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Student noticing of collaborative practices: exploring how college students notice during small group interactions in math

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Abstract

Over the last three decades, educational researchers and policymakers have increasingly promoted instructional strategies that centralize group work in mathematics. One difficulty teachers face in implementing group-based instruction in mathematics involves facilitating meaningful group interaction amongst students. In this paper, we explore student noticing as a novel strategy for supporting college students to collaborate effectively during group work in mathematics. First, we construct a noticing framework named *student noticing of collaborative practices* which provides a lens for "seeing" how students notice their collaborative practices. Then, we use the framework to explore how 25 college students noticed their collaborative practices in mathematics. After working on a novel mathematics task in groups, the college students listened to audiorecordings of their group interactions and responded to reflection questions about the effectiveness of their collaboration. We identified themes regarding how and what students noticed related to their collaborative practices. The findings reveal that students attended to many aspects of their collaboration, including their talking turns and propensity to listen to others. Students demonstrated a desire to change their collaborative practices in the future. The findings imply that teachers and researchers might leverage student noticing as a tool for improvement in mathematics group-based classrooms.

Keywords Mathematics · Student noticing · Collaborative learning · Group instruction

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1 Introduction

Over the last three decades, educational researchers and policymakers have increasingly promoted instructional strategies that centralize group work in mathematics. While there is variation between group-based instructional approaches in mathematics, the theoretical premise is consistent: students learn best by collaboratively problem solving with peers in small groups on novel and interesting mathematics tasks. The research surrounding group-based instruction in mathematics has been largely contradictory. Some research reveals that group-based instruction leads to high student achievement, strong peer relationships, meaningful participation, and positive motivation towards mathematics (Boaler, 2006, 2008; Boaler & Staples, 2008; Johnson et al., 2010; Liljedahl, 2016); however, some studies reveal contrasting results (e.g., Clark et al., 2012; Kotsopoulos, 2010).

One difficulty teachers face in implementing group-based instruction in mathematics involves facilitating meaningful and productive group interaction amongst students (Franke et al., 2007; Shaughnessy et al., 2021; Smith & Stein, 2011). Students do not naturally communicate in ways that help them learn from each other in mathematics—instead, they need to be taught how to communicate effectively (Campbell, 2021; Gillies, 2019; Sfard & Kieran, 2001; Webb, 2009). Researchers have identified several teaching strategies that support teachers to facilitate meaningful interaction amongst students in mathematics, including assigning group roles (e.g., Boaler, 2008; Johnson et al., 2010), instituting group rules (e.g., Hofmann & Mercer, 2016; Murphy et al., 2018), attending to group coordination (e.g., Campbell & Yeo, 2022), and explicitly teaching effective discourse moves (e.g., Campbell, 2021; Gillies, 2019; Johnson & Johnson, 2016). This research supports teachers to facilitate group-based instruction in mathematics, but there is still more work to be done.

In this paper, we explore student noticing as a novel strategy for supporting college students to collaborate effectively during group work in mathematics. Noticing refers to becoming intentionally aware of one's actions and ideas (Mason, 2002). Researchers have primarily applied this construct to teachers as an avenue to help them become aware of their in-the-moment instructional decisions (e.g., Jacobs et al., 2010; Sherin & van Es, 2005; Wieman & Webel, 2019). Studies have shown that teachers can engage in noticing by listening to or watching audio/video recordings of their teaching and reflecting on specific aspects of their teaching moves (e.g., Sherin & Russ, 2014; Walkoe, 2015). In this study, college students similarly engage in student noticing by listening to audio recordings of themselves interacting in groups and reflecting on specific aspects of their collaboration. We construct a noticing framework named *student noticing of collaborative practices* which provides a lens for "seeing" how students notice their collaborative practices. Then, we use the framework to explore how college students notice their collaborative practices while working on novel mathematics tasks. We hypothesize that when teachers provide opportunities for students to notice their collaborative practices, it can support learners to improve their group collaboration and awareness. We provide the relevant details in the following sections.

2 Literature review

2.1 Group-based instruction in mathematics

Group-based instruction is any type of instruction that centralizes group work. Over the last several decades, scholars have conceptualized various types of group-based instruction, including cooperative learning (Johnson et al., 2000; Slavin et al., 2003), collaborative learning (e.g., Palinscar et al., 1988), computer-supported collaborative learning (Dillenbourg & Fischer, 2007), complex instruction (Cohen, 1994), and dialogic instruction (e.g., Munter et al., 2015). While these types of group-based instruction differ in design, they are generally grounded in a similar premise: *students learn best through interaction with peers.*

This study is grounded in literature on the distinct nature of group-based instruction in contemporary mathematics classrooms. Contemporary group-based mathematics instruction is distinctive from other content domains. Whereas group-based instruction in other content areas promotes *discussion* in small groups, contemporary visions of group-based instruction in mathematics emphasize collaborative *problem solving* (e.g., Munter et al., 2015; Chan et al., 2018). The central idea is that students work in groups on non-routine mathematics problems to collaboratively construct knowledge through problem solving. These contemporary visions of group-based instruction in mathematics are consistent across the globe. Examples include Jo Boaler's work in England and the USA (e.g., Boaler, 1998, 2000, 2008), Peter Liljedahl's research in Canada (e.g., Liljedahl, 2016), and Man Ching Esther Chan and David Clarke's research in Australia and China (e.g., Chan et al., 2018). While scholars and practitioners use various teaching strategies for group-based instruction in contemporary mathematics classrooms, there is general consensus around the primacy of problemsolving.

