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The development of sociomathematical norms in the transition to tertiary exam-oriented individualistic mathematics education in an East Asian context

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Abstract
This study investigates social, mathematical, and sociomathematical norms perceived by college students in an engineering mathematics course and examines the students' sense of mathematics as signals of individual merit. Data sources include a survey and one-on-one interviews with 38 students. The findings help illustrate student perceptions of academic social norms in a large-lecture course represented by the acquisition model of learning in college, detached from communal and collaborative disciplinary practices. Findings provide insights into the local educational context of an East Asian country as a case study when exam-oriented mathematics is institutionalized as normalcy.

Keywords Social norms · Mathematical norms · Sociomathematical norms · Secondary-tertiary transition in mathematics education · Meritocracy · Exam-oriented education system

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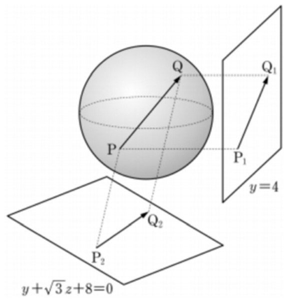
1 Introduction

Research indicates that students typically struggle to adjust to the rigor of college mathematics (Clark & Lovric, 2009; Dorier et al., 2000; Gueudet, 2008). Generally, students engage in more difficult and rigorous mathematics in college than in high school. During the transition, some students feel a new tone in the mathematics they are learning, distinct from what they found in high school. College students are effectively novices, who discover the culture of mathematicians and their communities (see novices in communities of practice described by Wenger, 1998). They become socialized into the disciplinary practices of mathematics, and prepare to contribute to the community. Examining how students embrace the rigor of college mathematics and socialize into mathematics, this study identifies sociomathematical norms in college as an important research topic; the norms are useful in identifying and interpreting how students and professors work together in mathematics classrooms as the learning community of the discipline to develop and sustain their belief systems (Presmeg, 2007). These belief systems might play an important role in students' study of mathematics, academic choices, and social and professional lives.

Extensive research has documented the culture of mathematicians (Bowers, 1985; Burton, 2009; Manin, 2007; Parameswaran, 2010; Wilson & Latterell, 2001), including *students* as novices within the culture (de Abreu et al., 2002; Inglis & Alcock, 2012). The research includes the discipline characteristics and mathematicians' and students' thought patterns, as revealed in mathematical arguments and proofs. One study examined sociomathematical norms demonstrated in a college-level linear algebra course (Rasmussen et al., 2015), describing four interactive constructs: classroom practices, disciplinary practices, individual thinking, and participation. Elrod and Strayer (2018) reported a college instructor's practice of shaping classroom culture to implement productive sociocultural norms, thereby building a community of learners. These studies examined various ways for students to engage in *communal* and *collaborative* mathematical practices and learn to develop and sustain productive norms in college classrooms. However, they did not consider the norms built from (and still evolving from) students' high school experiences and how students perceive these norms in relation to local contexts and cultures in society. Few studies have examined how college students enrolled in an engineering mathematics course with the exam-based large lecture format in East Asian contexts perceive their college mathematics classroom culture, especially using student voices as a source (e.g., Hernandez-Martinez et al., 2011). Little-explored topics include differences compared to high school classrooms; what it means to develop, sustain, or change norms; and how the presence or lack of meritocracy (Clycq et al., 2014) as a social system and sociocultural norms feed into the perception of norms after the transition.

The local context of the study This study aims to investigate classroom norms perceived by college mathematics students in Korea. In Korea, academic credentials function as key signals of individual merit in society; it is widely accepted that a diploma from a top university serves as the best access to top career pathways and high social status (Lew et al., 2011). Thus, mathematics is a key school subject in Korea, and students' mathematics scores in the national college entrance exam (see Fig. 1) directly affect their admission prospects into elite colleges. The term *hagwon* refers to for-profit private tutoring programs (Bray, 2014, p. 384); in 2009, expenditure on *hagwon* represented about 2% of the country's GDP, approximately 11% of the average household income per student (Choi & Choi, 2016; OECD, 2012). Despite increasing educational spending and high passion for

29. 좌표공간에서 구 $x^2+y^2+z^2=4$ 위를 움직이는 두 점 P, Q가 있다. 두 점 P, Q에서 평면 $y=4$에 내린 수선의 발을 각각 P_1, Q_1이라 하고, 평면 $y+\sqrt{3}z+8=0$에 내린 수선의 발을 각각 P_2, Q_2라 하자. $2|\overrightarrow{PQ}|^2-|\overrightarrow{P_1Q_1}|^2-|\overrightarrow{P_2Q_2}|^2$의 최댓값을 구하시오. [4점]



29. Point P and point Q are on the sphere $x^2 + y^2 + z^2 = 4$. Let P_1 and Q_1 be the points of intersection between the plane $y = 4$ and a perpendicular line from the point P and Q, respectively. Also, let P_2 and Q_2 be the points of intersection between the plane $y + \sqrt{3}z + 8 = 0$ and a perpendicular line from the point P and Q, respectively. Find the maximum value of $2|\overrightarrow{PQ}|^2 - |\overrightarrow{P_1Q_1}|^2 - |\overrightarrow{P_2Q_2}|^2$. [4 points]

Fig. 1 Example of a difficult test item (the correct response rate 11%) for high school students in the 2014 national college entrance exam in mathematics

education, emerging research (Lee et al., 2021; Pang & Seah, 2021) has indicated some Korean college students struggle to develop a sense of belonging in mathematics classes and in learning community of the disciplinary practice. It is widely accepted in the Korean research community that Korean students may perform well in mathematics exams while students' attitudes towards mathematics remain negative (see mathematics abandoners and feelings of learned helplessness described in Hwang, 2019; Pang & Seah, 2021). Korea's educational system uses "exam-oriented" (Ro, 2019) mathematics in college admission decisions. The implications of this policy could be that exam scores become a currency of merit that provides access to opportunity and status (Lee & Larson, 2000; Yoon et al., 2021) and college students may perceive mathematics learning as individualistic, competitive work.

