



Affect and emotions in mathematics education: toward a holistic psychology of mathematics education

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Abstract

Emotions and affect have become an area of interest to mathematics research. However, both tend to be intellectualized and approached as external to and separate from intellect. As far back as in the 1930s, Vygotsky considered the split between affect and intellect as psychology's greatest defect. To address that defect, over the last 18–24 months of his life, he sketched the barebones of a different psychology but died before he could develop his ideas into a theory of emotions. His ideas provided us with a roadmap for developing an approach in which intellect and affect are united to constitute a central part in the life of the whole person. Such a position, conceived here in dramatic terms, remains virtually unknown, in part because of the unavailability of specific texts and personal notes until a few years ago. In this study, we develop Vygotsky's seeds and go beyond what he actually stated. We focus our conceptualization of affect on mathematics classroom practices and exemplify our theoretical perspective with empirical data drawn from a number of studies.

Keywords Affect · Emotions · Dualism · Monism · Personality · Societal relations · Drama

1 Introduction

Mathematics education research historically has been preoccupied with reasoning at the expense of other aspects of students' and teachers' classroom experiences. More recently, however, a number of mathematics education researchers have shown considerable interest in the affective dimensions of mathematical learning—for example, the special issues of *Educational Studies in Mathematics*, 63(2) and 100(3) on affect in mathematics education (Batchelor, Torbeyns, & Verschaffel, 2019; Zan, Brown, Evans, & Hannula, 2006); and the special issue in *ZDM: Mathematics Education*, 49(3) (Schukajlow, Rakoczy, & Pekrun, 2017). In these

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engagements, however, affect tends to be approached in a dualist manner, which manifests itself as a parallelism of intellect and emotions. Thus, for example, emotions are thought of as “involv[ing] physiological reactions, as distinct from non-emotional cognition” and to be “consequences of cognitive processing” (Hannula, 2004, p. 108), where they “bias attention and memory and activate action tendencies” (p. 108). Thus, engagements with emotions tend to follow an intellectual route, clearly apparent in (a) discursive practice theoretical approaches (e.g., Evans, 2000), (b) systems of representation and communication (e.g., Goldin, 2004), (c) models that focus on self-evaluation and self-regulation (e.g., Malmivuori, 2006), and (d) (socio-)constructivist perspectives (e.g., Op’t Eynde, De Corte, & Verschaffel, 2006).

Whereas all the cited perspectives are distinct in approach, there is a significant point of convergence amongst them. At the heart of this work is the enlightenment idea of reason as universal, a priori, and substantive, and working from this premise, it is possible to propose a separation between affect and intellect. In the research on “affect traits,” constructs are developed as a means of capturing the non-rational aspects of subjectivity. Included in the suite of constructs are beliefs, attitudes, engagement, motivation, and anxiety (e.g., Clarke, 2015; Dowker, Cheriton, Horton, & Mark, 2019), all of which allow researchers to promote a cause-effect logic in descriptions of the performance of affected individuals. Examples of studies in mathematics education promoting such a logic may include those that (a) explore teachers’ beliefs about mathematics and mathematics education (Beswick, 2012); (b) show that open-ended investigations are effective in improving pre-service primary teachers’ attitudes toward mathematics (Bailey, 2014); (c) report primary school students’ enhanced engagement with mathematics when iPads were introduced (Attard & Curry, 2012); (d) explore the development of mathematics anxiety over time of a previously high achieving student (Heyd-Metzuyanim, 2015); and (e) develop a program to enhance teachers’ understandings of student motivation (Turner, Warzon, & Christensen, 2010).

In this article, we move away from the theoretical dualism that underpins contemporary engagements with affect—even in the case for embodiment (Sheets-Johnstone, 2009) and enactivist theories (Sheets-Johnstone, 2011)—to understand affect more holistically. To achieve the objective of developing a more unifying construct of affect, we are indebted to the later work of Vygotsky and, more specifically, to his Spinozist-Marxian work (Vygotsky, 1999), in which affect is *never external to intellect*. The argument, offered by Vygotsky as a more convincing explanation of affect, was driven by a need to solve the problem of the dualist approach, expressed in the separation of affect from intellect and in the separation of body from mind (Vygotsky, 1987). Exemplifying his proposal with the contrast of (a) hydrogen and oxygen and (b) water, he thus proposed moving from the analysis of elements of activity to one of units. A differentiation like this, according to Vygotsky, and more recently highlighted by a number of feminists (e.g., Butler & Scott, 1992; Code, 1995) is the “psychophysical problem” of psychology. More particularly, the dualist position manifests itself in the existence of two psychologies: scientific (physiological, biologically oriented) psychology and interpret(at)ive (idealist, mentalistic, metaphysical) psychology (Vygotsky, 1997).

