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Pre-paring for ambitious mathematics teaching: examining the role of video in supporting mathematics teacher educators

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Abstract
Mathematics content courses for prospective elementary teachers (PTs) should provide them with opportunities to deepen their understanding of and develop productive dispositions toward mathematics. Supporting mathematics teacher educators (MTEs) to learn how to develop practices that create rich learning opportunities for novice teachers comprises a challenge for our field. This analysis examines an unconventional professional learning experience for MTEs: viewing video of experienced teacher educators enacting the *same lessons* that they would enact with their own PTs. We examined how viewing videos embedded in a larger set of supports developed by the *Elementary Mathematics Project* (EMP) impacted MTEs' actions when planning or enacting the lessons. EMP also includes high-quality curriculum materials to use with PTs and instructional guides to support their enactment. Twelve MTEs were interviewed about their experience viewing video clips to help them to engage in preparation (Mason, 2011) as they prepared to enact EMP lessons. MTEs' noticings included how prospective teachers engage with the mathematics and one another and how the expert MTEs supported prospective teacher interactions and aspects of the classroom environment. MTEs connected the actions they identified to broader principles of teaching, offering a rationale for each and hypothesizing how it impacted the instruction. Importantly, through watching video, MTEs generated conceptual resources (e.g., identifying questions asked in the video that they wanted to try out), which they reported allowed them to respond to prospective teachers in the moment when teaching. Implications for using video to support MTE development are discussed.

Keywords Mathematics teacher educator · Prospective teachers · Mathematics content courses · Video to support professional learning · Mathematics teacher educator noticing

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Mathematics teacher educators (MTEs) are those instructors with primary responsibility for the mathematical training of prospective teachers. A subset of MTEs work extensively with prospective teachers who will become elementary teachers. For the purpose of this article, we refer to these prospective elementary teachers as "PTs." Suggestions for content courses for PTs include the following: a heavy emphasis on elementary mathematics; a focus on mathematical processes such as connecting, reasoning, generalizing, and justifying ideas; and models of effective teaching and developing productive dispositions toward mathematics (AMTE, 2017; CBMS, 2012; Kober, 2015). Teaching mathematics content courses for PTs requires extensive skill and specialized knowledge (Escudero-Ávila et al., 2021). Unfortunately, MTEs do not typically have professional learning opportunities or supportive curricular resources to expand and deepen their understandings (Masingila & Olanoff, 2021). This lack of professional learning opportunities and resources has left MTEs with few insights into the ideas and partial conceptions that PTs bring to these courses, resulting in them using less effective methods for teaching content to PTs.

In this paper, we report on an analysis from a larger project (the *Elementary Mathematics Project*, EMP) that developed educative curriculum materials for MTEs to use in content courses for PTs preparing for elementary teaching. Educative curriculum materials can include descriptions or annotated videos of classroom scenarios that "make central the work of enacting curriculum" (Ball & Cohen, 1999, p. 7). Our analysis contributes to the literature by exploring a case of how MTEs in the USA used videos embedded in educative curriculum materials to support their enactment of "ambitious" teaching (Lampert & Graziani, 2009). Ambitious teaching includes MTEs providing opportunities for PTs to engage with and discuss rich mathematical tasks, develop deep understandings of elementary mathematics topics, and productive dispositions toward mathematics (Gibbons et al., 2018; Schoenfeld, 2014). The EMP researchers created a collection of video clips—over 200, ranging from one to five clips per lesson—showing expert MTEs teaching the same EMP lessons that MTEs will teach to their own PTs. This study drew on a noticing framework proposed by Mason (2011) and analyzed how MTEs used these video clips in *pre-paration* to teach EMP materials. We accomplished this by examining what these MTEs noticed in the video clips and their reports of how these noticings supported considerations of possible actions to be carried out in the moment during EMP lesson enactments. The research question that guided our analysis is: How did the MTEs' noticings of the EMP classroom video clips support them to pre-pare to teach EMP lessons?

1 Who teaches elementary mathematics content courses for PTs in the USA

Much of what is known about US mathematics content courses for PTs, including the content, who teaches these courses, and the training and support MTEs receive, has been from two surveys of US higher education institutions focused on the mathematics training of PTs (Masingila & Olanoff, 2021; Masingila et al., 2012). The 2012 study was based on responses from 825 MTEs, while the 2021 study included responses from 413. MTEs who teach PTs include mathematicians, mathematics educators, graduate students, and classroom teachers. The majority of MTEs have advanced degrees (masters or doctorates) in mathematics or mathematics education but, because of their different professional backgrounds, may not be prepared for the work of teaching PTs.

While scholars recommend that MTEs possess elementary teaching experience because of the complexity of the topics in elementary mathematics (Bass, 2005; Seaman & Szydlik, 2007; Superfine & Pitvorec, 2021; Wu, 2009), only 10–25% of MTEs surveyed reported having taught elementary grades. Less than half of the institutions surveyed provided support for MTEs to learn about elementary mathematics. Furthermore, both surveys found that at the majority of institutions surveyed, courses were taught using a combination of lecture-based and activity-based approaches (Masingila & Olanoff, 2021; Masingila et al., 2012), indicating that the types of ambitious teaching suggested for content classes are less prevalent.

There are not many training models for MTEs, and most report learning to teach PTs "on the job" (Goodwin et al., 2014). Documented professional support includes conducting research in self-study models (Garcia et al., 2007; Taylan & da Ponte, 2016) and formal learning opportunities for colleagues through supported programs (Even, 1999; Superfine & Li, 2014; Superfine & Pitvorec, 2021). These models hold promise but only reach a small number of MTEs in the field. In the next section, we detail the current vision for teaching elementary mathematics content courses to PTs, thus constituting the types of learning opportunities for MTEs to create in classrooms while attending to goals for MTEs' development.