Because group-based instruction in mathematics classrooms is distinct, it is vital to study the social conditions of such instruction as a separate entity. Research on group-based instruction in other content domains do not, in large part, generalize to contemporary mathematics classrooms. For these reasons, we draw from literature directly related to mathematics as we discuss the missing links of prior research on group-based instruction in mathematics.

2.2 Missing links

One primary criticism of group-based instruction in mathematics is that effective group collaboration is difficult for teachers to facilitate (Franke et al., 2007; Hofmann & Ruthven, 2018). During group work, teachers must attend to groups' mathematical activity as well as their ability to collaborate effectively (Campbell & Yeo, 2022, 2023). Unfortunately, research reveals that students often struggle to communicate in ways that support their mathematical learning. For instance, Barron (2003) found in a study of 48 sixth grade learners that students with high prior achievement were sometimes unsuccessful at mathematical problem solving in small groups due to the quality of their communication. Furthermore, Sfard & Kieran, (2001) demonstrated through a case study analysis that collaborative problem solving in mathematics is an intricate process, and even well-meaning students can struggle to generate learning opportunities through collaboration. Other research reveals that students often create hierarchies while working in small groups in mathematics, with some students retaining intellectual authority and others assuming menial roles (Bishop, 2012; Wood, 2013).

Researchers have identified several teaching strategies for helping students to communicate in ways that support their learning in mathematics. Teachers can create group roles, institute group rules, and explicitly teach students promotive discourse practices (Boaler, 2008; Campbell, 2021; Hofmann & Mercer, 2016; Murphy et al., 2018). Group roles (e.g., recorder, task manager, presenter) support students to meaningfully participate in group problem solving (Boaler, 2008), while group rules (e.g., "respect all group members' opinions") support students to understand the standards of effective communication (Hofmann & Mercer, 2016). Explicitly teaching promotive discourse practices (e.g., "give reasons when you disagree") provides learners with a template for *how* to communicate in mathematics (Campbell, 2021). Such teaching strategies can mitigate the complexities of learning in group-based classrooms.

While prior research places the onus on teachers to facilitate meaningful collaboration amongst students in mathematics, it may be more beneficial to provide students with autonomy in determining how to improve their own collaboration. Such autonomy may have more lasting influence than receiving explicit direction from their teacher (Murphy et al., 2021; Yackel & Cobb, 1996). In the next section, we explore the noticing literature and reveal how student noticing can support learners to autonomously improve their communicative behaviors while working in small groups in mathematics.

2.3 Noticing in mathematics education literature

Educational researchers have leveraged the concept of noticing to support teachers or students in improving their teaching and learning practices (Campbell & Yeo, 2022; Goodwin, 1994; Jacobs et al., 2010; Lobato et al., 2012; van Es & Sherin, 2008). While conceptualizations of noticing in mathematics education literature are varied, noticing is grounded in a consistent premise. To illustrate this premise, let us consider a scenario. One can *notice* many things as they stare at a painting: an unusual color, a circular pattern, and a peculiar shape. While several aspects compete for one's attention within the painting, one only intentionally focuses on (i.e., notices) a subset of those aspects. The central idea is that noticing involves becoming intentionally aware of particular information, data, or activities amongst multiple sources (Sherin & Star, 2010). Therefore, researchers have examined what teachers or students notice amongst a variety of inputs in mathematics classroom settings (Wilkie, 2022).

Prior research has examined noticing from a variety of perspectives, including cognitive-psychological perspectives, socio-cultural perspectives, expertise-related perspectives, and discipline-specific perspectives (König et al., 2022).[1] This study is grounded in a discipline-specific perspective of noticing, which is a noticing perspective that focuses on practices for raising teacher or student awareness through systematic reflection (Mason, 2002, 2011). From this perspective, teachers or students focus their attention on specific aspects of their teaching or learning (usually by watching video records of themselves) and imagine how they might respond differently in the future. This study, being grounded in the discipline-specific perspective, provides participants with opportunities to engage in

[1] Although König et al.'s (2022) research refers to teacher noticing, these perspectives map nicely onto student noticing as well.

retrospective noticing whereby they systematically reflect on their collaborative practices by watching videorecordings of themselves interacting in groups.

