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Individual and group mathematical creativity among post–high school students

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Abstract
Promoting mathematical creativity is an important aim of mathematics education, which may be promoted by engaging students with open-ended tasks. Most studies of students' creativity have investigated the creativity of students working individually. This study concerns the mathematical creativity of students working as individuals as compared with those working in groups. Participants were 92 post–high school students, separated into two heterogeneous classes. Both classes engaged with the same three geometric open-ended tasks. For the first two tasks, one class worked individually, while the second worked in small groups of four to six students. For the third task, all students worked individually. Results were analyzed in terms of fluency, flexibility, and originality. No significant differences were found between classes for fluency and flexibility on the first task. However, for the second and third tasks, there were greater fluency and flexibility among those who worked or had worked in groups. For all three tasks, no significant differences between the classes were found regarding originality.

Keywords Mathematical creativity · Group creativity · Post–high school students · Open-ended tasks

1 Introduction

When discussing creativity in education, researchers are usually concerned with *mini-c* creativity, that is, creative and meaningful insights experienced by students as they learn a new concept (Kaufman & Beghetto, 2009). "Although students may not be discovering methods for solving problems that are … new to the larger field of mathematics, they are nonetheless creating novel solutions and ideas within their own learning trajectory" (Luria,

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Sriraman, & Kaufman, 2017, p. 1035). One way of occasioning mathematical creativity in the classroom is by engaging students with open-ended mathematics problems (Haylock, 1997; Kwon, Park, & Park, 2006; Silver, 1997). An open-ended problem does not have a single exact answer but rather a range of solutions (Silver, 1997). In other words, there is more than one correct final answer as well as several possible solution pathways. These problems may also be considered divergent production tasks, tasks that lead to fluency, flexibility, and originality (Haylock, 1997; Kattou, Kontoyianni, Pitta-Pantazi, & Christou, 2013).

Most studies of students' engagement with open-ended problems investigated the creativity of students working individually (Kwon et al., 2006; Tsamir, Tirosh, Tabach, & Levenson, 2010). While several studies have investigated collective mathematical learning, including the advantages of collective learning (e.g., Martin, Towers, & Pirie, 2006), few have focused on collective mathematical creativity in the classroom. Yet, collaboration, along with creativity, is considered an important skill for the twenty-first century (Partnership for twenty-first Century Skills, 2014). Can we integrate creativity along with group work in the classroom? This study concerns the following issues: examining evidence of mathematical creativity in a group setting, the mathematical creativity of students working as individuals compared with the mathematical creativity of students working in groups, and the possible impact of experiencing group work on the mathematical creativity of students even when they work individually.

2 Theoretical background

2.1 Characterizing and assessing mathematical creativity

Creative processes may be considered as processes involved in solving a problem and creating ideas and include being able to combine ideas or approaches in a new way, analyze a given problem in several ways, observe patterns, and use one's imagination (Kim, Roh, & Cho, 2016). A creative product is the solution to a problem or an idea that is often considered to be novel or original (Kim et al., 2016). Some mathematics educators refer to creativity in terms of both process and product. For example, Liljedahl and Sriraman (2006) associated school-level mathematical creativity with being able to view an old problem from a new angle, raising new questions and possibilities (creative processes), and unusual and/or insightful solutions to a given problem (creative products). Gómez-Chacón and de la Fuente (2018) claimed that creativity involves imagination (process) and original ideas (product). In this study, we analyze students' solutions to open-ended tasks. That is, we analyze products. Although research has not confirmed that creative products are always the result of creative processes, we regard the products of these tasks as expressions of creativity.

Gómez-Chacón and de la Fuente (2018) assumed that because ideas are generated by thinking and skills for thinking can be learned, mathematical creativity, including idea generation, can be learned. This study adopts the assertion that mathematical creativity is "an orientation or disposition toward mathematical activity that can be fostered broadly in the general school population" (Silver, 1997, p. 75). Silver (1997) goes on to elaborate the features of mathematical activity that a creative disposition would foster, such as formulating novel solutions, changing directions of focus, and producing a number of solutions or solution paths. These three features of creativity can help dispel the notion among students that mathematics is about following rules and that every problem has exactly one correct answer and one correct solution path. Encouraging students to view problems from different angles can encourage students to see connections between different representations and between different mathematical content areas (Leikin, 2009). They are especially relevant when solving open-ended tasks (Haylock, 1997; Jung, 2001; Levav-Waynberg & Leikin, 2012). While recognizing the quandary that exists between features of mathematical activity that are an outgrowth of creativity and features of creativity, we analyze the products of open-ended tasks in terms of fluency, flexibility, and originality. This is in line with previous research and with Silver's (1997) characterization of mathematical creativity.

Producing a number of solutions is related to idea generation and fluency. Fluency of an individual may be considered as the "continuity of ideas, flow of associations, and use of basic and universal knowledge" (Leikin & Lev, 2013, p. 184). It is considered relevant to creativity, especially for divergent-production tasks, because "fluent thinkers are able to generate many ideas, possibilities, and potential approaches to finding solutions to a problem" (Mann, Chamberlin, & Graefe, 2017, p. 58). It may also be that highly fluent students have a better chance of having high originality scores. For example, Runco and Albert (1985) found that originality may depend on fluency in nongifted populations, although not among gifted students.

Fluency is often measured by assessing the total number of unduplicated ideas generated (Jung, 2001). Within the context of mathematics, these ideas must also be in line with the rules and definitions of the specific domain and mathematically correct. For example, if the task is to identify among several natural numbers which number is the exception and why (Kwon et al., 2006), a reply stating that "nine is the exception because it is the only even number," would not be counted when considering the fluency of ideas because nine is not an even number.

Although fluency is an attribute to consider, it does not indicate whether a specific solution may be considered creative (Leikin & Lev, 2013). Thus, an additional hallmark of creativity is flexibility (Haylock, 1997; Jung, 2001; Leikin, 2009; Levenson, 2011). Flexibility may be thought of as the ability to modify one's way of thinking, or in Silver's words, "apparent shifts in approaches taken when generating responses to a prompt" (p. 76). Leikin (2009) found evidence of flexibility in individual students' solutions that employ strategies based on different representations (e.g., algebraic and graphical representations), properties, or branches of mathematics. Flexibility is sometimes considered to be the opposite of fixation, also thought of as mental rigidity (Haylock, 1997) or self-restrictions (Krutetskii, 1976). Levenson (2011), for example, demonstrated this type of flexibility in a fifth-grade classroom. Students were working on filling in a sequence of numbers that requires the use of "equal jumps" between elements of the sequence. During the lesson, most students stayed on this path. One student, however, suggested that a sequence may have unequal jumps, such as repeatedly adding some constant number and then multiplying by some number. That student diverged from the expected path, exhibiting flexibility. The above example also illustrates how mathematical creativity in the classroom is evidenced with respect to students' previous experiences and the performance of their peers (Leikin, 2009). In general, flexibility of an individual is measured by classifying students' solutions into categories and then counting the number of categories with correct responses (Kwon et al., 2006). This measure assesses the products but then infers from the products something about the creative process, such as one's ability to change directions when solving a problem.

Perhaps the most noted aspects of creativity are novelty and originality. In the classroom, a novel solution is one that is new to the classroom participants (Silver, 1997). Leikin (2009) measured the originality of a solution based on its level of insight and conventionality according to the learning history of the participants. For example, a solution based on a concept learned in a different context would be considered original but maybe not as original as a solution which was unconventional and totally based on insight. Levenson (2013) differentiated between the terms novelty and originality, stating that "Novel may refer to 'new' while original may refer to 'one of a kind' or 'different from the norm'... an idea, especially one raised in the classroom, may be new to a student, but if other students have the same idea, it may not be original" (Levenson, 2013, p. 271).

