



Teaching mathematics with technology: a multidimensional analysis of teacher beliefs

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Abstract

Teacher self-efficacy beliefs, epistemological beliefs, and beliefs about teaching with technology are regarded as crucial factors for teaching mathematics with technology. However, there is a lack of research that investigates these beliefs while taking into account the multidimensionality of the constructs. In this cross-sectional study with $n = 198$ upper secondary in-service teachers in Germany, we used multidimensional scales to measure teachers' i) beliefs about teaching with technology, ii) self-efficacy beliefs, and iii) epistemological beliefs. Furthermore, teachers were asked to self-report on their iv) implementation of technology with respect to different modes of technology use. The results uncover differential associations among the constructs and identify sub-dimensions that are especially central. In particular, three clusters of sub-dimensions can be reconstructed. One cluster reflects a broader set of sub-dimensions and is related to a more integrated and constructivist implementation of technology. In this cluster, self-efficacy can be identified as a central construct. A further cluster is related to using technology to support multiple representations which turned out to be independent of many other sub-dimensions. Finally, the third cluster comprises sub-dimension that can be interpreted as less central. This cluster in particular contains teachers' beliefs about the detrimental effects of teaching with technology, which points out that beliefs about the risks of technology use are less central than beliefs about the potential benefits of technology. The results can inform more differentiated approaches to teacher professional development related to teaching mathematics with technology.

Keywords Technology · Beliefs · Epistemological beliefs · Self-efficacy · Professional development

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1 Introduction

Research has shown that teaching with technology (e.g., function plotters or computer algebra systems (CAS)) can enhance the learning of mathematics, for example, by offering the potential to dynamically link different forms of representation and supporting more constructivist teaching approaches (e.g., Ball et al., 2018; Bray & Tangney, 2017; Drijvers, 2019; Drijvers et al., 2016; Hillmayr et al., 2020; Olsher & Thurm, 2021). However, technology will not unfold its potentials on its own, rather a didactically careful implementation is needed in order to realize its potentials and avoid negative effects (Jankvist et al., 2019). Despite the potentials outlined in the literature and increasing implementation in some countries (ibid.), technology is still often either underused or teachers do not exploit the potentials in a way suggested by research and policy (Bretscher, 2014, p. 43). Hence, it is of the utmost importance to thoroughly understand the factors associated with technology use (Clark-Wilson et al., 2014; Drijvers, 2019). One of the most important factors for teaching with technology that have been identified is teachers' beliefs, which play a crucial role since they frame, guide, and filter situations, actions, and intentions (Ertmer et al., 2015; Fives & Buehl, 2012; Thomas & Palmer, 2014). In particular, three dimensions of teacher beliefs are regarded as relevant for teaching with technology: i) beliefs about teaching with technology, ii) self-efficacy beliefs, and iii) epistemological beliefs (Ertmer & Ottenbreit-Leftwich, 2010; Thomas & Palmer, 2014; Tondeur et al., 2017). Previous research has described and reconstructed why and how these beliefs are present in teachers' individual belief systems (e.g., Duncan, 2010; Erens & Eichler, 2015) and operationalized and measured the extent of these beliefs (e.g., Kuntze & Dreher, 2013; Pierce & Ball, 2009). However, there is a paucity of research that investigates aforementioned dimensions of teachers' beliefs at a finer-grained level taking into account the multidimensional nature of the constructs (Ertmer et al., 2015; Philippou & Pantziara, 2015; Speer, 2008). Teachers' i) beliefs about teaching with technology, ii) self-efficacy beliefs, and iii) epistemological beliefs are multidimensional constructs with distinct sub-dimensions, but little is known about the differential relevance and interrelations of these sub-dimensions. However, such knowledge is crucial, as a more detailed view can provide valuable insights, for example, about particularly central sub-dimensions of teacher beliefs (e.g., Scherer et al., 2015; Scherer & Siddiq, 2015).

This study addresses this gap by using multidimensional scales to investigate the associations between teachers' i) beliefs about teaching with technology, ii) self-efficacy beliefs, iii) epistemological beliefs, and self-reported iv) modes of technology use on a finer-grained level. We expect to gain nuanced insights that will allow us to go beyond a coarse-grained view and deepen our understanding of the role of different sub-dimensions of teachers' beliefs in the context of teaching mathematics with technology. In particular, we expect to identify (sub)dimension of teacher beliefs that are especially central and to unpack interrelations between the different sub-dimensions. Results can help to inform a more differential design of professional development programs and teacher education approaches, which often lack the desired outcomes, probably due to a mismatch between the program design and teachers' actual needs (Driskell et al., 2016; Grugeon et al., 2010; Hegedus et al., 2017; Thurm & Barzel, 2020). The following theoretical background gives an overview of the different dimensions of teacher beliefs that are particularly relevant for teaching with technology. Next, the instruments that were used to assess teacher beliefs and technology use are described. Subsequently, we describe how classical correlation analysis as well as canonical correlation analysis were used to analyze the data, present the results of the analysis, summarize the key findings, and discuss theoretical and practical implications.

2 Theoretical background

Digital technology in the mathematics classroom comprises a plethora of different technologies. These range from general digital technology (e.g., word processing software) that can be used for communication, documentation, and presentation across different subjects to digital mathematical technologies such as function plotters, geometry packages, and CAS (Ball et al., 2018; Drijvers et al., 2016; Pierce & Stacey, 2010). In particular, mathematical multirepresentation tools (MRT) combine the capabilities of scientific calculators, function plotters, spreadsheets, statistic and geometry packages, and CAS. In this paper, unless stated otherwise, the term “technology” is used to refer to such digital mathematical technologies.

2.1 Modes of technology use

With respect to MRT, different modes of use have been described in the literature (e.g., Ball et al., 2018; Drijvers, 2018; Drijvers & Doorman, 1996; Pierce & Stacey, 2010). For example, the taxonomies of Pierce and Stacey (2010) and Drijvers (2018) map different modes of technology use that exploit the benefits of technology. These taxonomies do not cover all possible ways technology can be used in the mathematics classroom but focus on key pedagogical opportunities. Based on these taxonomies, we give an overview of common modes of technology use that have been identified in the literature (e.g., Ball et al., 2018; Drijvers, 2018; Drijvers & Doorman, 1996; Pierce & Stacey, 2010).

- *Using technology to support working with multiple representations*

It is widely agreed that flexibly changing between different forms of representations, like graphical, algebraic, and numerical representation, is crucial for comprehension of mathematical concepts (Duval, 2006). Technology can be used to support easy access to different forms of representations and to provide simultaneous access to multiple, linked representations in order to develop understanding of the subject at hand (e.g., Hegedus & Roschelle, 2013).

