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Different reading styles for mathematics text

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Abstract
A broad categorisation of different reading styles for mathematics text is generated in this research. The styles derive from those found in literature around academic reading skills. These styles are inductively refined using video transcripts of five specially chosen students studying out loud from a prescribed mathematics textbook. The context is a self-study mathematics course directed at high school mathematics teachers with weak content knowledge. Reading is understood as a transaction (enacted curriculum) between text (written curriculum) and reader. Reading styles are characterised in terms of depth of reading, focus on different components of text or not, connections within text or to prior knowledge, and performance on exercises. Five different styles of reading mathematics text are identified: close reading with strong connections, close reading with some connections, scanning, skimming, and avoiding. The different reading styles are also interpreted in terms of structure, voice, and genre of the textbook.

Keywords Mathematics reading styles · Mathematics textbook analysis · Written curriculum · Enacted curriculum · Learner's use of mathematical textbook · Form of address

1 Introduction

A textbook is a repository of mathematical knowledge, selected and organised systematically and pedagogically so as to provide access to key mathematical ideas. Kilpatrick (2014) proposes

> that throughout history, the principal function of mathematics textbooks has been to serve as repositories of authorized knowledge. The content messages they contain have been chosen by their authors to represent the most reliable and important mathematical knowledge that the culture possesses. (p. 6)

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Most textbooks contain explanatory text, worked examples, different representations of mathematical objects, and exercises with or without answers. At late high school and university level, they also include definitions, theorems, and proofs. A mathematics textbook may be electronic or hard copy. In this paper, the focus is on hardcopy textbooks.

The way learners may and do use mathematics textbooks as a basic resource for learning mathematics is an under-researched area in mathematics education (Rezat & Straesser, 2014). This may relate to methodological problems in researching an area involving activities which often take place in silence outside the classroom (Rezat, 2013) or it may be the result of an under-appreciation by mathematics educators of the opportunities for learning that many contemporary mathematics textbooks do provide to students.

In the context of the Third International Mathematics and Science Study (TIMSS 3), Valverde, Bianchi, Wolfe, Schmidt, and Houang (2002) assert that, in most classrooms, textbooks are "the physical tools most intimately connected to teaching and learning" (p. 3). Their use is ubiquitous throughout the world, and in many poorer communities, they are often the only material resource available to both teachers and learners. With this reality, it is unfortunate that many learners do not read or use their mathematics textbooks productively (Lithner, 2003; Österholm, 2008; Shepherd, Selden, & Selden, 2012). Helping learners to read mathematics productively is thus an urgent pedagogical imperative. And to be effective, such pedagogy needs to be based on research about the ways in which students do read and hence shape the mathematical text.

One type of such research (as in this article) is based on an interrogation of the types of interaction between a student reading the text and the pedagogic characteristics of the text. This form of interrogation leads to an understanding of different styles of reading mathematics text. In this article, the focus is on the reading of a section of a prescribed mathematics textbook within a self-study mathematics course for in-service or pre-service mathematics teaching graduates. The intention is to identify and characterise different styles of reading mathematics textbooks for self-study by students. A description of different reading styles (even if it does not describe every possible reading style) gives the educator or researcher an easily accessible framework within which to view the way different learners read and, where necessary, to consider remedial or appropriate responses to the style of reading. Knowledge of these different styles can also usefully inform the design and implementation of programmes or interventions dedicated to improving the mathematics reading skills of students and teachers. It can also inform the design of mathematics textbooks.

Because the empirical research takes place in a particular context with a particular textbook, this article does not present a comprehensive description of all possible reading styles; other reading styles may well present themselves with different students, with different textbooks, and with different reading goals. The article does, however, provide a useful and usable starting point for the development of a typology of mathematics reading styles. Although not empirically proved, my experience over many years of teaching mathematics also suggests that the five reading styles that it describes are widespread among mathematics learners.

This article is organised as follows: Having justified the need for research into different styles of reading of mathematics text, the article proceeds to a brief review of the existing research around the reading of mathematics textbooks. This is followed by the theoretical framework in which it is stressed that reading is an interaction between the reader and the text. The context of this research is then described. This is followed by a description of the methodology (primarily induction) of the research and how the data were collected (the methods). Analyses of both the written curriculum (the textbook) and the enacted curriculum (how the reader interacts with the text) follow. The article concludes with a summary of results and a discussion of the original contributions of the research as well as its limitations.