Scoring originality is often based on the relative scarcity of a solution. For example, Kattou et al. (2013) differentiated between solutions that appeared in less than 1% of the sample's solutions, between 1 and 5%, between 6 and 10%, between 11 and 20%, and more than 20% of the sample's solutions. Leikin (2009) assigned the highest originality score for solutions produced by less than 15% of the participants in the group. Levenson, Swisa, and Tabach (2018) measured students' originality according to the strategies students used when solving a problem. As such, their measurement of originality may be more in line with measuring creative processes than creative products.

The current study employs three open-ended tasks found to elicit fluency, flexibility, and originality in previous studies. The first task is based on Haylock's (1997) study, where 11–12-year-old students were asked to draw as many noncongruent polygons with an area of 2 cm^2 on a nine-dot cm^2 grid as possible. Haylock found that some solutions (e.g., a rectangle) were obtained by most of the students, whereas others (e.g., a nonparallelogram quadrilateral) were drawn by a few (7% of the students). The second task used in the current study is based on an activity presented to fifth graders working with a computer applet (Levenson et al., 2018). Relevant to our current study is how flexibility was measured. The fifth-grade students' solutions were sorted into four categories based on the class learning history and the computer context: triangles, rectangles, nonrectangles (and nontriangles) drawn solely on the grid lines and other polygons not drawn exclusively on the grid lines. The third task used in the current study is based on a task found in Klavir and Hershkovitz (2008) and Kwon et al. (2006). In those studies, students were presented with a set of numbers such as 1, 2, 4, 6, 8, and 12 and were asked which number is the exception and why. Each number could be considered as an exception for a different reason and for various reasons. In the above described studies, creativity was analyzed for individual students. In the current study, we also investigate the creativity manifested by individuals working in groups.

2.2 Creativity among groups of individuals

Previous research regarding group creativity is not decisive. Some studies have pointed out deficits. Studies of group-brainstorming, where each group member is explicitly requested to generate ideas, listen to the ideas of others, and combine or build on shared ideas (e.g., Osborn, 1957), show that the brainstorming does not always lead to more fluency. These studies have shown that the number of ideas generated by a group is not necessarily greater than if those same individuals sat on their own and the sum of all ideas was collected (e.g., Paulus, Larey, & Dzindolet, 2000). Reasons for this deficiency may be that individuals can find it difficult to generate their own ideas when carefully listening to the ideas of others. It may also be that even if the individual is attentive to other ideas, there is not enough incubation time to reflect on one's own ideas or to integrate one's own ideas with the ideas of others (Paulus & Yang, 2000).

Fluency is not the only measure of creativity. When considering flexibility, the diversity of group members may be both a hindrance and an advantage. Consider when a group of diverse individuals comes together to solve a problem (as in the workplace). Individuals with different backgrounds and knowledge may contribute different perspectives; hence, the group exhibits flexibility. However, if diversity is too wide, it might hinder individuals from understanding the different ideas that are raised, making it difficult to agree upon a solution (Kurtzberg & Amabile, 2001). Likewise, if a complex system, such as a classroom, is to sustain itself and move forward, then there needs to be diversity, as well as redundancy (Davis & Simmt, 2003). Diversity refers to the different ways members of the community contribute to finding solutions to a given problem, whereas redundancy refers to the similarities that enable the system to cope with stress, allowing for different members to compensate for others' shortcomings. For example, in Levenson's (2014) study of the collective creativity found in a sixth-grade classroom, diversity was found in the various solutions, acceptable and unacceptable, that arose during a whole classroom discussion. Some students hesitantly offered solutions, while others boldly stated their solution. Other students took the role of evaluator. Yet, students were also able to compensate for each other's deficiencies. For example, when a student put forth an incorrect or unacceptable solution, others, building on the idea, corrected the situation. Thus, diversity can be a benefit to promoting creativity.

The question of originality in a group setting is complex. Levenson (2011), in her study of the collective creativity in a fifth-grade classroom, raised the question of who is being creative. Can we say that an individual student who comes up with an original idea during a classroom problem-solving activity is being creative? Must we not take into consideration whole classroom interactions that lead up to that original solution? In her observations and analysis of classroom discussions, Levenson (2011) focused on students' interactions with materials, other students, and teachers and the ways in which "mathematical ideas… initially stemming from an individual learner, become taken up, built on, developed, reworked and elaborated by others…" (Martin et al., 2006, p. 156). Her answer to the question was that original solutions were the product of the group working collectively, and as such, the collective process led to collective originality. Similarly, focusing on shifts in knowledge, Hershkowitz, Tabach, and Dreyfus (2017) found that creative ideas raised in a whole class situation may filter down into a small group. When the idea is adopted in some sense by other students, it can serve as a milestone for further inquiry by the group.

In the above studies, as in the current study, we relate to the collaborative work of a group, where individuals are working together toward the same goal (as opposed to helping each other reach individual goals). As shown above, we see that although working in groups may be disadvantageous to the promotion of creativity, it can also be beneficial. What is missing from the above studies is a comparison of the mathematical creativity elicited by groups compared with that of individuals. What is also missing is the question of how working on open-ended tasks in a group may affect those individuals' creativity when they are not in a group.

2.3 Research aim and questions

The aim of this study is to investigate and compare the mathematical creativity of post–high school students working as individuals to those working in small groups, when solving open-ended geometry tasks. Three attributes of creativity are assessed: fluency, flexibility, and originality. Specifically, the study asks: (1) Is there a difference between the creativity evidenced by students who solve an open-ended task individually and those who solve the same task in small groups? (2) When all students work individually on a task, is there a difference between the mathematical creativity of students who have experienced solving similar problems in a group and those who have not had the group experience?

3 Methodology

3.1 Participants and context

Participants were 92 students enrolled in a preparatory course for taking college entrance exams. Over 90% of all college-bound students enroll in such courses. All students had passed their matriculation exams at the end of high school in one of three levels of mathematics and were high school graduates. Matriculation exams consist of questions that have one correct answer, although there may be several ways of approaching each question. Students spend most of their last 2 years in high school preparing for these exams and have very little experience, if any, in solving open-ended tasks. According to Kattou et al. (2013), content knowledge might be a prerequisite for creativity, but at the same time, familiarity with rules might suppress creativity. Accordingly, care was taken to divide the students into two heterogeneous classes, 45 in one class and 47 in another, making sure that the percentage of students who had graduated at a certain level of mathematics was the same in both classes and that a similar mix was present among students who worked in a group. The instructor was the same for each group and was the first author of this paper. At no point in the course were students given grades by the instructor. Participation was completely voluntary. Students were advised that the activities were a way of reviewing geometrical concepts and could help open their minds to new ways of thinking. Based on casual comments from students in the course, most had little experience in high school with group work in mathematics classes. That being said, during the preparatory course, students are encouraged to study together after lessons and study groups are often formed.

The mathematical context of the study was geometry. This context was chosen because all students, regardless of their matriculation level, had learned Euclidian geometry in high school. In the weeks prior to the study as well as during the study, lessons focused on attributes of two-dimensional figures, including triangles, quadrilaterals, and circles, as well as regular polygons. The instructor also reviewed and practiced with the students calculating the perimeter and area of each of those figures. From the beginning of the course, the instructor taught the same lessons to each class.