Drawing on student perceptions of norms in classrooms, this study analyzes how social, mathematical, and sociomathematical norms combine to create wide-ranging emotional and academic experiences in college mathematics classes against a backdrop of meritocracy in an East Asian context. Research (e.g., Di Martino, 2019; Di Martino & Zan, 2010) has shown that interpretative approaches involving essays, diaries, and interviews can be instrumental in analyzing students' narratives and developing authentic access to their views, allowing the researcher to investigate student beliefs, emotions, and behaviors. This study uses a survey instrument and in-depth interviews to assess college students' perceptions of patterned behaviors and expectations, facilitating a discussion about the reality of college mathematics classrooms. Our research question is: What do Korean college engineering students say about changes in social, mathematical, and sociomathematical norms between high school and college mathematics classrooms? The following question guided the discussion of the results. What does the norm perception implicate in the exam-oriented individualistic mathematics education?

2 Existing research

2.1 Transition from high school mathematics to tertiary mathematics

Research has shown that students encounter difficulties in their transition to tertiary mathematics. The nature of advanced mathematical thinking and students' cognitive discontinuities from school mathematics to more complex symbolism, abstraction, formal reasoning, and proofs characterize undergraduate mathematics classrooms (e.g., Tall, 1991). Regarding cognitive challenges in the secondary-tertiary transition, Selden (2005) described students' difficulties in reconceptualizing their mathematical knowledge. Alcock and Simpson (2002) also described student difficulties in adapting reasonings that had been effective and sufficient at the secondary level but not at the tertiary level.

Alternatively, some research underscored the importance of student affect in the secondary-tertiary transition and reflected on the complex nature of institutional transitions at individual and social levels. To better understand student affect in transition, researchers investigated college students' perceived changes in their learning experience regarding student perceptions of difficulties in transition (Hernandez-Martinez & Williams, 2013; Gueudet, 2008; Gueudet et al., 2016), student identities (Hernandez-Martinez et al., 2011), self-efficacy (Pampaka et al., 2011), and emotional responses to institutional changes (Clark & Lovric, 2008). Gueudet (2008) described students' perspectives in interpreting their difficulties in the tertiary transition. Clark and Lovric (2008) used anthropological theories to conclude that students experience emotional shocks when abruptly separated from secondary classrooms and incorporated into university classrooms—their individual ways of learning are interrupted and changed. Di Martino and Gregorio (2019) interviewed mathematics major students at a highly regarded university in Italy who performed well in secondary mathematics but struggled to adjust to sudden changes in teaching and learning norms in college mathematics. The didactic differences between secondary and university mathematics in Di Martino and Gregorio (2019) include fast paced and intensive college classes, higher expectations of autonomy in study habits, and relatively less support from professors or a lack of faculty interest in student difficulties. In particular, some students who dropped out of their programs (see also Hernandez-Martinez, 2016) mentioned that being compared with their peers lowered their confidence in mathematics and affected their self-worth. The same study also reported that social and academic support (i.e., sharing experiences of difficulty and attending study groups) helped students overcome psychological and academic challenges and served as a key factor affecting student success in tertiary mathematics transition. Successful math students in college experienced "the shift from viewing oneself as part of a competitive student community to a part of a sharing community" (Di Martino & Gregorio, 2019, p. 839).

2.2 Socialization into mathematics

Learning mathematics in a classroom setting is often theorized as a socialization process (e.g., Nunes, 1999; O'Connor, 1998). A student adds new meanings to an old one and builds knowledge through interactions with others or through stimuli in the classroom. Connections between new and old meanings are socially mediated, created, and extended. While socialization is concerned with the cognitive aspects of learning in general, the same term *socialization* can also refer to the process whereby students shape identity and form a sense of belonging in an academic discipline's community (Weidman, 2006). Socialization in mathematics occurs through mathematical learning, which—as has often been observed—helps to mold the novice learner into a mature, professional practitioner. Thus, students learn to talk and think like mathematicians (see O'Connor, 1998) and their sense of belonging to the disciplinary community develops as they clarify their intention to study mathematics. In the academic community, students—being new group members—are expected to learn the membership requirements, including rules or expectations that guide the community members' thinking and behaviors (Weidman, 2006; Winsløw et al., 2014). Additionally, a different construct of disciplinary practices (Rasmussen et al., 2015) is useful in characterizing the academic community and its rules and expectations. Disciplinary practices refer to ways in which mathematicians think and behave in their profession. For example, Rasmussen et al. (2005) listed defining, algorithmicizing, symbolizing, and theoremizing as examples of disciplinary practices among professional mathematicians. In this study, the process by which a student meets the social, cultural, and cognitive requirements of disciplinary practice is referred to as "socialization."