2 Literature review

Research around the use of mathematics textbooks by students is limited. Exceptions to this are Sierpinska (1997), Rezat (2006, 2008, 2013), Berger (2016, 2017), and various Swedish researchers such as Lithner (2003) and Österholm (2006, 2008). Sierpinska (1997) looks at the interaction between students, textbooks, and tutors in a linear algebra course in which the use of a textbook is privileged. She uses the construct "format of interaction" (Bruner, 1985) to unravel the mechanisms whereby mathematical meanings are made in these interactions. Rezat (2013) uses the theoretical framework of instrumental genesis within a socio-didactical tetrahedron model to develop a set of categorisations of students' utilisation of textbooks for self-regulated practice. Self-regulated practicing involves students' use of parts of the textbook to "improve" their mathematical knowledge; it is self-regulated in that the teacher does not explicitly tell students which parts of the textbook to use. Rezat identifies three different utilisation schemes within self-regulated practice: position-dependent practice (where students' choice is related to the teacher's previous actions), block-dependent practice (where students' choice is related to the focus of the particular block), and salient-dependent practice (where students' choice is related to the visual similarities of one task to another). Lithner (2003) looks specifically at how undergraduate students engage with prescribed mathematics tasks in a textbook and the nature of their reasoning. He distinguishes between different reasoning strategies: "plausible reasoning" where the focus is on the relevant mathematics (properties, definitions, and so on), "identification of similarities" in which the student focuses on surface similarities to worked examples or theorems in the text, and "established experience" when the reasoning is based on the reader's previous experiences. Although Lithner (2003) is looking specifically at students' engagements with prescribed exercises in the textbook, aspects of Lithner's categories of reasoning resonate closely with certain features of the reading styles in my study. This is discussed later.

Österholm (2006, 2008) interrogates the reading comprehension of high school and undergraduate mathematics students. In his 2006 paper, he compares students' reading of mathematics text with symbols, mathematics text without symbols, and history text. Using quantitative measures of the quality of students' written responses to the text, Österholm (2006) argues that the reading comprehension of a history text and of a mathematics text without symbols is similar. However, the reading comprehension of a mathematics text with symbols is different. Thus, it may be that "the reading process of the mathematical text with symbols is a special kind of comprehension process" which can be viewed as a "type of content-specific literacy skill" (2006, p. 340). Since mathematics, by its very nature, is and needs to be written with symbols, this argument points to the need for research on reading mathematics text specifically. Österholm's (2008) study takes up this point and asks whether students need to learn how to read mathematics texts specifically. His empirical research reveals the preponderance of several non-desirable reading strategies among predominantly high school students, when reading mathematics text. These strategies include focusing on key words when reading mathematics tasks and focusing primarily on the symbolic parts of text when reading mathematics exposition. In this regard, Österholm (2008) argues that students need to learn to read different types of mathematics text, such as exercises and exposition, all of which contain symbols. They also need to learn general literacy skills since focusing only on symbols can negatively affect reading comprehension.

With regard to the reading of mathematics textbooks specifically, Shepherd, Seldon and Seldon (2012) adapt the Constructively Responsive Reading Framework from literary theory to explore why many mathematics undergraduate students do not read their mathematics texts in a useful way. Their research is based on their observation of 11 undergraduate mathematics students reading a prescribed section of a mathematics textbook. All of these students were classified as competent readers according to an ACT test (ACT is a test used for evaluating college readiness in the USA). Despite this, all of the observed students struggled to complete relatively simple tasks related to their reading. Shepherd et al. suggest three factors that may contribute to the students' unproductive reading of mathematics text. These are insufficient prior knowledge; inattention to details in the text and students' lack of concern with their own errors and confusions. Although not a focus of my study, aspects, small or large, of these three factors are seen in the reading of all students in my study.

Berger (2016) develops a theoretical framework, specific to mathematics discourse, for exploring the relationship between enacted discourse (the student's way of reading the text) and the written discourse (the textbook). She combines this framework with that of Weinberg and Wiesner's (2011) notion of an empirical reader (the actual reader) and an implied reader (a hypothetical reader who reads productively for understanding) in Berger (2017). In Berger (2017), the reading activities of two students (the empirical readers) are analysed according to how they exploit opportunities for learning from the textbook, how they inject prior knowledge into their reading, and how they make connections between different parts of the textbook. Their ways of reading are compared to those of an implied reader. The analyses are used as a platform from which to highlight productive and less productive ways of reading to learn mathematics. This current article further expands Berger (2017) by developing broader categories for examining different styles of reading mathematics text. "Styles" relate to the styles of reading used in various academic skills' reading courses (BBC, 2011; BBC & British Council, 2008; Massey University, 2012). The emphasis in this article is on "styles" of reading rather than a comparison of productive or unproductive ways of reading (although these may be implicit in different styles of reading depending on what the goal of the reading is). As previously stated, a framework of "styles" provides an easily accessible way of examining (possibly with follow-up of pedagogic interventions) the reading of mathematics text.

In order to conserve space and promote coherence, relevant literature around the characteristics of mathematics textbooks is woven into Section 5.3.

3 Theoretical framework

On a broad level, this study is underpinned by a socio-cultural perspective whereby individual development and learning is understood as the individualisation of collective or social activity:

> Every function in the child's cultural development appears twice: first, on the social level, and later, on the individual level; first, between people (interpsychological) and then inside the child (intrapsychological). This applies equally to voluntary attention, to logical memory, and to the formation of concepts. (Vygotsky, 1978, p. 57)

The research focuses on the student's reading of a mathematics textbook. The textbook is a socially constructed artefact; accordingly, the social level takes the form of the written discourse in the mathematics textbook. In contrast, the individual level comprises the enacted discourse through which the particular student interacts with the written discourse. Learning takes place through the individual's thoughtful transactions with the written mathematical discourse.