3.2 Procedure

In all, the study took place over a 4-week period, with one activity being implemented each week. The study had three phases: an introductory phase (week 1), an intermediate phase (weeks 2 and 3), and an evaluation phase (week 4). Between activities, students continued to attend the preparatory course and lessons, including mathematics and geometry lessons, went on as planned. While this might be a limitation of the study, lessons not related to the study consisted of students solving exercises on their own and the instructor solving exercises on the board. Open-ended questions were not part of those lessons.

The aim of the introductory phase was to introduce students to the idea that a problem may be solved in several different ways and to give them practice doing so. During this phase, the instructor presented all students with a geometry task pertaining to finding the area of a triangle and explicitly requested students to solve the problem in as many ways as possible. After allowing several minutes for the students to solve the problem on their own, the instructor went over the results on the board, allowing students to show the different methods they used to solve the problem.

The intermediate phase took place during the following 2 weeks. During each of those lessons, students were presented with a geometry task, a different task for each lesson (see a description of each task in the following section). In one class ($N=45$), students worked individually, each on their own worksheet; hence, we call this Class I. In the second class ($N=47$), the students worked in small groups; hence, we call this Class G. There were between four to six students in each group, resulting in 12 groups. Each group was given one worksheet to work on together, although students were able to use their own writing utensils. Each group was video recorded. In each class, toward the end of each lesson, the instructor gathered up the worksheets and went over the activity together with the whole class, exposing everyone to the variety of solutions for the task.

The evaluation stage took place the following week. At this time, students in both classes worked individually on the task presented to them. This task also dealt with geometry and is described in the next section.

3.3 Tools

The tools included the video-recorded observations of the groups and the open-ended tasks. The researcher recording the groups focused, according to her discretion, on the worksheet and what was being written and erased or on the students speaking. Three open-ended tasks were employed in this study. Two were handed out during the intermediate stage, and one was handed out during the evaluation stage. The following is a description of each activity, the layout of the worksheet, and specific instructions.

The first task, called the “Geoboard” task (see Fig. 1) was based on Haylock (1997) (see Sect. 2.1). Students were reminded of the terms for using a geoboard, in that lines must be straight and must be anchored on “pegs” represented by the dots. Both sides of the worksheet were filled with “boards,” and students could request additional worksheets if they desired.

The second worksheet, called the “15 square units” worksheet, was essentially a plain piece of grid paper with the following instructions on top: “Draw as many different shapes as possible with an area of 15 units.” This was based on the task found in Levenson et al. (2018).

The third worksheet was handed out during the evaluation stage and was called the “The exception” worksheet (see Fig. 2). Unlike the first two tasks, this task does not request students to draw figures. We were concerned that on the one hand, students

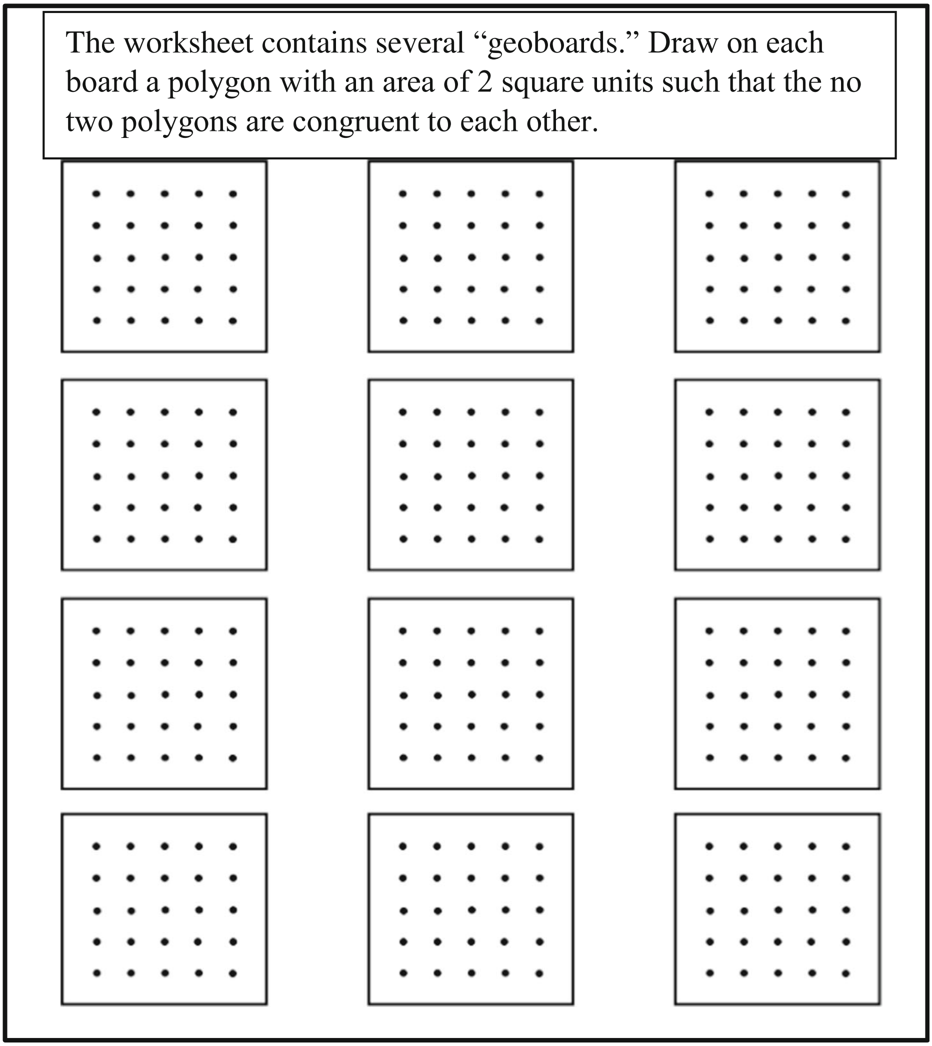

Fig. 1 Drawing polygons on "geoboards"

might not be interested in yet another drawing task and thus not participate as fully, and on the other hand, they might have had so much practice drawing that results would merely indicate a result of practice. We did consider a third drawing task, used by Levenson et al. (2018). However, results of that study indicated that it was not an appropriate task. Instead, we searched for another type of open-ended task, within the same content area, and adapted a task presented by Klavir and Hershkovitz (2008) and Kwon et al. (2006).

In adapting this activity to geometry, we chose figures that were familiar to the participants, where each figure could be considered an exception for different reasons. For example, only shape *d* (see Fig. 2) is not a pentagon, and only shape *a* is a regular polygon. However, shape *a* may also be considered an exception because it is the only one with all obtuse angles.

