



# Unleashing the potential: spatializing middle school mathematics for enhanced learning

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## Abstract

This study assessed the effectiveness of a spatialized mathematics intervention on the mathematical performance and spatial visualization skills of students from an underprivileged Indonesian community. Grade 8 students ( $N=407$ ) were assigned to one of twelve experimental classes, receiving the spatial mathematics intervention, or one of seven control classes, continuing with the standard mathematics curriculum. Prior to and following the intervention, a pre-test and post-test design was administered. The data were analyzed using Analysis of Covariance to ascertain the effect of the intervention. Results revealed that the students in the experimental group exhibited significant improvements in spatial visualization skills and mathematics performance (including both spatial and non-spatial competencies), compared to the control group. Importantly, the findings provide evidence that the transfer effects of the spatialized curriculum occur differentially for students of varying ability levels. The findings highlight the importance of spatial intervention in enhancing mathematics performance and spatial visualization skills. The study suggests potential avenues for a paradigm shift in mathematics education that recognizes and embraces the profound influence of spatial thinking.

**Keywords** Spatializing mathematics · Geometry · Mathematics performance · Spatial visualization skills

## 1 Introduction

Longstanding research in spatial reasoning, both within and beyond mathematics education, has emphasized its crucial role in teaching and learning mathematics, encompassing concept development to problem-solving (Battista, 2007; Clements, 2004; Clements & Sarama, 2020; DeSutter & Stieff, 2017; Gardner, 1984; Joag, 2016; Newcombe, 2010). However, translating these research findings into curricular initiatives has been relatively limited compared to the formal geometry curriculum. As the mathematics curriculum progresses through grade levels, spatial thinking tends to receive decreasing emphasis in curricular documents (Hawes et al., 2023).

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Recognizing this, researchers in mathematics education have independently pursued efforts to boost spatial reasoning through experimental research projects (Bruce & Hawes, 2015; Cheng & Mix, 2014; Hawes et al., 2015; Lowrie et al., 2017; Mulligan et al., 2020). These initiatives have sparked the idea of enriching students' spatial experiences within the mathematics curriculum (e.g., Davis and The Spatial Reasoning Study Group, 2015; Lowrie et al., 2018), thereby advancing the process of "spatializing" the mathematics curriculum. Newcombe (2017) argues not only for a reconsideration of curriculum design but also for improved teaching methods to enhance scientific and mathematical understanding through spatial skills. Similarly, Hawes et al. (2023) advocate for incorporating mathematical content that inherently involves spatial aspects or that can be approached using spatial strategies as a natural foundation for developing students' mathematical understanding and spatial reasoning skills.

Drawing on the spatial thinking conceptualization by The National Research Council (2006), we explore "spatializing the curriculum" as a means to provide customized curricular support for students in articulating space concepts, representation tools, and reasoning processes. Space concepts in the mathematics curriculum encompass attributes like position, location, and dimensionality (2D and 3D). Representation tools include elements such as drawings, diagrams, and models applicable in static and dynamic settings. Reasoning processes involve deriving inferences from given spatial information. By way of example, when teaching the surface area of a cube (space concept), students may actively engage in physically deconstructing the 3D cube into its 2D nets and vice versa. After performing these physical transformations through paper-based activities (representation tools), they later apply these skills interchangeably between 2 and 3D spaces through mental imagery. In this example, the reasoning process may involve making inferences about a specific 3D face of the cube and its position in the 2D net. These mental manipulations, arising from physical exploration, aim to enhance students' spatial visualization ability.

Efforts, both small-scale (e.g., Bruce & Hawes, 2015; Hawes et al., 2017; Patahuddin et al., 2018, 2020) and large-scale (e.g., Harris et al., 2023; Lowrie et al., 2018), have been made to integrate spatial elements into the curriculum. However, these attempts often served as momentary add-on interventions. Typically using an Experimental-Control design, previous studies have demonstrated that customized spatial activities could improve students' spatial ability (Hawes et al., 2017; Lowrie et al., 2017). In addition, the spatial activities improved students' performance in mathematics achievement with respect to missing term problems (e.g.,  $4 + \_ = 11$ ) (Cheng & Mix, 2014), symbolic comparisons (i.e., comparing pairs of Hindu-Arabic numerals) (Hawes et al., 2017), and in geometry tasks and word problems (Lowrie et al., 2019). Consequently, this suggests that spatial skills are malleable and can be developed with appropriate support even over a brief period.

This argument raises questions about the adequacy of spatial experiences in the current mathematics curriculum. Moreover, there is limited understanding of the transfer effects of spatial intervention programs onto mathematics understanding, i.e., how customized spatial activities lead to enhanced spatial reasoning and how this in turn supports mathematics achievement. We consider the transfer of learning to be in situations where the learning that takes place in one context (i.e., spatialized mathematics curriculum) influences performance in another context (i.e., mathematics assessment) (Perkins & Salomon, 1992). The current study aims to investigate the transfer effects of a 5-week spatialized intervention in a relatively large student sample. Empirical evidence has established that spatial ability comprises many dimensions like spatial visualization, mental rotation, and spatial orientation (Sorby, 1999). This study concentrates exclusively on spatial visualization as a recent spatial training meta-analysis has identified spatial visualization as a critical mechanism for supporting spatial transfer to mathematics (Hawes et al., 2022).

## 2 Spatial visualization and its relevance to the learning of mathematics

Spatial visualization refers to the manipulation of object figures into alternate spatial arrangements (Carroll, 1993; Lohman, 1993; McGee, 1979; Ramful et al., 2017). These spatial arrangements encompass actions such as mental folding (e.g., imagining the resulting shape when a net is folded to form a 3D shape), splitting or joining shapes, and envisioning how different parts of 2D shapes can assemble into a unified whole. This process may involve multi-step manipulations of spatial information, often of a complex nature (Linn & Petersen, 1985). Sorby (1999) identifies two main components of spatial visualization: (i) mental rotation (where the entire object is transformed through rotation) and (ii) mental transformation (where a part of an object undergoes some form of transformation).

We begin this section by identifying spatial visualization as a distinct type of spatial ability, seeking to outline its unique characteristics. Spatial visualization is a specific type of visualization related to spatial information. For example, while someone might mentally recreate the image of a tree, this differs from recreating or situating the tree's location in relation to surrounding objects in the mind's eye. Spatial visualization involves more intricate, dynamic manipulations (Kozhevnikov et al., 2002; Linn & Petersen, 1985). Although visualization ability might suffice for basic pictorial interpretations, spatial visualization is crucial for advanced spatial tasks within the mathematics curriculum. These tasks demand a nuanced understanding that surpasses generating images and involves manipulating multi-step spatial information (Arcavi, 2003; Clements & Battista, 1992).