On a finer level, the reading of mathematics textbooks is interpreted using Rosenblatt's (1982) theory of reading as a two-way interaction between reader and text in a particular cultural context. Rosenblatt's argument that both reader and text contribute to the reading experience is elaborated in the context of children's reading of literature:

> The words in their particular pattern stir up elements of memory, activate areas of consciousness. The reader, bringing past experience of language and of the world to the task, sets up tentative notions of a subject, of some framework into which to fit the ideas as the words unfurl. If the subsequent words do not fit into the framework, it may have to be revised, thus opening up new and further possibilities for the text that follows. This implies a constant series of selections from the multiple possibilities offered by the text and their synthesis into an organized meaning. (p. 269)

This relationship between reader and text is applicable to the reading of mathematics text by a student. Students read the contents of the textbook according to their prior knowledge, experiences, beliefs, and capabilities; as such, they give particular meanings to the text (which may or may not be what the author intended). In this way, the text offers up diverse opportunities for different interpretations and meanings.

In this article, the focus is on the enacted curriculum (the reader's responses) rather than on the written curriculum (the text). However since the act of reading is understood as a transaction between reader and text, it is necessary also to examine the written curriculum. For this purpose, Remillard's (2012) notion of "form of address" of curriculum materials is used. Form of address refers to the "physical, visual, and substantive forms it [the material] takes up, the nature and presentation of its contents, the means through which it addresses teachers" (p. 108). In the context of this particular study, the focus is more generally on readers (not specifically teachers). Form of address has five components: structure, look, voice, medium, and genre. These are further elaborated and applied to the relevant text in Section 5.3. This linking of the form of address of the textbook and the different reading styles is an original and important contribution of this article to the literature on reading mathematics text.

For the enacted discourse, the names and broad characteristics of the styles of reading are inspired by the characterisations of styles of reading used in various academic skills' reading courses (BBC, 2011; BBC & British Council, 2008; Massey University, 2012). These reading styles are skimming, scanning, intensive reading, and extensive reading. With respect to reading of mathematics text for self-study, the in-detail description of the styles and their characteristics are inductively developed in this article. The inductions are based on several observations of five different students reading mathematics texts.

According to BBC-British Council site on Teaching English (2008), skimming is "reading a text quickly to get a general idea of meaning". In contrast, scanning is "reading in order to find specific information, e.g. figures, names". Intensive reading is that which involves learners reading in detail with specific learning aims and tasks. Intensive reading can be compared with extensive reading, which involves learners reading texts for enjoyment. The categories of intensive reading, scanning, and skimming are potentially relevant for the reading of mathematics textbooks for the purpose of self-study. Formal academic research (as distilled in journal articles) on these reading styles does not seem to exist. Certainly, it does not exist in the scant literature around reading mathematics text: hence the need for induction and for a teasing out of these and other categories of reading styles relevant to mathematical textbooks.

4 Research questions

How do we categorise the different reading styles of students reading mathematics text from a mathematics textbook in a course designed for self-study through reading?

How do these different reading styles relate to the "form of address" (that is structure, look, voice, medium, genre) of the mathematics textbook?

4.1 Context

The research takes place within the context of Precalculus and Calculus modules which are part of a 1-year post-graduate degree (Honours level) programme in Mathematics Education for high school teachers at a South African University. Each module consists of one 3-h session per week for 11 weeks. I (the researcher) was the lecturer in charge of the modules. South African high school mathematics teachers often have weak mathematics-specific content knowledge (Adler & Davis, 2011). This is partly a result of teachers often having a degree or diploma in education and an absence of traditional university-level mathematics courses (a legacy of apartheid). Accordingly, the focus of these mathematics modules is on deepening and broadening mathematics content knowledge. For this reason, the in-service or pre-service teachers in the Honours course are conceptualised as mathematics learners. Another important aspect of the modules is an emphasis on self-learning. This emphasis derives from the idea that learning to use texts and other resources to learn or re-learn mathematical topics is necessary for a teacher who needs to keep on expanding and deepening her mathematical knowledge. In order to foreground the practice of self-learning, at the beginning of each module, all students (in this case, the teachers who are conceptualised as learners) are given a hand-out in which they are told exactly which part of the prescribed textbook to study for each weekly session. For the precalculus module, the textbook is Sullivan (2012). As students are repeatedly told, studying involves carefully working through definitions, theorems, proofs, and worked examples in the text, referring to other resources or other sections of the textbook if required, and doing and handing-in a set of prescribed exercises (taken from the back of the chapter). Thus "new" mathematical knowledge or revisited mathematical knowledge is accessed by the students through the prescribed textbook *prior* to the lecture.