2 Vision for teaching elementary mathematics content courses for prospective teachers

Reports from the past two decades have provided guidance to MTEs regarding what mathematical content knowledge elementary teachers should have in order to provide children with quality learning experiences (AMTE, 2017; CBMS, 2012; Ministry of Education of the People's Republic of China, 2011; Schmidt et al., 2013; The Australian Association of Mathematics Teachers, 2006). These courses should provide PTs with opportunities to engage as *learners* of mathematics (Kober, 2015). PTs must experience instruction that focuses on understanding and requires reasoning if we desire to have them engage future elementary students in such activities. PTs need opportunities to share their own ideas and build on those of others, working in collaboration with their peers as part of a learning community (NCTM, 2014). Such opportunities help PTs to develop positive dispositions about mathematics (Schoenfeld, 2014). Research has shown that supporting the development of a stance toward mathematics that highlights reasoning, community, and responsibility for learning and that anticipates PTs' future work in elementary classrooms holds promise as a training model (Gibbons et al., 2018; Kazemi & Wæge, 2015). This type of teaching and instruction in K-12 mathematics is often referred to as *ambitious teaching* and includes considerable amounts of small-group and whole-class discussions (Lampert & Graziani, 2009). We draw on this construct to further describe the use of this pedagogy with undergraduates.

Ambitious teaching "places multiple, competing, and often ambiguous demands" on MTEs (Ghousseini, 2017, p. 189). This is especially true when using discussions as a mode of instruction, because which ideas and questions will arise as PTs respond to each other during discussions is unpredictable. Fostering meaningful discussions entails multiple aims: recognizing the mathematical connections in PTs' ideas, guiding and extending PTs' reasoning, supporting PTs to engage with the reasoning of others, developing the skills required for dialogic discourse, positioning all PTs as authors of important ideas, and

ensuring that the whole class progresses toward disciplinary goals (Chapin et al., 2013; Ellis et al., 2019; NCTM, 2014; Smith & Stein, 2018). As MTEs work to attune themselves to PT sensemaking (Jacobs, et al., 2010; van Es & Sherin, 2002), they often encounter unexpected issues—such as confusion or unanticipated ways of thinking about a problem or solution method—that fall outside the planned "lesson image" (Rowland, et al., 2015). Furthermore, if MTEs lack familiarity with elementary education topics, it may impact their ability to respond by asking relevant questions or probing PTs' understanding.

Thus, an additional challenge is how to provide MTEs with support to develop the "in the moment knowledge" (Kennedy, 2016) required to teach PTs in ambitious ways, given that MTEs may have limited experience facilitating classroom discussions or managing group work. Even though many MTEs use "activities" to teach content, this does not guarantee they elicit student thinking or provide spontaneous rapid responses that further PTs' depth of knowledge (Jackson, et al., 2020). The scarcity of professional development programs designed to address the teaching of these courses for PTs further undermines the attempts to prepare well-qualified elementary teachers. In the following section, we consider the construct of *teacher noticing* as a way to support MTEs' capacity to make sense of events in teaching and strengthen their ability to respond in the moment as they enact ambitious teaching with PTs.

2.1 Teacher noticing and pre-paration for ambitious teaching

Teacher noticing is a construct that has gained traction in studying teaching and supporting teacher learning in the field of mathematics education (Choy et al., 2013; Jacobs & Spangler, 2017). It is described as the processes through which teachers manage the "blooming, buzzing confusion of sensory data" they encounter when trying to make sense of what is going on in the classroom (Sherin et al., 2011, p. 4). In the context of a classroom, many things happen at once, and teachers choose where to direct their attention and what to pursue at any given moment (Mason, 2002; van Es & Sherin, 2002). Mason (2002) says that "every act of teaching depends on noticing: noticing what children are doing, how they respond, evaluating what is being said or done against expectations and criteria, and considering what might be said or done next" (p. 7). Noticing is consequential for teaching because what teachers "do and don't see" shapes what they "do and don't do" (Schoenfeld, 2011, p. 228). Most authors consider noticing to involve two main processes; attending to particular events in an instructional setting and making sense of those events—which can involve interpreting (Sherin et al., 2011). Some researchers include deciding *how* to respond as an aspect of noticing (Jacobs et al., 2010).

For over four decades, John Mason has been engaging teachers in watching recordings of classroom activities to understand what they notice (Mason, 1982, 2002, 2021). This work led him to describe the discipline of noticing as "arranging to alert oneself in the future so as to act freshly rather than automatically out of habit" (Mason, 2011, p. 37). Mason argued that individuals can hone their discipline of noticing in order to be sensitized to notice something freshly for themselves, to become aware of useful distinctions and possible actions to initiate in the future, rather than acting out of habit.

Mason (2011) proposes the idea of *pre-paring* to notice in a future moment. Pre-paration involves getting to know the territory, affordances, and relationships that constitute the underlying structure, before addressing any specific consequence arising from and because of a situation. Through pre-paration, an individual can accumulate different desirable actions and become aware of situations in which these actions would be preferable,

providing them with instructional alternatives. Based on Mason's research, we suspect that MTEs can exercise noticing as an intentional practice to sensitize themselves to perceived future possibilities. This enables them to change their behavior and try out new instructional practices, instead of acting automatically out of habit when actually teaching.