Spatial visualization, as posited by cognitive psychologists and educational researchers, encompasses the manipulation and retention of sequential spatial information. This process involves coordinating various spatial configurations at successive stages to achieve specific objectives, such as constructing new spatial formations or selecting and matching given configurations. Carroll (1974) supports this view, emphasizing that spatial visualization entails serial operations. This definition aligns with cognitive psychology's perspective, while introducing an operational aspect beneficial for mathematics educators.

From a curriculum perspective, spatial visualization has been identified as integral to various mathematical concepts (see Ministry of Education Canada, 2005; Australian Curriculum Assessment and Reporting Authority [ACARA], 2010; National Research Council, 2006). For example, it plays a crucial role in understanding and working with symmetry, reflection, patterns, 2D and 3D shapes, and their relationships. Additionally, it is vital for grasping concepts related to the surface area, volume, and part-whole relationships of spatial configurations (Patahuddin, et al., 2019). These applications require the ability to store and manipulate spatial information, a skill central to the development of mathematical understanding and proficiency. Previous research indicates the positive influence of spatial interventions on both spatial and mathematical abilities (e.g., Gilligan et al., 2019; Lowrie et al., 2017, 2019). Nonetheless, prior studies predominantly feature Western contexts, leaving a study gap in non-Western contexts (Uttal et al., 2013). This study fills that gap by specifically focusing on an Indonesian context and forms the basis for the proposed research questions:

1. To what extent does a spatialized mathematics intervention program enhance students' spatial visualization ability?
2. Moreover, does the spatialized mathematics intervention program offer benefits for spatial skills across different student mathematics ability levels?

3. In what way does the spatialized mathematics intervention improve students' overall mathematics performance?
4. Additionally, does the spatialized mathematics intervention benefit mathematics performance for students of different mathematics ability levels?

By addressing these questions, the study aims to provide insights into the effectiveness of a spatial intervention for improving mathematical competencies and to understand the extent of their applicability across diverse student groups within underprivileged communities, particularly in the context of Indonesian education. Furthermore, there exists a notable gap in the literature concerning spatial reasoning within mathematics education, especially regarding evidence supporting transfer effects (e.g., Adams et al., 2023; Gagnier & Fisher, 2020; Resnick et al., 2020), thereby underlining the necessity of this research.

### 3 Context of the present study

Indonesian students consistently lag behind their international counterparts in mathematics performance, as evident from their poor performance in high-stakes tests like the Programme for International Student Assessment (PISA) and Trends in International Mathematics and Science Study (TIMSS). Only a small percentage of Indonesian students achieve minimum proficiency levels in mathematics (The Organisation for Economic Co-operation and Development [OECD], 2019). Moreover, specific regions, such as the West Nusa Tenggara province, consistently exhibit low academic performance, particularly in mathematics, positioning them among the lowest-performing regions in the country (The Indonesian Ministry of Education and Culture [MoEC], 2018). It is also worth noting that West Nusa Tenggara has the sixth-lowest Human Development Index (HDI) out of the 34 provinces in Indonesia. The lower HDI in West Nusa Tenggara is associated with lower life expectancy, educational duration, and income levels compared to other regions—factors that likely influence educational outcomes (Statistics Indonesia, 2020). Nationally, middle school students in West Nusa Tenggara province rank as the second lowest among all 34 provinces in Indonesia in terms of overall subjects assessed in the standardized national exam (2017/2018), and specific to mathematics, they rank as the third lowest (The Indonesian Ministry of Education and Culture [MoEC], 2018).

This research, which formed part of the doctoral research of the lead author, addressed the ongoing challenge of Indonesian students' underperformance in mathematics relative to their global counterparts. Concentrating on the West Nusa Tenggara Province, known for persistently low mathematics achievement, the study evaluates the efficacy of a spatialized mathematics intervention program with respect to its impact on students' mathematics performance and spatial visualization ability.

### 4 Method

The research design followed a randomized experimental-control approach, comparing the effectiveness of the spatialized mathematics intervention with regular mathematics instruction.

## 4.1 Participants

The study involved 10 schools located across Lombok Island in West Nusa Tenggara, Indonesia. The schools represented a range of mathematics performances within the West Nusa Tenggara region, from relatively high (Mataram City District = 2nd) to low (West Lombok District = 7th). Schools were randomly assigned to either the experimental group (six schools) or the control group (four schools).

The study's emphasis on the adolescent age group, specifically middle school students aged between 13 and 15 years, is strategically aligned with the critical period of increased complexity in mathematics education (Schielack & Seeley, 2010). It is during these years that students encounter more advanced mathematical concepts and transition to using textbooks and technology-based resources. Incorporating spatial skills during this stage is particularly advantageous, as spatial visualization proves most helpful when encountering novel and complex content (Lowrie & Kay, 2001).

Teachers were recruited through regional school supervisors (superintendents) in three districts: West Lombok, East Lombok, and Mataram City. The superintendents reached out to local schools to find volunteers, and interested principals were provided with detailed information about the study. A total of 10 teachers participated, with one teacher in each school, six in the experimental group, and four in the control group. In the experimental group, teachers implemented the spatialized mathematics intervention within their mathematics classes, while teachers in the control group conducted regular mathematics classes. All participating teachers provided written consent to participate in the program.

The study encompassed 19 classes consisting of Grade 8 students from the ten schools, with each school having one or two classrooms. The experimental group comprised twelve classrooms from six schools, and the control group comprised seven classrooms from four schools. To ensure the independence of the two treatment groups, classrooms within the same school were assigned to the same group, either all in the experimental group or all in the control group. The data analysis followed a 3:2 ratio between the experimental and control groups.

A total of 407 Grade 8 students (mean age = 13.79, S.D. = 0.78) participated in the study. The experimental group consisted of 260 students, while the control group comprised 147 students. Written consent was obtained from all parents or guardians of the participating students.

## 4.2 Experimental design

### 4.2.1 Intervention group

**Teacher workshop: collaborative lesson planning and micro-teaching** In a focused 4-day workshop (approximately 20 h), six in-service teachers were prepared to implement the "Spatializing Mathematics" intervention. The session was led by the lead author. Teachers were provided with extensive guidelines and a comprehensive nine-lesson plan. These lessons were previously developed in collaboration with Indonesian mathematics educators. The workshop aimed to acquaint teachers with the project, their roles in data collection, and the integration of spatial skills in mathematics.

On day 1, the project was introduced and spatial skills within Indonesia's mathematics curriculum were identified and discussed; demonstrations of spatial visualization techniques were also explored. The second day featured video analyses of Indonesian geometry classes, highlighting examples of spatial visualization in the curriculum, and assisting teachers in understanding its practical application. On the third and fourth days, teachers, guided by the lead author, thoroughly reviewed the lesson plans, focusing on content, terminology, and structure. Their feedback prompted minor adjustments to enhance practicality and applicability.