In the mathematics self-study course, intensive reading of relevant portions of a mathematical textbook is required. The reader needs to focus on making sense of what they are reading, on bringing prior knowledge to bear on the reading, and on actively engaging in solving exercises and worked examples in the text. The reader also needs to make connections between different theory sections, between theory and examples, and between different representations of the same mathematical object (e.g., diagrams or graphs and verbal definitions of mathematical objects). Also, the reader needs to know when she needs to pay attention to something in the text.

5 Methodology

5.1 Overview

As has been indicated, there is an absence of research on reading styles in mathematics education. Accordingly, the starting point for the categorisation of different styles is the categories of styles found in literature around academic reading skills. These categories of style (skimming, scanning, and intensive reading) are refined using the constant comparative method (Glaser & Strauss, 1967) as elaborated within the inductive methodology of Grounded Theory and applied to the mathematics domain. Specifically, the styles are constructed using the inductively derived categories of reading (extent of reading, focus, connections, performance). Combinations of different features within these categories are used to construct five different reading styles. See Table 1 where the different styles are summarised according to various features of the categories. Each different style is also illustrated via a narrative description of each of the five chosen readers; each of these readers typifies one of the five styles.

Different forms of data (transcripts, interviewer notes, students' solutions, and writings) were collected. This contributes to the triangulation of data sources, so enhancing the credibility of the research. Also, the iterative process of description and analysis of data by the researcher provides some form of triangulation of data analysis across time. That is, a time period of several weeks elapsed between iterations: this allowed for revisions and a fresh look at the data over time by the researcher.

As stated, the categorisation of styles is based on a long-accepted typology of reading skills in the academic reading literature. However, given that only five students were observed in this study, it cannot be claimed that the five mathematics reading styles are the only possible mathematics reading styles. Other reading styles may be possible given different students, different texts, and different reading goals.

With regard to the written curriculum, Remillard's (2012) construct of "form of address" is the primary tool used to deconstruct the textbook. In accord with the underlying notion that reading is an interaction between the reader and the text, the categories of reading are linked specifically to aspects around the structure, genre, medium, look, and voice (Remillard, 2012) of the book.

5.2 Collection of data

Five students from the Precalculus and Calculus module agreed to be video-taped while studying a prescribed section of a chapter (the sub-chapter) from the prescribed textbook. These students were purposefully chosen according to the researcher's informal observations about the varying ways in which these students seemed to approach studying in the course. Ideally, and as discussed with the students in weekly sessions, studying meant carefully reading and trying to make meaning of all sections in the prescribed reading. For the video-

Table 1 Summary of reading styles

		Close reader with strong connections	Close reader with some connections	Scanner	Skimmer	Avoider
Category of reading	Features					
Extent	Skims over text initially.	Yes.	Yes.	No.	No.	Yes.
	Reads text carefully.	Yes.	Yes.	No.	No.	No.
	Skims over Exercises first.	No.	No.	Yes.	Yes.	No.
Focus	Pays attention to worked examples (WEs).	Yes.	Yes.	Sometimes. Exercise-driven.	Sometimes. Exercise-driven; usually unproductive.	Sometimes—if "new".
	Pays attention to theorems, definitions, etc.	Yes.	Yes.	Sometimes. Exercise-driven.	Sometimes. Exercise-driven; usually unproductive.	Sometimes—if "new".
	Pays attention to proofs.	Yes.	Yes.	Occasionally. Exercise-driven.	No.	Sometimes—if "new".
	Pays attention to warnings.	Yes.	Yes.	No.	No.	No.
Connections	When reading text, makes connections.	Yes—within text and to prior knowledge.	Yes—within text and to prior knowledge.	No.	No.	Occasionally.
	When doing exercises, refers back to theory, proofs, or explanatory text.	Yes—explicitly.	Yes—implicitly.	Sometimes.	Occasionally but usually unproductive.	Sometimes.
	When doing exercises, refers back to Wes.	Sometimes.	Sometimes.	Sometimes.	Sometimes but usually unproductive.	Sometimes.
Performance	Able to do most exercises.	Yes.	Yes.	Yes—explanations are all procedural.	No.	No.

taped session (also called "interview") presented in this article, students were asked to study sub-chapter 5.5, "Properties of Logarithms" (Sullivan, 2012, pp. 296–304) as they would in preparation for class and to talk out loud as they did so. They were also given a set of exercises at the back of this sub-chapter as was the case for their weekly sessions. This sub-chapter addresses operations with logarithms and change of bases. It was chosen because it looks at mathematical concepts with which the students are familiar, from a more advanced perspective. For example, most students in the course know about procedures for working with logarithms but are not familiar with proofs of theorems around logarithms (the properties of logarithms). Each video-taped session was at most one and a half hours long.

Aside from the transcripts of each video session, the researcher wrote a set of notes during each video session, noting points of interest in the session. The writings and solutions to exercises of each student were photocopied after the session. These field notes and student solutions are used in the interpretation of the transcripts.