- *Using technology to support discovery learning*

Technology can be used to assist students in exploring, discovering, and developing mathematical concepts on their own, for example, by investigating regularity and variation (e.g., Hoyles et al., 2013). This can be achieved, for example, by generating examples and exploring patterns, providing students with the opportunity to learn mathematics as a constructive activity.

- *Using technology to support individual learning*

Furthermore, technology can be used to facilitate more individualized learning, for instance, by offering different solution pathways for a given task. For example, the teacher may encourage students to use technology to find their own way to solve a task, to self-check solution approaches, and to compare and evaluate individual solution approaches.

- *Using technology to support practicing*

Digital technology can not only be used when acquiring new mathematical knowledge but can also be useful for deepening and practicing skills acquired previously, which is important—for example—for fostering mathematical principles at a more basic level (Drijvers, 2015; Hillmayer et al., 2020; Bokhove, 2011).

- *Using technology to support reflection*

Technology is not the best tool to be used in every situation and can also foster misconceptions, for example, by misleading graphical outputs (Jankvist et al., 2019; Mitchelmore & Cavanagh, 2000; Ward, 2000). Therefore, technology can serve as means for reflection in the classroom, for example, on the limitations of technology, on misleading output, or on the question when to use technology and when not. Such reflection can strengthen students' mathematical understanding and can lead to interesting mathematics (Mitchelmore & Cavanagh, 2000).

Again, we want to point out that these modes of technology use are “not complete and that categories are not mutually exclusive” (Drijvers, 2018, p.233). However, they cover important modes of technology use in which “mathematical understanding and technical skills come together” (Drijvers, 2019, p. 167; Artigue, 2002, Hitt & Kieran, 2009). Yet, the potentials of technology reflected across the different modes will not unfold on their own. Rather, it is well established that teachers' beliefs play a crucial role (Ertmer et al., 2015; Thomas & Palmer, 2014).

2.2 Teacher beliefs and technology integration

There is a consensus among researchers that teacher beliefs “designate individual, subjectively true, value-laden mental constructs” (Skott, 2015, p. 19). However, there are multiple different definitions of teacher beliefs and different views on how to distinguish beliefs from attitudes, values, or worldviews (Philipp, 2007; Skott, 2015). In this study, the term “belief” is used according to the broadly accepted definition proposed by Philipp (2007, p. 259) who defines beliefs as “psychologically held understandings, premises, or propositions about the world that are thought to be true.” Teacher beliefs are important as they act as “filters for interpreting their experiences, frames for addressing problems they encounter, and guides for actions they take” (Levin, 2015, p. 49; Schmidt et al., 2007; Fives & Buehl, 2012; Fives & Gill, 2015). This assertion particularly holds true in regard to the implementation of educational technology (Ertmer et al., 2015; Ertmer & Ottenbreit-Leftwich, 2010; Thomas & Palmer, 2014; Thurm, 2018). In fact, Ottenbreit-Leftwich et al. (2010, p. 1322) hypothesized that “perhaps the largest barrier to student-centered technology integration is teacher belief.” However, teacher beliefs are not a unidimensional construct. Rather, they have been differentiated into various dimensions and sub-dimensions, and it has long been understood that some teacher beliefs are more important than others and that teacher beliefs are organized in a differentiated belief system (Ertmer et al., 2015; Ertmer & Ottenbreit-Leftwich, 2010; Rokeach, 1968; Thompson, 1992). In the following, we will elaborate on the role of three dimensions of teacher beliefs that are regarded as particularly important for technology use in the mathematics classroom: i) beliefs about teaching with technology, ii) self-efficacy beliefs, and iii) epistemological beliefs (Ertmer & Ottenbreit-Leftwich, 2010; Thomas & Palmer, 2014).

2.2.1 Beliefs about teaching with technology

Teacher beliefs about teaching with technology refer to “beliefs about the role of technology in learning” (Goos & Bennison, 2008, p. 105). These beliefs comprise, for example, beliefs about the detrimental or beneficial effects of teaching with technology (Pierce & Ball, 2009). Research indicates that these beliefs influence teachers’ technology use (e.g., Duncan, 2010; Erens & Eichler, 2015; Hennessy et al., 2005). For example, Hennessy et al. (2005) used focus group interviews with teachers and found that technology use was related to “recognizing the educational value and believing in the transformative potential of the technology” (p. 185). However, apart from the general notion that positive beliefs about teaching with technology are beneficial for teaching mathematics with technology, there is a lack of research that takes into account the distinct sub-dimensions of teacher beliefs. Research has recurrently identified various distinct sub-dimensions of teacher beliefs about teaching with technology, which will be described in the following:

- *Beliefs about the role of technology to support discovery learning* refer to beliefs about the value of technology to support student exploration of mathematical concepts, for example, by generating and investigating multiple examples (e.g., Doerr & Zangor, 1999).
- *Beliefs about the role of technology to support multiple representations* refer to beliefs about the value of technology to dynamically link different forms of representations like table, graph, and algebraic expression (e.g., Duncan, 2010; Patterson & Norwood, 2004).
- *Beliefs about time needed to teach with technology* refer to beliefs that teaching with technology requires additional time, for example, for teaching students the handling of the software (e.g., Pierce & Ball, 2009; Simonsen & Dick, 1997).
- *Beliefs about the loss of by-hand skills* refer to teacher beliefs about the consequences of technology use on students’ basic by-hand skills like graphing or solving linear or quadratic equations (Erens & Eichler, 2015; Handal et al., 2011; Simmt, 1997).
- *Beliefs about mindless working when teaching with technology* refer to teacher beliefs that technology use will lead to mindless “button pushing” and is rather a substitute for thinking than a support for understanding (Handal et al., 2011; Pierce et al., 2009; Simonsen & Dick, 1997).
- *Beliefs about the time point of technology use* refer to teacher beliefs about whether technology should only be used after students have achieved conceptual mastery of the mathematics without technology or whether technology should also be used, for example, at the beginning of the learning process (Fleener, 1995; Özgün-Koca, 2010).

Clearly, these sub-dimensions are not exhaustive for teacher beliefs about teaching mathematics with technology but comprise consistently identified sub-dimensions from literature. However, little is known about the differential relevance of these sub-dimensions. Goos and Bennison (2008) indicated that particular sub-dimensions of teacher beliefs may be more central than others, showing that teachers’ beliefs about the role of technology to support discovery learning were much more strongly associated with teachers’ self-reported technology use than teachers’ beliefs about the effects of technology on students’ attitudes. These results point to the urgent need to investigate differential associations in more detail (Erens & Eichler, 2015; Ottenbreit-Leftwich et al., 2010).