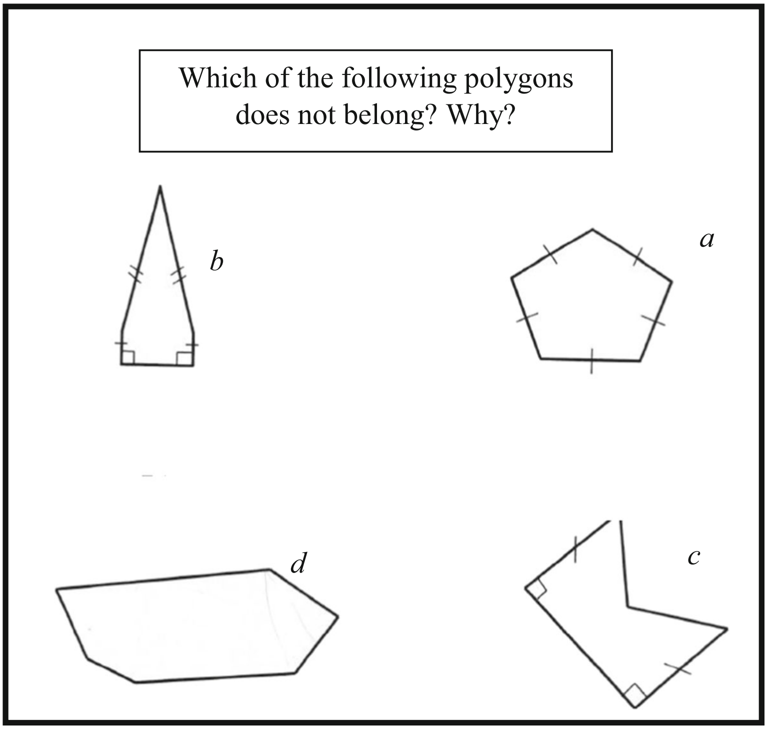

Fig. 2 The exception worksheet

3.4 Data analysis

Video recordings were transcribed by the first author of this paper. In order to ascertain participants' involvement in the group work, we tallied the number of speaking turns for each participant and also attempted to see how much each participant worked on the worksheet. This second analysis was unreliable as the camera sometimes moved off the worksheet in order to capture a student talking. Video recordings were also used to validate analysis of the written work. At times, the written work was either unclear or was insufficient when categorizing solutions. Thus, written work was reviewed and analyzed while listening and watching the video recordings (see, for example, the category of "counting" for the second worksheet).

The products of the students' work exhibited on each worksheet were analyzed in terms of fluency, flexibility, and originality. The fluency score for each worksheet was based on the number of correct (noncongruent) answers for that worksheet. The mean number of correct solutions was then calculated separately for Class I and Class G. To determine flexibility, solutions from all the worksheets of that activity were gathered and categorized. Categories were first determined based on the specific shapes that were reviewed in the course and their attributes. We then conducted an inductive analysis to determine additional categories. Categorization of all solutions was carried out independently by the first two authors. Disagreements were discussed until a consensus was reached. The flexibility score accorded to a worksheet was determined by the number of categories found on that worksheet. Details of category analysis are given below for each worksheet. Originality was based on both frequency and insight. In general, a solution that was present in 15% or less of the worksheets was considered original. Insight was based on the specific activity and will be discussed in the findings.

Previous studies (e.g., Leikin, 2009) combined fluency, flexibility, and originality scores into one creativity score. Among creativity researchers, great debate has surrounded how these three scores should be combined. For example, should the originality score be divided by the fluency score (Plucker, Qian, & Wang, 2011)? Among mathematics educators, similar debates occurred, for example, regarding which of the three components should be assigned the greatest weight when measuring creativity (Leikin, 2009). This study, however, relates to each component separately. For each class, the range of scores is also given.

3.4.1 "Geoboard" solution categories

Correct solutions for this activity (see Fig. 3) were first categorized into polygons that were reviewed during the course: triangles, squares, rectangles (that were not squares), parallelograms (that were not rectangles or rhombuses), deltoids (that were not parallelograms), and trapezoids.

Polygons not reviewed during the course but that students drew on their worksheets were quadrilaterals (that were not in any of the previous categories), nonregular pentagons, and nonregular hexagons. In total, there were nine categories.

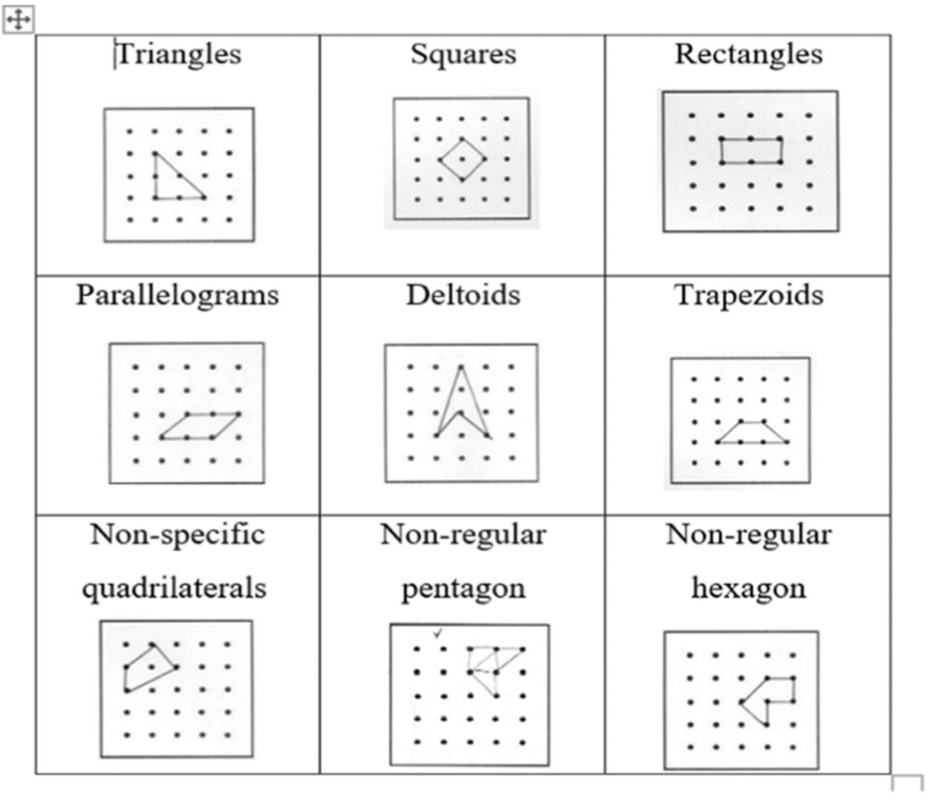

Fig. 3 Examples of geoboard solution categories

3.4.2 "15 square units" solution categories

As with the "geoboard" activity, solutions were first categorized according to shapes that were reviewed during the course before the study. In addition to those mentioned in the first activity, participants drew circles, sectors, regular pentagons, and regular hexagons. In total there were 11 categories based on figures reviewed in the course (see Fig. 4). As can be seen from Fig. 4, the writings on the drawings point mostly to the students' knowledge of calculating areas of geometrical shapes. Even when calculations are not explicit, markings point to this strategy. For example, on the nonsquare rhombus, one can see the markings on the diagonals, signaling the use of the area formula for a polygon with orthogonal diagonals. On the drawing of the regular hexagon, the hexagon was divided into six congruent equilateral triangles, and the side of the triangle is marked as $\sqrt{\frac{10}{\sqrt{3}}}$, which is the correct length of the side of a regular hexagon with area 15 units squared. The regular pentagon was divided into five triangles. A student marked the base and height of the triangle as 3 and 2, respectively. Interestingly, for the nonsquare rhombus, the drawing dimensions are in line with the grid lines. That is, one diagonal is actually 10 units long, and the other 3 units long. However, for the pentagon, this is not the case. The markings on the drawing of the pentagon do not reflect actual units as seen by the grid lines.