5.3 Analysis: written curriculum

Since this article is concerned with how students *read* a specific sub-chapter of the textbook for self-study, the unit of analysis for form of address (Remillard, 2012) is the relevant sub-chapter used in the empirical study (sub-chapter 5.5) together with the introduction to the whole textbook. According to Remillard, form of address has five components: structure, look, voice, medium, and genre.

5.4 Structure

The structure of the curriculum materials refers to the ways in which the various components of the textbook are organised, the mathematical content included or excluded, and the emphasis of the content (Remillard, 2012). Sierpinska (1997) refers to mathematical content as the mathematical layer of a textbook: it comprises definitions, theorems, proofs, explanatory text, different representations of mathematics objects, worked examples, and exercises. Valverde et al. (2002) developed a well-regarded framework for examining the structural features of mathematics textbooks. In that study, textbooks from 48 educational systems (mostly countries) were analysed.

Using Rezat's (2013) distinction between the three structural levels of a textbook, that is, the book level, the chapter level, and the lesson level, the textbook (Sullivan, 2012) in my study can be deconstructed as follows: there are 14 topics in Precalculus, each of which constitutes a chapter. Each chapter consists of sub-chapters (the textbook lessons). There are 10 appendices devoted to assumed prior knowledge (e.g., Algebra Essentials) and to extensions (e.g., Complex Numbers). Each level comprises different "blocks" (i.e., components), each with its own pedagogical purpose.

Similar to Rezat (2006) in his analysis of the structure of German mathematics textbooks, Valverde et al.'s (2002) "rhetoric of textbook lessons" (p. 140) with its construct of building blocks is used to describe the structure of sub-chapter 5.5 (the textbook lesson) in the Precalculus textbook (Sullivan, 2012). Sub-chapter 5.5 has seven blocks, each with its own pedagogical purpose. The blocks are Learning Objectives, Introductory task, Exposition, Kernels, Worked examples, Warnings, and Exercises. These blocks, except for the Learning Objectives (beginning of textbook lesson) and Exercises (end of textbook lesson), occur several times throughout the lesson. The blocks, other than Learning Objectives and Warnings which are geared towards the self-studying student, are the same as the blocks Rezat (2006) delineates.

Sub-chapter 5.5 begins with a list of Learning Objectives. This is followed by an Introductory Task and Exposition. The Exposition consists of several sub-blocks such as explanatory text, theorems (which in this case are properties of logarithms), and proofs. Many of the properties and statements of theorems are placed in boxes so as to highlight their importance; that is, they are "Kernels". The Exposition and Kernels are followed by several illustrative Worked Examples. This pattern of Introductory Task, Exposition, Kernels, and Worked Examples repeats itself several times during the course of the textbook lesson. After each set of Worked Examples, there is a heading "New Work" which points the reader to Exercises at the end of the sub-chapter, relevant to the particular property being illustrated in the Worked Examples. Various "Warnings" are also given to the reader. These point to common errors in using the properties of logarithms. Sierpinska (1997) argues that these types of warnings are an acknowledgment of the possibility of divergent meanings in the mathematics textbook. The content section concludes with a summary of properties of logarithms (Kernel) and a historical feature discussing the invention of logarithms (Exposition). After this, there is an Exercise block headed "Assess your Understanding". This comprises five sub-blocks: Concepts and Vocabulary section contains short-answer questions around key concepts; Skill Building has exercises which give an opportunity for further practice of problems similar to those in worked examples; Mixed Practice asks questions which require the reader to connect different concepts; Applications and Extensions relate to real-world problems or extend concepts addressed in the sub-chapter; Explaining Concepts requires a verbal explanation or generation of an example of a particular concept.

5.5 Look

"Look" refers to the visual appearance of the material. As with other sub-chapters in this textbook, the text in Sub-chapter 5.5 is presented in a large font, well-spaced, and with large margins in which the reader can write notes to herself, or in which the Warnings are presented. Kernels are highlighted in yellow or blue and boxes are used to give focus to the contents of the kernel. Each Worked Example is clearly distinguished from the next by a line and a new number. The Exercises, that is, the assessment questions, are clearly presented in sections with a heading according to their purpose, for example, Skill Building.

5.6 Voice

Voice refers to the ways in which the author(s) of the textbook represent themselves and how they communicate with the reader. In Sullivan (2012), these voices differ according to whether they are speaking in the pedagogical or mathematical layer. In the mathematical layer (i.e., the textbook lesson), explanatory text and information is frequently addressed to the reader and also reveals the presence of an author through the pronoun "we". For example "*We* shall derive properties (3), (5) and (6) and leave the derivation of property (4) as an exercise" (p. 298, my italics). In contrast the statements of theorems, questions in worked examples and exercises use the more impersonal voice of traditional mathematical texts where reader and author are hidden by the absence of personal pronouns. For example, Worked Example 7 (p. 301) reads "Approximate $\log_2 7$. Round the answer to four decimal places".