The study prominently featured microteaching trials, wherein teachers led short, focused sessions on spatial visualization, using the researchers' lesson plans, to small-group interactions with peers. Immediate feedback followed each session, enhancing pedagogical skills and allowing for the adaptation of lessons to specific classroom needs. The microteaching method has been shown to be an effective way to help pre-service teachers in learning and reflect upon effective teaching practices (Bell, 2007). The workshop concluded with teachers being equipped with revised, practical lesson plans and tangible teaching aids, ensuring effective implementation and maintaining the quality of the intervention.

**Assessing fidelity of implementation** It was important that teachers implemented the intervention according to the material provided, so a fidelity form was devised that focused on context, content, and competence. The context section gathered information about the class setup, the teachers' planning and preparation for each session. The content section evaluated whether the lesson plan was implemented as intended and whether teachers felt more effective in their teaching approach. The competence section examined how teachers engaged with students in the classroom and how they responded to student questions and contributions. Observations were conducted by the research team during class sessions to monitor teacher adherence to each section.

To ensure fidelity in implementation, each lesson was monitored through observations and video recordings. Six research assistants were assigned to observe 12 classrooms, with each assistant being responsible for two classrooms. The observations used both static and dynamic video recorders, and the fidelity form was completed for each lesson. The results indicated that all teachers adhered to the lesson plans, following the sequences and activities outlined. Additionally, it was observed that teachers effectively delivered the designated mathematical tasks during the lessons.

**Spatializing mathematics intervention** The Spatializing Mathematics intervention incorporated explicit spatial visualization (ESV) activities into the mathematics curriculum, specifically focusing on the concepts of surface area and volume of prisms and pyramids within the geometry strand. It diverged from traditional drill-and-practice methods, opting for a more holistic approach that enriched the curriculum without compromising the integrity of the mathematical discipline.

The ESV intervention began with an introductory lesson on various types of prisms and pyramids. This lesson used a tactile-based game called "Bag of Tricks" (Patahuddin et al., 2015) and finished with a worksheet about the characteristics and elements of prisms and pyramids. In the bag of tricks game, a model of a 3D object was placed in an opaque bag and one student, with closed eyes handled the 3D model. With this activity, students were expected to create a visual prototype that directed their thinking (Clements, 2004) so that they could identify the characteristics of the 3D object. The second lesson focused on the concept of the surface area of a cube, and engaged students in activities where they

mentally unpacked a cube (3D) into cube nets (2D) and then drew what they visualized on paper. The third lesson involved constructing prisms by using multiple cube units through cube games. In this activity, students constructed cube blocks based on the instruction given. Students took turns as the instructor or as the one who received the instruction. Note that both situations required students to visualize either the given instructions or when giving instructions.

**Table 1** Structure of the spatializing mathematics intervention




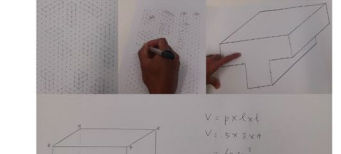
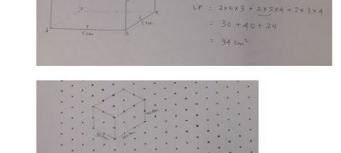
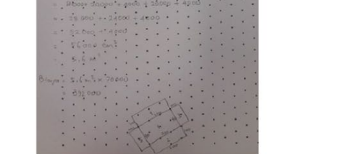
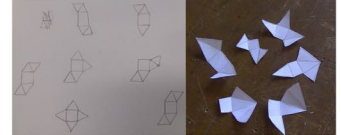
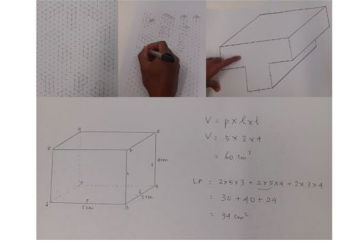
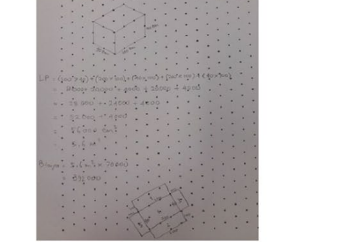
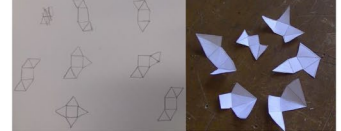
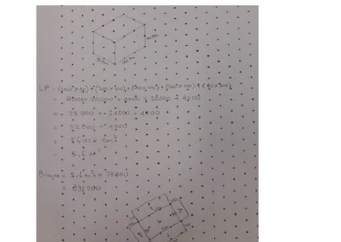
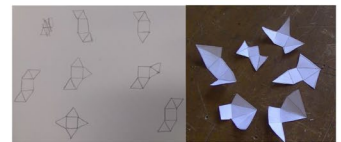
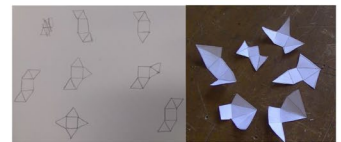
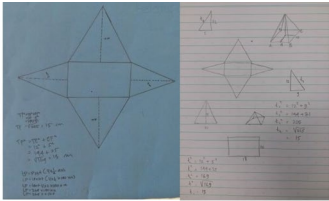
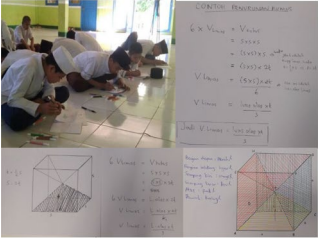
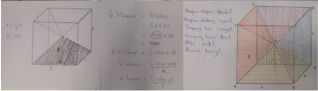
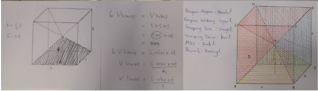