2.3 Norms

2.3.1 Normalcy

Norms in the classroom exist in the form of "behavioral regularities, patterns of sanctioning, and institutionalized practices and rules" (Morris et al., 2015, p. 1). Norms can refer to required patterns of behavior or personal expectations, which are repetitively structured over a long period through interactions (Bicchieri, 2006). Adhering to a norm may mean behaving normally, like everyone else because it is considered abnormal (or "weird," "uncomfortable," or "not okay") to break the norm. Morris et al. (2015) argue that norms exist subjectively in the form of perceived descriptive norms, perceived injunctive norms, and personal norms. Perceived descriptive and injunctive norms function as a device to guide a student's thinking and behavior and an analysis of these two norms can identify cultural dynamics at the micro (how they begin and are implemented) and macro level (how they spread and continue to evolve) (Kredentser et al., 2012; Morris et al., 2015).

2.3.2 Norms in the mathematics classroom

As students find means of functioning as members of their disciplinary practice in the mathematics classroom, they think and behave in adherence to written and unwritten social or mathematical rules, conventions, and expectations associated with the discipline. All members become a collective social unit that develops, negotiates, or reinforces (Cobb, 1999; Saxe et al., 2009). In the mathematics classroom, these norms are categorized as social, mathematical, and sociomathematical (e.g., Cobb & Yackel, 1996; Sekiguchi, 2005; Voigt, 1995). They can be influenced by a specific teacher or classroom; however, this study hypothesizes that these norms extend beyond the school/institution or mathematics as a "discipline."

Social norms In social situations involving human activities, people (actors) refer to rules that guide their thinking and behavior (Cialdini & Goldstein, 2004). This study defines social norms as the rules or expectations that guide students' academic thinking and behavior when interacting with their peers and the instructor (e.g., justifying opinions or interpreting peer ideas in Cobb & Yackel, 1996). Teachers and peers may either approve or disapprove of behaviors under a social norm (Terry & Hogg, 2001; Yanovitzky & Rimal, 2006). However, a social norm is not necessarily reflective of mathematical content.

Mathematical norms Mathematical norms relate to the rules or conventions by which one approaches mathematics through reading, writing, and speaking. This is widely referred to as "professional norms," which is similar to metaknowledge and forms important cultural knowledge of mathematical activities (Zandieh & Rasmussen, 2010). Sekiguchi (2005) used "mathematical norms" to describe standards in mathematical activities, but the term was indistinguishable from sociomathematical norms. In this study, mathematical norms are operationally defined as norms that participants perceive as injunctive norms for communicating mathematics, through course texts or instructors' speech or writing.

Sociomathematical norms In mathematics educational literature, "sociomathematical norms" describe what are collectively approved to be mathematically valid, useful, effective, and efficient norms in the classroom setting (Cobb, 2000; Yackel & Cobb, 1996; Yackel et al., 2000). A primary assumption underlying sociomathematical norms is that mathematical learning entails the transformation of individual construction through "mathematical enculturation" (Bishop, 1988). Students and instructors collectively evaluate mathematical arguments/solutions to develop a consensus about the quality of mathematical reasoning as valid, sound, efficient, complete, complex, similar, or different (Yackel et al., 2000). Sociomathematical norms represent the process of intra-objectifying mathematical arguments/solutions through "joint labor" between the instructor and students in the classroom. Radford (2016, p. 5) describes it best:

> The joint labor-bounded encounters with historically constituted mathematical knowledge materialized in the classroom common work are termed processes of objectification. … [The students and teachers] produce subjectivities, that is to say, singular individuals in the making. This is why, from this perspective, processes of objectification are at the same time processes of subjectification.

Yackel and Cobb (1996) suggested that sociomathematical norms are not prescribed rules introduced into the classroom from the outside; rather, these shared understandings are shaped by and evolve alongside students and instructors through classroom discussions. Further, an established link appears in college mathematics education literature among student identity (Wood, 2013), the sense of belonging (Solomon, 2007), and the productive mindset (Boaler, 2016). This may indicate that classroom norms and their relationship to socialization experiences (van Oers, 2001)—whether successful or not—have substantial importance for tertiary mathematics education.

2.4 Situating the study

This study uses the abovementioned theories of socialization and the three categories of norms in the classroom as a framework. Through this framework, we posit that students and instructors may mirror or develop various social norms broadly for teaching and learning mathematics. That said, while our assumption is a local examination of the norms in the case study (Yin, 1994), it could inform specific cultural and collective learning experiences of high school and college mathematics. Further, it allows for the sociocultural assessment of

Table 1 The operational definitions of the three norms

Norms	Definitions	Students' sample comments in the study
Social norms	Rules and expectations relative to student belief system towards others in the classroom and learning through interaction	"If I fail on a test, I think it is my fault. Studying is like a battle with myself." (ST K28-questionnaire)
Mathematical norms	Rules and expectations relative to student belief system specifically about mathematics	"[Mathematics] requires us to memorize problem types and the best possible solution in exams." (ST L4-questionnaire)
Sociomathematical norms	Rules and expectations relative to student belief system towards others teaching and learning mathematics as a community	"A theorem is there for us to understand. Nobody can prove it, of course. Then the professor proves while we watch him. That's how it works in the math class—high school teachers teach concepts, and we do problems and college professors teach us proofs and we get lost." (ST K2-interview)

the transition problem—how social and academic experience in the mathematics classroom helps shape students' senses of who they are (*student identity*) and how they engage in learning, how collective experience (or a lack thereof) supports student learning to overcome challenges (*the productive mindset*), and how classroom cultures and contexts enable students to participate in learning and develop a sense of community (*belonging*) in college mathematics classrooms. We argue that student voices describing their perceptions of the three categories of norms in their mathematics classrooms could best construct a critical narrative about social interactions in exam-oriented individualistic mathematics education in East Asia. Here, "exam-oriented individualistic" refers to a meritocratic educational system where the individual student's mathematics exam scores serve as merit. This will include interactions within academic and social normative contexts for students in two distinct but closely related classroom communities (i.e., high school and college) and show how collective classroom practices in college are closely connected (or not) to the disciplinary practices in mathematics.