Scholars have argued that using videos can cultivate and refine teachers' ability to notice and help them learn to make sense of the complex practice of teaching (Borko et al., 2005; Dreher et al., 2021). In our work, we wanted to understand how videos of classroom activity could provide opportunities for MTEs to pre-pare for teaching EMP lessons in their own classrooms, with their own PTs. The purpose of developing our video library was to give MTEs images of the type of ambitious teaching we believe would be most productive in supporting PT learning. In our analysis, we wanted to explore how the video could provide a structure for working on the discipline of noticing and how this would support MTEs to notice aspects of ambitious teaching intentionally. They would then be able to draw on what they noticed, or "marked" (Mason, 2002), with the goal that, in future moments as they are teaching the EMP lesson, they would respond freshly rather than react habitually to the situation of teaching and learning while it develops.

2.2 Video in supporting mathematics teacher educator development

Studies conducted over the last two decades have shown video to be a promising tool for supporting K-12 mathematics teachers' professional learning and development (Blömeke et al., 2015; Borko et al., 2005; Dreher et al., 2021; Kaiser et al., 2017; Yang et al., 2021). Video[1] can be paused or rewound, allowing viewers to watch a segment multiple times. Viewing video allows teachers to see multiple models of teaching and consider the difficulties the videoed teachers encountered in implementing new instructional practices.

Watching video has supported teacher learning as they assimilate images of ambitious teaching strategies (e.g., how to probe for student thinking) and identify areas they want to improve in their own teaching (e.g., considering what amount of explanation must or should come from the teacher) (Gaudin & Chaliès, 2015; Seidel et al., 2011; Zhang et al., 2011). In addition, researchers found that video supported teachers to closely examine students' mathematical understandings and consider the importance of attending to student ideas during instruction (Borko et al., 2005; Gröschner et al., 2018). Analyzing video also impacted practice by supporting teachers to develop an interpretative perspective that related instruction to students' thinking and learning opportunities (Jeschke et al., 2019; Kaiser et al., 2015) and increased the extent to which teachers were able to enact ambitious teaching practices (van Es & Sherin, 2010).

Relatively few studies examine how video has been used as a professional learning tool to support the development of MTEs. One such study conducted by Superfine and Li (2014) explicitly used recordings of PTs engaged in a mathematics class to support MTEs as a part of a larger professional development design. MTEs engaged in doing mathematics together, explored potential tasks to use with PTs, examined PTs' mathematical work, and analyzed video of classroom teaching featuring MTEs teaching PTs. The facilitators pressed the participants to analyze the questions asked by the MTE in the videos, which they felt would provide participants "an opportunity to reflect on the ways in which one might support preservice teachers in mapping between mathematical representations" (p. 128).

[1] Video-based professional learning can include teacher's own classroom video, peers' classroom video, or commercially published video (see Zhang et al., 2011).

None of the studies we found had MTEs or K-12 teachers examine videos of instruction in which the lesson recorded was the *same* as the lesson they were getting ready to teach. Our study considers how MTEs viewing video of expert MTEs, situated in the context of a larger set of supports, helped them pre-pare to enact instructional practices that aim to engage in ambitious teaching of PTs. Thus, we were curious how MTEs marked events as they examined video clips and whether they reported noticing aspects of ambitious teaching in a sufficiently salient way to initiate an action in their future teaching (Mason, 2002).

3 Context of the EMP project

For the past decade, the EMP project has focused on how PTs in the USA are prepared to teach mathematics. To this end, EMP researchers developed curriculum materials that emphasize the development of mathematical knowledge for teaching (see also Chapin et al., 2021). The seven units, each consisting of six to 11 lessons, focus on mathematics topics found in all curricular materials for PTs and are vital for beginning K-5 teachers to understand and know how to teach.[2] EMP is a problem-based curriculum; mathematics problems focus on sensemaking and generalizations, connections among ideas, and mathematical structure. The problems are presented in recurring cycles of engagement for small and whole group work, including extensive discussions, to support PTs in reasoning and thinking deeply about mathematics. Our premise was that as PTs participated in the activities and discourse, the shared knowledge generated by the community of learners would be appropriated as their own (Chapin et al., 2021). Because the curriculum was designed for ambitious teaching and deviates from the norm in the USA (Masingila & Olanoff, 2021), EMP authors developed detailed, educative support materials for MTEs, including Instructor Guides and video clips, to guide the planning and enactment for each lesson.

3.1 Instructor Guides

The Instructor Guides (one per lesson) provide information about (1) particular concepts and skills related to the content; (2) PTs' common errors and partial conceptions; (3) suggestions for supporting PTs with limited prior knowledge; (4) questions to stimulate discussion and probe PTs' understanding; (5) best practices for engaging learners in inquiry; (6) information on representations, models, and examples used in the curriculum and how they can be used in the elementary grades; and (7) intuitive ideas held by elementary school children on the topics.

3.2 Video clips

The video clips show expert MTEs[3] teaching undergraduate PTs in elementary mathematics content courses that comprised part of the PTs' teacher education program: none of the interactions were staged. The videos were edited to create one to five video clips per lesson,

[2] The topics are Number Concepts, Addition & Subtraction, Multiplication & Division, Elementary Number Theory, Fractions, Geometry, and Geometric Measurement.

[3] Two expert MTEs are featured in the videos. One expert has over 10 years experience teaching elementary grades, a 40-year career as a mathematics educator teaching undergraduates, and has written curriculum for elementary and middles grades. She is the primary author of EMP. The second expert has middle school teaching experience, taught undergraduate mathematics content courses for over 10 years, is an author of the EMP materials, and trained alongside the first expert.