Lesson descriptions	Integration of spatial visualization	Example of activities/students' work
<p><b>Lesson 1-Introducing prisms and pyramids</b> Introducing prism and pyramid through guessing shape game.</p>	<p>Visualizing/mentally constructing a prism/pyramid (numbers of vertex/edge/side required to form, for instance, a triangular prism).</p>	
<p>Learning the elements of 3D shapes (i.e., flat surface): vertex, edge, side, face diagonal, space diagonal, and cross-sections of the opposite edges.</p>	<p>Visualizing/mentally constructing face diagonals, space diagonals and cross-sections of the opposite edges.</p>	
<p><b>Lesson 2-Drawing a cube net</b> Unpacking a cube into a net.</p>	<p>Mentally decomposing a cube (3D) into a cube net (2D).</p>	
<p>Discovering other formations of a cube net.</p>	<p>Visualizing/ mentally constructing the different formations of a cube net, then drawing it on a paper.</p>	
<p>Finding a cube net pattern.</p>	<p>none</p>	
<p><b>Lesson 3-Construct prisms from the cube units</b> Drawing different forms of prisms (constructed from four cube units) on isometric paper.</p>	<p>Visualizing/mentally constructing the prism to be formed from four cube units then drawing it on isometric paper.</p>	
<p>Game of cube construction.</p>	<p>Visualizing the construction that was directed by other students to form a prism.</p>	
<p><b>Lesson 4-Constructing different prisms with the same volume but different surface area or the same surface area but different volume</b> Constructing various prisms with the same surface area and different volume (and vice versa).</p>	<p>Visualizing various possible formation of prisms with the same surface area but in different volume or vice versa (i.e., students calculate the surface areas and volumes of the formations to check that they have the same area and different volume or vice versa)</p>	
<p>Calculating the surface area and volume of a prism (with measurement).</p>	<p>Visualizing/decomposing all surface parts of the cuboid to calculate its surface area and volume.</p>	
<p>Calculating the surface area and volume of a composite prism (without measurement).</p>	<p>Describing the steps to find the total surface area and volume of the composite prism (utilizing spatial languages)</p>	
<p><b>Lesson 5-Designing a glass aquarium in a prism shape</b> Designing a glass aquarium that consists of a 3D drawing of the aquarium, the net of the aquarium and its detailed measurements to make the best construction (minimum cost with the largest possible volume).</p>	<p>Composing a prism by drawing it on isometric paper; Decomposing the prism into a 2D net; Calculating the total surface area and volume of the prism designed.</p>	
<p><b>Lesson 6-Drawing a pyramid net</b> Decomposing a pyramid into a net; discovering another formation for a net of a square pyramid.</p>	<p>Mentally decomposing a square pyramid (3D) into a net of a square pyramid (2D); Visualizing/mentally constructing the different forms of a square pyramid net, then draw it on a paper.</p>	
<p>Finding a square pyramid net pattern.</p>	<p>none</p>	

Table 1 (continued)

Lesson descriptions	Integration of spatial visualization	Example of activities/students' work
<b>Lesson 7-Finding the surface area of the pyramids</b> Drawing a net of the rectangular pyramid (in real size).	Decomposing a rectangular pyramid into the 2D net; Visualizing a right-angle triangle to get the slant height of the triangular faces then calculate it by using Pythagorean theorem; Calculating the total surface area by adding the area of 4 triangles and rectangular base.	
<b>Lesson 8-Finding the volume of the pyramid through cube construction</b> Fitting square pyramids into a cube.	Composing square pyramids to form a cube (mentally/visualizing how many square pyramids can fit to form a cube).	
Shading the pyramids inside the cube.	Shading the pyramids with different colours; exchange with a friend then identify the colours and position (front, back, left & right side, top, down) of pyramids within the cube.	
Looking at the relationship within the volume of a cube and pyramids.	Conservation of volume: If the height of a cube is twice the height of a pyramid and the area of one side of the cube is equal to the base area of the pyramid, then the volume of the cube is equal to six times the volume of the pyramid. Therefore, the volume of the pyramid = $1/3$ of the base area of a pyramid times its height	
<b>Lesson 9-Building a triangular pyramid (tetrahedron)</b> Tetrahedron puzzle as opening activities. Building the tetrahedron from straws.	Mentally manipulating two pentahedrons to form it into a tetrahedron. Mentally constructing a tetrahedron from two equilateral triangles made from straws and one additional straw. Then try to check it by building it physically, name it as Tetrahedron 1.	
Calculating the surface area and volume of a tetrahedron. Building a bigger Tetrahedron.	Calculate the surface area and volume of Tetrahedron 1. Building a bigger tetrahedron from 4x Tetrahedron 1 then calculating its surface area and volume.	

The fourth lesson entailed drawing activities and calculating the surface area and volume of prisms. Students were tasked with drawing different prisms using a specific number of cube blocks to showcase prisms with equal volumes but varying total surface areas, and vice versa. In this activity, students were not equipped with concrete cube blocks. The absence of concrete material (e.g., cube blocks) required students to perform mental constructions of the cube blocks which they then drew on isometric paper. In addition to mental constructions and drawing, students were required to calculate the surface area and volume of each drawing they made. The fifth lesson involved a group project where students applied their knowledge of surface area and volume to design an aquarium, aiming to create the best possible design for the given task.

Lessons six to nine concentrated on pyramids, examining their surface area and volume. Activities included: mentally decomposing a square pyramid (3D) into its possible nets (2D) and then drawing the nets on paper; calculating the surface area of a pyramid by adding the area of the square base and four triangles; and exploring the concept of volume whereby students were instructed to visualize the number of square pyramids that could fit together to form a cube when the base area of the square pyramid was equal to the area of one side of the cube. The final lesson incorporated interactive games for constructing a tetrahedron, thereby enabling students to apply learned formulas in a hands-on manner.

Overall, the ESV intervention actively engaged students through a series of thoughtfully designed lessons that stimulated spatial visualization skills. The approach combined games, collaborative activities, and practical applications, providing a comprehensive learning



experience. The tasks that did not start with concrete manipulatives required mental construction and drawing 3D shapes on isometric paper. The use of such spatial tools provided opportunities for students to exercise their spatial visualization ability (Ben-Chaim et al., 1985; Newcombe, 2017). The structure of these lessons is illustrated in Table 1.

#### 4.2.2 Control group

The control group paralleled the experimental group, with both focusing on the same mathematics topics: surface area and volume of three-dimensional objects. The key distinction between these groups lies in the teaching methodology. Unlike the experimental group, which incorporated explicit spatial visualization (ESV) training into the curriculum, the control group followed a more common approach in Indonesia.

Due to resource limitations, direct classroom observations were not conducted for the control group. However, the teachers in this group reported adhering to the standard curriculum as prescribed by the Ministry of Education, specifically following the guidelines in the mathematics textbook (As'ari et al., 2014). This textbook served as their primary instructional resource.

The teachers in the control group used physical models of various prisms only for demonstrations, a common practice in mathematics education in Indonesia. Importantly, the total instructional time for the control group was kept consistent with that of the experimental group, ensuring a fair comparison in terms of exposure to the subject matter.

The specific content taught to the control group is outlined in Table 2. This provides a clear and detailed account of the common teaching approach used, offering a basis for comparing the effectiveness of the ESV intervention against conventional teaching methods. The inclusion of this control group is crucial for assessing the impact and potential advantages of the innovative teaching strategies employed in the experimental group.

It is worth mentioning that teachers in the control group did not receive any workshops or professional development specifically related to spatial visualization, ensuring that the business-as-usual class remained unaffected by the lesson designs employed in the ESV intervention.

**Table 2** Topics for the unit 'flat surface 3D objects' in the business-as-usual maths class

Lesson	Topics
Lesson 1	Determining the surface area of cubes and cuboids
Lesson 2	Determining the surface area of prisms
Lesson 3	Determining the surface area of pyramids
Lesson 4	Determining the volume of cubes and cuboids
Lesson 5	Determining the volume of prisms
Lesson 6	Determining the volume of pyramids
Lesson 7	Determining the surface area of composite 3D objects
Lesson 8	Determining the volume of composite 3D objects
Lesson 9	Estimating the surface area and volume of composite 3D objects

### 4.3 Measurement of students' spatial visualization skills and mathematics performance

Two key instruments were employed: the SRI-Spatial Visualization (SRI-SV) and the Mathematics Test (MT). The SRI-SV was used to directly assess the spatial visualization skills that were targeted through the intervention program. The MT was included as a measure of the transfer of spatial visualization skills to broader mathematics, with the measure including explicitly spatial and not explicitly spatial content.