Scholars have created many frameworks to examine student and teacher noticing, but one influential framework that contributes to the conceptual grounding of this study is Jacobs et al.'s (2010) *professional noticing of children's mathematical thinking* framework. Jacobs and colleagues' (2010) framework describes how teachers notice students' mathematical thinking, and the framework consists of three components: (1) attending to learners' mathematical strategies; (2) interpreting students' mathematical understanding; and (3) deciding how to respond. The first component, attending to learners' mathematical strategies, refers to teachers becoming aware of the mathematical details of students' work (verbal, written, or otherwise). The second component, interpreting students' mathematical understanding, refers to teachers interpreting the nature of students' mathematical understanding based on the mathematical details of learners' work. The third component, deciding how to respond, refers to teachers deciding how to respond on the basis of their interpretations. These three noticing processes occur almost in tandem when teachers make in-the-moment decisions; however, teachers can contemplate each of these processes separately when reflecting on video recordings of their practice (Sherin & van Es, 2005). Research reveals that supporting teachers to reflect on the noticing processes can support their teaching practice (e.g., Sherin & Han, 2004). That is, teacher noticing can be improved by changing what teachers pay attention to or how they analyze it.

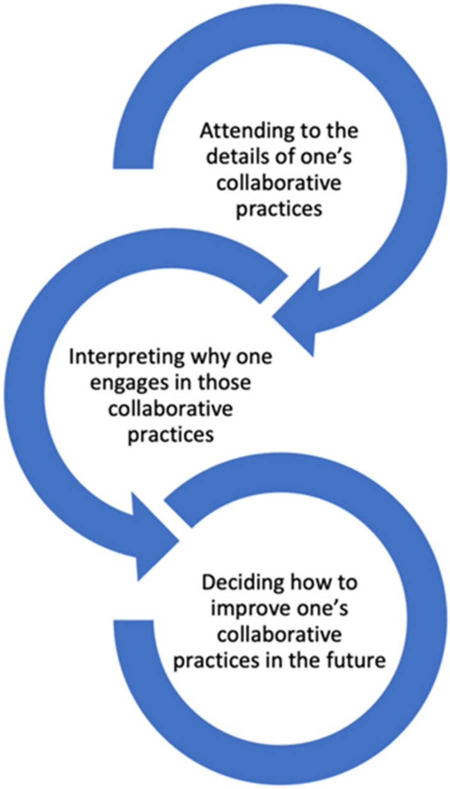
While a body of studies has examined teacher noticing processes in mathematics classrooms (e.g., Jacobs et al., 2010; van Es et al., 2022), student noticing is gradually receiving research attention. Researchers often focus on what students notice mathematically and how this noticing relates to their learning (e.g., Hohensee, 2016; Lobato et al., 2012, 2013). For instance, researchers can examine the mathematical details that students focus on as they solve mathematics problems to make inferences about how they learn. Researchers have found that what students notice can account for differences in the nature of students' mathematical activity (Lobato et al., 2013).

Research reveals that student noticing is an important construct for examining mathematical activity; however, there is limited research exploring student noticing as an intentional process to support students in improving their collaborative practices in mathematics (i.e., the nature of their social interactions with others). In this study, we are interested in how college students notice phenomenon related to the nature of their social collaboration with others while working on mathematics tasks. In the next section, we describe a framework that describes how students might notice their collaborative practices in group-based instruction in mathematics.

3 Conceptual framework

Jacobs and colleagues (2010) constructed a useful and simple framework that provides a lens for understanding how teachers notice students' mathematical activity according to three processes: attending, interpreting, and deciding. In this study, we leveraged these three processes to construct the *student noticing of collaborative practices* framework (see Fig. 1). Similar to Jacobs et al.'s framework, the student noticing of collaborative practices framework provides an analytic lens by which researchers and teachers can examine how students notice their collaborative practices in mathematics classrooms. The student noticing of collaborative practices framework consists of the following: (1) attending to

Fig. 1 Student noticing of collaborative practices



the details of one's collaborative practices; (2) interpreting why one engages in those collaborative practices; and (3) deciding how to improve one's collaborative practices in the future. First, by watching video or audiorecordings of themselves, students attend to specific aspects of their collaborative practices. Then, learners interpret why and how they engaged in such practices. Finally, students decide how they might improve their collaborative practices in the future. For instance, consider a student who, after listening to an audio recording of themselves interacting in a group in mathematics, attends to the fact that they often interrupted other group members. The student may interpret that they did this because they were trying to actively engage with other group members' ideas by responding quickly and attentively; however, the student acknowledges that they interrupted others before letting them finish talking (a phenomena the student was not aware of before listening to the recording). Now, the student decides in the future that they can improve their collaboration by listening to other group members' strategies and providing time for other group members to finish their ideas before responding. This realistic example illustrates the process of attending, interpreting, and deciding, and this specific pattern (interrupting others) was prevalent in our data. The framework informs our analysis by providing a lens to "see" how students notice their collaborative practices during group work in mathematics.