Here, we take up the notion that affect is integral to intellect; intellect is one of the consequences of the affective nature of an organism’s life (e.g., Leont’ev, 1981). This notion provides a counterpoint to contemporary interpretations of Vygotsky’s work within mathematics education. Typically, within mathematics education, distinctions drawn between lower (bodily) and higher (mental) functions serve to endorse “the old Cartesian dualism” (Zavershneva, 2010b, p. 72). Of significance to our theoretical development is the idea that affect is an active state. According to Vygotsky in his *The Teaching about Emotions* (Vygotsky,

1999), the essence of human nature is passion, not in the contemporary sense of the word, but as experienced in our “being affected” and in our affects, as we journey through life on a path to freedom. Psychologically, this integration of intellect and passion is best understood in terms of drama, because each of its phases has practical, intellectual, and affective qualities (Politzer, 1929; Roth, 2019; Vygotsky, 1989), an issue that we exemplify and explain below.

Through his more comprehensive understanding of Spinozism and the unity and identity¹ of affect and intellect, the later Vygotsky (1999) (a) rejected the physiological approach to affect, such as that exemplified in the approach of the psychologists W. James and C. G. Lange and (b) dismissed the meaning-oriented, descriptive, and interpretive psychological approach exemplified, for example, in the works of the philosophers W. Dilthey and M. Scheler. In his innovative theory of development, Vygotsky—as did a number of other social theorists (e.g., Foucault, 1988; Lacan, 1977; Nietzsche, 1967)—took as his starting point the centrality of “will” for understanding human behavior. This leads us to a construct familiar to many mathematics education researchers, namely the notion of activity and, more specifically, descriptions such as *affective-volitional* (Vygotsky, 1987) or *emotional-volitional* in character (Bakhtin, 1993). However, the key to understanding the notion of activity in the way that Vygotsky intended (and the point that many researchers have missed) is that the unity is expressed as a paradox. Put simply, the claim that activity is a unity of affect and will is a deceit, principally because such an expression reduces “feelings to a purely cognitive process” (Vygotsky, 1999, p. 176).

In Vygotsky’s proposal, which we take as our starting point, the human subject is not only an agent but also has a connotation of being subjected to. On the one hand, human beings master passions through concepts, that is, they become free subjects of action. On the other hand, human beings are subjected to passions to the extent that mastering the passions also means yielding to them. As a result, the notion of the human subject is internally contradictory: it is endowed with agency while, at the same time, subjected to the conditions of its agency (cf., Walshaw, 2014). Thus, passions are never completely mastered. As Vygotsky explained, “freedom [comes through] affect in the concept” (in Zavershneva & van der Veer, 2018, p. 209), which is realized in the word—its auditory part *manifesting affect* and its super-auditory part *manifesting the ideal concept*. Simultaneously, the verbal expression manifests the unity of the different perspectives of participants in a social relation during mathematics lessons (Roth, 2016). Freedom comes from assuming necessity, arising from the multiple determinations of the word (Vygotsky, 1999). However, if we can no longer conceive of the human subject as an affective and willing subject, able to act in an autonomous fashion, we might begin to wonder, like Radford (2012): “How can the modern subject be the locus of meaning, feeling and intentionality if it has to talk, feel and intend through thoughts and words that are not its own?” (p. 106). More specifically for mathematics education research, how might we investigate emotions when talking *about* an emotion—in questionnaire or interview—is not the same as emotion? Thus, in one short sweep of the Vygotskian pen, contemporary approaches to affect within mathematics education are placed under erasure. Once old orthodoxies are disturbed, we are presented with a new set of challenges relating to the analysis of affect.

¹ In dialectics, two opposing terms (e.g., intellect and affect) not only form a unity but also are identical such that there is difference at the heart of identity (cf. Marx & Engels, 1975). The expression “unity and identity” (Vygotsky, in Zavershneva & van der Veer, 2018, p. 245) reflects the dual meanings of Marx’s German *Einheit* and Vygotsky’s Russian *edinstvo*.

Vygotsky's envisioned theory of emotions is one of the most complex and differentiated on offer. Even so, it remains one of the most undeveloped areas of inquiry. Indeed, it has been observed that "a cultural-historical theory of emotions. . . has yet to be created" (Zavershneva, 2010b, p. 81). In this article, we seek to contribute to the creation of such a theory for the psychology of mathematics education. We do that by re-articulating perspectives on affect that arise from the seeds Vygotsky offered during the last months of his life but never had the opportunity to develop (Zavershneva, 2010a).

2 The drama in/of mathematics lessons

In searching the literature for exemplary case studies where sufficient detail is provided to be usable for exemplifying the theoretical proposals made here—i.e., providing evidence for on gestures, facial expressions, pitch and speech volume, body positions and orientations, and so forth—we found only one (Roth & Radford, 2011). Most studies we found are of the descriptive/interpretive type, which Vygotsky critiqued in that they attempt to derive claims about emotions that occur *during* the doing of mathematics from after the fact interviews. Fewer studies collect data while students do mathematics, for example, by recording comments students make. Thus, for example, Walshaw recorded her student Rachel talking into a tape recorder saying:

[2.31] I don't get it. I don't GET it. What am I... What do I make it...?