The pedagogical layer comprises advice given to the reader (student and teacher separately) at the front of the book as well as warnings about common errors and directives such as "New Work" which are woven into the mathematical layer of the sub-chapter. Specifically, in the front of the book, there is a section headed "To the Student" in which the student is told that "this textbook was written with you in mind" (Sullivan, 2012, p. iv). The student is given advice on how to use the book and is urged to read the text *before* class so that they can ask questions in class about what they do not understand in the text. Although the frequent warnings in the text do not use explicit pronouns such as "you", they are clearly talking to the student. For example, a typical warning reads: "In using properties (3) through (5), be careful about the values that the variable may assume …" (p. 299). Similarly, the highlighted phrase "New Work" indicates which exercises are relevant to the content and is directed to the student (and teacher).

5.7 Medium

Sullivan (2012) is a hardcopy textbook. Additional student resources such as student solutions' manual, video lessons, and online courses related to the textbook are available and listed at the front of the book. Due to financial constraints, we did not use any of these extra resources in the course.

5.8 Genre

According to the online Merriam-Webster Dictionary (2016), genre is "a category of artistic, musical, or literary composition characterized by a particular style, form, or content". Remillard does not actually define genre; rather she indicates that "it has implications for the expectations teachers bring to a curriculum resource that influence the way they engage it" (p. 113). In the educational arena, Schleppegrell (2004) explains that "genre" is a term used to refer to a particular type of text or discourse. Within the school or mathematics context, there are many different genres. As such "students need to learn about the genres of schooling and the purpose for which they are useful" (Schleppegrell, 2004, p. 83). In the mathematics context, Gerofsky (1999) gives the example of mathematical word problems as a specific genre with specific characteristics. She also argues that "the use of a generic form may bring with it intentions that are not exactly the same as those of the current writer or speaker" (p. 37). This points to the difficulties students may encounter when reading mathematics text, which itself contains various sub-genres such as Theorems, Worked Examples, Exercises, and so on. These sub-genres correspond to the seven blocks and sub-blocks mentioned in Section 5.4 above.

When discussing "genre", it is pertinent to engage with Österholm and Bergqvist (2013) who argue that many claims in the research literature around the special linguistic properties of mathematical texts, such as compactness and complexity, are vague and/or not based on empirical or logical arguments. Österholm and Bergqvist also demonstrate, through linguistic analyses of school-level mathematics and history textbooks, that there is little statistical evidence for asserting that mathematics texts are more compact or complex than history texts. Both these aspects of Österholm and Bergqvist's arguments (i.e., regarding the quality of much research around linguistic properties of mathematics texts and their own empirical research) may be used to argue that there is no difference in genre between, say, history text and mathematics text. But this argument runs contrary to everyday experience: most people who look at, say, a secondary level mathematics textbook can very quickly detect that it is a mathematics textbook and not a history textbook. Österholm and Bergqvist do acknowledge that pervasive claims (for which as yet there is little or no evidence) about the special linguistic properties of mathematics text may be valid when looking at a higher-level textbook (such as in this study) or mathematics texts on particular topics. Indeed it is pertinent to suggest that a different sort of linguistic analysis (probably using linguistic properties other than compactness, complexity, and use of pronouns) and/ or a structural analysis may reveal statistical and solid evidence that indeed a mathematics text does has its own unique genre.

Besides the mathematics text, there is the question of the mathematics textbook as an entity with a specific genre. In this regard, a mathematics textbook contains pedagogically selected and organised mathematical knowledge and assessment activities, each expressed according to conventionally accepted genres (or sub-genres), such as that of the theorem statement, proof or worked example, and so on. In this regard, Sullivan (2012) is classified as a hardcopy book containing mathematical content around precalculus concepts. It is written for use by students and lecturers involved in an undergraduate precalculus course. Furthermore, aspects of its "voice" (discussed above) qualify it as a mathematics textbook geared towards self-study.

5.9 Analysis: enacted curriculum

The transcripts of all five learners are analysed in terms of the ways in which students read the sub-chapter (the text) and do the prescribed exercises which appear at the end of the sub-chapter. The analysis focuses on what and how the students read, what connections they make, and on how they solve the exercises.

The constant comparative method (Glaser & Strauss, 1967) is used as the primary method of analysis. More specifically, the analysis of data consisted of four major iterative steps. First, there was the descriptive level: the researcher read through the transcript of each interview together with field notes and the student's photocopied solutions. This allowed the researcher to make notes on the transcripts of the video-sessions (e.g., reader writes Property 1 $\left(a^{\log_a M} = M\right)$ next to proof of Property 6; reader paraphrases proof of Property 3). During this descriptive level, four broad analytic categories relating to how the components of the textbook were being used as a resource for learning were inductively generated. The researcher then re-read the transcripts and applied the analytic categories (e.g., *focuses* on proof of Property 5; *connects* solutions of exercise X to property 3). Short narrative descriptions of each student's activities while reading the textbook together with a summary table of reading styles were then produced by the researcher. All these steps were repeated several times, so as to refine the descriptions, the analytic categories, the summary narratives, and the table. Finally, a set of different styles of reading was generated for different patterns, or revealed during categorisation (see Table 1).