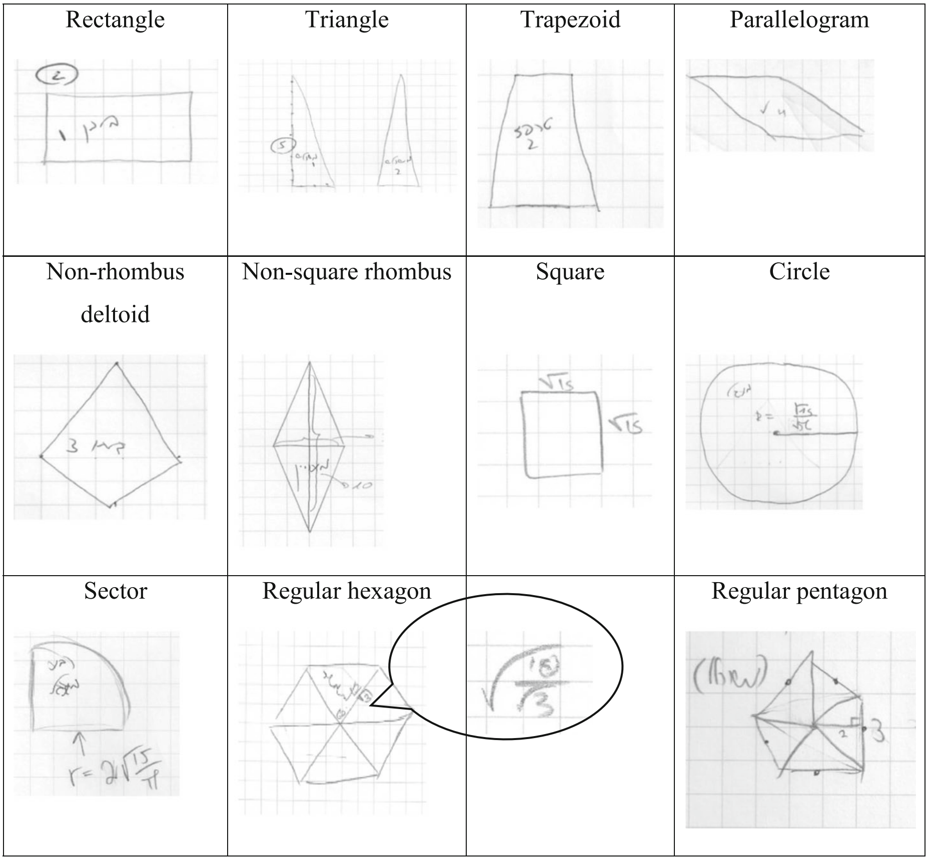

Fig. 4 Shape categories based on figures reviewed during the course

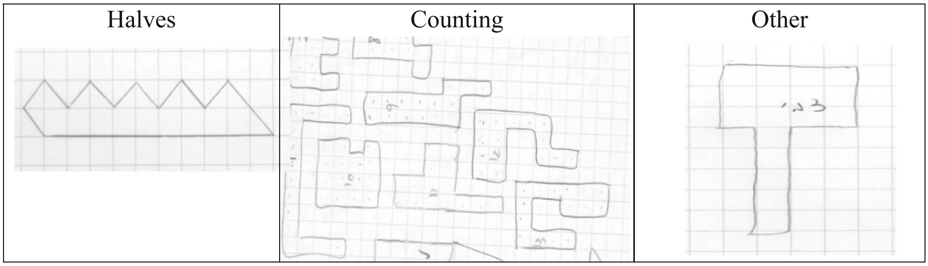

Fig. 5 Additional shapes not reviewed during the course

Additional figures, not reviewed during the course, were also drawn (see Fig. 5). These were sorted into three additional categories based on students' strategies as seen in the drawings on the grid paper, in conjunction with data collected from the observations. The first category, called "halves," was based on drawings where students constructed 15 square units, by using half units (see Fig. 5). The second category, called "counting," was based on drawings where a dot made by a pencil was found in each grid square and where it could be heard on the video that a student counted 15 grid squares (see Fig. 5). To distinguish between categories, none of the shapes shown above in Fig. 4 were included in the category of counting. Finally, the last category, consisting of all other valid drawings, was called "other." In this category were additional shapes not reviewed during the course, some of which were shapes constructed from other shapes (like the octagon in Fig. 6).

Altogether, there were 14 categories of solutions. Based on this categorization, a flexibility score was configured. In a similar manner to that of the geoboard worksheet, if a worksheet included three triangles, one rectangle, two parallelograms, one regular hexagon, and seven "counted" figures, a flexibility score of five (out of a possible 14 categories) was given to that worksheet.

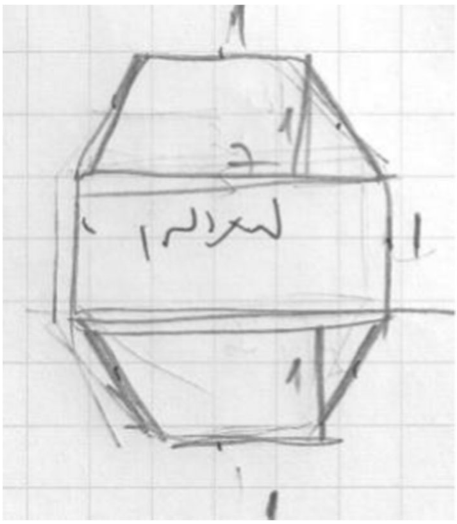

Fig. 6 Constructing an octagon from two trapezoids and a rectangle

3.4.3 "The exception worksheet" solution categories

For this activity, categorization was based on participants' reasons for why they thought a certain shape was an exception (see Table 1). Some of the categories seemed to have their basis on attributes of shapes reviewed in the course, such as the sum of the interior angles of a polygon. Other types of reasoning, such as "deconstructing the shape," arose from the data. As with previous tasks, the flexibility score was based on the number of categories found on each worksheet. Regarding categorization, note that three reasons refer to diagonals, but only one was placed in the category of diagonal-based reasons. The instructor of the course pointed out, and the other researchers concurred, the two other examples mentioned diagonals, but the focus was on other elements. For example, although the last example in Table 1 mentions the word diagonals, the emphasis is on the star shape produced by drawing diagonals. This reason was accompanied with a drawing by the student (see Fig. 7).

Table 1 Examples and frequency of categories for the "Exception" worksheet

Category	Example
Shape name	"Shape *d* is a hexagon."
Regular shape	"Only shape *a* is regular."
Angle-based reasons	"Shape *c* is the only one with an angle greater than 180°."
Side-based reasons	"Shape *d* has the most amount of sides."
Deconstructing the shape	"Shape *b* is the only one that can be deconstructed into a rectangle and triangle by drawing a diagonal."
Diagonal-based reasons	"Shape *c* is the only shape with a diagonal passing outside of the shape."
Vertices-based reasons	"Shape *d* has the most amount of vertices."
Concaveness	"Only shape *c* is concave."
Circle-based reasons	"Shape *a* is the only one that can be circumscribed within a circle."
Perimeter and area calculations	"Shape *a* is the only one where the area and perimeter can be calculated if you know the length of one of the sides."
Imaginative	"If you draw the diagonals in shape *a*, you get a star."