[2.36] It's stupid. Can't do THAT.[2.37] Can't do THAT. (Walshaw, 1999, p. 207)

In such instances, too, the student is making comments about the present state of affairs—not being able to do the task. It is an intellectual assessment, a form of intellect, rather than a manifestation of affect in the form of affect. However, as Vygotsky (1999) following Spinoza suggests, there is no way we can get from the material body ("material substance") to thought ("thinking substance") or from thought to the material body. *Interaction* between the two merely is a revival of a Cartesian dualism.

In another example, Roth and Radford (2011) describe the case of three fourth-grade students in a French immersion classroom working on an algebra task. Their study is instructive in the present context because it contains aspects that go against the grain of often-reported cases and there is evidence in the case study that language is an issue for some students. The task presents the story of a girl, who receives a piggybank with \$6, and who decides to save \$3 each week. The students are asked to model the story for the first six weeks using goblets and differently colored chips. In a second step, they are asked to fill a table of values that already contains some prompts concerning the contents of each cell (e.g., "+ 6" below "1" of week 1). Below the row containing the number of the week, the curriculum intends students to write the amount of money in the piggybank in the form $3 + 6$, $3 + 3 + 6$, $3 + 3 + 3 + 6$ etc. The intended contents of the third row is a multiplicative form of the second: $3 + 6$, $2 \times 3 + 6$, $3 \times 3 + 6$ etc.

The three students Thérèse, Mario, and Aurélie respond very differently to this task. Thérèse's greater confidence is seen in the fact that (a) the other two students turn to her to ask what she had done, (b) Mario tells the teacher that Thérèse is finishing on her own, and (c) Thérèse is filling part of Aurélie's table of values. Mario begins the task, but then turns for help, initially toward Thérèse, then calls the teacher. He repeatedly emphasizes his lack of understanding, which the teacher treats as a request to help. In a protracted exchange between

the teacher and Mario, both parties manifest frustration and act upon the frustration manifested by the other. Eventually, after completing the fourth cells of the second and third row of the table of value, the teacher leaves the group after having said, “I think you understand now.” Mario completes the remaining two cells of row two and three after which he states, “Me, I now understand.”

From early on in the authors’ account, Aurélie is struggling with the task, manifesting emotion in bodily form and voice: She throws herself against the back of the chair to lounge in apparent disengagement with the task, and she pounds the desk top with her right hand in what can be seen as frustration. Prior to the lesson fragment below, she has already said in what the authors characterize as “plaintive voice” that “it doesn’t make sense” (Roth & Radford, 2011, p. 167). She also said, with “plaintive” intonation, “What are you doing Thérèse?” and with “lamenting” voice, “We have no idea what you are doing” (p. 167). During that time, Thérèse laughs and manifests “confidence,” inviting others, “Aw, chuggy, just copy me” (p. 167). She has already finished filling up her table of values. The following exchange ensued, here presented in both the original (mixed-language) talk and the English translation.

2.1 Lesson fragment²

29a A: ((*head on desk*))

b ((*lifts head, plaintive*))

c j=cOMprENds pA::s. je vais jamAIs [comprEN]dre.
 ((*I understand not; I will never* [understand]))

30 (0.84)



31a T: tiENs (0.30) faut que tu fasse (1.41) trois plus six (0.60) yup.
 ((hold have to you do three plus six yup))

b (1.79)

c [t’écris] whatever.
 (([ya write] whatever, in English))
 ((*Aurélie pounds on desk*))



² From Roth and Radford (2011, p. 161 French, p. 167 English). Transcription was augmented with permission. The transcription conventions used here include (0.87) = pauses in seconds; ((pounds)) = transcriber’s description; “=” = latching of sounds; :: = lengthening of sound, 0.1 s per colon; [com] = shading indicates correspondence with image to the right; tiENs = capitalization marks emphasis (by means of intonation or intensity); [] = brackets in consecutive lines indicate overlapping speech; . = punctuation marks intonation, period for strongly falling.