The SRI-SV, a component of the larger SRI developed by Ramful et al. (2017), is tailored for students aged 11–13 years. This subtest includes 10 items designed to evaluate abilities such as mental unpacking, object slicing, and paper folding. For the study's Indonesian context, the SRI-SV was translated into Indonesian, with appropriate modifications to symbols and names. Figure 1 illustrates examples from both the original and the Indonesian-adapted versions of the instrument.

The MT was designed to assess the mathematics performance of students. It aligns with the curriculum standards for Indonesian Grades 7–8. The test featured a combination of questions from the Indonesian national exam (Ujian Nasional; 67%) and based on the Australian National Assessment Program-Literacy and Numeracy (33%), following guidelines from the Australian Curriculum Assessment and Reporting Authority [ACARA] (2010).

The MT was divided into two sections: items requiring ESV (M-ESV) and items devoid of ESV (M-NSV). The M-ESV section constituted 43% of the test, with 13 items focused on topics such as surface area and volume. The M-NSV section made up the remaining 57%, with 17 items covering areas such as numbers, algebra, and statistics. To illustrate the diversity of the test items, Figs. 2 and 3 provide examples of items that require ESV skills and those that do not, respectively.

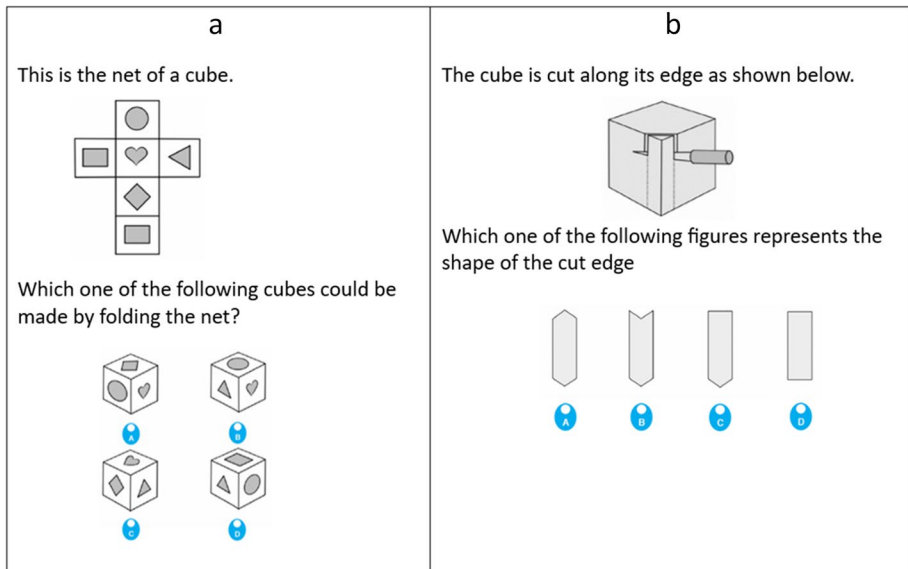


Fig. 1 Example of spatial reasoning test items

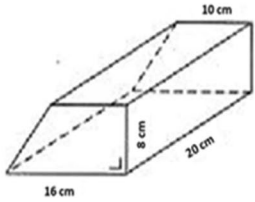
a	b
<p>A square pyramid has a length of the base edge 12 cm and slant height 9 cm. If 10 m of wire is provided, how many square pyramid frames can be made?</p> <p>a. 8 square pyramid frames b. 9 square pyramid frames c. 10 square pyramid frames d. 11 square pyramid frames</p>	<p>See the following right trapezoidal prisms. The surface area of this shape is ...</p> <p>a. 170 cm<sup>2</sup> b. 800 cm<sup>2</sup> c. 1088 cm<sup>2</sup> d. 1152 cm<sup>2</sup></p> 

Fig. 2 Example of items involving explicit spatial visualization

a	b
<p>A car covers the distance from city A to city B within 1.2 hours with the speed of 80km/hour. To make the distance reachable within 60 minutes, then the speed needs to reach ...</p> <p>a. 96 km/jam b. 72 km/jam c. 66 km/jam d. 62 km/jam</p>	<p>What is the solution for the following equation?</p> $\frac{1}{3}(x + 5) = \frac{1}{2}(2x - 1)$ <p>a. <math>-\frac{13}{4}</math> b. <math>\frac{7}{4}</math> c. <math>\frac{7}{4}</math> d. <math>\frac{13}{4}</math></p>

Fig. 3 Example of items that do not involve explicit spatial visualization

To ensure that both instruments (the translated SRI-SV and MT) were clear and suitably challenging for Grade 8 students in Indonesia, they were validated by local mathematics experts and experienced senior mathematics teachers. For scoring, each item in both tests was straightforwardly marked: one point for a correct answer and zero for an incorrect response.

#### 4.4 Procedure

Initially, students were given the SRI-SV and MT 3 days before the classroom intervention. About a week following this intervention, a post-test was conducted for all students using the same assessment tools as the pre-test.

Both groups studied the topic of surface area and volume of 3D objects for 5 weeks. This included twice-weekly double periods (80 min each), totaling approximately 720 min of instruction. The primary difference between the groups was their curriculum: the experimental group followed the spatialized mathematics curriculum (outlined in Table 1), while the control group adhered to the MOEC textbook curriculum (detailed in Table 2).

## 4.5 Data analysis

The study employed descriptive statistical analysis for pre-test and post-test on each item within the SRI-SV and MT across both the experimental and control groups. Comparative analysis of pre- and post-test performance was undertaken using a *t*-test, stratified by different group categories. To ascertain the effectiveness of the spatializing mathematics intervention, an analysis of covariance was conducted on the post-test scores, with the inclusion of pre-test scores as covariates. This approach, in line with the methodologies outlined by Field (2009) and Maxwell & Delaney (2004), was implemented to control for any potential systematic biases between the two groups. Furthermore, a detailed examination of the data was conducted based on the varying initial ability levels of participants, aiming to provide insights into how students of different academic proficiencies benefited from the ESV intervention.

## 5 Results

The results of the study are presented in two sections. The first section presents the effect of the intervention on spatial visualization skills. Pre-test and post-test differences were compared between the two groups for the SRI-SV and at an item level. Further analysis was conducted to assess the impact of the ESV intervention based on students' initial mathematics ability levels.