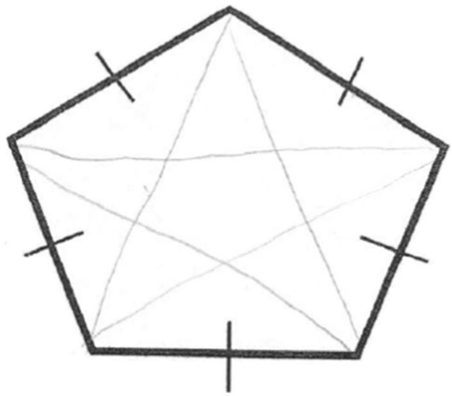

Fig. 7 A star made by drawing diagonals of a pentagon

4 Findings

4.1 Group observations

Findings from the video-taped observations showed that indeed the individuals in the groups collaborated with each other and that it was not the case that one student did all the work for an entire group. Evidence of this was found in several areas. For example, during the first activity, one participant was observed holding the pencil, taking responsibility for drawing, while other participants leaned over the paper and were seen and heard to contribute to the discussion. In that group, out of 175 speaking turns, the distribution of turns among the participants was 31% (for the pencil holder), 25%, 9%, 13%, 9%, and 13%. While the number of turns for each participant may not be equal, it is evidence that all participated and were attentive. Among all groups, the least amount of participation by an individual was 7%. Additional evidence of group work can be seen by looking at the writings of the group worksheet. Note in Fig. 8 that the calculations are written from different sides of the paper, some facing the "right" way, and some facing upside-down. Evidence of group work was also found in the questions group members asked one another. For example, one student said to his group mates, "Ok. We have a

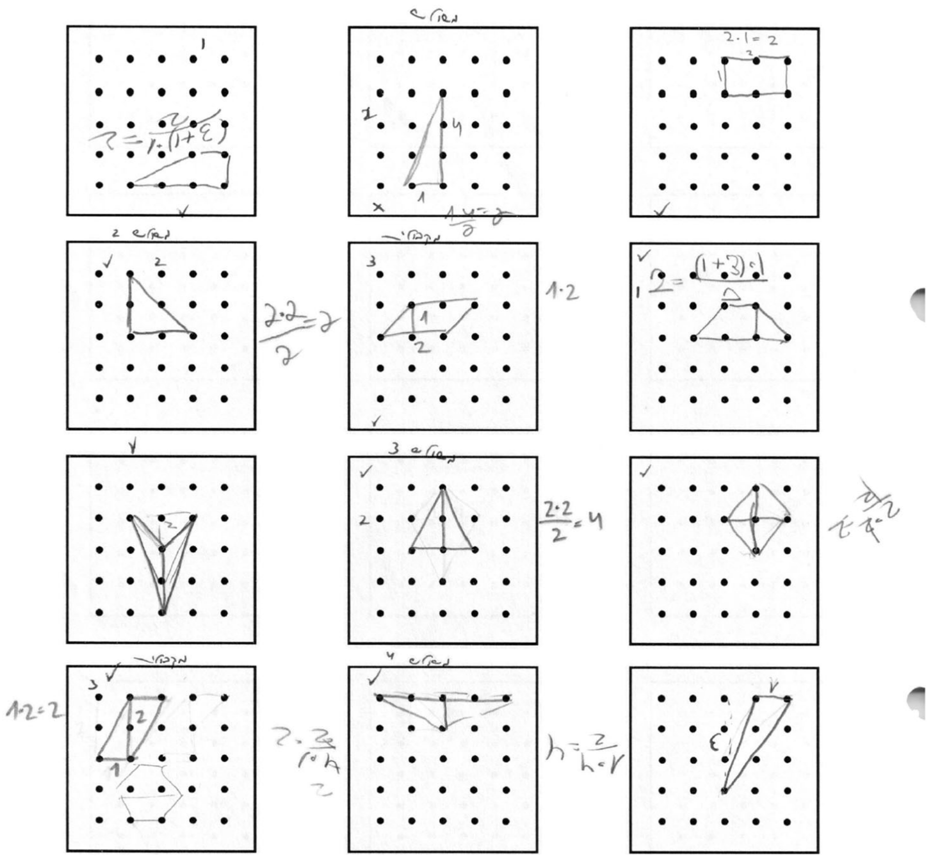

Fig. 8 Working on the geoboard activity from different sides of the paper

rhombus, triangles, a rectangle. Do we have a square?" Students in the group also offered words of encouragement, such as "Guys, this (pointing to the worksheet) is the result of working together. In my life, I never would have been able to do this myself."

In the following sections, fluency, flexibility, and originality scores are presented for each worksheet separately. The scores for those working individually are then compared with the group scores.

4.2 General findings

Results regarding fluency, flexibility, and originality for all three tasks are presented in Table 2. Independent samples t tests were used to compare the means between classes. Significant differences of $p < .01$ are noted by a double asterisk (**).

4.3 The "geoboard" worksheet

As seen in Table 2, no significant differences were found between Class I and Class G on any of the three creativity measurements for the first task. Regarding originality, we note that 33 different solutions were found. Recall that a solution was considered original if it was found in less than 15% of the worksheets. A total of 19 original solutions fit this definition. Yet, of the 19 solutions, 14 were pentagons or hexagons. Due to multitude of ways these figures could be drawn, it is not surprising that so many different ones appeared in less than 15% of the worksheets. The other five original solutions were an obtuse triangle, a nonspecific quadrilateral, and three different deltoids (two concave and one convex). Each original solution was accorded one point. Thus, if three original solutions were found on a worksheet, the originality score for that worksheet was three. The range of originality scores for Class I was from 0 to 5, while for Class G, the range was from 0 to 2. The mean number of original solutions for Class I was 1.36 (SD = 1.25), and for Class G was .83 (SD = 0.83). Comparing means using an independent samples t test, no significant differences were found between the classes in terms of originality scores.

Table 2 Fluency, flexibility, and originality scores per task, per class

	Class I M (SD), Range	Class G M (SD), Range	t value	Eta-squared
Task 1: Geoboard				
Fluency	7.51 (3.04), 0–12	8.42 (1.8), 5–11	NS	
Flexibility	5.20 (1.80), 0–8	5.67 (0.89), 4–7	NS	
Originality	1.36 (1.25) 0–5	0.83 (0.83), 0–2	NS	
Task 2: 15 square units				
Fluency	9.38 (5.11), 3–30	19.08 (13.02), 5–52	4.03**	0.228
Flexibility	4.89 (1.86), 1–9	7.08 (1.44), 5–9	3.71**	0.200
Originality	0.44 (0.69), 0–3	0.67 (1.07), 0–3	NS	
Task 3: Exception worksheet				
Fluency	1.90 (1.43), 0–5	3.00 (1.39), 0–6	3.44**	0.135
Flexibility	1.60 (1.22), 0–5	2.82 (1.14), 0–5	4.56**	0.215
Originality	0.38 (0.84), 0–3	0.66 (1.26), 0–5	NS	

4.4 The “15 square units” worksheet

This task has an infinite number of solutions. Thus, not surprisingly, the fluency scores were greater for this task than the previous task. As shown in Table 2, fluency and flexibility scores were significantly greater in Class G than for Class I.

Regarding originality, taking into consideration the great number of solutions produced for this activity (over 50 solutions on one of the worksheets), basing originality on each separate figure would have led to a great many original figures. Thus, for this activity, originality was based on the frequency of a category occurring among the worksheets, as opposed to a single solution.