6 Results

6.1 Categories

Four analytic categories were generated regarding the student's use of the textbook as a resource for learning:

Extent of reading: Did the student carefully read through the whole sub-chapter? Which sections, if any, did she skip? Did the student accurately paraphrase what she was reading?

Focus: To which structures did the student pay attention (e.g., worked example, proofs of property (theorem), exercises, warning, etc.)?

Connections: Did the student make connections to prior knowledge and/or between different parts of the text?

Performance: How did the student do the exercises at the end of the chapter? Did she make connections to the text or prior knowledge when doing exercises?

The extent of reading category and the connections category relate primarily to how the student engages with the "structure" of the book. The focus category relates to how the student uses the book as a resource for learning, that is, the "genre" of the book, and on how the student engages with the pedagogical layer, that is, the "voice" of the book. The performance category relates specifically to one structure, the exercises, in the book; it provides a measure of how successful the reading (as a form of self-study) has been for the particular student. The look and medium of the textbook are implicit in the description, analyses, and categorisation of reading style and are not commented on further.

6.2 Categorisation of different styles

The primary purpose of this research is the generation of analytic descriptions of different reading styles for mathematics text. In this research, five different styles of reading—close reading with strong connections, close reading with some connections, scanning, skimming, and avoiding—were observed.

"Close reading with strong connections" is characterised by a thorough reading of the text in the sub-chapter, as well as the making of explicit connections to prior knowledge or to other parts of the text. Close reading of the text is closely aligned with the category of "intensive reading" as described in the literature on reading skills. An indicator of a comprehensive reading is the accuracy of paraphrasing or explanations of the text by the student as she reads. Close reading with strong connections is also evidenced by the correct solving of assessment problems (exercises) in which conceptual justifications and explicit connections are made to the text. Although solving of exercises with strong connections may be also related to good prior knowledge, if the student struggles with the assessments or solves problems without any conceptual connections or justifications, the reading style cannot be classified as close reading. The use of justifications and connections to the mathematics text resonates with Lithner's (2003) notion of plausible reasoning. Plausible reasoning involves "argumentation that is relatively stable mathematically" (p. 38); it involves an analysis of the underlying mathematical characteristics of the exercise.

"Close reading with some connections" is similar to "close reading with strong connections". The difference is that the reader does not make explicit connections to the text when doing the exercises. However, she does make explicit connections to prior knowledge or structures in the text when reading the text and she is able to conceptually justify her solutions to exercises when asked to do so.

"Scanning" is characterised by the reader looking for specific keywords or information in the text and being able to use this information productively. A scanner is able to solve many problems in the exercise section but, given the lack of attention to depth and detail in the reading, these solutions focus on procedural justifications rather than conceptual justifications or connections. This approach to solving exercises relates to Lithner's (2003) category of reasoning based on identification of similarities. With this type of reasoning, the strategy is to identify similar surface features of the exercise and an example or theorem encountered earlier in the text.

"Skimming", like scanning, is characterised by the reader looking for specific keywords or information in the text. However, unlike the scanner, the skimmer is not able to find appropriate structures (e.g., worked examples or properties) in the text or is unable to use the appropriate structure in a productive way. Accordingly, the skimmer is not able to solve many problems in the exercise section. Strategies used in both skimming and scanning resonate with two separate strategies identified by Österholm (2008): searching for keywords and focusing on mathematic symbols at the expense of the written words.

"Avoiding" is peculiar to mathematics reading: theory and proofs are avoided. All attention is focused on procedures. When solving the exercises, few if any connections are made to theorems in the text.

Of course, no real-world reading style is "pure"; there may be several elements of different reading styles evident in one reader and one reading. For example, a skimmer, scanner, or avoider may read a particular section of the text closely and with accurate paraphrasing. However, and as with most inductively derived distinctions of real-world phenomena, the categorisations are based on *predominant* activity during the reading. The response to questions in the exercises further supports the classification of the reading style.

6.3 Narrative illustration of the styles

Close reader with strong connections Abby is a high-performing mathematics student. She and Solly (see below) are the top mathematics performers in the Honours class. Abby has previously studied 2 years of undergraduate pure mathematics in a B.Sc. degree and she has a 1-year post-graduate Certificate of Education specialising in mathematics. Abby is currently teaching at a high school in Johannesburg. She teaches the laws of logarithms and how to do computations with logarithms but very few proofs of properties of logarithms. Accordingly, Abby regards this part of the textbook (proofs of properties) as new to her; she regards some of the procedural work, for example, adding two logarithms, as familiar.

Abby's approach is to start off by "skimming to see what I know and what I don't know. That's always how I start with these" (line 4). After this skim-through, she carefully reads through the entire chapter, that is, text, theorems, proofs, definitions, and worked examples, highlighting what is new or unfamiliar to her. She pays particular attention to the "new" work, paraphrasing the proofs of properties which are new to her. She also pays some attention to the properties and problems with which she is already familiar: "I do look at them. It's nice. I think I'm just checking if I gave this to my own students if it would help them understand" (line 61).

