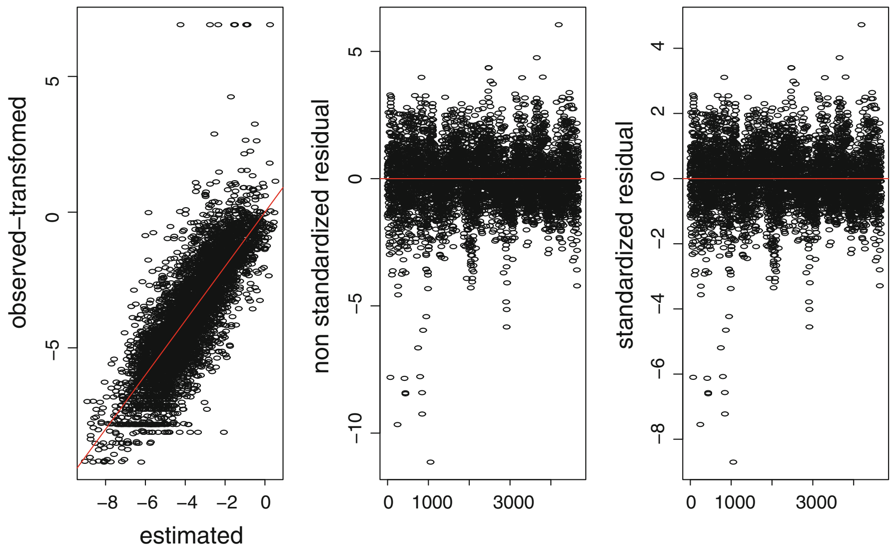

Fig. 4 Bayesian residuals (non-standardized and standardized) of the linear model

governs the ignition as well as the fire size probabilities. The posterior mean for the variance ($1/\tau_1$) of this process was 7.765 and posterior standard deviation 0.1373.

The adequacy of the model was studied through the analysis of the Bayesian residuals from the linear model and the analysis of the Bayesian estimate of the ROC curve from the logit model.

The analysis of the standardized residuals (Fig. 4) obtained from the model shows that there is a good fit.

The box plot (Fig. 5) of the posterior predictive mean and observed percentage of burned area shows, as expected, an underestimation of large values of burned area fraction. This suggests that grid cells displaying extensive burning may need to be modeled separately, using extreme value theory. Other explanation for this underestimation is that the model is not capturing all the space-time variability. The second set of box plots in Fig. 6 clearly shows, that an improvement is obtained with the introduction of both the spatial and temporal random effects through the processes V_1 and δ_1.

However, these random effects may not be sufficiently rich enough to capture all the space-time variations. Therefore, other modeling alternatives, such as the model given in (3) should be considered.

The posterior mean of the spatial random effects is displayed in Fig. 7.

The area under the ROC curve (Fig. 8), which results from the logistic model for the probability of ignition, is 0.96. Predicting a fire when the probability of ignition is above 0.5, gives as result a sensitivity of 0.94, a specificity of 0.81, a positive predictive value of 0.92 and a negative predictive value of 0.85. These numbers reveal a good performance of the model.

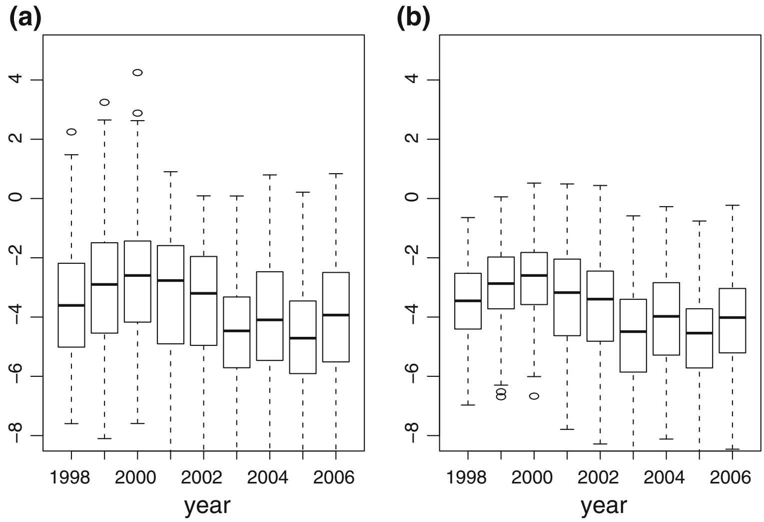

Fig. 5 Box plot of (**a**) observed and (**b**) posterior predictive mean of burned area by time