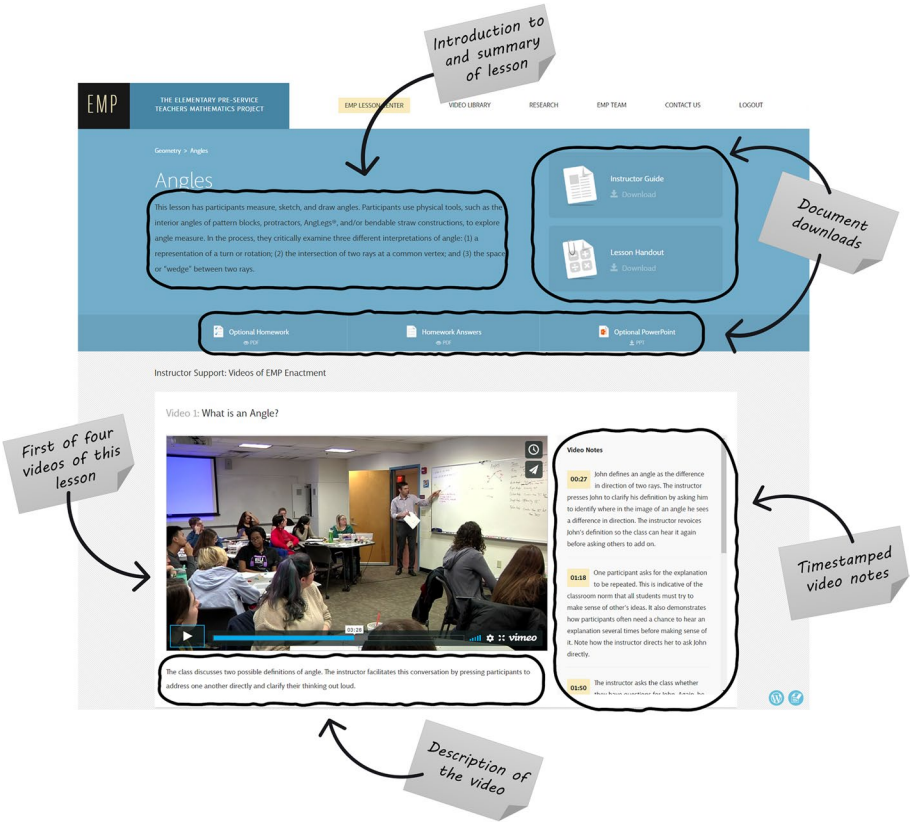

Fig. 1 Annotated Image of the EMP-developed webpage for the lesson, "Angles"

each clip ranging in length from 1 to 12 min. Interactions were selected to provide images of how expert MTEs taught the content using ambitious teaching practices. We included video of common problems of practice that emerged as we worked to support PTs' learning (e.g., when no PT ventures a response to a question or when one offers only a partial explanation). Commentary accompanied each video clip to provide context, highlight particular actions and questions made by the MTE or PTs, or provide the expert's explanations of instructional decision-making. Figure 1 shows an annotated image of the webpage for the "Angles" lesson and exemplifies what the participants saw when navigating to a lesson page. Each lesson webpage features an introduction and a summary of the lesson, document downloads (including the Instructor Guide and lesson handout), video clips of the lesson enacted by expert MTEs, and, for each clip, a description with commentary.

4 Methods

Throughout the paper, we have used the term "mathematics teacher educator" to describe the individuals who teach mathematics content courses. In this section, we make a distinction between the MTEs who were filmed and the MTEs who were a part of the study. We

refer to the experienced MTEs from the EMP video as "expert MTEs" and the participants of the study as "participants."

4.1 Participants

In this study, we were interested in understanding how MTEs' noticings of the EMP classroom video clips supported them to pre-pare to teach EMP lessons. The participants in our study came from a convenience sample, selected from field testers who volunteered to use materials during the 2018–2019 academic year who indicated that they watched the videos and found them useful in their planning. Of the 69 field testers, 58 reported on an end-of-semester survey about the materials that they watched videos from the EMP website, and of these, 39 reported finding these videos either *useful* or *very useful*. The majority of field testers reported being knowledgeable about topics in elementary mathematics. Furthermore, the group broadly shared a philosophy about supporting PT learning. However, they varied in experiences in implementing ambitious teaching practices. Thus, our sample was selected from the 39 field testers based on their self-reporting of their instructional practices when teaching content courses to PTs prior to using the EMP materials. All reported that they used a combination of lectures and activities; four of the participants indicated they had used discussions to help students make sense of concepts and procedures. We also considered the type of institution where they taught (i.e., public, private, two-year, four-year, doctoral programs) and the number of times they had taught a content course to PTs in selecting the sample. We were interested in a maximum variation sampling method in background and experience. We invited 15 to be interviewed, of which 12 agreed to participate. See Table 1 for details.

4.2 Data collection

All of the semi-structured interviews took place over a two-week period directly after the academic year the participants had field tested EMP materials. For some participants, five months had passed since they had taught the EMP materials, while for others, only two weeks. Each interview lasted 35–90 min and was conducted and recorded by one of the first three authors via Zoom video conferencing. Two of the authors were the expert MTEs in the videos; they did not interview participants who watched videos in which they were the expert MTE. We first asked participants to describe how they typically planned for an EMP lesson. We then discussed their use of video and asked how they selected clips to watch, when during the planning process they watched them, whether they had certain intentions or goals when watching videos, and whether and how the videos helped them enact EMP lessons. We further probed by asking for specific instances when video clips were particularly useful and if participants tried anything new or different in their practice in relation to something they saw in the video.