The subsequent section shifts the focus to the effectiveness of the intervention on mathematics performance relative to the business-as-usual control group. The MT results were separated into two categories for a more nuanced analysis: (i) items requiring ESV (M-ESV) and (ii) items devoid of ESV (M-NSV). This categorization aids in assessing the transfer effects of the intervention. Furthermore, an investigation into the impact of the intervention on M-ESV and M-NSV items was conducted, taking into account the students' mathematics ability levels as determined by the pre-test.

Before proceeding with the main analyses, preliminary *t*-tests were performed to assess any pre-existing differences between the experimental and control groups. These preliminary tests revealed no statistically significant differences between the groups in their pre-intervention spatial visualization scores ( $t(401) = -1.37, p = .17$ ), overall mathematics scores ( $t(401) = 1.02, p = .31$ ), scores in mathematics items involving ESV ( $t(401) = .48, p = .63$ ), and scores in mathematics items lacking ESV ( $t(401) = 1.13, p = .26$ ).

### 5.1 Pre- and post-intervention test results: spatial visualization

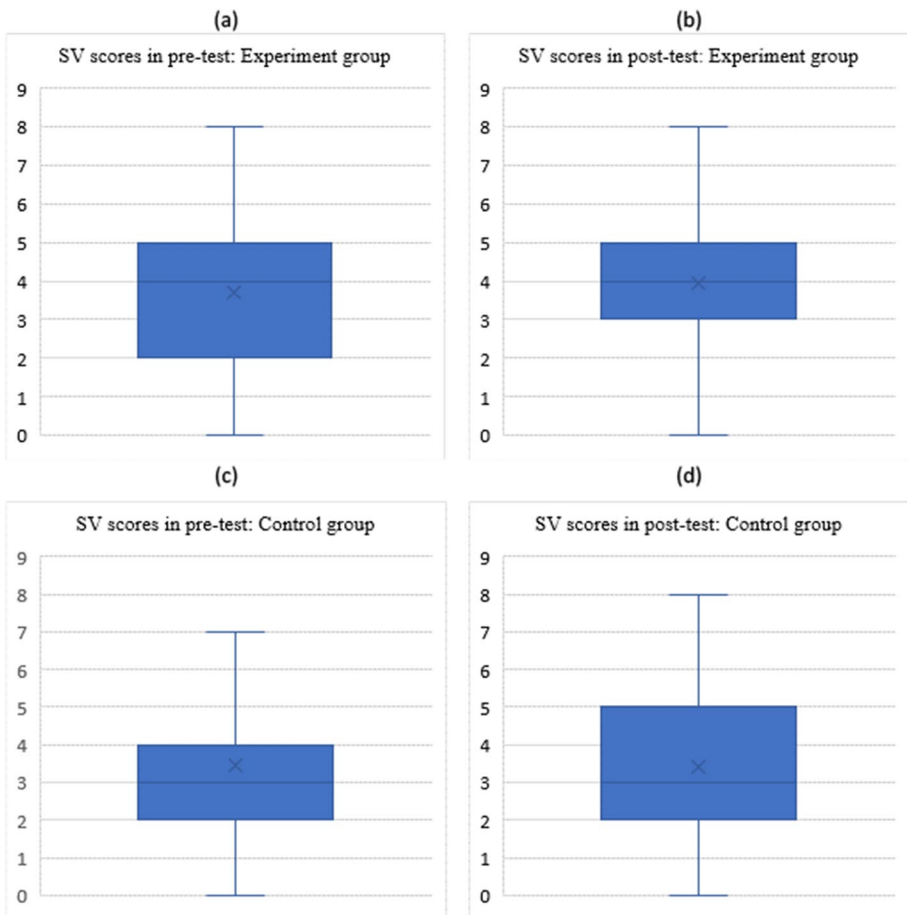
Table 3 illustrates the comparative performance of the experimental and control groups in the spatial visualization test, with the experimental group achieving higher scores in both pre- and post-test assessments. During the pre-intervention phase, the differences in scores between the two groups were not statistically significant. However, post-intervention, these differences became significant, with the experimental group showing notably higher scores.

Figure 4 depicts the distribution of the SRI-SV scores (after omitting four outliers) for both control and experimental groups. Notably, the skewness of the score distribution exhibited a change following the intervention, indicating a shift in performance metrics.

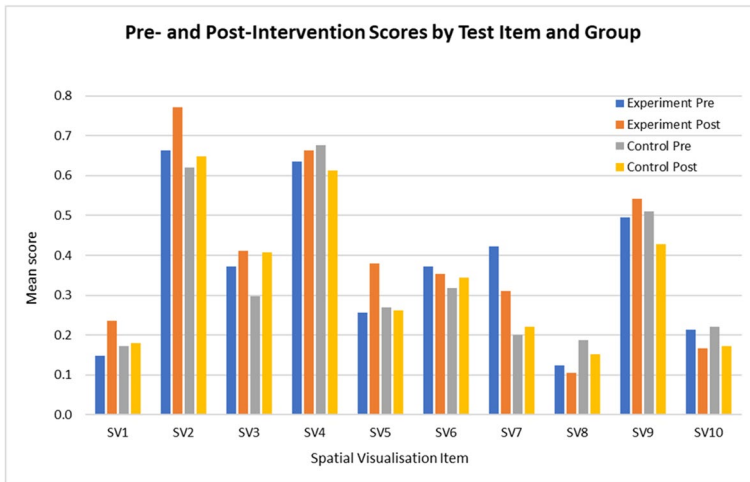
**Table 3** Descriptive statistics for SRI-SV scores across the entire sample

Treatment	Test	Mean	SD	Min	Max
Experimental group	Pre-Intervention	3.70	1.67	0	8
	Post-Intervention	3.94	1.84	0	8
Control group	Pre-Intervention	3.47	1.59	0	7
	Post-Intervention	3.43	1.84	0	8

Figure 5 presents the pre- and post-intervention mean scores for each spatial visualization test item, comparing the experimental and control groups. The experimental group showed an increase in mean scores from the pre-test to post-test. In contrast, the control group’s mean scores remained relatively stable, with a slight decrease from the pre-test to the post-test. Specifically, the experimental group demonstrated notable improvements in items SV1, SV2, and SV5. Conversely, the control group showed a decrease in performance in items SV4 and SV9. Some items, such as SV6 and SV10, showed minimal change in mean scores for both groups. Overall, the experimental group demonstrated



**Fig. 4** Students’ performance in spatial visualization test by treatment



**Fig. 5** Comparison of mean scores by test item and group, before and after the intervention

higher spatial visualization post-test scores compared to the control group following the business-as-usual mathematics class, and the difference in mean was significant,  $F(1,400) = 5.26, p = .022, d = 0.28$ .