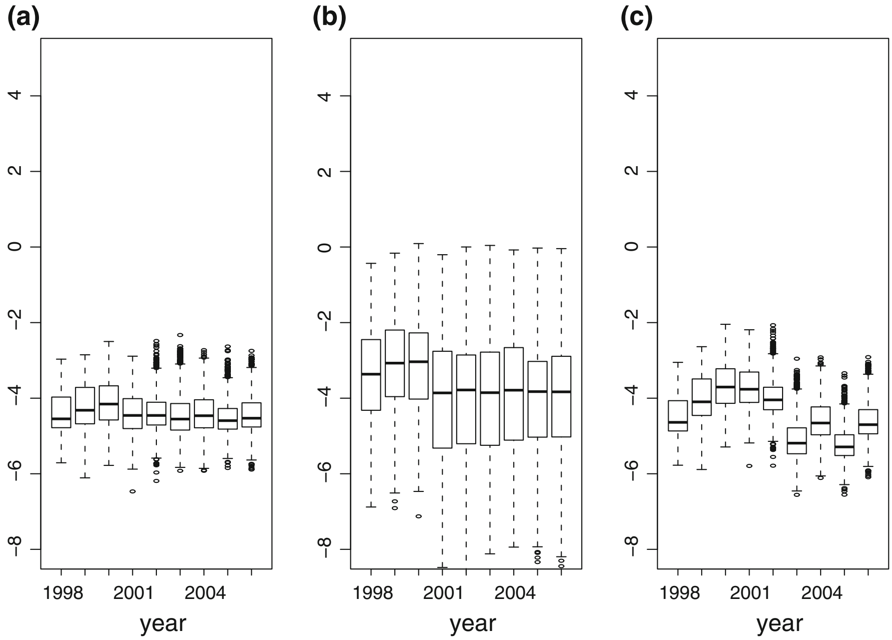

Fig. 6 Box plot of the posterior predictive mean of burned area by time. **a** Without both random effects, **b** with only spatial effect, **c** with only temporal effect

Fig. 7 Posterior mean of the spatial random effects

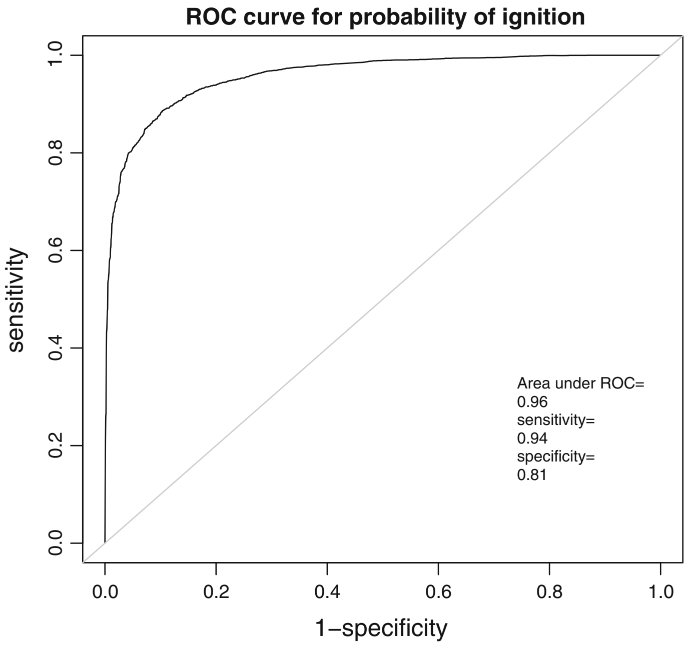

Fig. 8 ROC curve for probability of ignition

4 Conclusions

We believe that our model brings substantial improvements on previous spatial analysis of fire data in many aspects.