3 Affect in concept

The transcription of the lesson fragment manifests both intellect, in the form of assessments of the task status and the potential future state of affairs (turn 29c), and affect, in bodily form, including the resting head (turn 29a) and the hand pounding the desk (turns 29c and 31c). When Aurélie says, “Je ne comprends pas [I don’t understand],” she is intellectualizing a fact; she is not manifesting affect itself. It is only in the intellectualized form that her concrete, practical relation to the mathematical objects in the here-and-now of her situation can be likened to other situations where she or other persons use the same phrase (cf. Dewey, 1938). The statement itself does not imply an affect and could easily appear in an expert’s think-aloud protocol. But affect here is expressed through the modulation of the sound body of the words in *that* concrete situation. It is only when “feeling, sensation and emotion” are “identified and described in terms of the immediate presence of a total qualitative situation”—here Aurélie doing mathematics—that our inquiry becomes adequate to the phenomenon (Dewey, 1938, p. 68). Thus, the modulation of her voice, though apparent in the transcription, is even better represented in the output from the sound analysis. In the published transcription, turn 29c can be heard and is described as “plaintive.” Our re-analysis shows that the pitch sharply rises from 175 Hz at the beginning of the phrase to 500 Hz, moves between 217 and 320 Hz in the middle part, and drops to 120 Hz in the final part where the sound of the hand pounding the desk disturbs the voice signal (Fig. 1). Scherer (1989) has demonstrated that such modulations in pitch—pitch mean, range, variability, and contour—correlate with displeasure/disgust and grief/desperation. This correlation is further supported in Aurélie’s case by other information in the voice not represented here, such as changes in the first and second formants (i.e., maxima in the sound spectrum). That is, while the supersensible (“meaning”) part of the phrase manifests an intellectual, reflexive assessment, its sensible part, the sound, manifests certain form(s) of affect. However, pitch is only one aspect of voice that a full consideration of the affective (emotional)-volitional dimension of activity considers. There are other aspects of the voice that we hear as manifestations of affect. Physical behavior—such as Aurélie’s pounding of the fist on the desk, throwing herself “demonstrably” into the back of the seat, and her slouching as if sleeping—as well as facial expressions (e.g., grimacing, blushing), all may be treated as communicating specific affects generally (e.g., Planalp, 1999) and in mathematics classrooms specifically (e.g., Roth, 2011).

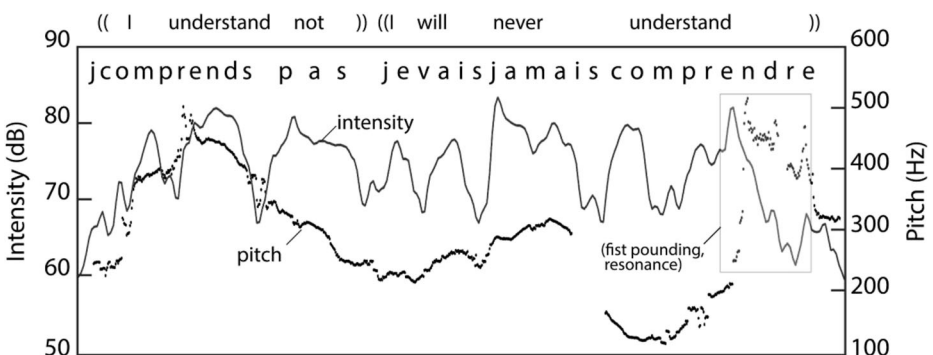


Fig. 1 Output of the sound analysis (PRAAT) showing speech intensity and pitch, words, and instant when the fist hits the desk

In the phrase “Je ne comprends pas” (Fig. 1), the intellectual (suprasensible) part does not refer to affect, which manifests itself in the sensible (sound) part of the speech. This is so because “the word absorbs intellectual and affective content from the entire context in which it is intertwined. It means both more and less than it does when we view it in isolation” (Vygotsky, 1987, p. 276). Because of the sound, the word means more than it “does when we view it in isolation” (p. 276), all the while meaning less because its dictionary sense has no bearing on the situation. In any case, the fragment and its analysis underscores the point that the children’s words are not connected to the event, in which they are caught up and which they (have to) live through, unless “wishes, feelings, and emotional reactions are linked with it” (p. 119). Ontogenetically, vocal productions and the first words of the child have affective functions in her life and in the relations with her parents. There are indeed several analyses of situations in which adults or youths repeat the same word (e.g., a swear word or “penis”) multiple times (6, 10) in which the dictionary sense is completely irrelevant and only the sound, as a form of affect-laden evaluation matters (Roth, 2015; Vološinov, 1930; Vygotsky, 1987). It is only afterward that the sound, as word, also comes to have a supersensual, ideal dimension: “purely emotional speech [becomes] objective speech” (p. 103).

In our analysis, we are mindful of the need to avoid making too much of the reported correlations or attempt to make conclusive (verbal) statements about the affect expressed. This is so because the associations are always based on correlating *manifestations* that are *external to* the thinking body in which affect and intellect are united. Such external associations fail to capture the core of the phenomenon (Dewey, 1938; Hegel, 1807). Instead, the unity and identity of affect and intellect exists in the sound-word itself, irreducibly consisting of sensible material and suprasensible form. This unity, namely, the relationship between thinking (suprasensible) and speech (sensible), is both the central issue in and the *via regia* [royal road] to overcoming the psychophysical problem (Vygotsky, 2010). It is precisely this problem that impedes an appropriate understanding of the relation between affect and volition toward and cognition of mathematics.