Because we were interested in understanding how the MTEs' noticings supported their pre-paration for teaching, the interviewer and participants watched one or two EMP video clips together with us during the interview (four participants opted to watch two clips, and eight participants opted to watch one). Before starting the video, we said, "I am curious to hear what you are attending to and noticing as we watch it" and indicated that they could either pause and talk about the lesson during the viewing or wait until the video was over to tell us what they had observed. We then probed them to elaborate on what they had learned about PTs, the MTEs' roles, and whether they learned something new about the

Table 1 Self-reported information about participants

	Highest level of education	Number of content courses taught prior	Prior instructional practices	Two- or four-year institution	Reported amount of video watched	Reported usefulness of video
Jade	Doctorate in Mathematics Ed	1–3	Lecture with some activities	4	At least one video per lesson	Useful
Francesca	Master's in Ed. Psychology	1–3	Lecture with some activities	2	Some or all videos for each lesson	Useful
Rosa	Doctorate in Elementary Ed	1–3	Activities with some discussions	4	Some or all videos for each lesson	Very useful
Katie-Lynn	Master's in Mathematics Ed	4–6	Activities with some discussions	4	A few videos or parts of videos	Very useful
Meili	Doctorate in Mathematics Ed	4–6	Activities and discussion	4	Some or all videos for a lesson	Very useful
Kiri	Doctorate in Mathematics Ed	7–10	Equal lectures and activities	2	A few videos or parts of videos	Useful
David	Doctorate in Mathematics Ed	11–15	Activities, some lecture, some small group discussions	4	Some or all videos for each lesson	Very useful
Lettie	Doctorate in Mathematics Ed	11–15	Activities, lectures, and small group work	4	Some or all videos for each lesson	Very useful
Kendra	Doctorate in Mathematics Ed	11–15	Activities with some lecture	4	A few videos or parts of videos	Useful
Ryan	Doctorate in Mathematics Ed	11–15	Lectures with some activities	4	A few videos or parts of videos	Useful
Lucia	Master's in Mathematics Ed	16 +	Mainly activities but few group discussions	4	Some or all videos for each lesson	Useful
Rhiannon	Master's in Mathematics	16 +	Mainly activities	2	At least one video per lesson	Somewhat useful

mathematics related to that lesson. We also asked them to reflect on when they originally watched the video and consider whether and how their noticings had influenced their planning or enactment. When participants mentioned trying something new in their class based on what they had viewed in the video, we asked them to specify that with which they had experimented, why, and what had resulted.

4.3 Data analysis

The authors of this paper were members of the EMP curriculum development or research teams, each carrying out various roles on the project. This suggests a potential for bias in our interpretation of the data; however, our analysis is not focused on whether the videos were useful. Instead, we wanted to understand how the video clips may have pre-pared participants to teach and in what ways. Following each interview, we wrote memos documenting the ways the participants described how they used video clips to support their planning and teaching, as well as what they noted observing[4] when watching the video. Once the interviews were transcribed, researchers collectively developed an initial coding scheme based on the memos to understand what the participants noticed. To deepen the coding scheme, researchers added codes based on the literature about ambitious teaching (Kazemi et al., 2009; Lampert & Graziani, 2009) since the participants had limited experience enacting ambitious teaching practices. Each of the four authors read approximately 75% of the data set to refine and expand codes; we then finalized the coding scheme and used it to code the entire data set (see Appendix). We developed a set of codes to describe MTEs' actions, such as MTEs' avoiding *telling* too much about the mathematics to the PTs, mathematical examples that MTEs helped generate, or how MTEs built on PTs' thinking. Since participants noticed what the PTs were doing, we developed codes to describe PTs' actions, such as PTs' explanations, their errors, or how attending to the filmed PTs' responses allowed them to anticipate their own students' responses. Codes related to the classroom environments and norms category included building community and supporting equitable discourse.

Codes were applied at the level of the participant's turn of talk or at the sentence level if turns were large; the turn of talk or sentence could be assigned more than one code. For example, when describing what she was attending to in a video clip, one of the participants said:

> I think I was mindful of the questioning, you know? I mean [the expert MTE] constantly asked students what they think. And without jumping in. And not really, not even giving it away, whether they're right or wrong. [The expert MTE] goes to a different person. And then you let them basically validate… they have enough confidence that they eventually figure it out themselves.

We coded this excerpt as MTEs' not telling too much, MTEs' stepping in and out of the conversation, equitable participation, and shifting authority to PTs. All four authors participated in coding using Dedoose qualitative analysis software, with two coders assigned to each interview and care taken to systematically vary the coding partnerships so that each coder was paired with other coders for a similar number of interviews. After

[4] As Sherin and Starr (2010) have pointed out, we assume that educator noticing is available for verbalization and participants are discussing with us what they consciously notice in the videos viewed.

coding segments of data, the two coders came together to build consensus by discussing and resolving any differences in their coding assignments. Next, we explored patterns and themes in the data. We began by using Dedoose to generate a table that allowed us to see which codes were applied most. We then created a new set of memos to capture what participants reported noticing in relation to ambitious teaching that had the potential to support them in future moments while enacting EMP lessons as they responded in new ways to teaching while it unfolded.

5 Findings

In this section, we report on what participants noticed as they watched video clips they had selected as impactful and commented on what they observed. Participants made numerous comments about the actions of the PTs[5] and the expert MTEs. As participants examined video clips, their noticings[6] were in relation to what PTs were doing and saying, aspects of the classroom environment, and what the expert MTEs were doing and saying. We organize this section to describe their noticings (van Es & Sherin, 2002) and how participants connected their noticings to principles of ambitious teaching and learning (Lampert & Graziani, 2009), and in doing so, discussed their perceived value.

5.1 Attending to PTs' reasoning

All participants noticed how PTs responded to particular mathematical ideas and activities when viewing clips. A majority of participants repeatedly remarked that they paid close attention to PTs' speech to hear what a "typical" response might be. For example, Rosa, indicated that when watching the clips, she was "really focused on the questions the students had. I would think, 'Gee, I wouldn't think they would think that!' …I learned an incredible amount from the students in terms of their mathematical reasoning." Francesca commented, "I loved hearing [the PTs'] explanations. The explanations were helpful in that I really didn't know how they were going to explain." Lettie indicated that the explanations she noticed on the clips, "allowed me to really anticipate, so I could go in [my classroom] better prepared." The participants felt they gained insight into PTs' understanding of topics, reasoning about specific content, and what type of arguments they might use.