The objective of this study is to explore how college students notice their collaborative practices as they work in small groups in mathematics. In particular, we explore what students attend to in relation to their collaborative practices, how they interpret those attendings, and how they decide to improve their collaborative practices in the future. While this study focuses specifically on college students, we hope that scholars extrapolate the instructional procedures and findings of this study to further explore student noticing in school contexts in the future.

4 Methodology

The study took place at a university in Eastern Canada. Participants were 25 preservice teachers (Grades K–12) who were enrolled in a mathematics education course. While the participants were future teachers, they were positioned as learners of mathematics for the purposes of this study. We were interested in how they noticed their collaborative practices in small groups as mathematics learners; therefore, we refer to the preservice teachers as students where appropriate.

The mathematics education course was a 13-week course designed to support preservice teachers to learn mathematics pedagogy for K–12 classrooms. The course was problem-based, meaning that students learned about teaching and learning mathematics *through* engagement with complex problems and tasks. As part of an assignment in the course, the preservice teachers engaged in a reflection activity wherein they listened to audio recordings of themselves working in small groups on complex mathematics tasks. First, students were placed into groups of 4 or 5 to work on a problem-solving task. The task was to divide an L-shaped figure (see Fig. 2) into four equal sections. Each group's verbal conversation was audio-recorded as they worked on the task for 10 min. Most groups finished within the 10-min time frame. After completing the task, students were asked to listen to the audio recordings of themselves interacting in groups and respond to the following prompts:

(1) Did anything surprise you about how you communicated with others? Describe.
(2) Is there anything you might change about how you collaborate with others in the future?

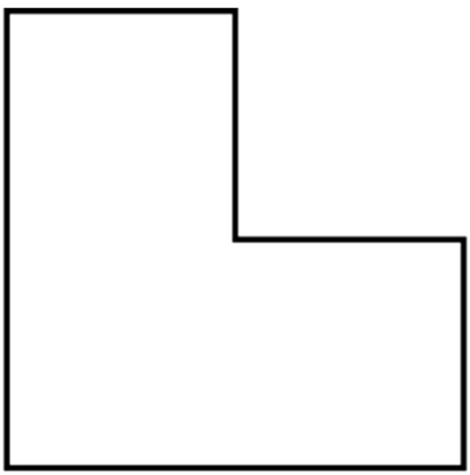


Fig. 2 L-shaped figure task

(3) How do you think your group collaborated? Was everyone equally involved?
(4) What can your group do to improve their collaboration?

We chose these four prompts because they are open-ended, allowing students to write about collaborative practices that were important to them and because they orient students towards reflection and future change. This orientation towards reflection and future change is consistent with our noticing framework (attending, interpreting, deciding). Students were asked to individually write their response to the four questions in no less than 1 page double-spaced.

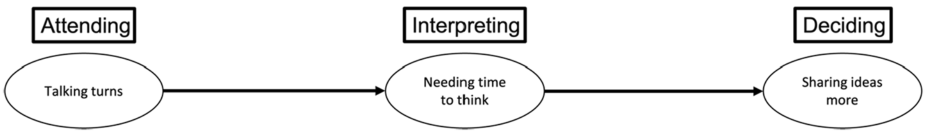
Students' responses to the four reflection prompts were used as the data source for this analysis. To analyze the data, we used a thematic approach (Nowell et al., 2017). First, we coded student's responses to the four reflection prompts according to their attendings, interpretations of those attendings, and decisions for how to improve in the future. Each code contained the marker (attending), (interpreting), or (deciding). The following excerpt shows an example of how we coded students' written reflections:

> **Melissa** I was kind of surprised by how little I spoke during the process of solving the problem [*talking turns (attending)*]… I am someone that likes to express my thought process and help people figure out their thought processes, but by allowing them time to think [*needing time to think (interpreting)*]… In the future, I would probably give a bit more of my opinion and try to involve myself by asking to have the visual [*sharing more ideas (deciding); sharing resources (deciding)*].

We coded "I was kind of surprised by how little I spoke…" as *talking turns (attending)* because Melissa attended to how often she spoke during problem solving. We coded "I am someone that likes to express my thought process…" as *needing time to think (interpreting)* because she interpreted that the reason she spoke infrequently is because she and her group members needed time to think. Finally, we coded "In the future, I would probably give a bit more of my opinion and try…" as *sharing more ideas (deciding)* and *sharing resources (deciding)* because she decided that she wanted to share her ideas more and have more access to the physical group resources in the future.

After the coding process, we examined the data to look for similarities across codes. We clustered similar codes under a single theme (Nowell et al., 2017). For example, we collapsed the codes "inequitable use of resources" and "sharing resources" into a single theme called "sharing resources."

Finally, after constructing themes, we created noticing chains to describe how students' attendings, interpretations, and decisions were connected to one another. For instance, one of Melissa's noticing chains might be:



In the findings section, we only considered noticing chains that were exhibited by two or more students. For instance, we do not consider the noticing chain exhibited by Melissa in the findings because she was the only student who exhibited this complete chain. The noticing chains illustrate how and why students become aware of certain phenomena, as well as how such awareness might support them to improve their collaboration in the future. It is important to note that noticing chains are constructed based on students' written statements and may or may not reflect actual changed behavior. In the next section, we share our thematic findings, along with the noticing chains.