To explore growth in spatial visualization skills, a two-way ANCOVA was performed. This analysis aimed to assess the impact of the intervention on different student performance levels within the treatment group. Student performance levels were determined based on their scores in the mathematics pre-test, used as a proxy for their mathematical ability. The students were classified into low, average, and high categories, depending on their scores being below the lower quartile, between the lower and upper quartiles, or above the upper quartile, respectively. Table 4 illustrates that students across all three categories—high, average, and low—benefited from the spatialized curriculum in the experimental group. However, the two-way ANCOVA revealed no statistically significant interaction between the treatment and the students' performance levels in relation to their spatial visualization scores, while controlling for the pre-test spatial visualization ( $F(2,396) = 0.398, p = 0.672$ ). Given the absence of a significant interaction between treatment and student performance levels, further analysis within these specific groups was deemed unnecessary. Instead, a paired t-test was conducted to examine the differences between pre-test and

**Table 4** Pre-and post-intervention mean spatial visualization scores for low, average, and high performing students

Treatment	Category	Pre-test <i>M (SD)</i>	Post-test <i>M (SD)</i>
Experimental	High ( $N = 78$ )	4.22 (1.53)	4.82 (1.39)
	Average ( $N = 94$ )	3.56 (1.71)	3.65 (1.92)
	Low ( $N = 86$ )	3.38 (1.67)	3.45 (1.85)
Control	High ( $N = 48$ )	3.48 (1.54)	4.04 (1.75)
	Average ( $N = 58$ )	3.60 (1.52)	3.38 (1.80)
	Low ( $N = 39$ )	3.26 (1.74)	2.74 (1.79)

post-test scores in spatial visualization. This approach provided a more direct assessment of the intervention's impact on spatial visualization skills across the student groups.

The paired *t*-test indicated a statistically significant difference between the pre- and post-test SRI-SV scores for high-performing students in both the experimental group ( $t(77) = -3.061, p = .003$ ) and control group ( $t(47) = -2.527, p = .015$ ). However, no significant differences were noted between the pre- and post-SRI-SV scores for the average-performing students in both the experimental group ( $t(93) = 1.938, p = .066$ ) and the control group ( $t(57) = 0.935, p = .354$ ). Similarly, no significant differences were observed between the pre- and post-test SRI-SV scores of the low-performing students in both the experimental group ( $t(85) = -0.332, p = .741$ ) and control groups ( $t(38) = 1.938, p = .060$ ). These findings suggest that the intervention's effectiveness in enhancing spatial visualization skills may be more pronounced among high-performing students in the context of this underprivileged community.

## 5.2 Pre- and post-intervention test results: mathematics

In this section, the findings addressing the second research question are presented, focusing on how the ESV intervention impacted students' overall mathematics performance and its effectiveness across different student mathematics ability levels.

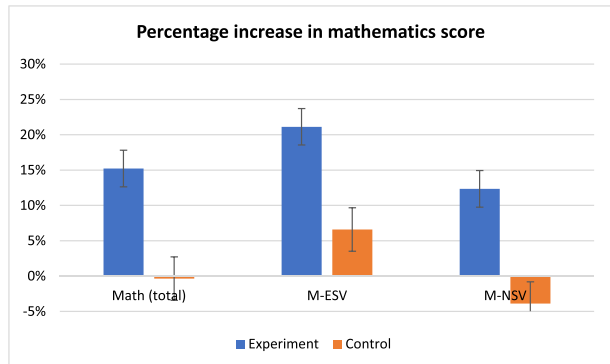
Table 5 shows the mean and SD of the scores in the MT as a whole separated by the test items that consist of ESV and NSV. The ANCOVA analysis result showed significant differences in mathematics performance after the intervention between the experimental and control groups, favoring the experimental group,  $F(1,400) = 18.20, p < .001, d = 0.30$ . The percentage gain in students' performance from the pre-test to the post-test is depicted in Fig. 6.

To further understand the growth in mathematics performance, a two-way MANCOVA was conducted. This analysis aimed to examine the impact of the intervention on different student performance levels within the treatment group, specifically focusing on mathematics items that explicitly involve spatial visualization (M-ESV) and those that do not (M-NSV). The results of the two-way MANCOVA showed that there was no statistically significant interaction between the treatment and students' performance levels in terms of their scores on both M-ESV and M-NSV items, even after controlling for the pre-test mathematics scores ( $F(4,790) = 0.416, p = 0.797$ ). Consequently, further analysis within these combined groups was deemed unnecessary. Instead, separate analyses were conducted on students' ESV and NSV scores, as detailed in the subsequent sections. This approach allows for a more nuanced understanding of the intervention's effects on distinct aspects of mathematics performance.

**Table 5** Means and standard deviations for measures categorized by treatment

Treatment	Measure	Pre-test <i>M</i> (SD)	Post-test <i>M</i> (SD)
Experiment	Mathematics (total)	10.45 (4.05)	12.04 (4.03)
	M-ESV	3.55 (1.75)	4.30 (1.85)
	M-NSV	6.89 (2.90)	7.74 (2.81)
Control	Mathematics (total)	10.86 (3.74)	10.82 (4.03)
	M-ESV	3.64 (1.72)	3.88 (1.80)
	M-NSV	7.22 (2.64)	6.94 (2.81)

**Fig. 6** Percentage increase for the experimental and control groups in the mathematics test



### 5.2.1 Items with spatial visualization embedded within mathematics (M-ESV)

The ANCOVA analysis indicated a significant performance improvement in the experimental group compared to the control group on the M-ESV items, with  $F(1, 400)=6.712$ ,  $p<0.01$ , and a small effect size ( $d=0.23$ ). Furthermore, when categorizing students by mathematics ability level (high, average, and low), the analysis revealed near significant differences for high-performing students between the experimental and control groups,  $F(1, 123)=3.576$ ,  $p=0.06$ , and for average-performing students,  $F(1, 149)=3.666$ ,  $p=0.06$ . However, among low-performing students, no significant difference was observed between the groups,  $F(1, 122)=0.976$ ,  $p=0.325$ .

### 5.2.2 Items with no explicit spatial visualization embedded within mathematics (M-NSV)

For the M-NSV items, the ANCOVA analysis revealed a significant advantage for the experimental group compared with the control group,  $F(1,400) = 15.82$ ,  $p < .001$ ,  $d = 0.28$ . Similar to the previous analysis, students were categorized by their mathematical ability level. Significant differences were observed between the experimental and control groups among high-performing students,  $F(1, 123) = 10.382$ ,  $p = 0.002$ ,  $d = 0.56$ , and average-performing students,  $F(1, 149) = 4.559$ ,  $p = 0.034$ ,  $d = 0.37$ . However, for low-performing students, no significant difference was found between the groups,  $F(1, 122) = 1.532$ ,  $p = 0.218$ .