Four categories appeared in less than 15% of the worksheets: regular hexagons (4%), halves (4%), sectors (2%), and regular pentagons (2%). In addition, two categories, squares and circles, were considered to be original based on insight because the worksheet was on grid paper, which was thought to lead students to calculate areas based on the units of the grid paper or at least to base shapes on rational numbers. However, unlike the “geoboard,” which had strict rules about drawing lines between “pegs,” this activity did not have strict rules, although the same rules might have been assumed by some students. Students breaking away from the grid lines were considered to have insight. Taking this into consideration, drawing a square was considered insightful because in order to come up with a 15-square-unit area, the side had to be an irrational number (i.e., $\sqrt{15}$) and obviously not aligned with the grid. The square was found in 21% of the worksheets. The same, of course, can be said for the circle, which was found in 16% of the worksheets. The drawing of the pentagon in Fig. 4 was also not in line with the grid units. In other words, those students who broke away from the grid lines came up with original solutions. Altogether, six categories were considered original categories. No significant differences were found between the classes in terms of originality scores.

4.5 The exception worksheet

During the third stage of the study, all students worked individually on their worksheets, allowing us to compare the fluency, flexibility, and originality of participants who had worked individually on the previous two worksheets (those in Class I) to those who had experienced group work (in Class G). Due to several absent students on the day this worksheet was handed out, 40 students from Class I and 38 students from Class G participated.

A response consisted of both a specific shape considered to be the exception, as well as the reason for claiming that shape to be the exception. Thus, if a student responded that shape *d* (see Fig. 2) does not belong because it is the only hexagon and then also wrote that shape *d* is the exception because the sum of its interior angles is greater than the others, this was counted as two separate responses. A total of 25 different responses were found. As shown in Table 2, fluency in Class G was significantly greater than in Class I. A reduction in effect size for fluency between the second and third tasks was observed, though the effect size remained substantial. Regarding flexibility, note that the large effect size obtained on the second task was maintained in the third task, even though Class G changed its mode of work to individuals.

In considering originality, categories of reasoning that appeared in less than 15% of the total number of responses were considered original. The reason for considering original categories as opposed to original individual reasons was because out of 25 different specific reasons, 24 appeared in less than 15% of the worksheets. Thus, the frequency of each category was calculated, and the following categories of reasoning were found to be original: diagonal-based reasoning (13%), vertices-based reasoning (5%), concaveness (4%), circle-based reasoning (3%), perimeter and area reasoning (1%), and imaginative reasoning (1%). No significant differences were found between the classes in terms of originality scores.

5 Summary and discussion

All three open-ended tasks related to geometrical figures and their properties, a topic recently reviewed in the course. Results of the first task did not show differences between those working individually and those working in groups in any of the three aspects of mathematical creativity investigated. Results of the second and third tasks showed greater fluency and flexibility among those that worked or had worked in groups, but no differences in originality. Why were differences found in fluency and flexibility, but not for originality? Why were differences found in the second task, but not in the first, even though the groupings of students were the same for both? Why were differences found in the third task, even when all students worked individually? This section discusses these questions.

There are several possible reasons for why no differences in originality were found between those working as individuals and those working in groups. As reviewed in the background, originality may be characterized by a unique way of thinking, that is, thinking that is different from the group, retaining individuality. Coming up with an original solution can involve illumination, a process whereby an unconscious idea comes to the fore of the mind, often associated with an AHA! experience (Liljedahl, 2013). This is an individual experience. Originality may also be associated with high mathematics content knowledge (Van Harpen & Presmeg, 2013). Indeed, many of the original solutions in this study, such as coming up with a regular hexagon of area 15 and knowing that a regular pentagon can be circumscribed in a circle, may be ascribed to geometrical knowledge that goes beyond properties normally taught in school. In addition, these original solutions were found in maybe one or two worksheets. Further evidence that originality is a more internal characteristic than fluency and flexibility was found by Levav-Waynberg and Leikin (2012). In their study, high school geometry students who engaged with multiple solution tasks increased their fluency and flexibility, but not their originality.

Yet, if originality stems from the individual, one might think that originality could be depressed in a group setting. First, as pointed out in the background, there is less time for incubation and more distraction (e.g., Paulus & Yang, 2000). Second, group dynamics, such as rudeness, having to save face, and status concerns, may suppress originality (Chiu, 2008). Although rudeness was not observed in this study, we find that another contribution of this study is that an individual's ability to express original mathematical thinking is not necessarily suppressed by the group.

There are several known benefits to working in a group. However, some of the beneficial aspects may take time to develop, hinting at why differences in fluency and flexibility between the classes were not found in the first task. Yackel and Cobb (1996), for example, discuss the "evolution of students' understanding of what counts as an acceptable mathematical explanation and justification" (p. 467). Evolution is a process that develops over time, in this case, through interactions between participants in the classroom. In Levenson's (2011) study, collective mathematical creativity was found among students who had been learning together as a class for years and had been with the same class teacher for months. Yet, in the current study, students did not have a long period of time between tasks in which they could work out norms and understandings. Instead, we turn toward theories that view collective learning and understanding as an improvisational process, one in which the flow of the participants is unpredictable and emerges from the actions of the participants (Sawyer, 2004). If improvisation is to be productive, it is necessary to develop a "group etiquette" (Francisco, 2013; Martin et al., 2006), a way of being in the group. The notion of learning as an improvisational process fits with the current study in that the exchanges between group participants could not be predicted. It could be that while working together on the first task, students were busy developing a group etiquette, possibly different etiquettes for different groups, but one that may have allowed the groups to be more productive when working on the second task.

Regarding the differences in fluency and flexibility between the classes on the second task, we first note that the third task was different from the first two, and findings might be a result of this difference. That being said, one of the benefits of group learning and group problem solving is exposing students to a variety of ideas and problem solving strategies, providing them with opportunities to critically examine ways of reasoning, and build a deeper understanding of mathematical concepts (Francisco, 2013; Goos, Galbraith, & Renshaw, 2002). Thus, it might simply be that those who worked in the group gained additional mathematical knowledge that served them later when working alone. However, in both classes, the instructor went over the task solutions after the worksheets were handed in, essentially exposing all students to a wide variety of solutions and strategies.

In addition to increased mathematics knowledge, group work may have other benefits, benefits that are specifically gained during collaboration but may remain with the individual when working alone. One of those benefits is an increase in metacognitive control behaviors, such as noting the conditions of a specific problem, analyzing appropriate plans, and taking account of progress made (e.g., Goos et al., 2002). In a study of undergraduate students working on combinatorial problems, where some worked in pairs and some worked individually, those working in pairs not only showed a higher degree of control than those who worked individually but also had more correct solutions (Eizenberg & Zaslavsky, 2003). Analyzing the results of that study from the perspective of creativity, more correct solutions may be interpreted as higher fluency. Similar results were found among eighth graders, with the addition of greater flexibility among students working collaboratively than among students who worked individually (Kramarski & Mevarech, 2003). While this study did not focus on types of group interactions, observations did show that the individuals were collaborating with each other to achieve a common goal. Acknowledging that the students' group experiences in this study were relatively brief, this study may be considered as a first step in extending those previous studies, relating group work, individual work, and creativity. A next step would be to first investigate students' fluency, flexibility, and originality on an individual basis before they work in groups, and then again after the group experience, with larger sample sizes and perhaps with students learning in different educational settings.

An additional direction would be to investigate further the group creativity processes, including developing measures for assessing an individual's contribution to a group process, as well as focusing on processes that are retained when working alone. Presmeg (2003) called for a need "to investigate more deeply the reflexive interplay between individual learning and group processes in mathematics classrooms." Although progress has been made, this paper shows that there is a continuing need to investigate these processes as they may also affect mathematical creativity.

References

Chiu, M. M. (2008). Effects of argumentation on group micro-creativity: Statistical discourse analyses of algebra students' collaborative problem solving. *Contemporary Educational Psychology*, *33*(3), 382–402.