5 Findings

Table 1 reveals the thematic findings for what students attended to related to their collaborative practices in mathematics, their interpretations for why they engaged in those collaborative practices, and how they decided to improve their collaboration in the future.

Table 1 Thematic findings

Theme	Description	Frequency of occurrence
Attending		
Talking turns	Attending to the number of times one spoke during problem solving	18
Sharing ideas	Attending to the number of ideas one shared during problem solving	6
Group rapport	Attending to the quality of the group's collaboration and teamwork	4
Specific math moments	Attending to specific mathematical moments related to the groups' problem-solving efforts	4
Listening to others	Attending to one's propensity to listen to their group members	5
Interrupting others	Attending to how often one interrupted while others were speaking	3
Interpreting		
Personality	Interpreting that one's limited talking turns (for instance) were due to their personality	7
Competence or confidence	Interpreting that one refrained from sharing ideas (for instance) because they viewed themselves as incompetent or non-confident	6
Comparison to others	Interpreting that one refrained from sharing ideas (for instance) because there were other people in the group more capable of completing the task	3
Trouble explaining	Interpreting that one refrained from talking (for instance) because they experienced difficulty explaining their ideas	2
Being recorded	Interpreting that one did not talk much (for instance) because they were being audio-recorded	2
Deciding		
Sharing ideas more	Deciding to share more ideas in future group interactions	6
Listening to others	Deciding to intentionally listen to others' input in future group interactions	2
Equitable Participation	Deciding to promote equitable participation amongst ALL group members in future group interactions	6
Sharing resources	Deciding to share resources (e.g., allowing each group member to see the task sheet) in future group interactions	14
Confidence	Deciding to be more confident in future group interactions	5

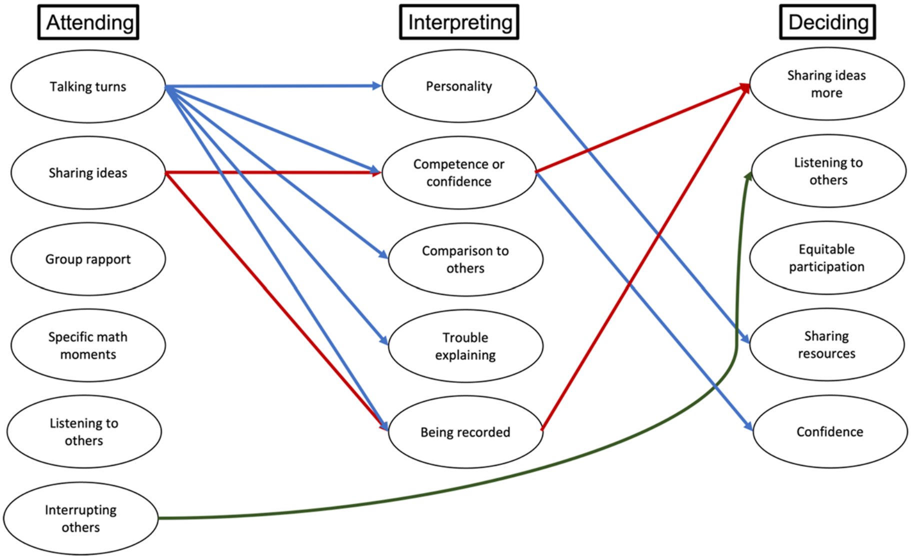


Fig. 3 Noticing chains

Table 1 also reveals how many students revealed each theme within the data (frequency of occurrence). The frequency of occurrence is important for two reasons. First, it provides evidence for the themes we constructed by revealing how many times the theme occurred within the data. Second, it reveals which themes are most prevalent in the data. Figure 3 reveals the noticing chains, which describe how students' attendings, interpretations, and decisions were connected. The coloured arrows represent each connection. For instance, sharing ideas (attending), competence or confidence (interpreting), and sharing ideas more (deciding) are connected by a red arrow and represent a complete chain. Some themes are not part of a chain. For instance, listening to others (attending) does not have any arrows protruding from it. This suggests that students attended to their propensity to listen to others, but they did not provide an interpretation for the quality of their listening. We organize our interpretation of the findings as follows. First, we exemplify the noticing chains (Fig. 3) via selected excerpts from students' written responses. These selected excerpts are intended to represent the larger sample of students. We choose to exemplify the complete chains, or chains that are linked from attending to deciding because these chains represent the process by which students make decisions for future change.[2] Following our exemplification of the noticing chains, we share examples of the remaining themes (Table 1) that are not identified in a complete noticing chain.