In our analysis, we do not have to impute (interpretively speculate) the “meaning” Aurélie makes. Indeed, observers do perceive the effect of Aurélie’s verbal and physical behavior on others. That is, anyone can notice its interactional relevance. In her reply, Thérèse offers help, telling her friend what she has to do, that is, “three plus six” (turn 31a). This is what Aurélie will record a few seconds later into her table of values. Investigators might too quickly continue, failing to attend to the fact that we observe affective forms in the case of Thérèse, too. This is immediately apparent when we gloss turn 31 as a manifestation of *compassion*, that is, as manifestation of an affective feeling (passion) with (com-) Aurélie. Thérèse’s helping gesture is documentary evidence of compassion (cf. Mannheim, 2004). But this compassion is the result of having been affected (or should we say infected?) by the passion of her peer. How might we understand this? Consider the following exchange, which constitutes both a partial re-transcription of the lesson fragment and an augmented version thereof. The transcription is augmented because it includes actions whose occurrence is necessary for the event to have unfolded in the way that it did. However, it is those actions that are typically omitted in general research practice.

3.1 Partial retranscription

For Thérèse to be able to act on what Aurélie has said, she has to have actively attended to what her friend has said. That is, her response begins by actively attending to what the other

says, and it ends with her articulating what needs to be done. But while actively listening, Thérèse cannot know what will be said until the saying has ended. She is affected all the time during which she actively attends. This affectation derives not merely from the content of the words—because the phrase remains incomplete and indeterminate until Aurélie has stopped speaking—but also from the various material expressions made available (pounding, voice quality). Thérèse *can be* affected precisely because she has the word in *common* with Aurélie, so that the word is a reality for the two of them, whereas it would be impossible for one person (Vygotsky, 1987). The word, materially a sound wave, is the resonance phenomenon connecting the two persons. Thérèse's and Aurélie's intersubjectivity therefore is founded on the interobjectivity of the sensible sound-word and the sensibility of the material setting.

Moving our understanding forward, it is helpful to consider that, in responding, Thérèse cannot know the effect the response will have until she can see or hear it in what Aurélie does or says. Any thinking and feeling of Thérèse associated with her talk in the partial transcription (which we never access itself) begins with her attending to Aurélie and ends when she can perceive the effect of her action—a Spinozist-Marxian position (e.g., Il'enkov, 1977). Vygotsky (1987) already takes up this point in asking for an “analysis of thinking [which] presupposes that we identify its motive force” (p. 50). Our analysis shows that the stimulus at play leading to the statement “you have to do three plus six” lies outside of Thérèse, *affecting* her as she is attending to the speech of Aurélie; but this stimulus comes from within the situation as a whole that unfolds because of a motive force over which nobody has complete control (e.g., because nobody knows what the next person will say). This relation to others and the affective-volitional character of the context constitute who Aurélie is and can be (Bateson, 1979; Collins, 2004). Our analysis thus allows us to realize that “thought has its origins in the motivating sphere of consciousness, a sphere that includes our inclinations and needs, our interests, and impulses, and our affect and emotion” (Vygotsky, 1987, p. 282). Thought, from the very beginning, is affective through and through. The augmented aspect of this partial transcription also makes apparent that participation means more than agency, as it is typically defined; it also includes passivity and passion. Thérèse has no control over what she is going to hear or perceive in other modalities: she is affected by what is coming at her—Aurélie's words, body movements, voice, and writing. Thérèse is affected *before* she could construct “meaning” of anything said, because the content of a phrase, and thus its “meaning,” is available only after her speaking has ended.

Thérèse can be *compassionate* because she is affected by the manifestations of passion of her friend. Empathy presupposes a feeling of the pathos (suffering) of the other. The relation of the two girls exists precisely in the materiality of their co-presence, which they perceive and feel in various ways, including the sounds they hear, with which they resonate or from which they feel repelled, and that *also* manifest one or another form of intellectual content. If it eventuates that Aurélie was comforted, then the origin of this changed affect is the material relation.

If Thérèse can be compassionate because affect and emotions are public, so can researchers: Roth and Radford (2011) describe many of the ways in which affect manifests itself in the behavior of Aurélie and Mario (prosody, body movements, facial expressions, etc.). Important here is the realization that emotions are not the researchers' interpretations; they do not require special efforts to be located. Instead, others in the setting attend to the manifestations of affect and volition in the behavior of their protagonists. Indeed, it has been suggested that emotions are private only in pathological cases (Dewey, 1934/2008), and some sociological theories are built entirely on the affective-volitional nature of human relations at large, which are produced

in and through interaction ritual chains (e.g., Collins, 2004). In the drama of a relation with others, “each attitude and gesture, each sentence, almost every word, produce[s] more than a fluctuation in the intensity of the basic emotion. ... a change of shade and tint in its quality” (Dewey, 1934/2008, p. 50).