All the participants indicated that they watched video clips of content they were less familiar with to better understand how students might respond to questions or what their explanations of a mathematical idea might sound like. For example, in relation to the angles lesson, David said:

> I had not focused on this concept before so I guess I didn't really know what my students would say when asked that question. We would kind of use angles but we never took the time before to actually talk about what an angle is. So I didn't really have a background about what my students would even say... Watching students similar to

[5] In this section, we continue to refer to the students in these courses as prospective elementary teachers (PTs). Often participants refer to these individuals as "students," which is reflected in the quotes.

[6] We will use "noticings" but in using this phrase, we mean what Mason (2011) has further explained as marking: "something [that] may be noticed and sufficiently salient to be able to initiate a remark about" (p. 41).

> mine wrestle with that particular idea was really useful in terms of what my students might do.

As seen in David's remarks, the video supported him to pre-pare to teach a lesson in which PT's were asked, "What is an angle?" Attending to PTs' reasoning in the video gave him ideas about how his PTs might respond and enabled him to pre-pare possible responses.

The videos helped participants to anticipate PT's partial conceptions so that they could consider what ideas they would want to clarify when launching the problems or where they might want to dwell during a discussion. For example, Meili said:

> The Instructor Guide talked about how students might think this, or some students might have this misconception. It gives you this warning. But the video is real. We can see what students are thinking, what they're doing. For example, [in the lesson] where you have a rectangular piece of paper and you're building, you're making a cylinder, and then drawing the net for it… If you've never taught this lesson, you would think, 'Oh, it will be very easy.' But then if you watch the video before you teach it for the first time, you can see, wow, it's really hard for them to wrap the circle at the end and realize the volume will be different for it. Most people assumed that with the same piece of paper the volumes should be the same, surface area should be the same.

Meili's comments show how the video clips brought to life the struggles PTs can have with specific content by showing what they are thinking and how they are engaging with mathematics. As can be seen in the next quote, she described how the video clips assisted her in better understanding PTs' thinking and pressing them to think more deeply about the content.

> Another [example] is the one where you use square tiles and find the perimeter of a shape. If we add one more tile [to the shape], do you always add one to the perimeter or not? So if you didn't see the video… you might have a discussion that is pretty good, but watching the video you can see there are *many* different ways of thinking to justify why the perimeter doesn't always increase, sometimes it doesn't change... So if I didn't watch the video, I might be happy with one method. But after watching the video I saw there's a student talking about several different ways to look at the same notion. So my students did not come up with all the different ways of thinking. But I introduced them and asked, 'What is this? What is that?'

In Meili's remarks, we can see that by watching the video clips, she sensitized herself to respond in new ways when she enacted the EMP lesson; gaining images of what mathematical reasoning is possible supported her to notice in the moment that important mathematical ideas were not being shared in her classroom. She equipped herself to bring those ideas up to the class for consideration.

5.2 Establishing classroom environments that support PTs' development

All participants noticed that certain norms had "been established" in the videoed classrooms to allow the social and mathematical interactions they observed. Most of their observations focused on the creation of a respectful environment and the development of a learning community which had the potential to support equitable participation among PTs.

For example, six participants mentioned PTs' willingness to ask questions or seek clarification when they did not understand an idea, a level of comfort participants highlighted as an important aspect of a classroom environment that supports PTs to engage with mathematics. Katie-Lynn noticed how the expert MTEs provided PTs with the agency to have ownership over the mathematics and reasoning.

> …the mindfulness of making students, you know, have more control in the classroom that they're running the show. And that [MTEs] need to work with the students to get feedback from peers. I think it also showed the environment was really supportive of helping students work toward that understanding…what that environment looks like.

Katie-Lynn's noticing of PTs' agency led to her thinking about her role in supporting the development of agency, both through her actions and her creation of a supportive environment. Another participant, Kendra, noted how she got ideas from the videos on how to help PTs feel competent. In discussing the expert MTEs actions, she highlighted the importance of building a community where individual thinking is recognized and valued.

> Some of the things that were really cool were that the instructor always started with the students' proposed definition. After restating it, he would ask other students to... ask a question or clarification... let that idea float for a while and... have the rest of the students sort of chime in to help clarify his thinking... It makes the students feel validated and creates an atmosphere where everybody's thinking is valued.

Every participant noted ways in which the expert MTEs had shifted the authority in the classroom from the MTE solely leading to "handing over power" to the PTs. For example, David noticed how the expert MTE often asked PTs to go to the front of the room and "become the teacher" to explain their mathematical thinking: "[The expert MTE's] strategy of putting someone in front, explaining to the class, is not something I do often. So I did do that once or twice [in the past]. And that kind of spurred me on to do that." Sharing authority in the classroom is one aspect of ambitious teaching. Another participant, Ryan, noticed that the expert MTE did not do most of the talking. "[The expert MTE's] facilitation of the discussion was primarily through questioning. And that's definitely something I think that's good. That is a great model to follow to end up trying to do less telling and more questioning." Questioning can lead to PTs greater participation in the mathematical reasoning, another component of shared authority.

Eight participants noted the large number of PTs that participated in the classroom discussions, another feature of classrooms in which PTs have agency and authority. Kiri stated, "everyone participates," and Ryan noted that PTs "learn from sharing ideas." Rosa mentioned that PTs "build off of each other" and Lettie commented that the PTs are "forming understanding collaboratively" (Schoenfeld, 2014).