Davis, B., & Simmt, E. (2003). Understanding learning systems: Mathematics education and complexity science. *Journal for Research in Mathematics Education*, *34*(2), 137–167.

Eizenberg, M. M., & Zaslavsky, O. (2003). Cooperative problem solving in combinatorics: The inter-relations between control processes and successful solutions. *The Journal of Mathematical Behavior*, *22*(4), 389–403.

Francisco, J. M. (2013). Learning in collaborative settings: Students building on each other's ideas to promote their mathematical understanding. *Educational Studies in Mathematics*, *82*(3), 417–438.

Gómez-Chacón, I. M., & de la Fuente, C. (2018). Problem-solving and mathematical research projects: Creative processes, actions, and mediations. In N. Amado, S. Carreira, & K. Jones (Eds.), *Broadening the scope of research on mathematical problem solving* (pp. 347–373). Cham, Switzerland: Springer.

Goos, M., Galbraith, P., & Renshaw, P. (2002). Socially mediated metacognition: Creating collaborative zones of proximal development in small group problem solving. *Educational Studies in Mathematics*, *49*(2), 193–223.

Haylock, D. (1997). Recognizing mathematical creativity in schoolchildren. *ZDM Mathematics Education*, *27*(2), 68–74.

Hershkowitz, R., Tabach, M., & Dreyfus, T. (2017). Creative reasoning and shifts of knowledge in the mathematics classroom. *ZDM Mathematics Education*, *49*(1), 25–36.

Jung, D. I. (2001). Transformational and transactional leadership and their effects on creativity in groups. *Creativity Research Journal*, *13*(2), 185–195.

Kattou, M., Kontoyianni, K., Pitta-Pantazi, D., & Christou, C. (2013). Connecting mathematical creativity to mathematical ability. *ZDM Mathematics Education*, *45*(2), 167–181.

Kaufman, J., & Beghetto, R. (2009). Beyond big and little: The four C model of creativity. *Review of General Psychology*, *13*(1), 1–12.

Kim, M. K., Roh, I. S., & Cho, M. K. (2016). Creativity of gifted students in an integrated math-science instruction. *Thinking Skills and Creativity*, *19*, 38–48.

Klavir, R., & Hershkovitz, S. (2008). Teaching and evaluating 'open-ended' problems. *International Journal for Mathematics Teaching and Learning*, *20*(5), 23.

Kramarski, B., & Mevarech, Z. R. (2003). Enhancing mathematical reasoning in the classroom: The effects of cooperative learning and metacognitive training. *American Educational Research Journal*, *40*(1), 281–310.

Krutetskii, V. A. (1976). *The psychology of mathematical abilities in schoolchildren*. J. Teller, J. Kilpatrick, & I. Wirszup (Eds.). Chicago, IL: The University of Chicago Press.

Kurtzberg, T., & Amabile, T. (2001). From Guilford to creative synergy: Opening the black box of team-level creativity. *Creativity Research Journal*, *13*(3 & 4), 285–294.

Kwon, O. N., Park, J. S., & Park, J. H. (2006). Cultivating divergent thinking in mathematics through an open-ended approach. *Asia Pacific Education Review*, *7*(1), 51–61.

Leikin, R. (2009). Exploring mathematical creativity using multiple solution tasks. In R. Leikin, A. Berman and B. Koichu (Eds.) *Creativity in mathematics and the education of gifted students* (pp. 129-135), Sense Publishers.

Leikin, R., & Lev, M. (2013). Mathematical creativity in generally gifted and mathematically excelling adolescents: What makes the difference? *ZDM Mathematics Education*, *45*(2), 183–197.

Levav-Waynberg, A., & Leikin, R. (2012). The role of multiple solution tasks in developing knowledge and creativity in geometry. *The Journal of Mathematical Behavior*, *31*(1), 73–90.

Levenson, E. (2011). Exploring collective mathematical creativity in elementary school. *Journal of Creative Behavior*, *45*(3), 215–234.

Levenson, E. (2013). Tasks that may occasion mathematical creativity: Teachers' choices. *Journal of Mathematics Teacher Education*, *16*(4), 269–291.

Levenson, E. (2014). Investigating mathematical creativity in elementary school through the lens of complexity theory. In Ambrose, D., Sriraman, B. and Pierce, K. M. (Eds.), *A critique of creativity and complexity-Deconstructing clichés* (pp. 35-52). Rotterdam, the Netherlands: Sense Publishers.

Levenson, E., Swisa, R., & Tabach, M. (2018). Evaluating the potential of tasks to occasion mathematical creativity: Definitions and measurements. *Research in Mathematics Education*, *20*(3), 273–294.

Liljedahl, P. (2013). Illumination: An affective experience? *ZDM Mathematics Education*, *45*(2), 253–265.

Liljedahl, P., & Sriraman, B. (2006). Musings on mathematical creativity. *For the Learning of Mathematics*, *26*(1), 17–19.

Luria, S. R., Sriraman, B., & Kaufman, J. C. (2017). Enhancing equity in the classroom by teaching for mathematical creativity. *ZDM Mathematics Education*, *49*(7), 1033–1039.

Mann, E., Chamberlin, S. A., & Graefe, A. K. (2017). The prominence of affect in creativity: Expanding the conception of creativity in mathematical problem solving. In R. Leikin & B. Sriraman (Eds.), *Creativity and giftedness: Interdisciplinary perspectives from mathematics and beyond* (pp. 57–76). Cham, Switzerland: Springer.

Martin, L., Towers, J., & Pirie, S. (2006). Collective mathematical understanding as improvisation. *Mathematical Thinking and Learning*, *8*(2), 149–183.

Osborn, A. F. (1957). *Applied imagination*. New York, NY: Scribner's.

Paulus, P. B., Larey, T. S., & Dzindolet, M. T. (2000). Creativity in groups and teams. In M. Turner (Ed.), *Groups at work: Advances in theory and research* (pp. 319–338). Hillsdale, NJ: Hampton.

Paulus, P. B., & Yang, H. (2000). Idea generation in groups: A basis for creativity in organizations. *Organizational Behavior and Human Decision Processes*, *82*(1), 86–87.

Plucker, J. A., Qian, M., & Wang, S. (2011). Is originality in the eye of the beholder? Comparison of scoring techniques in the assessment of divergent thinking. *The Journal of Creative Behavior*, *45*(1), 1–22.

Presmeg, N. (2003). Creativity, mathematizing, and didactizing: Leen Streefland's work continues. *Educational Studies in Mathematics*, *54*(1), 127–137.

Runco, M. A., & Albert, R. S. (1985). The reliability and validity of ideational originality in the divergent thinking of academically gifted and nongifted children. *Educational and Psychological Measurement*, *45*, 483–501.

Sawyer, R. K. (2004). Creative teaching: Collaborative discussion as disciplined improvisation. *Educational Researcher*, *33*(2), 12–20.

Silver, E. (1997). Fostering creativity through instruction rich in mathematical problem solving and problem posing. *ZDM Mathematics Education*, *3*, 75–80.

Tsamir, P., Tirosh, D., Tabach, M., & Levenson, E. (2010). Multiple solution methods and multiple outcomes – Is it a task for kindergarten children? *Educational Studies in Mathematics*, *73*(3), 217–231.

Van Harpen, X. Y., & Presmeg, N. C. (2013). An investigation of relationships between students' mathematical problem-posing abilities and their mathematical content knowledge. *Educational Studies in Mathematics*, *83*(1), 117–132.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education*, *22*, 390–408.

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