5.1 Talking turns→personality→sharing resources

The first noticing chain links talking turns (attending), personality (interpreting), and sharing resources (deciding). Students often attended to the number of times they

[2] We do not exemplify the chain through "being recorded" because this chain is unlikely to be present in group collaboration when audio recorders are not used.

spoke during problem solving, and they interpreted that the frequency of their talk was related to their personality. Surprisingly, students who attributed their personality to the frequency of their talk believed that they could improve their future collaboration via sharing resources as a group. Kaye reflected on her talking turns and how she could improve her collaboration in the future:

> **Kaye** I am very in tune with the fact that I am not the group leader or spokesperson, however, I can be in those roles when I need to be. I always come off as a tad quiet and reserved but I don't feel that this is a bad thing. Every person in a group plays a role and even though I am quieter I still participate and contribute. I am a real thinker and evaluator and tend to observe others in my own thought process. I have a tendency to really examine my thought process and need to feel confident in my answers before I put my idea out there for my group members…We should have only used one piece of paper instead of working in different smaller groups we should have had one person holding the marker and had group member roles taking turns with our ideas.

Kaye did not talk often in her group, and she attributed this to her personality traits of being a "quieter" person and a "thinker." She believed that she could improve her collaboration if her group shared the physical resources (paper and marker).

5.2 Talking turns→competence or confidence→confidence

The second noticing chain links talking turns (attending), competence or confidence (interpreting), and confidence (deciding). Some students attributed the frequency of their talk to their competence or confidence (e.g., "I am bad at math, so I will not share my ideas as much"; "I am not sure if this is correct, so I will not share my idea"). Students exhibiting this pattern decided that they could improve their collaboration by exhibiting more confidence in future interaction. Jean's reflection exhibits this pattern:

> **Jean** What I had noticed that surprised me is how quiet I was. I thought that I knew how to do it but once I saw how we were supposed to solve the problem I realized that I was wrong and then in a way I got embarrassed and stayed quiet for the first part of the problem…I think that in the future I need to be more comfortable and share my thoughts even if they are wrong. I know that it is ok to be wrong and learn from that, and I am ok being wrong but if I know I am wrong then I don't want to share my thoughts. That is one thing that I need to work on. Because if I am wrong, maybe a part of my thought process could spark another idea in the group and get closer to solving the solution…

Jean did not often talk, and he attributed this to losing confidence after he realized he did not understand how to solve the problem. He states that his embarrassment kept him from contributing. Jean decided that he needed to be confident and share his ideas in the future even if his ideas are wrong.

5.3 Sharing ideas→competence or confidence→sharing ideas more

The third noticing chain links sharing ideas (attending), competence or confidence (interpreting), and sharing ideas more (deciding). Some students attended to how many ideas they shared during problem solving. "Sharing ideas" refers to the number of intellectual contributions the student offered—not simply the number of times they spoke. Students attributed the frequency of their sharing to their competence or confidence. Students who attributed the number of times they shared ideas to competence or confidence decided that to improve their collaboration, they should share more ideas. Alisha reflects on her experience:

> **Alisha** When listening back to the recording, I found that though I participated in the discussion frequently, I did not often contribute any ideas of my own…If I am not quite sure about my answer, I usually keep it to myself…In the future, I will try to add my ideas to the group discussions more often.

Alisha realized that she talked frequently during problem solving, but she did not often share ideas of her own. She attributed this to her confidence ("If I am not quite sure about my answer…"). She decided that, despite her limited confidence, she should add ideas to the group discussion more often in the future.

5.4 Interrupting others→listening to others

The fourth noticing chain links interrupting others (attending) to listening to others (deciding). This chain did not have a middle link revealing students' interpretations for why they interrupted others. Some students attended to how often they interrupted others during problem-solving, and they decided that they should consciously listen to their group members in future interaction. Braden's reflection reveals this chain:

> **Braden** I interrupted and cut people off more than I was aware of…As mentioned before, I did cut my group members off, I think in the future I will hold back in adding my ideas or reiterating what my group said in my own words until they are completely finished their thought.

Braden realized that he interrupted others during group work, and he decided that in the future, he should refrain from talking and listen to others until they completed their thoughts.

5.5 Thematic findings

In this section, we share examples of the remaining themes (those that have not been described via the noticing chains above) for each category—attending, interpreting, and deciding. We share one example for each theme, acting as a representative case for other responses that fit under the same thematic heading.

There were six themes categorized under "attending to the details of one's collaborative practice," three of which have not yet been described via noticing chains. They are group rapport, specific math moments, and listening to others. Students who attended to group rapport focused on the quality of their group's collaboration and teamwork. Nathan reflected this in his writing:

Nathan I am also shocked to hear how much fun we had as a group. I remember it being fun but listening back I notice that we all had good rapport and laughed a lot.

Nathan believed his group exhibited good rapport by having fun and laughing a lot. Some students attended to a specific mathematical moment that occurred during group work. That is, they attended to a specific mathematical conversation or problem-solving effort that affected the group collaboration. Daniel revealed this type of attending in his writing:

Daniel Initially, I was stuck on the idea of triangles being the shape to solve the problem. I tried showing my work on our handout, yet it was obvious that this was incorrect. I never said that triangles were the only answer; however, I never deviated from my initial thinking, and when teammates asked if I thought it was triangles I said, "I feel it has to be triangles."