2.2.2 Self-efficacy beliefs about teaching with technology

Self-efficacy beliefs (also called “confidence”) refer to “beliefs in one’s capabilities to organize and execute the courses of action required to produce given attainments” (Bandura, 1997, p. 3). Hence, self-efficacy beliefs do not reflect the actual ability of a person but rather describe how a person perceives his or her ability. With respect to teaching mathematics with technology, studies have shown that self-efficacy for teaching with technology is related to teaching mathematics with technology. For example, Doerr and Zangor (2000) and Cavanagh and Mitchelmore (2003) found that low self-efficacy beliefs led to tightly structured teacher-centered technology use, while Clark-Wilson and Hoyles (2019) identified low self-efficacy as the likely reason that teachers did not enact their intended lesson plans for using technology in their mathematics classrooms. While these studies support the general notion that self-efficacy beliefs are important for teaching mathematics with technology, there is a lack of research that takes into account the distinct sub-dimensions of teacher self-efficacy beliefs. The review by Philippou and Pantziara (2015) on mathematics teachers’ self-efficacy research highlighted that self-efficacy is a multidimensional construct and varies across tasks and domains of functioning and that there is an urgent need for research studies that take this multidimensionality seriously. For example, Scherer and Siddiq (2015) showed that the differentiation of general computer self-efficacy into three factors provided a much more detailed view than unidimensional approaches. Also, teaching mathematics with technology is not a unidimensional construct. Rather, literature has consistently highlighted the following two domains of functioning (e.g., Drijvers, 2019; Drijvers et al., 2010; Leung & Baccaglini-Frank, 2016).

- *Task design and selection* (e.g., Leung & Baccaglini-Frank, 2016)

Task design and selection is a crucial domain, as realizing the potentials of technology for the learning of mathematics requires appropriate tasks (Hitt & Kieran, 2009; Leung & Baccaglini-Frank, 2016). In particular “a great majority of the old problems, which teachers used in the past, become inadequate in technological environments” (Hitt & Kieran, 2009, p. 122).

- *Lesson design and implementation* (e.g., Drijvers, 2019; Drijvers et al., 2010).

Tasks will not exert their potential on their own; rather, teachers must also have the self-efficacy to design and implement appropriate lessons (Drijvers et al., 2010; Thomas & Palmer, 2014). Teaching with technology challenges the way in which teachers orchestrate learning and requires new ways to set up appropriate didactical configurations and implement them in the classroom.

Again, these two domains of functioning are not exhaustive, but they cover consistently identified domains described in literature. Yet, little is known about the differential relevance of these sub-dimensions, and more research is urgently needed that takes these sub-dimensions into account to provide a more fine-grained perspective.

2.2.3 Epistemological beliefs and technology integration

Epistemological beliefs refer to beliefs about the nature of knowledge and learning (Ernest, 1989; Hofer & Pintrich, 1997; Lunn et al., 2015). With respect to mathematics teachers, epistemological beliefs are distinguished into beliefs about the nature of mathematics and beliefs concerning the acquisition of mathematical knowledge (Felbrich et al., 2008). This

partition can be further refined. Beliefs about the nature of mathematics can be classified into two main orientations (Dunekacke et al., 2016; Felbrich et al., 2008):

- *Static perspective (rules and procedures)*
This perspective entails formalism- and scheme-related views and characterizes mathematics as a collection of terms, rules, and formulae which has an axiomatic basis and is developed by deduction.
- *Dynamic perspective (inquiry)*
This perspective characterizes mathematics as process- and application-related. Mathematics is seen as a science that consists of problem-solving processes and the discovery of mathematical structures and regularities.

Beliefs about the learning of mathematics have been conceptualized by distinguishing between the following two perspectives (Barkatsas & Malone, 2005; Felbrich et al., 2008; Perry et al., 1999):

- *Instructivist perspective (teacher direction)*
Learning mathematics is seen as a teacher-centered process, where the teachers' role is mainly transmitting mathematical knowledge to the students, whereas students obey the instructions.
- *Constructivist perspective (active learning)*
The constructivist perspective highlights the active involvement of the students. Teachers support mathematical learning if they provide a learner-centered environment in which students have ample opportunities to construct their own mathematical meanings.

With respect to technology integration, research indicates that these beliefs influence how and when technology is used (Erens & Eichler, 2015; Misfeldt et al., 2016; Tondeur et al., 2017). In particular, teachers with stronger constructivist beliefs and a dynamic perspective on mathematics tend to use technology more often in constructivist ways (Erens & Eichler, 2015; Misfeldt et al., 2016; Sinclair & Wideman, 2009; Stacey et al., 2002; Tharp et al., 1997). In addition, teachers with marked instructivist beliefs tend to have a restricted image of the potential of the use of technology for mathematics teaching and learning (ibid.). However, apart from the general notion that epistemological beliefs are related to constructivist/instructivist ways of technology use, there is a lack of research that takes a more nuanced view of the role of the different sub-dimensions of teachers' ii) epistemological beliefs.

3 Research questions and hypotheses

Researchers warned that research on teacher beliefs is typically too coarse grained, focusing broad dimensions of teacher beliefs as opposed to taking a more fine-grained point of view (see Section 2; Speer, 2008; Ertmer et al., 2015). The aim of this study is to provide a fine-grained analysis and to extend previous studies by using multidimensional scales to simultaneously assess i) beliefs about teaching with technology, ii) self-efficacy beliefs, and iii) epistemological beliefs and to relate these beliefs to teachers' self-reported iv) modes of technology use (see Fig. 1). By this, the study aims to uncover structural relations among the sub-dimensions and to identify sub-dimensions that are especially central. In particular, the study investigates the following three research questions:

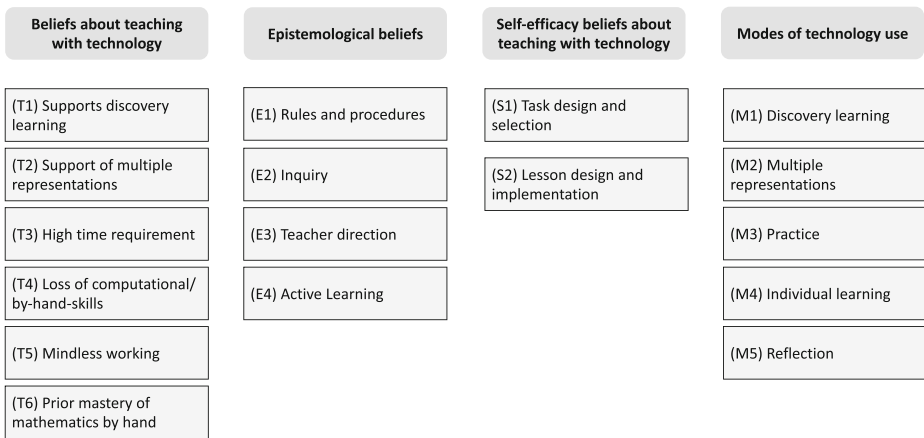


Fig. 1 Sub-dimensions in the context of teaching with technology

- RQ1: To what extent are different sub-dimensions of i) teacher beliefs about teaching with technology associated with different sub-dimensions of self-reported iv) modes of technology use?
- RQ2: To what extent are different sub-dimensions of ii) teacher self-efficacy beliefs associated with different sub-dimensions of self-reported iv) modes of technology use?
- RQ3: To what extent are different sub-dimensions of iii) teacher epistemological beliefs associated with different sub-dimensions of self-reported iv) modes of technology use?

Since existing research literature which takes into account the distinct sub-dimensions of teachers' beliefs and technology use is sparse (see Sections 2.2.1, 2.2.2, and 2.2.3), it is not viable to derive hypotheses for these research questions. Hence, this study has an exploratory nature.

4 Method

To measure teachers' i) beliefs about teaching with technology, ii) self-efficacy beliefs, iii) epistemological beliefs, and iv) modes of technology use, quantitative questionnaires were used, which will be described in Section 4.1. Participants and data analysis will be subsequently described in Section 4.2, respectively.

4.1 Instruments

In the following, we describe the instruments used in this study and give sample items for each scale. The complete instrument is available from Thurm (2020).

4.1.1 Beliefs about teaching with technology

To measure teacher beliefs about teaching with technology, we used a multidimensional questionnaire (Thurm, 2017; Klinger et al., 2018; see Table 1) with six subscales that cover common sub-dimensions of teacher beliefs about teaching with technology (see Section 2.2.1). Scales (T1) and (T2) address the potential benefits of technology use (e.g., technology

Table 1 Scales and sample items to measure i) beliefs about teaching with technology

Scale (number of items)	Sample item
(T1) Supports discovery learning (5)	<i>MRT enable students to explore mathematical concepts (e.g., meaning of parameters) on their own</i>
(T2) Support of multiple representations (4)	<i>An important advantage of MRT is the opportunity to quickly change between forms of representations, like algebraic expressions, graphs, and tables</i>
(T3) High time requirement (3)	<i>The use of MRT costs valuable time that is subsequently missing in the mathematics classroom</i>
(T4) Loss of computational/by-hand skills (4)	<i>By using MRT, students forget procedures and algorithms (or do not learn them at all)</i>
(T5) Mindless working (5)	<i>If students have access to MRT, they think less</i>
(T6) Prior mastery of mathematics by hand (4)	<i>Students should know the mathematical procedures thoroughly before they are provided access to MRT</i>

supports discovery learning), whereas scales (T3), (T4), and (T5) refer to perceived barriers of technology use (e.g., technology leads to a loss of by-hand skills). Scale (T6) evaluates whether teachers believe that technology should be used only after mathematics has been understood without the use of technology. The items were tested through cycles of cognitive interviews (Peterson et al., 2017), with teachers and experts to ensure face and content validity. Table 1 provides an overview of the subscales as well as sample items. Each item was rated on a 5-point Likert scale ranging from 1=“strongly disagree” to 5=“strongly agree.”

4.1.2 Self-efficacy beliefs about teaching with technology

Teachers’ self-efficacy beliefs were captured in the two domains (S1) task design and selection and (S2) lesson design and implementation (see Section 2.2.2). Items were generated following Bandura’s “guide for constructing self-efficacy scales” (Bandura, 2006) and previous research (e.g., Thomas & Palmer, 2014). The items were subsequently refined through cycles of cognitive interviews (Peterson et al., 2017) with teachers and experts to ensure face and content validity. Teachers rated their self-efficacy on a scale ranging from 0 to 100. Such a scaling is particularly recommended for capturing self-efficacy beliefs because of its higher accuracy compared to using Likert-scale measures (Bandura, 2006; Maurer & Andrews, 2000). For the analysis of the data, the answers were rescaled to 0–5. Table 2 gives a sample item for each subscale.

4.1.3 Epistemological beliefs

Epistemological beliefs (see Section 2.2.3) were measured using the scales from the international TEDS-M study (Blömeke & Kaiser, 2014; see Table 3). Subscales (E1) and (E2) focus on teacher beliefs about the nature of mathematics; one subscale refers to the static perspective (E1, “mathematics as rules and procedures”), and one subscale refers to the dynamic perspective

Table 2 Scales and sample items to measure ii) self-efficacy beliefs about teaching with technology

Scale (number of items)	Sample item
(S1) Task design and selection (4)	<i>I can design tasks for use with MRT</i>
(S2) Lesson design and implementation (4)	<i>I can design and implement lessons that support discovery learning using MRT</i>

Table 3 Scales and sample items to measure iii) epistemological beliefs

Scale (number of items)	Sample item
(E1) Rules and procedures (6)	<i>Mathematics is a collection of rules and procedures that prescribe how to solve a problem</i>
(E2) Inquiry (6)	<i>If you engage in mathematical tasks, you can discover new things (e.g., connections, rules, concepts)</i>
(E3) Teacher direction (8)	<i>Pupils learn mathematics best by attending to the teacher's explanations</i>
(E4) Active learning (6)	<i>Teachers should encourage pupils to find their own solutions to mathematical problems even if they are inefficient</i>

(E2, “mathematics as process of inquiry”). Subscales (E3) and (E4) focus on teachers’ beliefs about the learning of mathematics. While subscale (E3) focuses on learning mathematics through teacher direction, subscale (E4) targets teachers’ beliefs about learning mathematics through active learning. Table 3 provides sample items for all scales. Responses were given on a 6-point Likert scale ranging from 1= “strongly disagree” to 6= “strongly agree.”