4 A *via regia* to affect in the psychology of mathematics education

The purpose of this study is to contribute to the creation of a cultural-historical theory of affect and emotion for the psychology of mathematics education by elaborating on some intuitions Vygotsky had for the development of such a theory. The preceding analyses exemplify how such a theory works itself out in the concrete case of an elementary school mathematics classroom—though, precisely because the theory is stated abstractly, it may also be useful for understanding affect in lessons with different subject matter (e.g., science, technology, or engineering). In summary, the psychology of mathematics education to which we contribute here views affective social relations generative of every human being: “*a relation generative of man is nothing other than the affective, sense-giving relation of our animal forebears, in the first instance, toward one another*” (Mikhailov, 2001, p. 26, underline added). Life in society means relations with others. These relations consist of the “collision” or “struggle” of differences—for example, the different ways of seeing and understanding the mathematical task on the part of Aurélie, Mario, and their teacher—and thus constitute *drama* (Politzer, 1929; Vygotsky, 1989), where “drama” denotes the relational nature of knowledgeability and human behavior, a fact dissociated from the romantic overtones of the term (Merleau-Ponty, 1945; Politzer, 1928). The multiple viewpoints that come with participation in the drama of life give rise to “a binocular view in depth. The double view *is* the relationship” (Bateson, 1979, p. 133), and the double view is inherently dramatic. The double view means dramatic contradiction, which constitutes the above-noted *moving force* of activity; it also is the moving force of human development (Bateson, 1979; Veresov, 2004). The psychology in terms of active relations therefore takes us straight into a psychology in terms of drama, where affect and intellect form a unity and identity in the word, which indeed *is* the relation between two people (Vygotsky, 1987). It is in the sound-word (and its relation to thinking) that the solution to the psychophysical problem is to be found (Vygotsky, 2010).

4.1 The relational origin of the affective personality

Students often find mathematics teachers distant and stern, and they may even fear them (e.g., Walshaw, 1999). We urge mathematics educators to give consideration to the point that all relations (including those with teachers) are affective volitional in nature (e.g., Bakhtin, 1993; Collins, 2004). The dramatic nature of the relation with the other—for example, a student with her teacher—is inherently affective and, therefore, emotionally tinged drama (Veresov, 2004). The experience of the practical, intellectual, and affective relation subsequently becomes an aspect of the personality. The compassion manifested in the preceding lesson fragment may indeed reproduce and cement the friendship relation between the two girls.

According to Vygotsky (1989), all higher psychological functions, generally, and mathematical reasoning, specifically, exist in human, society-specific (societal) relations that make the dramatic life of humans. Thus, they are to be studied not by identification of traits or attributes but, rather, by means of a psychology of mathematics education situated within a

more general concrete human psychology that focuses on the whole person. We envision here a concrete psychology of mathematics education that—all the while deconstructing the drama of human relations in a mathematics lesson—moves toward categories that are themselves dramatic, and these categories imply the totality of the person–(social, material) environment relation. Taking the psychology of mathematics education as drama allows us to think in new ways and theorize individuals appearing in existing studies, such as Walden and Walkerdine’s (1982) Kathy or Walshaw’s (1999) Rachel, who are afraid and do not or no longer approach the teacher when they have a question. When Rachel responds to her classmate’s invitation to ask the teacher saying “NO. ‘Cos I’m SCARED of that woman. She confuses me more” (p. 206) and then explains by telling how her last interaction with the teacher had ended, she brings to bear a past dramatic collision to the present instant for the purpose of anticipating the possible intellectual and active outcomes of approaching the teacher. Similarly, Kathy chooses not to queue for help from the teacher, as was the custom in her classroom, even though Kathy shows significant distress at the repeated failures to find an answer to a question on her work card. Walden and Walkerdine describe Kathy as repeatedly being rebuked by the teacher for wasting time asking for help, in a context when, for another student, jumping the queue invariably brought immediate assistance.