David focused on a specific moment related to partitioning the L-shaped figure into four sections. He believed that the group needed to partition the shape into four triangles, and he "never deviated from [his] initial thinking." He believed this specific mathematical moment affected his group's collaboration. Finally, students attended to their propensity to listen to other group members. Chris revealed this theme in his writing:

Chris In listening to our recording, it was evident to me that I was talking more than almost everyone else, but that I was intentionally taking the time to listen to any opinions or ideas that others wanted to share, which is essentially how I am trying to train myself to behave in those situations.

Chris indicated that while he talked more than others, he also intentionally listened to others' ideas.

There were five themes categorized under "interpreting why one engages in those collaborative practices," three of which have not yet been described via noticing chains. They are comparison to others, trouble explaining, and being recorded. Students' reasons for engaging in particular collaborative behaviors were sometimes due to their tendency to compare themselves to others. Raheema reflects this in her writing:

Raheema The first thi[n]g I did, the moment I knew who my group members were, was that I, unconsciously, identified myself as the weakest/slowest member. I literally asked myself 'what am I doing here'?

Raheema refrained from talking often or sharing ideas in her group, and she seemed to attribute this to her competency in comparison to others. In addition to comparing oneself to others, students also stated that their difficulties in explaining ideas contributed to certain collaborative behaviors. Donnie reflects this in his writing:

Donnie I struggled with explaining why I thought we should try a particular method or give meaningful suggestions to the group, with the rapid development of the group's progress.

Donnie experienced difficulty verbalizing his reasoning to his group, which limited the number of ideas he shared with the group. Finally, students believed that being recorded contributed to their collaborative behaviors. Kourtney's writing reveals this theme:

Kourtney To be honest, I found it intimidating to be recorded, and I think that contributed to how much I offered to the group.

Kourtney was intimidated by being audio-recorded, which limited how much she participated in her group.

There were five themes categorized under "deciding how to improve one's collaborative practices in the future," one of which has not yet been described via noticing chains: equitable participation. Students decided that to improve collaboration in the future, they needed to promote opportunities for all group members to participate. Felicia's writing reveals this theme:

> **Felicia** In the future, I would like to continue growing in my teamwork abilities. I will continue to work on decreasing my tendency to be overinvolved in the task and allowing others to contribute with their talents as well. When there are periods of silence, I will let others fill it sometimes rather than chipping in myself every time. There were a couple of moments when I prompted a teammate; that is something I would like to do more often.

Felicia believed that she needed to allow others to contribute by prompting them to share their ideas. Overall, she wanted to work on her "teamwork abilities."

6 Discussion

In this paper, we constructed a noticing framework named *student noticing of collaborative practices*, and we used the framework as a lens to "see" how college students noticed their collaborative practices as they worked in groups on a mathematics problem. We constructed themes revealing what students attended to while reflecting on their collaborative practices, their interpretations for why they engaged in those practices, and how they decided they might improve their collaboration in the future. In what follows, we discuss the contributions of this study and its relevance for future research and practice in mathematics education.

The first contribution of our study is the conceptualization of a framework (*student noticing of collaborative practices*) for understanding student noticing in contemporary group-based mathematics classrooms. While prior studies primarily leveraged noticing as a construct for analyzing teacher practice in mathematics (e.g., Jacobs et al., 2010; Sherin & Russ, 2014), our study leveraged student noticing as a construct for analyzing *student* practice. Furthermore, the student noticing literature has primarily focused on students' noticing of mathematical activity (e.g., Lobato et al., 2012, 2013), but our study provides a new lens for understanding how students notice *collaborative* mathematical activity. Future research might use the student noticing of collaborative practices framework to analyze collaboration in group-based instruction in mathematics.

The framework provided a unique lens to uncover several interesting findings. First, students seemed to attend to negative aspects of their mathematical collaboration rather than positive aspects. For instance, 18 out of 25 students attended to the number of times they spoke, and their interpretations often suggested that they were not pleased with the frequency of their talking turns. Focusing on negative aspects of collaboration supported students to think about how they might improve in the future; however, it may be beneficial for students to consider both negative and positive aspects of their collaboration while noticing. If students continually focus on negative aspects of their collaboration, it may negatively influence their identity (Heyd-Metzuyanim, 2015). Our findings suggest that students may need support in noticing positive aspects of their collaboration.