4.1.4 Modes of technology use

With respect to modes of technology, we focused on the different modes of technology use outlined in Section 2.1. These modes comprise technology use for discovery learning (M1), technology use for linking multiple representations (M2), technology use for practicing (M3), and technology use to support individual learning (M4). In addition, category (M5) concerns whether technology use is subject to critical reflection. This raises the question of how to capture these modes of technology use. Direct observation of teachers’ practice is extremely difficult over prolonged time, especially for a large number of teachers, and may be flawed by teachers changing their teaching practice because they know they are being observed (social desirability bias). Self-reports about teachers’ practice are much easier to collect but may also suffer from a social desirability bias (this issue can be alleviated by ensuring anonymity to the participants) and carry the risk that teachers may not be able to accurately self-report their practice (hence self-reports could also be thought of as “teachers’ espoused beliefs about their classroom practice”). However, reliability of teachers’ self-reports, which are frequently used in teacher surveys in the context of teaching mathematics with technology (e.g., Goos & Bennison, 2008), can be improved by focusing on discrete questions about *frequencies*: “Teacher surveys that ask behavioral and descriptive, not evaluative, questions about [...] teaching have been shown to have good validity and reliability” (Desimone 2009, p. 190). Taking all these arguments together, we decided to collect teachers’ self-reports (for pragmatic reasons) while focusing on *frequency* with respect to the modes of technology use (in order to increase reliability).

Items were generated and refined through cycles of cognitive interviews (Peterson et al., 2017) with teachers and experts, to ensure face and content validity. The interviews showed that teachers recapitulated their teaching during the last months across their classrooms to answer the items. In particular, teachers stated that their answer would depend on the particular school year (e.g., in lower school years, teachers often did not use technology because students have limited access to technology). This was in contrast to the items of the other scales i)–iii) which teachers answered generically. We take this as an indication that items pertaining to the iv) modes of technology use do indeed measure a different construct than items of the scales i)–iii). To avoid ambiguity, we focused the items towards a specific class of the teacher by adding the following phrase at the beginning of questions about iv) modes of technology use:

Table 4 Scales and sample items to measure iv) teachers technology use

Scale (number of items)	Sample item
(M1) Discovery learning (5)	<i>How often were MRT used to explore mathematical relationships?</i>
(M2) Multiple representations (5)	<i>How often were MRT used to link graphical and algebraic representation?</i>
(M3) Practice (4)	<i>How often were MRT used to practice mathematical content?</i>
(M4) Individual learning (3)	<i>How often were MRT used to support students to find their own solution approaches to solve a task?</i>
(M5) Reflection (5)	<i>How often did you discuss the limitations of MRT?</i>

“The following questions relate to your lessons in the introductory phase in this school year” (the “introductory phase” is the first year of upper secondary school in Germany). Scales (M2), (M4), and (M5) were rated on a 5-point Likert scale with the following response categories: “almost never,” “once or twice a quarter,” “once or twice a month,” “once a week”, and “almost every lesson.” Scales (M1) and (M3) were rated on a 5-point Likert scale with respect to the proportion of technology use. Table 4 gives a sample item for each subscale.

4.2 Participants and data analysis

Data were collected in the Federal State of North Rhine-Westphalia in Germany. All upper secondary schools in this state were contacted and asked to invite their teachers from the first year of upper secondary school to participate in the research study (the main topics covered in this school year are basic properties of power, exponential and sine functions, basic understanding of the concept of derivative, and differential calculus of polynomial functions). Teachers who signed up subsequently received a pen-and-paper version of the previously described questionnaires. Anonymity was guaranteed. $N = 198$ teachers completed the questionnaire. The average age of the participating teachers was approximately 43 years, which closely matches the average of 45 years of teachers in North Rhine-Westphalia. Fifty-four percent of the participants were female, which reflects the average number of female teachers in North Rhine-Westphalia. Although these statistics do not necessarily imply that the sample is representative, they do suggest that the sample closely reflects the characteristics of the population as a whole with respect to gender and age and thus very likely also with respect to teaching experience.

Associations between teacher beliefs and self-reported modes of technology use were analyzed by means of correlation analysis. Since the relationship between teacher beliefs and technology use is still unclear and very likely reciprocal (Goos & Bennison, 2008; Sinclair & Wideman, 2009), correlation analysis has an advantage over regression analysis in that it does not require specification of the dependent and independent variables. We calculated pairwise correlation coefficients between all subscales of i) beliefs about teaching with technology, ii) self-efficacy beliefs about teaching with technology, iii) epistemological beliefs, and self-reported iv) modes of technology use. Correlations were calculated using Mplus (Version 7.31) on a latent level, thus providing more accurate correlation coefficients by taking measurement errors into account. To account for deviations from multivariate normality, we used the robust MLR-estimator which is implemented in M-Plus (Muthén & Muthén, 2010; Yuan & Bentler, 2000). Correlation size was interpreted according to empirical guidelines for psychological research (Hemphill, 2003; small <0.20 ; medium = 0.20 to 0.30 , and large >0.30).

In addition, data was analyzed using canonical correlation analysis (CCA), which investigates lower dimensional projections of the data structure to discover and quantify associations between

two sets of variables (Härdle & Simar, 2015; Sherry & Henson, 2005). For CCA “the underlying principle is to develop a linear combination of each set of variables in a manner that maximizes the correlation between the two sets” (Hair et al., 2019, p. 28). CCA can unpack ways in which two sets of variables are related, the strengths of the relationships and the nature of the relationships. In the CCA, one set of variables comprised the subscales capturing teachers’ beliefs (beliefs about teaching with technology, self-efficacy beliefs, epistemological beliefs), whereas the other set of variables comprised the subscales measuring teachers’ self-reported modes of technology use. CCA was performed using the R package “CCA” (González et al., 2008). We calculated p-values to test the number of canonical relationships (also called canonical functions) using the robust F approximations of Pillai’s trace test statistic (Finch & French, 2013; Olson, 1974). To interpret the results, we calculated the canonical loadings which measure the linear correlation between the original observed variable and the variable sets canonical variate. Loadings greater than or equal to 0.45 are relevant, loadings greater than or equal to 0.55 are good, and loadings greater than or equal to 0.63 are very good (Comrey & Lee, 1992; Sherry & Henson, 2005).

5 Results

Sections 5.1, 5.2, and 5.3 present the results of the pairwise correlation analysis. Section 5.4 presents the results of the CCA.