5.3 MTEs role in supporting PTs' learning of mathematics

Participants routinely noticed how the expert MTEs in the videos gave the PTs opportunities to make sense of the mathematics for themselves, a hallmark of ambitious teaching. One way in which participants marked that the expert MTEs accomplished this was through the types of questions they asked. Eleven of the twelve participants noted the questions that probed PTs' thinking, pushed them to justify their claims, or encouraged discussion of the mathematical idea by opening up the topic rather than funneling toward one answer.

Nine participants noted the role of "talk moves" (ways to engage all students in an academic discussion; Chapin et al., 2013) used by the expert MTE to facilitate discussions. Participants noted the following talk moves: expert MTEs restated or revoiced PT ideas, they pressed a PT for clarification or reasoning, they brought in other PTs' ideas, and they made space for many PTs to speak and share by having them talk to each other in pairs and small groups. For example, Kendra shared: "I thought that was a good teacher move… having a student explain again, then really emphasizing that they should turn and talk to their neighbors. And I found it helpful that [the expert MTE] asked [a PT] to rearticulate before asking others to think about it again." Francesca reported how viewing the video deepened her understanding of why an MTE might use talk moves with PTs: "I think you can read about the talk moves and support and that thing but when you see it in action. I think helps deepen your understanding and see the usefulness and value of them." Francesca saw the function of using talk moves to support PTs; beyond reading about the ambitious teaching concept, she saw the MTE experts bring these ideas to life and noted how useful these instructional moves can be to support PTs to deepen their understanding of mathematics.

Lucia indicated that viewing the video helped her reflect on her own instructional practice. She noted particular talk moves that the expert teachers employed and used these noticings to pre-pare for enacting classroom discussions with her PTs:

> ...the main help from the videos for me was reminding me not to just say, 'That's right,' and go on. Reminding me to say, 'Okay, do you agree or disagree with this? Okay, can I hear from someone else? Can you tell me what they said in your own words?'

All participants noted that their observations supported them to reflect on their role in their mathematics classrooms. For example, David commented:

> It's not a *sage on the stage* type... It's developing scaffolding conversations with the group, and the students themselves are asking questions among each other. And [the expert MTE] is probing with questions, but by no means is he telling information. He's engaging students in conversations to lead to a mathematical concept.... He's literally sitting with a group of students having a conversation, with another student standing in front of the class kind of modeling mathematics.

Participants also noted other instructional moves that the expert MTEs made, including carefully monitoring the discussion and stepping in at critical moments, not affirming whether a response was right or wrong, not providing too much information to students, and moving to the back of the classroom to support student-led presentations (something Meili reported trying in her own classroom). Most participants indicated that these observations "reinforced" for them what their role was. For example, Lettie reported, "You don't want to tell them the answers. You don't want to provide too much information. You want them [PTs] to kind of understand it and form that understanding on their own." Here, Lettie is connecting the moves that MTEs made to supporting PTs' sensemaking of the mathematics.

Eleven participants commented on specific questions that the expert MTEs asked PTs. For example, Jade said she noted in a video clip of a place value lesson:

> I think that last question [the MTE] asked. …she had that student explain what was happening with the one on the right. And then she poses you know, we're trading and we're repackaging. And even using that language that's been used in previous les-

sons, but you're not changing the quantity. So, what's happening? She says, 'What's happening in your strategy?'

Jade later went on to say that the video clips helped "to enhance my questioning techniques." She described a process of taking notes about the video clips and recording them directly into her Instructor Guide, which she referred to as she enacted EMP lessons. She showed the interviewer her notes, written in purple ink, and said: "I would have asked this question, why does this work in addition and subtraction? So that was a word-for-word question that [the expert MTE] had asked." Other participants mentioned writing down the questions that the expert MTE had asked. In these instances, the viewing video supported them to try out new questions with their PTs, questions that they felt would help deepen PTs understanding of the mathematics.

6 Discussion

For the MTE community, teaching mathematics content courses that deepen the knowledge of PTs while modeling effective teaching methods poses many challenges. Few professional learning opportunities have been created to support MTEs (Masingila & Olanoff, 2021). Thus, insight into opportunities that can be provided to support MTEs' teaching and development is critical in assisting them in understanding the complexity of elementary mathematics topics, the partial conceptions PTs have about mathematical concepts, and effective methods for teaching mathematics to PTs. In this analysis, we examined how video clips embedded in an educative curriculum provided a structure for MTEs to pre-pare to teach PTs using ambitious teaching practices. That is to get to know the territory, the affordances, and the relationships which constitute the underlying structure of EMP lessons before trying to address any specific action arising as they enact ambitious teaching. Among other things, participants noticed the video recorded PTs' questions, reasoning, and claims about specific content, errors and partial conceptions, their affect while participating in the lesson, and how they engaged with each other and the mathematics. Participants also noticed aspects of the classroom environment that they felt developed norms to support PTs to engage in discussions about mathematics in particular ways. Noticings regarding the expert MTEs that were made by participants included how they facilitated discussions and how they shifted mathematical authority to the PTs.

We found that participants' noticings and the remarks they made about the video clips fell into similar categories as K-12 teachers' noticings. Sometimes participants noted what the expert MTEs or PTs were doing or saying, observing merely what was taking place. This is similar to what Mason (2002) referred to as "accounting of" the classroom activity and what others call "attending" (Sherin et al., 2011). Other times participants noted why the expert MTE or PTs took a particular action, explaining why the action may have been productive for the PTs in the videoed classrooms or for PTs more generally. Noticing why a particular action was productive is similar to what has been referred to as "making sense" of the events or "interpreting" them (van Es, 2011), which involves evaluation on the part of the participant (Mason, 2011). In these remarks, participants connected their noticings of the classroom events to broader principles of ambitious teaching (van Es & Sherin, 2008). Connecting to broader principles of ambitious teaching allowed the participants opportunities to

reflect on the underlying rationale for different actions and glean new insights into the benefits of those instructional practices (Munter & Wilhelm, 2020). Participants may have seen a familiar form but discovered a new function of an MTE or PT action when they closely observed how the activities and interactions unfolded in the video.