Social norms, mathematical norms, and sociomathematical norms are operationalized as indicated in Table 1. We assume that the *mathematical* norms in this study should be primarily about one's engagement with the subject, not involving peers through interaction. We advance a comprehensive version of the notion of sociomathematical norms, not limited to normative aspects of mathematical argumentation (i.e., Yackel & Cobb, 1996) and to discussions in the college mathematics course. Rather, the norm could be useful when it refers to the perceived rules and expectations that students find relative to a belief system toward others teaching and learning mathematics as a collective unit.

3 Methods

3.1 Participants

College students (total = 292: sophomores and juniors; 53 females and 239 male) from a private highly selective university in a large metropolitan city in Korea participated in the study. The university has a homogeneous student body at similar academic and socioeconomic levels. The study was conducted over three semesters (five course sections, from Spring 2019, Fall 2019, and Spring 2020) in the course Engineering Mathematics 3 as part of a grant research program to improve student retention and learning in introductory engineering courses. All students were enrolled in STEM programs at the university and had completed Engineering Mathematics 1 and 2. The study recruited voluntary participants (292 students from 414 registered students) with non-disclosure agreements and a beverage coupon was offered upon completion of the study. The course's primary teaching mode was lecturing with recitation sessions and the syllabus covered topics in ordinary differential equations (ODEs), linear algebra (vectors, matrices), and vector calculus (differentiation and integration of vectors), all of which have applications in engineering fields.

3.2 Data collection

Our data includes the participants' written and verbal statements in response to a questionnaire ($n=292$) and semi-structured one-on-one interview ($n=38$). The questionnaire presented three prompts for response: (1) Describe *mathematical* norms in your engineering mathematics course. How are they different from your high school math classes? (2) Describe *social* norms in your engineering mathematics course. How are they different from your high school math classes? (3) Describe *sociomathematical* norms in your engineering mathematics course. How are they different from your high school math classes? For each, the participant was asked to indicate whether the same norm had also been implemented in high school.

Before taking the questionnaire, the participants watched a video clip (runtime 12 m:33 s) describing the three norms. This information was necessary as, in our pilot study, most participants were unclear about the meanings of the norms, and we wanted the participants to have a shared baseline knowledge. We ensured that the video content had no verbal or non-verbal cues including examples that might influence participants. In the video, social norms were explained (2 m:44 s) as a set of written or unwritten rules, guidelines, or expectations regulating general academic thinking or behaviors for peer or instructor interactions in the classroom; mathematical norms (1 m:37 s) were described as a set of rules or conventions regarding engagement in mathematics through reading, writing, and speaking mathematics; and sociomathematical norms (4 m:56 s) were described as collectively evaluating mathematical arguments/solutions in the classroom setting and developing consensus about the quality of mathematics thinking and reasoning as valid, sound, efficient, complete, complex, similar, or different. Additionally (3 m:16 s), the participants were told that describing norms could help to interpret the meaning of classroom learning as individuals or as a community and to understand how one's beliefs, attitudes, and values might develop over time in the community.

Individual student interviews were arranged when (1) written responses to survey items needed clarification or elaboration, (2) verbal responses were preferred or requested, and (3) conflicting responses were provided by the same respondent. Initially, an invitation was offered to 51 students, and 38 interviews via Zoom or in-person were completed successfully. The mean interview time was 32 min (range 17–66 min). Our probing questions during the interview included opening questions such as "What do you remember about norms in high school and college?" and "What has changed in your idea about norms in the math class since you finished the class?", and follow-up questions such as "Did you accept the norm?", "Is it a good/bad norm for you?", "What happened?", "How so?", "What is your guess about where the norm has come from?", "Can you tell me more about …?", and "Is it your own idea or do you think your peers or professor would agree?". The interviewer actively listened to the interviewees' responses to help them feel comfortable sharing their ideas and experiences.

3.3 Data analysis

A ratio in percentage of each norm was calculated. First, we calculated the percentages of respondents who reported a norm in college. We also calculated how many of the respondents who reported the norm also identified it in high school classrooms. Then, we determined major norms in the college classroom by the majority of the respondents, ranging from 83% to 48% of respondents (see Tables 3, 4, and 5). Norms reported by 20% or fewer respondents were excluded from further analysis.

The study analyzed written data from open-ended survey items and transcribed audiotaped interview data. Following the procedures of analytic induction (Taylor & Bogdan, 1998) and thematic analysis (Boyatzis, 1998), the qualitative data were analyzed to identify norms, propose themes, and decide on final themes. For example, participants' descriptive or declarative statements about a norm (e.g., "we should," "I think the class did," "everybody did") were reviewed independently by three researchers to describe the main idea with preliminary descriptive codes (e.g., "valuing test scores," "math thinking is important," "formula is important," "teachers explain theorems," "students are competition-driven"). The descriptive codes were then sorted and grouped with proposed themes (see Table 2). Disagreements were resolved through discussions regarding thematic points that emerged and final themes were decided through consensus.