5.1 Results on teachers’ i) beliefs about teaching with technology

Table 5 reports the results of the correlation analysis. The reliability coefficients of all scales were good, ranging from 0.78 to 0.93. Differential associations are clearly visible across Table 5, since certain self-reported iv) modes of technology use are particularly related to certain sub-dimensions of teachers’ i) beliefs about teaching with technology. In particular, self-reported use of technology for discovery learning (M1) correlated with four sub-dimensions of teachers’ i) beliefs about teaching with technology, namely beliefs about discovery learning (T1, $\rho=0.32^{**}$), beliefs about the high time requirements of teaching with technology (T3, $\rho=-0.28^{**}$), beliefs about the loss of skills (T4, $\rho=-0.21^*$), and beliefs that mathematics should be mastered by hand prior to technology use (T6, $\rho=-0.22^*$). The self-reported use of technology to support more individualized learning (M4) correlated with three sub-dimensions of teacher i) beliefs about teaching with technology, namely (T1, $\rho=0.26^{**}$; T2, $\rho=0.19^*$; T3, $\rho=-0.24^{**}$). Remarkably, the self-reported use of technology for reflection (M6) was completely unrelated to teacher i) beliefs about teaching with technology. Furthermore, it is striking that beliefs about the loss of skills (T4) and beliefs that mathematics concepts/procedures should be mastered prior to technology use (T6) were only marginally related with self-reported modes of technology use and that the risk of mindless working (T5) did not show any significant correlations with self-reported modes of technology use. Table 5 reports the results of the correlation analysis between teachers’ iii) epistemological beliefs and self-reported iv) modes of technology use. The reliability coefficients of the scales ranged from 0.60 to 0.88. No significant correlations were found between beliefs about the nature of mathematics (E1, E2) and self-reported modes of technology use. However, beliefs about the learning of mathematics (E3, E4) were associated with some self-reported modes of technology use. The belief that learning mathematics is best achieved by direct instruction (E3) was negatively correlated with the self-reported use of technology for discovery learning ($\rho=-0.20^*$). In addition, the belief that learning mathematics is best achieved by active learning

Table 5 Results of the pairwise correlation analysis between teacher beliefs and teacher practice

	(M1) Discovery learning	(M2) Multiple representations	(M3) Practice	(M4) Individual learning	(M5) Reflection	Mean (SD)	alpha
i) Beliefs about technology							
(T1) Supports discovery learning	<u>.32**</u>	.10	.16	<u>.26**</u>	-.10	3.39 (0.84)	.87
(T2) Support of multiple representations	.11	<u>.34**</u>	.11	<u>.19*</u>	-.04	3.88 (0.82)	.86
(T3) High time requirement	<u>-.28**</u>	<u>-.06</u>	<u>-.26**</u>	<u>-.24**</u>	.00	2.63 (1.22)	.92
(T4) Loss of comp./by-hand- skills	<u>-.21*</u>	-.02	<u>-.04</u>	<u>-.15</u>	.04	3.81 (0.90)	.86
(T5) Mindless working	<u>-.14</u>	.04	-.02	-.05	.02	3.48 (0.93)	.89
(T6) Prior mastery of math. by hand	<u>-.22*</u>	.06	-.13	-.04	.01	3.27 (1.24)	.93
ii) Self-efficacy beliefs							
(S1) Task design and selection	<u>.43***</u>	<u>.23*</u>	<u>.44***</u>	<u>.39***</u>	<u>.29***</u>	2.63 (1.27)	.92
(S2) Lesson design and implementation	<u>.56***</u>	<u>.32**</u>	<u>.45***</u>	<u>.52***</u>	<u>.29***</u>	2.94 (1.11)	.90
iii) Epistemological beliefs							
(E1) Rules and procedures	-.13	.11	-.03	-.14	-.02	3.51 (0.75)	.60
(E2) Inquiry	.01	.01	.05	.11	.15	5.09 (0.59)	.70
(E3) Teacher direction	<u>-.20*</u>	-.01	-.14	-.11	-.17	2.21 (0.56)	.64
(E4) Active learning	<u>.08</u>	.08	<u>.22**</u>	<u>.27**</u>	.10	5.25 (0.56)	.76
Mean (SD)	2.98(0.99)	2.65 (0.97)	3.24 (0.92)	2.86 (0.96)	2.70 (0.98)		
Cronbach's alpha	0.85	0.88	0.88	0.78	0.81		

*p< .05; **p< .01; ***p< .001; correlations above 0.2 (medium) are underlined

(E4) was associated with higher levels of self-reported technology use for practicing (M3, $\rho=0.22^{**}$) and individual learning (M4, $\rho=0.27^{**}$). Remarkably, self-reported use of technology to support multiple representations (M2) and self-reported use of technology to support reflection (M5) were not related to epistemological beliefs at all. In summary, beliefs about the nature of mathematics were unrelated to self-reported modes of technology use, while more constructivist beliefs about the learning of mathematics showed significant associations with some of the teachers' self-reported modes of technology use.

5.2 Results on teachers' ii) self-efficacy beliefs

Table 5 reports the results of the correlation analysis between teachers' ii) self-efficacy beliefs and self-reported iv) modes of technology use. The reliability coefficients of all scales were good, ranging from 0.78 to 0.92. It can be clearly seen from Table 5 that teachers' self-efficacy was positively correlated with self-reported iv) modes of technology use. However, the correlations varied considerably across sub-dimensions. Self-efficacy for task design and selection (S1) correlated particularly highly with the self-reported use of technology for discovery learning (M1, $\rho=0.43^{***}$) and practice (M3, $\rho=0.44^{***}$). Self-efficacy for lesson design and implementation (S2) showed the strongest correlation with self-reported use of technology for discovery learning (M1, $\rho=0.56^{***}$) and individual learning (M4, $\rho=0.516^{***}$). In contrast, both self-efficacy scales had much lower correlations ($0.23^* \leq \rho \leq 0.32^{**}$) with self-reported use of technology to support multiple representations (M2) and self-reported use of technology for reflection (M5). Hence, differential associations can be observed in two ways: some self-reported modes of technology use are particularly strongly related to self-efficacy beliefs, and self-efficacy for lesson design and implementation (S2) was more strongly related to self-reported modes of technology use than self-efficacy for task design and selection (S1).

5.3 Results on teachers' iii) epistemological beliefs

Table 5 reports the results of the correlation analysis between teachers' iii) epistemological beliefs and self-reported iv) modes of technology use. The reliability coefficients of the scales ranged from 0.60 to 0.88. No significant correlations were found between beliefs about the nature of mathematics (E1, E2) and self-reported modes of technology use. However, beliefs about the learning of mathematics (E3, E4) were associated with some self-reported modes of technology use. The belief that learning mathematics is best achieved by direct instruction (E3) was negatively correlated with the self-reported use of technology for discovery learning ($\rho=-0.20^*$). In addition, the belief that learning mathematics is best achieved by active learning (E4) was associated with higher levels of self-reported technology use for practicing (M3, $\rho=0.22^{**}$) and individual learning (M4, $\rho=0.27^{**}$). Remarkably, self-reported use of technology to support multiple representations (M2) and self-reported use of technology to support reflection (M5) were not related to epistemological beliefs at all. In summary, beliefs about the nature of mathematics were unrelated to self-reported modes of technology use, while more constructivist beliefs about the learning of mathematics showed significant associations with some of the teachers' self-reported modes of technology use.