The second interesting finding was the "noticing chains" that describe how and why students became aware of certain phenomena, as well as how they envisioned changing their mathematical collaboration in the future. Noticing chains are helpful for future research and practice because they support scholars and teachers in understanding why students engage in certain collaborative behaviors in math class. Prior research reveals *that* students often struggle to collaborate effectively in mathematics (Bishop, 2012; Wood, 2013), but our study provides insight into *why* students might struggle to collaborate effectively in mathematics. For instance, the noticing chain (sharing ideas→competence or confidence→sharing ideas more) reveals that students often refrain from sharing ideas because they believe that they lack the competence or confidence to share ideas. Interestingly, the noticing chains reveal that students can internalize their interpretations (e.g., "I did not share ideas much because I wasn't as smart as other group members") to envision how they might improve in the future (e.g., "I will confidently share more ideas in the future").

The third interesting discovery was the thematic findings which revealed the nature of students' noticing. Students attended to many aspects of their communication including interrupting others, listening to others, and group rapport. These themes reveal that students can notice distinct aspects of their communication while problem solving in mathematics. We also constructed themes related to students' interpretations and decisions for how to respond. These thematic findings may support researchers and scholars toward understanding what and how students notice in relation to group collaboration in mathematics.

The fourth finding of our paper relates to unique patterns of collaborative noticing that are directly tied to group learning in mathematics. As discussed earlier in this paper, group-based instruction in mathematics is distinct from group-based instruction in other content domains, and therefore, it should be studied as a separate entity. Our findings illuminate unique patterns of collaborative noticing that are specific to mathematics. For instance, some students in our data attributed their lack of talking turns to a perception of their abilities in comparison to other group members. Raheema wrote, "The first thi[n]g I did, the moment I knew who my group members were, was that I, unconsciously, identified myself as the weakest/slowest member." Students' propensity to compare themselves to others, while somewhat prevalent in other content domains, is magnified in mathematics classrooms. This is because students often view mathematics as a performance subject where the goal is to score high marks on standardized tests (e.g., Boaler, 2022). Students also attributed the frequency of their talking turns to the difficulty they experienced in explaining their ideas. This phenomenon (difficulty explaining one's ideas) is common in mathematics classrooms. Another interesting theme related to students' attending was "specific math moments." We did not ask students to reflect on a specific mathematical conversation or problem-solving effort, but four students referenced a specific math moment in relation to their group's collaboration. This indicates that students perceive that the mathematical thinking of the group can affect the group's collaboration.

Because of the distinct nature of group-based instruction in mathematics, we suspect that many of our findings will not replicate to other content domains. Nevertheless, researchers might leverage the *student noticing of collaborative practices* framework to study student collaboration in other settings. It would be interesting to understand which findings persist across all content domains and which findings are specific to mathematics classrooms.

The next step for research on student noticing of collaborative practices is to determine whether students' noticing supports them to engage in different collaborative behaviors in the future. For instance, future research should empirically examine whether students who noticed that they used limited talking turns change their behavior to use more talking turns in future group interactions. In fact, we are in the process of conducting small-scale research related to this very phenomenon, and preliminary findings are promising (Campbell et al., 2022). That is, our preliminary findings reveal that noticing can lead to changed behaviors. Still, there is a need for large-scale research to determine whether changed behaviors persist over time. Research should further determine how often students need to engage in noticing to influence their behavior (e.g., once per month; twice per semester).

One primary implication for practice is that teachers can use noticing as a tool for improving student collaboration during group-based instruction in mathematics. Similar to the instructional procedures of this study, teachers can ask students to watch or listen to short video/audio recordings of themselves interacting in groups and respond to reflection questions. We suggest that teachers use short clips (about 10 min) rather than long ones because it is easier for students to reflect meaningfully on short clips. Reflection questions should be related to students' individual collaboration, and it should prompt them to critically evaluate their collaboration (e.g., "What surprised you about how you collaborated with others?"). It remains to be determined how often students should engage in noticing exercises to promote lasting changes in their mathematical collaborative practices.

Student noticing as a tool for improving collaborative practice stands in stark contrast to other teaching strategies that promote effective collaboration in mathematics. Teaching strategies such as constructing group rules (Hofmann & Mercer, 2016; Murphy et al., 2018), assigning group roles (Boaler, 2008; Johnson et al., 2010), attending to group coordination (e.g., Campbell & Yeo, 2022), and explicitly teaching discourse practices (Campbell, 2021; Gillies, 2019; Johnson & Johnson, 2016) are teacher-centered approaches to promoting effective collaboration in mathematics. Student noticing is student-centered because students make decisions for how to improve their collaborative practice via intense reflection. The student-centered nature of student noticing may have more lasting effects on student collaboration than teacher-centered approaches.

In conclusion, this study contributes to research on group-based instruction, specifically as it relates to student collaboration in mathematics. The primary limitation of this study relates to our sample. We used a small sample of college students with the hope that the findings from our sample might extrapolate to other mathematics settings (including school settings). School-aged students may or may not exhibit similar patterns of noticing. Further research is needed. We hope that scholars, researchers, and stakeholders will build on the findings of this study to further improve group-based instruction.

Data availability The dataset analyzed during the current study are not publicly available due to continued analysis and privacy of participants but are available from the corresponding author on reasonable request.

Declarations

Conflict of interest The authors declare no competing interests.

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