Unclear statements were clarified during the interview to improve the accuracy of the codes and to refine the context for our interpretation of the themes. For example, the statement "my professors were like a human textbook" was initially coded as "sociomath; teachers use textbook" and revised later as "sociomath; teachers promote formalism" because the participant added, "my professors always wanted the answers to be perfect in logic, symbols, quantifiers and all; he was just like another textbook that presents theorems and facts with little feeling or emotion."

4 Analysis of students' perceived norms

We present identified norms from students' responses in the open-ended survey items and interview data regarding social norms, mathematical norms, and sociomathematical norms.

4.1 Social norms

Our analysis initially yielded 38 microsocial norms, which were eventually merged with similar norms. Five major social norms (Table 3) were chosen from 16 norms. These social norms are most strongly perceived by the participants as the rules and expectations in their mathematics courses from their college classroom experiences. The most prevailing (82%) social norm in the participants' experience as mathematics students was that their highest priority in going to school was to achieve academic excellence. A representative student

Table 2 Examples of participant comments and final themes from the thematic analysis

Norm	Representative participant comments (translated)	Extracted codes	Proposed themes	Final themes
Social	"It was something I had to understand at home. I think I basically retaught myself." (ST O3-questionnaire) "If I fail on a test, I think it is my fault. Studying is like a battle with myself." (ST K28-questionnaire) "Teachers do their job. The class was like a stage where I performed the art of my brain power. The show was entirely up to me and my skill levels." (ST J2-interview)	Review task at home and alone Self-sense of accountability Study as battle Divide work between teachers and students Individual excellence	Students have a strong sense of individual responsibility in studying Students consider studying to be an individualistic action	Accepting individual responsibility to develop knowledge and skills
Mathematical	"We had to memorize the type of problem and memorize the shortcut with formulas to save time and do it fast and correctly in the test." (ST L15-questionnaire) "I admired those who could solve such a complex problem three or four times faster than me. I began to wonder about what their secret is." (ST KA4-questionnaire) "It didn't matter how well you understood the concept front or back. If you can't solve a problem fast and correctly in a test, you are a loser after all." (ST K32-questionnaire)	Memorize problem-solving methods Using formulas as shortcuts Solving exam problems quickly and correctly is an important skill Self-efficacy relative to problem-solving speed and accuracy	Students consider mathematics in terms of solving problems quickly and precisely with memory Students develop self-efficacy relative to exam-based mathematical performance	Performing fast and precise calculations

Table 2 (continued)

Norm	Representative participant comments (translated)	Extracted codes	Proposed themes	Final themes
Sociomathematical	"The math class always starts with a theorem. While I am confused with notations, you have a wave of graphs, examples, calculations, and complex ideas." (ST PA1-questionnaire) "Professors like to recall theorems and start explaining usually with proofs. That's when I am totally lost. Sometimes drawing a graph helps, but most of the time the graphs make it worse." (ST H1-interview) "It is almost predictable that theorems are presented to us, and the professor proves it or/and gives examples. The book has related simple exercise problems with graphs." (ST L5-interview)	Theorem as the main part of a lesson Excessive or extensive learning material in limited time Proofs and graphs of a theorem as intellectual challenge Theorem, examples, graphs, and exercise problems as the main staple of the class	Theorem is the most essential part of learning mathematics in college Theorem, examples, graphs, and exercise problems combine to create students' learning experiences in college-level mathematics	Mathematical presentation beginning with a theorem followed by examples, including graphs and exercise problems

Table 3 Ratio of social norms perceived by students in the classroom

Description of the norm: In (any) classroom/school …	College	High school*
1. Setting the highest priority as academic excellence	82%	100%
2. Determining one's status in the classroom based on test performance	80%	100%
3. Enforcing compulsory class attendance	68%	86%
4. Accepting individual responsibility to develop knowledge and skills	65%	51%
5. Identifying family wealth as a key factor to gaining academic resources and high performance	64%	72%

*The percentage expresses the ratio of those students indicating a specific norm for high school in the sample of those who indicated the same norm in college.

comment was "if you ask me why I go to a math class every day, it would be doing well in the exam and getting a top course grade" (ST L2-interview). The high school percentages ranging 51%–100% indicate the ratio of participants who agreed that the pertinent college norm was also valid in their high school classroom. The fourth norm, "Accepting individual responsibility to develop knowledge and skills," means that the participants felt that the study of mathematics is a personal endeavor, executed by individually studying a textbook or watching extra video material, rather than attaining knowledge through the resources, support, and communication available from the classroom community. A representative student comment was "[I believe] one very unique aspect of learning math is you are on your own, it is a lonely battle to understand your math book—sometimes you are very lucky to find a video on the Internet that does a good job of explaining the concept you're confused about" (ST P5-interview). The fifth norm, "Identifying family wealth as a key factor toward gaining academic resources," indicated that the participants felt they could benefit from for-profit tutoring services, but their families could not afford the service while their wealthier peers could. A representative student comment was "some say all you need to study math is your clear mind, paper, and a pencil. I dare to disagree. I think what you really need is to be born into a rich family who can find the best tutor out there who can help you with your homework and tests. I think you would be able to guess my last test score if I tell you how much money my parents make in a year" (ST K6-interview).