## 6 Discussion

### 6.1 Spatializing mathematics: impact on grade 8 students

This study examined the impact of a spatialized mathematics intervention over 5 weeks on the spatial visualization skills and mathematics performance of Grade 8 students in a disadvantaged community in West Nusa Tenggara, Indonesia. Drawing upon theories of spatial visualization (Carroll, 1993; Lohman, 1993; McGee, 1979; Ramful et al., 2017), the aim of the intervention was to investigate the transfer effects of spatialized mathematics curriculum on student learning. Consistent with findings by Hawes et al. (2017), Battista



et al. (2018), Lowrie et al. (2019), Sorby & Veurink (2019), and Lowrie & Logan (2023), our results indicated significant improvements in both spatial visualization skills and mathematics performance in the experimental group compared to the control group, which followed the traditional curriculum.

The enhanced performance in spatial visualization and mathematics in the experimental group is attributed to their engagement with spatialized mathematics activities, as suggested by Sorby (2009). This aligns with the idea that direct experience with spatial tasks is crucial for developing spatial competencies. For instance, working with nets of 3D objects likely facilitated understanding of surface area problems (refer to the range of activities in Table 1), echoing the findings of other researchers (e.g., Hawes et al., 2017; Lowrie et al., 2019).

The study also highlighted a notable trend; namely, that students with higher mathematical abilities improved in spatial visualization skills more significantly than their peers. This observation aligns with the symbiotic relationship between mathematical ability and spatial visualization as suggested by Harris et al. (2023). Intriguingly, this improvement was not exclusive to the spatializing curriculum, as high-achieving students in the control group also demonstrated enhanced spatial visualization skills, albeit without exposure to the spatialized curriculum. This suggests that high-ability students might potentially benefit from any educational environment in the development of their spatial skills, including traditional learning methods. Such an insight underscores the need to consider inherent student capabilities when assessing the effectiveness of specific pedagogical approaches.

Regarding overall mathematics performance, the experimental group outperformed the control group in mathematical tasks explicitly requiring spatial visualization (M-ESV) and in those that did not (M-NSV). This suggests that the spatialized curriculum facilitated learning in spatially oriented tasks and other areas of mathematics, such as symbolic algebra and numerical statistics. However, the progress varied across different ability levels, with high and average-ability students showing more significant improvement, a finding consistent with Adams et al. (2023).

The mechanism behind the transfer from spatialized learning to non-spatial mathematics tasks remains a subject for further investigation. This study's findings align with Cheng & Mix (2014), Hawes et al. (2017), and Lowrie & Logan (2023), indicating that transfer is more likely when learning methods and approaches are similar to the tasks in the transfer setting. This implies that understanding the task, developing strategies, and visualization are crucial in solving both M-ESV and M-NSV tasks, rather than mere reliance on formulaic approaches. In this sense, students might utilize visualization strategies or approaches to understand the task rather than (only) searching for a formula that matches the task or question.

## 6.2 The role of spatializing the mathematics curriculum

This study makes significant contributions to the spatializing mathematics curriculum in three primary areas. Firstly, it introduces a novel teaching approach by embedding spatial visualization activities directly into mathematics lessons. This method contrasts with previous studies, such as those by Freina et al. (2017) and Hawes et al. (2015), which treated spatial training as a supplementary component. Our approach enriches the student experience by integrating spatially rich activities into the core curriculum, enhancing their learning without disrupting the established curriculum.

Secondly, this research is pioneering in examining the efficacy of a spatialized mathematics intervention in a non-Western context, employing a randomized controlled design. Although previous studies like those by Bruce & Hawes (2015) and Hawes et al. (2017) focused on

spatializing the curriculum in underprivileged Canadian communities, our study extends this concept to disadvantaged communities, demonstrating substantial improvements in mathematics for high- and average-achieving students. It is important to note that this was not the case with low-achieving students. Consequently, more work needs to be done on intervention programs in these highly disadvantaged communities, so that such programs do not leave this cohort even further behind. To this point, the efficacy of the program will be further enhanced with additional support for those students who are most vulnerable mathematically, possibly through further implementation support for the classroom teacher.

Thirdly, this study offers new advances in how interventions can be designed in school settings. This is the first study to include both spatial skills (for student learning) and spatial tools (for teacher presentation of materials) in the development of an intervention program. This dual approach to intervention design provided an approach to spatializing the curriculum (focusing on spatial visualization skills) while encouraging teachers to implement the program to embody and represent the materials in overtly spatial ways.

## 7 Conclusion, limitations, and future directions

This study contributes significantly to the expanding research on the benefits of spatializing the mathematics curriculum, standing out as one of the few to integrate spatialized content in mathematics instruction (i.e., Bruce & Hawes, 2015; Harris et al., 2023; Hawes et al., 2017; Lowrie et al., 2018; Patahuddin et al., 2018, 2020) and evaluate its effect across various mathematics topics. It underscores the significance of students' mathematical ability levels as a key factor in the transfer of learning.

As evidence for the effectiveness of spatialized curriculum in contexts such as disadvantaged communities where mathematics performance is below national and international benchmarks, this program's educational implications are substantial. The development of spatial skills offers a unique opportunity to significantly enhance student learning. However, it is crucial to acknowledge certain limitations in this study. One such design limitation related to the researcher's presence in the classroom. Each lesson in the experimental group was monitored through observations and video recordings to ensure the fidelity of implementation. By contrast, the observations and recordings were absent from the control group. Fidelity was limited to teachers documenting their practices via textbook guidelines as their primary instructional resource, which adhered to the standard curriculum in Indonesia (As'ari et al., 2014).

This investigation has added to the recent research on employing spatial reasoning interventions to support transfer to mathematics learning. In particular, this study demonstrated that transfer was able to occur in situations where an emphasis on spatial skills was outside the parameters of typical practices. The efficacy of the program was likely supported by the fact we introduced spatial tools as a way of supporting teachers' instructional approaches to content relatively new to them. In conclusion, this study provides evidence for the efficacy of spatialized mathematics interventions in enhancing both spatial visualization skills and overall mathematics proficiency. The differential impact based on mathematical ability levels and the exact mechanism of transfer to non-spatial tasks are areas warranting further research.

**Authors' contributions** Destina Wahyu Winarti: conceptualization, methodology, formal analysis, and investigation; writing—original draft preparation—reviewing and editing, funding acquisition, project administration. Sitti Maesuri Patahuddin: conceptualization, methodology, formal analysis, writing, reviewing and

editing, supervision. Tom Lowrie: conceptualization, methodology, formal analysis, reviewing and editing, supervision. All authors read and approved the final manuscript.

**Data Availability** The data obtained during the current study is not available for public as it contains confidential information in relation to participants. However, corresponding author could provide information on how to obtain access to the data on request.

## Declarations

**Ethics approval and consent to participate** The research on which this manuscript was based was conducted during doctoral studies at the University of Canberra, supported by the Australia Award Scholarship. Any opinions, findings, conclusions, or recommendations expressed in this material are those of the authors and not necessarily those of the funding agencies. I thank the participating teachers, students, Ajay Ramful for the insightful comments, and the reviewers of this manuscript.

**Conflict of interest** On behalf of all authors, the corresponding author states that there is no conflict of interest.

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