5.4 Results of the canonical correlation analysis

The CCA resulted in the identification of two significant canonical relationships (Table 6). The first canonical relationship had a canonical correlation of 0.58, while for the second canonical

Table 6 Test of the number of canonical relationships

Canonical relationships	Corr.	F	df1	df2	p
1	0.58	2.34	60	730	0.0000
2	0.44	1.75	44	740	0.0022

relationship, the canonical correlation was slightly lower, at 0.44. Table 7 presents the canonical loadings obtained from the CCA for these two canonical relationships. For emphasis, canonical loadings above .45 are underlined in Table 7 following recommendations for interpreting loading size (see Section 4.2).

Looking at the loadings for canonical relationships 1, one sees that relevant variables for the canonical variate representing teacher beliefs were primarily self-efficacy beliefs ($S1=-0.73$, $S2=-0.93$), beliefs about discovery learning ($T1=-0.50$), and beliefs about the high time requirements for teaching with technology ($T3=0.50$). Regarding the set of variables representing teachers self-reported technology, discovery learning ($M1=-0.86$), practice ($M3=-0.79$), and individual learning ($M4=-0.78$) were the primary contributors to the canonical variate. Moving to the second canonical relationship, which is independent from the first canonical relationship, the loadings in Table 6 show that the only variable of relevance in the set representing teacher beliefs was teachers' belief that technology supports teaching with multiple representations ($T2=-0.69$). For set representing teachers' self-reported modes of technology use, the self-reported use of technology to support multiple representations ($M2=-0.35$) was contributing most to the canonical variate. Looking across both canonical relationships, it can be seen that particular sub-dimensions contribute only little. For example,

Table 7 Canonical loadings of the two significant canonical relationships

	Canonical loadings for the two significant canonical relationships	
	1	2
i) Beliefs about teaching with technology		
(T1) Supports discovery learning	<u>-0.50</u>	-0.06
(T2) Support of multiple representations	<u>-0.25</u>	<u>-0.69</u>
(T3) High time requirement	<u>0.50</u>	-0.03
(T4) Loss of computational/by-hand skills	0.28	-0.03
(T5) Mindless working	0.14	-0.16
(T6) Prior mastery of mathematics by hand	0.23	-0.20
ii) Self-efficacy beliefs		
(S1) Task design and selection	<u>-0.73</u>	0.08
(S2) Lesson design and implementation	<u>-0.93</u>	-0.10
iii) Epistemological beliefs		
(E1) Rules and procedures	0.15	-0.28
(E2) Inquiry	-0.12	-0.10
(E3) Teacher direction	0.24	-0.19
(E4) Active learning	-0.37	0.05
iv) Modes of technology use		
(M1) Discovery learning	<u>-0.86</u>	0.15
(M2) Multiple representations	<u>-0.51</u>	<u>-0.81</u>
(M3) Practice	<u>-0.79</u>	0.11
(M4) Individual learning	<u>-0.78</u>	-0.11
(M5) Reflection	-0.43	0.02

For emphasis, canonical loadings above $|.45|$ are underlined

looking at i) beliefs about teaching with technology, T4, T5, and T6 contribute only little, while for iii) epistemological beliefs, E1, E2, and E3 contribute only little.

6 Discussion

In this study, we focused on relating teacher i) beliefs about teaching with technology, ii) self-efficacy beliefs, iii) epistemological beliefs, and teachers self-reported iv) modes of technology use. Little research has investigated this relation while taking into account the multidimensionality of the constructs. We have presented results from a cross-sectional study with $n = 198$ upper-secondary teachers in which we employed multidimensional scales to investigate the differential associations between different sub-dimensions of teacher beliefs and self-reported modes of technology use. In the following, we will discuss the results (Section 6.1, 6.2), outline the limitations of the study (Section 6.2), and summarize the results and conclude with suggestions for further studies (Section 6.3).

6.1 Differential association between teacher beliefs and self-reported classroom practice

Previous research has documented the importance of beliefs about teaching mathematics with technology, self-efficacy beliefs, and epistemological beliefs (e.g., Erens & Eichler, 2015; Ertmer & Ottenbreit-Leftwich, 2010; Philipp, 2007). The results of our study are completely in line with these previous results. However, our study goes beyond this coarse picture and uncovers a more nuanced and differential picture. In particular, the results show that the different sub-dimensions are particularly related, while others may be less central. Based on the analysis in Sections 5.1, 5.2, 5.3, and 5.4, we can put forward three clusters of sub-dimensions (see Fig. 2).

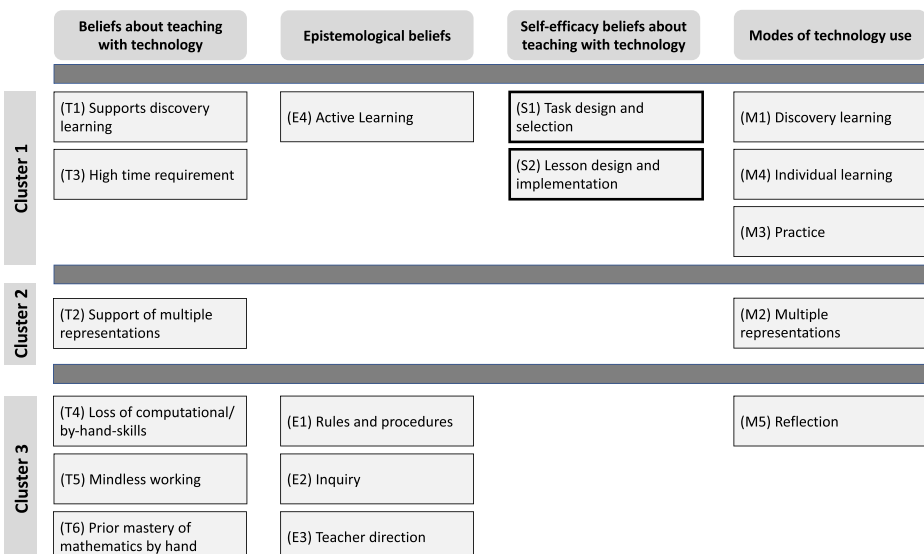


Fig. 2 Structure of the sub-dimensions based on the CCA and the correlation analysis

6.1.1 Cluster 1

The first canonical relationship obtained from the CCA and also the correlation analysis shows that a larger set of eight sub-dimensions are inherently intertwined. These sub-dimensions are linked to a more integrated and more constructivist way of teaching with technology (e.g., beliefs about the potentials of technology for discovery learning, beliefs about active learning, self-reported use of technology for individual learning and discovery learning). One striking result for this cluster is the highlighted role of self-efficacy, in particular self-efficacy for lesson design and implementation. This can be related to the fact that using technology in more constructivist ways such as for discovery learning is often less controllable. Students generate a wide range of different solutions and ideas, which requires teachers to deal with unplanned situations and to spontaneously react (e.g., Clark-Wilson & Hoyles, 2019; Dunham & Dick, 1994). This is particularly challenging if teachers are more used to tasks and teaching settings which have more a procedural focus than an explorative one.