In Rachel’s as in Kathy’s situation, we observe the young women (girls) in their relations with teachers, other students, or parents at home. These relations are part of dramatic events. We then enact a dramatic psychology of mathematics education, understanding personality, generally, and emotions, specifically, in terms of the ensemble of dramatic relations that these individuals have experienced. Because specific experiences—those that are referred to as *an* experience (Dewey, 1934/2008)—also serve to anticipate future situations that the individual goes and has to go through, we understand the young women’s reluctance in terms of reluctance to live through the same or similar kind of affective-volitional, dramatic relations that they have lived through before. We wish to emphasize that the unity of the dramatic experience is not practical, intellectual, or emotional; it does not exist in the form of representations that are so central to a number of current conceptualizations of affect and emotion in mathematics education. Rather, as in drama, person and environment cannot be teased apart, constituting, instead, an irreducible unity and identity: In theatrical terms, there always is a front (outer) and back (inner) stage. Thus, both the origin and terminus of Thérèse’s phrase have been identified to lie in the environment, so that the accompanying thinking also has moved from (and has qualities of) outside to inside to outside. This is precisely how the unity is articulated in a Spinozist-Marxian approach, such that “*mind is possible only at the borderline where there is a continual coming and going of one into the other, at their dynamic interface*” (Mikhailov, 2001, p. 20, original emphasis).

4.2 Toward a psychology of mathematics education in terms of drama

Central to the preceding analysis of the episode from one mathematics classroom is its affective-volitional coloring. In the account, we do not make conjectures about Aurélie’s or Thérèse’s private mind or feelings. We do not focus on the individual. Instead, we investigate the affective-volitional relation as it plays itself out in public. The case of the two girls exemplifies that precisely because passions are public that others can sense them and be compassionate. The contents of the relation consist of mathematical and mathematics-related phrases. Actions, passion, and compassion are united (ever since Aristotle’s poetics and ethics) in drama and dramatic conceptions of life (Ricoeur, 1984). Our conception of self therefore is

dramatic. For example, in the preceding sections, we have reported on an event of understanding less after asking the teacher for help (Rachel); we have found ourselves in the line waiting for attention only to see someone else by-pass the line and being dealt with immediately (Kathy); we have had an open ear for offers to help when we are in despair (Aurélié), and we have assisted another who apparently needed help to be able to continue (Thérèse). Where should we look to find how to account for the experience, that is, to an appropriate psychological theory of mathematics education? It has been suggested that turning to individual psychology or sociology will not help further our understanding (Bateson, 1979; Mead, 1972; Dewey, 1938). We suggest—following similar moves of others (e.g., Dewey, 1934/2008; Politzer, 1929; Vygotsky, 1989)—a turn to drama. We do so because the affective (emotional)-volitional nature of activity makes its experience a unitary event.

Approaching mathematical teaching and learning through a psychology of drama inherently comes with the important aspects of Vygotskian psychology (Roth, 2019). Drama always involves relations with others, and it is precisely in the physical relation with others that higher psychological functions and personality have their genetic origin. Drama involves affect and emotion, which therefore no longer have to be theorized separately because every aspect of an activity is affective through and through—not in the least because we are affected by the other (e.g., Thérèse by Aurélié's distress, Aurélié by Thérèse's assistance). Will, too, is integral to drama (Nietzsche, 1967; Politzer, 1929), such as when we want to complete a task and understand what it is that we are to learn (e.g., Mario); will is affective and emotive in nature because it is concerned with the movement of the event toward an outcome hoped for or feared (Dewey, 1934/2008). Whereas learning in the case of desired outcomes is expansive and tinged with positive emotions, in the case of feared outcomes (e.g., failure) learning will be defensive, and students do whatever it takes to avoid failing in mathematics, including just dropping the subject (Holzkamp, 1993). In Aurélié's case, she ended up copying the contents of the table of values from Thérèse, after the latter had filled in the two cells of the first column and after Aurélié had added " $3 + 3$ " to the second-row cell of the second column following Thérèse's instruction (extended transcription).

An important aspect of the episode from the fourth-grade mathematics lesson is that it shows how affect and emotion change in the course of activity. Thus, Mario, even though his behavior initially manifested negative affect, ends up manifesting positive emotions. That is, the actions themselves, thought marked by negative affect, turn into a positive affective state. On the other hand, Aurélié did nothing but copy the contents of Thérèse's table of value into her own. The editors of *The German Ideology* (Marx & Engels, 1978) note—in a phrase that Vygotsky (1987) takes up almost identically—that the decisive causes for the change of intellectual needs, interests, and inclinations of feelings are grounded in the material life of the society. Without participation in this life—here the mathematics classroom—feelings will not change (see Aurélié), but they may change to more positive values when students, despite frustrations (and perhaps helplessness), engage in the task. Of course, the opposite is possible too, leading to an aggravation of the despair, which is what Aurélié anticipates in saying that she will never understand. These relationships become apparent when we conduct analysis into complex units of mathematical activity, which make apparent the *inner* relationships between (a) thinking and (b) needs and inclinations, on the one hand, and between the concrete activity of the person and thinking, on the other hand. In the same way, Aurélié's teacher manifests varying forms of affect, which include the emotional expression of frustration (when Mario does not understand and she so far has failed to help him) and satisfaction (when she tells Mario that she now thinks he understands). Again, the variation arises from their *joint*

activity that has come with a change of Mario's power to act, and the expansion and reduction of agency and the anticipation of these tendencies manifest themselves in positive affective forms. When agency is stifled, such as for Aurélie, negative manifestations of affect are observed.