4.2 Mathematical norms

Our analysis initially yielded 21 mathematical norms. We merged similar norms to produce a final list of five (Table 4). The most prevailing mathematical norm amongst participants was that students were expected to perform rapid calculations. Notably, the fourth and fifth norms have a high college ratio and a low high school ratio, indicating that the pertinent mathematical norms apply to college mathematics but not necessarily to high school mathematics. The fourth norm, "Theorems have different uses—to be proved or useful in problem-solving," means that participants noticed some theorems were useful in engineering problem-solving tasks, but they were not required to study to prove them; and vice versa. Referring to a test problem, one participant stated, "I didn't know how to prove the Gershgorin circle theorem and the similarity theorem … all I had to know was how to calculate the radius and draw the discs. And if the discs do not overlap, I know the eigenvalues are distinct… not exactly sure why similar matrices have the same eigenvalues, but we have to memorize these things to solve exam

Table 4 Ratio of mathematical norms perceived by students in the classroom

Description of the norm: In mathematics …	College	High school*
1. One should perform fast and precise calculations	83%	92%
2. One should master notations and formulae	80%	90%
3. Graphs of functions should be memorized	76%	76%
4. Theorems have different uses—to be proved or useful in problem-solving	51%	10%
5. Proofs begin by stating the definition	51%	5%

*The percentage expresses the ratio of those students indicating a specific norm for high school in the sample of those who indicated the same norm in college.

problems" (ST K3-interview). For the fifth norm, a representative student comment was "It took me only two, three math classes in college to realize that all theorems start with definitions. My high school math teachers never asked us about definitions from a theorem, maybe some math vocabulary, of course though. These days, I pay more attention to definitions and notations when I get confused with a theorem, it is like going back to the basics" (ST K6-interview).

4.3 Sociomathematical norms

Our analysis initially yielded 43 sociomathematical norms; we merged similar norms to produce a final list of nine (see Table 5). The most prevailing sociomathematical norm was that mathematical presentation (i.e., teacher explanations) in the classroom should begin with theorems, followed by examples including graphs and exercise-based problems. Notably,

Table 5 Ratio of sociomathematical norms perceived by students in the classroom

Description of the norm: In the math classroom …	College	High school*
1. Mathematical presentation begins with a theorem followed by examples including graphs and exercise problems	72%	30%
2. Students compete in exams by solving complex mathematical problems; students' knowledge is evaluated by professors (teachers)	70%	90%
3. Showing worked-out solutions is preferred above verbalizing thinking and reasoning in the classroom discussion	70%	90%
4. Problem-solving is posed by the professor (teacher) and students solve the problems after the class	65%	33%
5. Professors (teachers) attend to students' accurate use of definitions and math terms	63%	60%
6. Student participation in the classroom is mostly responding to professors' (teachers') questions	57%	90%
7. Technology is useful in visualizing complex graphs of functions	57%	75%
8. The complete solution set is useful in learning to structure mathematical arguments	48%	33%
9. Memorizing definitions is a popular way to check student understanding	48%	85%

*The percentage expresses the ratio of those students indicating a specific norm for high school in the sample of those who indicated the same norm in college.

the second and third norms were commonly observed in both college and high school classrooms, while the first norm was more relevant in the college classroom. Similarly, the ninth norm, "Memorizing definitions is a popular way to check student understanding," was more relevant in the high school classroom. The fourth norm, "Problem-solving is posed by the professor (teacher) and students solve the problems after the class," means that the instructors rarely solve problems in the college classroom to show students but still actively assign challenging problems as homework. Here, students may not have as much time to solve problems with the instructor as they did in high school. Furthermore, a similar ratio, if low, indicates different norms between high school and college. For example, the eighth norm, "The complete solution set is useful in learning to structure mathematical arguments," as a college norm (48%) means that the participant needed a more detailed example as they learned to write proofs while the same norm did not apply as much in high school mathematics (33%). Presumably, this was because high school mathematics did not require them to write proofs, rather than that the participants did not need detailed solutions in high school:

> I always appreciate worked-out solutions and I learn a lot from the way the author put together the solution with one math idea leading to the next and so forth. But I began to appreciate more of those in college texts because textbook examples are never enough for me to master the logic and the notation. … The irony is, when I don't need a good solution in high school, the textbook provides too many [solutions], when I actually need a good solution in college, the textbook doesn't bother to offer one in the back. (ST K7-interview)

5 Changes in students' perceived norms from high school to college

5.1 Student perception of social norms

Our findings concerning student perceptions of social norms (i.e., norms 1 and 2, see Table 3) suggest that the notion of classroom learning as "credentialing" academic excellence remains unchanged from high school to college (Collins, 1979). These norms have potential implications for using mathematics test scores as a signal of student merit. A possible consequence of such merit-centric practice is that students gain status in the classroom mainly through their test scores, including attendance grades (i.e., norms 2 and 3). This starkly contrasts research on student participation in collective knowledge creation (e.g., Cobb et al., 2001) as learning in the classroom community (see the participation metaphor for learning/knowledge in Sfard, 1998; Paavola & Hakkarainen, 2005; Lave & Wenger, 1991; Krummheuer, 2011). This finding confirms the idea of knowledge as a social commodity (see the acquisition metaphor of learning/knowledge elaborated in Sfard, 1998; Paavola & Hakkarainen, 2005) from which a high score is a token of individualistic ethos (cf. egalitarian and cooperative approach described in Gosine & Islam, 2014) to preserve intellectual capital to build status. Such strongly held belief in school meritocracy could serve as a system-justifying tool (Wiederkehr et al., 2015) with a deficit view toward low-performing students. This study confirms a clear counterposition to students' belief in math scores as meritocratic means of equal opportunities in any school system: students perceive family wealth (see norm 5) as a tool enabling access to tutoring outside of the classroom (see also Park, 2008) and even associate it with prestige (Lee & Shouse, 2011). This finding conflicts with Quaresma (2017) about high-performing students ($n = 24$, ages 12 – 17) in flagship public schools in Chile. The students believed excellence is a function not of socioeconomic factors but efforts and work alone. In this study, the students thought that merit did not entirely contingent upon individuals' mathematical talents and efforts.