6.1.2 Cluster 2

The second canonical relationship obtained from the CCA and also the correlation analysis shows that using multiple representations is largely independent of the other sub-dimensions, in particular from more constructivist oriented sub-dimensions. This is very interesting, because if this sub-dimension is independent of other sub-dimensions, it can be addressed and changed more easily. For example, a PD program addressing teaching with multiple representations would need to heavily address time requirements, as this is not related to teaching with multiple representations. But why is teaching with technology to support multiple representations a more independent construct? It can be hypothesized that supporting multiple representations with technology fits more easily into a large variety of classroom routines and does not necessarily require changes along multiple dimensions of teacher beliefs and practice compared to more disruptive practices like discovery learning or individual learning. Therefore, it does not necessarily require much additional time or self-efficacy as teacher routines are disturbed less.

6.1.3 Cluster 3

Finally, some sub-dimensions contributed only marginally to the two canonical relationships, identified in the CCA and showed only small correlations in the correlation analysis. These sub-dimensions can therefore be considered as less central. These sub-dimensions comprise, for example, teachers' epistemological beliefs about the nature of mathematics (e.g., E1, E2, E3) and teacher beliefs about the risks of technology use (i.e., loss of skills, mindless working). The finding that risks of technology use showed only little associations is particularly striking since beliefs about the risks of technology were well pronounced in the sample. It can be concluded that *risks of technology* may carry less weight than teachers' beliefs about the *potential benefits* of teaching with technology (e.g., the support of multiple representations, the support of discovery learning) which were related to teacher self-reported modes of technology use (see above cluster 1 and cluster 2). The reason for this could be that teachers may be convinced that technology is beneficial and, on the other hand, be aware of the risks but believe that their teaching with technology is good enough to overcome these risks.

In summary, these three clusters (or sets) of sub-dimensions which are depicted in Fig. 2 can clearly help to provide a more differential view on the sub-dimensions and help to reduce the complexity of the sub-dimensions which were investigated in this study.

6.2 Practical implications

The results clearly call for a more differential approach for professionalizing teachers. For example, sub-dimensions of cluster 3 can be seen as less central and can possibly play a minor role in professional development. The affordance of technology to support multiple representations (cluster 2) is particularly suited for beginners or teachers with more reserved views on more integrated and more constructivist ways of teaching with technology. Sub-dimensions of cluster 1 can then slowly be interwoven at later stages of professional development. At this stage in particular, self-efficacy for lesson design and implementation has to be addressed. This would entail placing a stronger focus on allowing teachers to gain specific mastery experience in implementing technology, as this is the most powerful source of teacher self-efficacy (Bandura, 1997; Klassen & Usher, 2010). These suggestions are in line with observations that technology integration is an evolutionary rather than a revolutionary process, where teacher change occurs in small steps (Ertmer & Ottenbreit-Leftwich, 2010). Hence, the results of our study can support more focused approaches to PD that are better aligned with teachers' actual needs.

6.3 Limitations

Of course, the results of this study are subject to limitations. A general challenge is that beliefs are not directly observable entities and have to be inferred from sources such as questionnaires or interviews. Due to the large sample size, the data collection in this study relied on quantitative questionnaires, and the questionnaire data were not triangulated with data from interviews or classroom observations. In particular, there remains a risk of circular findings with respect to associations between beliefs about teaching with technology and self-reported modes of technology use, even though survey measures are likely reliable if the survey data focus on how often certain strategies or practices are used (Desimone et al., 2010), as was the case in our study. Further limitations are that other factors, such as teachers' knowledge and the school context, were not considered and that the sample was not a random sample, even though the sample means of age and gender matched the population means. Furthermore, it must be noted that the scales used to assess epistemological beliefs showed only average reliability and that constructivist beliefs were strongly favored over instructivist beliefs, a phenomenon that has also been observed in other studies (e.g., Blömeke & Kaiser, 2014). Despite these limitations, the study provides valuable insights that can help to unpack the complexity of teachers' beliefs with respect to teaching with technology.

7 Conclusion

To our knowledge, this is the first quantitative study that investigates teacher beliefs in the context of teaching mathematics with technology use on a finer-grained level, taking into account the multidimensionality of the constructs. The results highlight the differential structure of teachers' beliefs with respect to teaching with technology. In particular, we identified three clusters of sub-dimensions that can inform a more focused professional development.

Furthermore, the results of this study indicate that self-efficacy is a central construct for higher level aspects of teaching mathematics with technology and that beliefs about potential benefits of technology use and beliefs about the time requirements of implementing technology may outweigh the importance of teachers' beliefs about negative effects of technology use. Our study extends previous research (e.g., Erens & Eichler, 2015; Goos & Bennisson, 2008; Tharp et al., 1997) by providing a more nuanced picture and is in line with recent studies in areas other than teaching mathematics with technology, which highlight the importance of taking a multidimensional approach for investigating teacher beliefs (Perera et al., 2019; Scherer & Siddiq, 2015). The results of this study should encourage future studies to take a multidimensional approach when investigating teacher beliefs and technology integration in the mathematics classroom.

Availability of data and material All authors have ensured that all data and materials as well as software applications or custom code support the published claims and comply with field standards.

Code availability Not applicable.

Author contribution All authors whose names appear on the submission made substantial contributions to the conception or design of the work; the acquisition, analysis, or interpretation of data; or the creation of new software used in the work; drafted the work or revised it critically for important intellectual content; approved the version to be published; and agreed to be accountable for all aspects of the work in ensuring that questions related to the accuracy or integrity of any part of the work are appropriately investigated and resolved.

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Declarations

Conflict of interest The authors declare no competing interests.

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