Drama also implies experience, with its practical, intellectual, and affective qualities. Mario's apparent intention to complete the algebra task already is dramatic, as is every action accomplished toward the realization of this goal, since achieving it means expanding control over the conditions (Holzkamp, 1993). A cultural-historical activity approach implies that this control cannot ever be complete. In this endeavor, affect and emotions play a central role. This is not so because they somehow influence intellect from the outside but because every aspect of the drama in the mathematics classroom is shot through with affect and emotions—though they may in some instances be more neutral. Emotions are important because “all emotions are qualifications of a drama and they change as the drama develops” (Dewey, 1934/2008, p. 48). Even more importantly, “experience is emotional but there are no separate things called emotions in it” (p. 48). Extracting emotions from activity and experience means completely destroying the unity within which they have their important role and function.

An important aspect of experience is that we are not only the active agents but also are undergoing events over which we never have absolute control. In every experience, there is “an element of undergoing, of suffering in the large sense” (Dewey, 1934/2008, p. 47–48). Whether the sense of undergoing is pleasurable or painful depends on the particularities of the situation. Even though the three students in the fourth-grade algebra class do not talk about their experiences, do not make thematic their affectations and emotions, they still manifest affectively what is happening to them in ways that others can perceive to lie along a continuum between the extremes of pleasure and pain, clearly expressed in Aurélie's apparent frustration in not understanding, Mario's frustration and his ultimate content. When Walshaw's (1999) Rachel talks into her recorder saying, “I don't get it. I don't GET it.. .. It's stupid. Can't do THAT” (p. 207), she also manifests passion, undergoing a situation over which she does not have control. She undergoes her own inability to understand what the task is asking her to do. In the same way, Aurélie is undergoing her lack of understanding, a situation from which she does not see a way out, and, indeed, from which she anticipates never being capable of getting out. It is this undergoing aspect of drama and experience that remains undertheorized in our field so much concerned with agency and constructive agents. A drama always occurs in the here and now: it is always situated. Unlike existing research, which (almost) exclusively focuses on (cold) cognition and mathematical content knowledge, the dramatic perspective makes thematic practical doing, the role of cognition, and the affective tinge as manifestations of an irreducible whole.

5 Coda

In this study, we articulate perspectives that elaborate the monist position that the late Vygotsky sketched in his notebooks and some of his latest texts (*Thinking and Speech, The Teachings of Emotion*). For a psychology of mathematics education, the approach we develop means that we cannot merely investigate thinking and affect during a specific task, or during a mathematics lesson. Rather, we need to take into account the individuals' lives as a whole. Vygotsky explicitly rejects considerations of thinking on its own, as we tend to find it in research on mathematical learning, and orients us toward investigating “the relationship that links [the

individual's] thought to the dynamics of behavior, to the concrete activity of the personality” (pp. 50–51). Integral to the whole person are affects and emotion. Indeed, “our affects make it clear to us that we, together with our body, are one being” (Vygotsky, 1999, p. 164).

Theoretical ideas, by themselves, hold little currency. Operationalizing new ideas, however, is not an easy task. As a starting point, it means reflecting on the limits and constraints of our traditions. By venturing into less familiar ground, mathematics educators might find the conceptual tools needed to offer explanations for the dramatic activities that take place in mathematics classrooms—those that are troubling as well as those that are joyful. Such ventures mark the place where new and more productive and enriching classroom practices become possible. The starting points for positive turns in the affective-volitional relation between students and mathematics may not lie with specific mathematical tasks. It may hinge around dialog between a teacher and her student in a context in which the teacher, building on a deeply caring relationship and with a predominant concern about how the student is being affected in the classroom, works with the student in find ways that will enhance her relationship to mathematics.

Vygotsky had made a decisive advance in overcoming the psychophysical problem in his attempt “to revive Spinozism in Marxist psychology” (Vygotsky, in Zavershneva & van der Veer, 2018, p. 209). He held Spinozism to be a parallelist approach, and thus considered it to be a flawed. Instead, he turned to a Marxist conception where the unity and identity of opposites (i.e., a contradiction) is the driver (force) of activity. But this is also where Vygotsky's functionalist unit of intellect and affect ultimately falls short: he continues to conceive of affect and intellect substantively—as substances, essences. Neither substances alone nor their dialectical contradiction leads to the movement inherent in life (James, 1909). Thus, rather than beginning an integrative theory in terms of substances, a theory is required that has as its minimum unit the event, which includes in itself novelty and is extended in space and time (Mead, 1932; Whitehead, 1919). A theory for mathematics and the mathematical mind working on an evental theory is currently in development (Roth, 2019, forthcoming).

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