Notably, students felt more individual responsibility (i.e., studying alone) in college than high school (see norm 4). It is unclear from the data exactly what normative aspect of high school mathematics instruction contributes to this perception. Possibly, Korean high school students remain in the same classroom throughout the day, generating a sense of community (while college students meet briefly only during college classes). What is clear is that such perception of the norm in the college classroom indicates students exercised individual efforts to understand material outside the classroom community, facilitated through venues such as for-profit education and training providers. Participants stated, "if I don't get the concept in the class, which is the case most of the time, I have to watch paid lecture videos or YouTube clips about the material alone at night" (ST L58-questionnaire) and "There is nothing much I get out of this class. I feel like I teach myself or learn with my tutors and videos, go to the class to sign in for attendance, and take the exam to get the professor's blessing."

5.2 Student perception of mathematical norms

Our findings about student perceptions of mathematical norms suggest that the common high school notion of mathematics as procedures and calculations (i.e., norm 1 in Table 4) persists in college. One participant stated, "I felt nice about solving one application problem symbolically on a test with an incredibly long solution, but I am still unclear about the concept" (ST O1-interview).

A relatively low perception (vs. norms 1 and 2) regarding the need to memorize the graphs of functions in high school (norm 3) suggests that students may feel that college mathematics presents advanced functions with more complex graphical representations. Participants also mentioned the role of definitions (i.e., Vinner, 1991) in mathematical arguments as the normative aspect of disciplinary practice, relative to the low perception of the practice in high school mathematics (norm 5). Participants indicated their perception of theorems as proof tasks or as useful formulae in problem solving, but they perceived little about the uses of theorems in high school mathematics (norm 4). The second norm, in tandem with norms 1 and 3, indicates that participants approached the mathematical norm with a traditional *learner*'s perspective, whereby they expect to perform and be evaluated. The students described the norms referring to their actions such as memorizing (norm 3), doing (norm 1), using (norm 4), and mastering (norm 2).

5.3 Student perception of sociomathematical norms

Our findings about student perceptions of sociomathematical norms suggest that students are cognizant of differences in instruction styles between high school and college, especially regarding how information is presented. In the participants' experiences, college courses are centered around the discussion of theorems, with an initial formal definition followed by examples, diagrams, and applications (norm 1). We note that the ratios of norms 1, 2, and 3 in college are similar, but for high school norms, norm 1 stands out. This suggests that the first norm is perceived to be a mark of change from high school to college in students' classroom experiences. College professors appear to have a strong presence in the class as experts/authority figures, which high school teachers lack (norms 4 and 5); the norms indicate that the professors challenge students with difficult problems (norm 4) and make judgmental statements (norm 5). The eighth norm, "The complete solution set is useful in learning to structure mathematical arguments," implies that students recognize the normative aspect of making mathematical arguments as a convention of college mathematics. We also suspect that the participants had few opportunities (or little patience or support from the instructor) to reorient their intellectual faculties to bridge the gap between the practico-technical block and the technologico-theoretical block (the duality mentioned in Winsløw, 2008 and de Vleeschouwer, 2010 as a major factor of student difficulty in secondary–tertiary transition). One participant noted that "in high school, I never had to write a math argument on my own. I did some proofs but mostly it was filling in the blanks with expressions. It is not that they taught proof writing or writing good solutions in college. I just had to buy worked-out solutions and try to imitate their writing style (ST R1-interview)." The role of the instructor as described by van Oers (2001, p. 59)—to "demonstrate the tools, rules, and norms that are passed on by a mathematical community"—was withheld in this instance, and the worked-out solutions (norm 3) could not fill the void. Using technology in the classroom could have supported student learning (norm 7), but the study confirmed that students used technology individually. One participant said, "I watched [my classmate] use WolframAlpha to plot a particular solution to the PDE and thought I better look at the technology. […] Nobody told me about it, but I finally began to understand the general solution and particular solutions as graphs (ST P3-interview)". Another participant added, "my tutor was a PhD student and he showed me how to make a moving graph of the surface in GeoGebra with gradient vector del *f* of *x* and *y*. That helped me understand what it meant when they say the del *f* is the direction of steepest ascent. […] I wish I could show this to the class, but there is no space for that because the class is just too big and the professor always has just too many topics to cover" (ST P2-interview).

6 Our interpretations

Initially, the study embarked upon an investigation to explore student-perceived norms (social, mathematical, sociomathematical), seek a better understanding of college students' socialization into mathematical practices, and find ways to support student learning in the transition between high school and college mathematics. The participants' norms confirmed three out of four elements of the professional mathematicians' disciplinary practice in Rasmussen et al. (2005): (1) defining (math norm 5 and sociomath norm 9), (2) symbolizing (math norm 2), and (3) theoremizing (math norm 4 and sociomath norm 1). However, the participants demonstrated little sense of learning through collective inquiry in the classroom, with "[the] invitation to attend to the sensuous manners in which teachers and students bring mathematical ideas to the fore and produce mathematical meanings (Radford, 2016, p. 2)." We conclude that the norms we found are very telling about the participants' perceptions of social norms and their individualistic ethos about studying engineering mathematics: the purpose of mathematical learning is to gain high knowledge and skills in the subject so they can perform better on exams, the results of which signal their personal merit and status. Bauersfeld (1980) wrote:

> [Mathematics] is mediated through parents, playmates, teachers. The student's reconstruction of mathematical meaning is a construction via social negotiation about what is meant, and about which performance of meaning gets the teacher's (or the peer's) sanction. How can we expect to find adequate information about teaching and learning when we neglect the interactive constitution of individual meanings? (p. 35)

In this study, a different sociomathematical norm from Cobb and his colleagues emerged; good study habits at the individual effort level (outside the classroom community, but potentially with the support of paid tutoring) "[constituted] individual meanings" (Bauersfeld, 1980, p. 35) and served as a major source towards acquiring mathematical knowledge. The participants may have internalized the acquisition of merit and status in the classroom—poignantly captured in the metaphor of knowledge acquisition (Sfard, 1998)—as the key (perhaps the only) purpose of taking the class. The emergence of different social and sociomathematical norms in the study from those in Cobb et al.'s research indicates that students' classroom learning relates little to interacting with peers, creating common knowledge, or contributing to the class (as a community of inquiry, Biza et al., 2014). Alternatively, there are ample perceived norms that illustrate learning as a competition to acquire knowledge and individual merits. We conclude this is the main characteristic of exam-oriented (meritocratic) individualistic mathematics education in Korea compared to the communal or collaborative nature of learning mathematics in the literature. Relevant examples demonstrating this include "my classroom status came from my high test scores" (ST J2-interview), "it was my merit, my effort to do well in the exams" (ST H9-questionnaire), "there was a clear divide between the rich classmates and those from poor families" (ST K65-questionnaire), "brutal competition in exams" (ST L5-interview), and "my participation in class helped no one because it was done basically to boast how much more I knew than my classmates" (ST P1-interview). The courses used exam and test grades as the primary evaluator of student understanding. Many of the norms indicate good study habits and academic skills as positive behaviors and desired skills in the mathematics classroom. For example, the perceived norms suggest that the participants' social status in the classroom reflected their test performance and study habits, granting them identity as model students. Some participants said, "I am a model student, and my professor speaks highly of me in the class about my performance in the test" (ST H1-interview) and "my classmates like me since I am a hard-working student" (ST K5-interview).

7 Conclusions and implications

Our findings confirmed student struggles in college mathematics (Alcock & Simpson, 2002; Selden, 2005) and shed light on the students' reconstruction of social meaning of studying mathematics, contributing to the literature (Clark & Lovric, 2008; Di Martino & Gregorio, 2019; Gueudet, 2008) regarding student perceptions of tertiary transitions in mathematics. First, our analysis revealed that students who have long endured competitive learning environments where the sole criterion for excellence is timed complex problem solving (e.g., Leung, 2001—about the East Asian educational context—and more specifically Dawson, 2010—regarding the case of private tutoring) perceived their mathematics learning detached from disciplinary practice. This points to the development of different kinds of sociocultural norms in the school context, which may not truly reflect "the cultural historical dimension of [mathematicians'] practices" (van

Oers, 2001, p. 59). Since East Asian countries share Confucian pedagogic commonalities such as test-driven educational systems (Zeng, 1999), it is important to keep investigating student perceptions of sociocultural norms among students in other East Asian countries and students with different cultural backgrounds. Second, this study describes different sociomathematical norms in exam-based large engineering mathematics lecture courses from those found in the sociomathematical norm in Cobb et al. (2001). Meritocracy, as found in the social norms of exam-oriented undergraduate engineering mathematics, persisted in collegiate mathematics and served as a tool for the privilege which might hinder establishing the community in classrooms, especially when departments believe in the value of collective learning community.

We caution that our findings should not be generalized to all students learning collegiate engineering mathematics courses, let alone pure mathematics. The cultural context of this study is engineering mathematics students at one particular university and cannot be described as a "case" of an East Asian country, as Korean, or the subject discipline/community as that of "mathematics." We also note that the instructors' opinions on sociomathematical norms could have provided different viewpoints in balance with the students' voices. We recognize this is an important limitation of this study. Nonetheless, our findings can provide a critical understanding of factors hindering students' productive socialization into mathematics in exam-based individualistic contexts and wider cultures and communities. Further, our findings are not meant to draw a negative picture of traditional lecture-based instruction, which has proven effective in covering extensive materials in a relatively short period and in organizing complex information so that students can quickly acquire the necessary knowledge, relative to reform-based mathematics instruction, with productive and authentic (not prescribed) sociomathematical norms. We also note that this study does not suggest that the *large* lecture class format produces negative (mathematical or social) outcomes in students. Indeed, mathematics is a multi-disciplinary field with multiple cultures/communities and a variety of (sometimes conflicting) purposes. Instead, this study highlights how students speak of the kind of mechanisms—whether culture or policy—that fail to empower the college classroom community to implement student-centered, socially constructed agentic learning of mathematics in college and beyond.

We recommend that future research should produce case studies on classroom communities in college with sociomathematical norms that support or hinder a successful transition from high school to college (e.g., the teaching of an undergraduate abstract algebra course described in Fukawa-Connelly, 2012) and facilitate student learning so students can develop a balanced perception of learning as knowledge acquisition and participation. To broaden our findings, future studies may compare the perceived norms between other factors, including undergraduate vs graduate students, other mathematics majors, other majors in the engineering field, and arts and humanities majors. This is a step in the right direction for our field to better understand the rich and complex nature of some general and particular configurations and norms of one of many cultures and communities of mathematics.

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Data availability The datasets generated during the current study are available from the first author on reasonable request.

Declarations

Conflict of interest The authors declare no competing interests.

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