Arcsine transformation and the standard generalized linear model are not adequate for data sets showing unusually high number of zeros. The composite model given in Sect. 3.1, together with $\log\left(\frac{Y^+}{1-Y^+}\right)$ transformation seems to be an appropriate modeling strategy.

As it was clearly shown (see Figs. 5 and 6), temporal and spatial latent factors are highly significant, and it is possible to improve upon these results by using dynamic spatial latent factor, as suggested in (3). This observation leads to the conclusion that models built around independent residuals will result in high bias and under estimation of variability.

Finally, significant latent temporal-spatial dependent random factors indicate that there are other unobserved explanatory variables which affect the ignition probabilities as well as the percentage of land burned. These findings should encourage further research into looking for other significant environmental determinants of observed pyrogeographical patterns.

References

Anselin L, Griffith D (1988) Do spatial effects really matter in regression analysis? Pap Reg Sci Assoc 65:11–34

Archibald S, Roy DP, van Wilgen BW, Scholes RJ (2009) What limits fire? An examination of drivers of burnt area in southern Africa. Glob Chang Biol 15(3):613–630

Banerjee S, Gelfand A, Finley AO, Sang H (2008) Gaussian predictive process models for large spatial data sets. JRSS B 70:825–848

Banerjee S, Carlin B, Gelfand A (2004) Hierarchical modeling and analysis for spatial data. Chapman and Hall, London

Bartholome E, Belward AS (2005) GLC2000: a new approach to global land cover mapping from earth observation data. Int J Remote Sens 26:1959–1977

Breckle SW (2002) Walter's vegetation of the earth—the ecological systems of the geo-biosphere. Springer, Berlin

Fotheringham AS, Brunsdon C, Charlton M (2002) Geographically weighted regression. John Wiley, New York

Gelfand A, Schmit A, Banerjee S, Firmans C (2004) Nonstationary multivariate process modeling through spatially varying coreginalization. TEST 13:263–312

Giglio L, van der Werf GR, Randerson JT, Collatz GJ, Kasibhatla P (2006) Global estimation of burned area using MODIS active fire observations. Atmos Chem Phys 6:957–974

Krawchuk MA, Moritz MA, Parisien M-A, Van Dorn J, Hayhoe K (2009) Global pyrogeography: the current and future distribution of wildfire. PLoS ONE 4(4):e5102. doi:10.1371/journal.pone.0005102

Legendre P (1993) Spatial autocorrelation: trouble or new paradigm? Ecology 74:1659–1673

Lunn DJ, Thomas A, Best N, Spiegelhalter D (2000) WinBUGS—a Bayesian modelling framework: concepts, structure, and extensibility. Stat Comput 10:325–337

Ntzoufras I (2009) Bayesian modeling using winBUGS. John wiley and Sons, New York

Russell-Smith J, Yates CP, Whitehead PJ, Smith R, Craig R, Allan GE, Thackway R, Frakes I, Cridland S, Meyer MCP, Gill AM (2007) Bushfires down under: patterns and implications of contemporary Australian landscape burning. Int J Wildland Fire 16:361–377

Sá ACL, Pereira JMC, Mota B, Charlton M, Fotheringham AS, Barbosa PM (2009) Pyrogeography of sub-Saharan Africa: spatial non-stationarity of fire-environment relationships. J Geogr Syst under review

Sanderson EW, Jaiteh M, Levy MA, Redford KH, Wannebo AV, Woolmer G (2002) The human footprint and the last of the wild. Bioscience 52(10):891–904

Spessa A, McBeth B, Prentice C (2005) Relationships among fire frequency, rainfall and vegetation patterns in the wet-dry tropics of northern Australia: an analysis based on NOAA-AVHRR data. Glob Ecol Biogeogr 14(5):439–454

Thomas A, Best N, Lunn D, Arnold R, Spiegelhalter D (2004) GeoBUGS User Manual, version 1.2. http://www.mrc-bsu.cam.ac.uk/bugs/winbugs/geobugs12manual.pdf

Xie PP, Arkin PA (1997) Global precipitation: a 17-year monthly analysis based on gauge observations, satellite estimates, and numerical model outputs. Bull Am Meteor Soc 78:2539–2558

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