All participants' remarks indicated that, at times, they were quite struck by a particular incident and recorded it so that they could access the incident in the future when they enacted the EMP lessons (Mason, 2002). These noticings broadened the participants' reflections on their own instructional practices, providing them with alternative actions and responses that might be retrieved in the moment when teaching. All participants reported having tried out particular questions or actions in their classrooms based on what they noticed in the video clips. Likewise, all participants mentioned that their students' struggles were similar to the struggles of the PTs they observed in the videos. These noticings supported teaching in new ways using a collection of alternative actions and an awareness of situations in which these teaching moves could be applied (Mason, 2002).

Through watching video, all participants indicated generating conceptual resources, such as identifying questions used by the expert MTEs that they wanted to try out, and these resources allowed them to try out new or deepen existing ambitious instructional practices (Horn & Little, 2010; Lampert & Graziani, 2009). All participants stated that the videos had proven useful, no matter which aspects of practice were "available for individuals to see and experience" based on their "horizons of observation" (Horn & Kane, 2015, p. 377). Each of our participants came with a different background and set of experiences for ambitious teaching. Ryan had mainly used lecturing, so he noticed how the expert MTEs supported student-led presentations and attended closely to the questions the expert MTEs asked. Others had not taught some of the elementary mathematics content before or had taught the content in a way that limited their insight into PTs' understanding of each topic, so they attended closely to PTs' mathematical reasoning. Meili and Katie-Lynn had the most experience with ambitious teaching practices and noticed more subtle aspects of ambitious teaching, highlighting refinements they wanted to incorporate into their own teaching. Future research could examine: As MTEs' practices shift over time, how does what they observe in classroom videos change? Could the videos be helpful to other MTEs teaching content courses to PTs not using the EMP materials, and if so, how?

The development of PTs' mathematical knowledge for teaching is important since PTs will go on to teach many children and shape their mathematical identities and intellectual development. Thus, developing professional learning opportunities for MTEs and how to offer these at scale is critical. Our analysis showed that video, as a medium embedded in an educative curriculum, offers a potentially productive tool to hone MTEs' noticings. Furthermore, what participants noticed enabled them to consider options for deepening and broadening their own instructional practices. Video provides a cost-efficient way to support multiple MTEs; they can choose when and where to access them, to view the videos either alone or with colleagues, and can re-watch them any number of times. While there is much to learn about MTEs' professional learning, video appears promising, particularly when embedded in educative materials, which provide MTEs opportunities to investigate practice as they pre-pare for teaching, take these noticings to catalyze new instructional moves or practices within their own classroom during lesson enactment, and continue to reflect post-enactment.

Appendix

Codebook

	Codes	Description
MTEs' Moves	Connecting ideas	How the expert MTE connects mathematical ideas
	Use of talk moves	Expert MTE's use of talk moves, either specific ones (e.g., revoicing, rephrasing, etc.), or general moves for facilitation robust discussion
	Examples	How the expert MTE uses mathematics examples
	Explanations/Thinking	Explanations provided by the expert MTE, or the expert MTE's own mathematical thinking
	Use of vocabulary/language	The expert MTE's use or treatment of particular vocabulary
	Questions	Specific questions asked by or questioning practices used by the expert MTE
	Not telling too much	Restraint shown by the expert MTE in a moment when the expert MTE does not "give away" too much mathematical information, rather, allowing the PTs space to come to conclusions on their own
	Representing PTs' thinking	How the expert MTE represents a PT's mathematical thinking, likely with writing on a whiteboard
	Shifting authority to PTs	How the expert MTE's actions shift mathematical authority to the PTs in the classroom
	Stepping in/out of the conversation	Highlights how the expert MTE removes herself or himself from the conversation, or re-enters it, at a specific moment
	Use of tools	How the expert MTE uses a mathematical tool, such as manipulatives

	Codes	Description
PTs' Moves	Use of tools	How PTs use a mathematical tool, such as manipulatives
	Anticipating PTs' responses	PTs' responses to the mathematics and how observing these responses helped them anticipate how their own PTs might respond
	Use of vocabulary/language	The PTs' use or treatment of particular vocabulary
	Math is challenging	Highlights that mathematics can be difficult for PT learners
	Connecting ideas	Highlights PTs connecting mathematical ideas
	Errors/misconceptions	Highlights incorrect or incomplete thinking or reasoning demonstrated by the PTs in the videos
	Explanations/thinking	Discusses explanations provided by PTs, or the PTs' mathematical thinking
	Questions/examples	Observes questions asked by or examples created by PTs
Managing time & space	Attending to format (Small Group/ Whole Class)	Makes comments about the small group or whole class context of mathematical or pedagogical activity
	Board-work (logistical)	Observes the ways in which the board in the classroom is being used to support classroom mathematical activity
	Instructor Position	Discusses the physical location of the expert MTE within the classroom
	Pace of lesson	Makes any mention of the timing or pacing of the lesson
	Wait time	How the expert MTE uses wait time during a classroom discussion
Norms	Building community	Observes either the presence of an established community within the classroom, or moves which contribute to its creation
	Equitable participation	Observes either the presence of an equitable participation within the classroom, or moves which create opportunities for this type of participation
	Respectful discourse	Observes either the presence of respectful discourse within the classroom, or moves which support this type of communication

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Declarations

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