Abby accurately paraphrases much of what she reads. She explicitly relates what she is reading or doing to relevant theorems and worked examples in the chapter or to prior knowledge or to the content that she teaches at high school. For example, she usually writes down in the textbook the property of logarithms that is used by the proof, worked example, or exercise that she is reading or doing. She does each of the more challenging worked examples on her own and then compares her solutions to those of the textbook. Abby also reads and frequently highlights the warnings (about common errors when working with logarithms). In this way, Abby is in tune with the pedagogical voice of the text.

Abby only starts doing exercises when she has read the entire sub-chapter. She is able to do all exercises very well, justifying her mathematical activities by referring to the relevant property of logarithms. She makes conceptual connections to prior knowledge or to what she has read in the sub-chapter. She also makes many connections to her own teaching experiences around logarithms. When answers to questions are available in the textbook, Abby checks her answers against those in the textbook.

In her reading, Abby engages adequately with all the structures of the sub-chapter and with the text's pedagogical voice. Her reading style is thus appropriate for the genre of this specific textbook. It exemplifies that of a "close reader with strong connections".

Close reader with some connections Solly's biographic profile is similar to that of Abby: he is a high-performing mathematics student and he also teaches at a high school in Johannesburg. Like Abby, Solly teaches logarithms at school with particular focus on computations and very little theoretical content. Unlike Abby, Solly has studied mathematics in an education degree rather than in a science degree.

Like Abby, Solly first skims through the entire sub-chapter. During the initial skim, Solly puts stars next to work which is new to him (e.g., several theorems) and all the warnings. After the skim, Solly reads through all the text very carefully, especially proofs of all theorems and all warnings which he previously skimmed. He frequently and accurately paraphrases and explains what he is reading. Solly clarifies why he reads so carefully: "From my experience of reading mathematical textbooks, the definitions and some explanations, they hold some key information that you need to take into account if you are to successfully complete the given exercises" (line 17). Solly tries to do worked examples which look familiar to him, on his own; he checks his final solution against the text. For worked examples based on text or properties with which he is not familiar, he first reads carefully through the worked example before attempting the example on his own. Like Abby, while reading proofs or doing worked examples, Solly notes explicitly which properties of logarithms he is using and any relevant warnings. Unlike Abby, Solly does not explicitly refer to his teaching experience as he reads. After reading the sub-chapter, Solly does the exercises. He does most of these correctly but, unlike Abby, does not make explicit connections to properties in the text, to prior knowledge or to his teaching experience. However, when asked, he can explain his solutions using connections to the text or prior knowledge.

As with Abby, Solly's careful reading of the warnings and directives indicates his sensitivity to the pedagogical voice of the text. He also engages with all structures of the book. His reading thus aligns with the genre of this specific textbook. Accordingly, Solly's reading style exemplifies that of a "close reader with some connections".

Scanner Paul is an average performing students on the Honours mathematics course. Paul has an undergraduate degree in Education, specialising in Mathematics Education. He is a full-time pre-service student who has not yet taught in schools. Near the beginning of the reading session, Paul tells the researcher that he is familiar with the content around logarithms.

Paul starts off by skimming through a few exercises. He then looks back at the text (explanatory text, theorems, worked examples, etc.) according to the focus of the exercise: "I'm just going to the place on the notes based on the headings on the question that I'm going to do" (line 7). (In this statement, Paul is referring to the way he moves from the exercises to the relevant section in the text). For Paul, it seems that the reading of the text is driven by the exercises. That is, he only reads text when he thinks that it will help him solve a particular exercise. This type of scan reading is made explicit in the following two utterances:

> Paul: "At first I look at the questions and then from there I went back to look at the…I looked at the properties basically" (line 23). A little later (line 126), the researcher asks: "So you wouldn't necessarily read through the proofs of the theorems unless you needed them. Is that right?" Paul replies: "Yes. Ya" (Lines 127).

Paul does not paraphrase the text, nor does he explain worked examples. He is able to do most exercises correctly but he does not relate (in writing or speech) any of his solutions to properties of logarithms. When asked to explain a solution to an exercise which he has written, Paul's explanations are extremely superficial. For example, in one exercise (Q96) Paul is required to use the property "if $\log_a M = \log_a N$ then $M = N$". He does use this property (implicitly) but when asked to explain why it was okay to just "drop" the logarithms, he argues (line 89) "like I just used this idea here… you introduce the log to both sides. So these they are exactly the same thing that I used here. So like when I drop it, it's like when I introduce. And then I can just also drop as long as I have it also on both sides". Paul is arguing that since you can always "insert" a log on both sides of an equation, so you can "drop" or remove a log on both sides of an equation.

Paul tells the researcher that much of the work is familiar to him. Yet, he often does not recognise theorems in the textbook. Indeed he sometimes states that a particular theorem "looked unfamiliar" (lines 26, 30) at first. Nonetheless he is able to use all these and other pertinent properties of logarithms, implicitly, while doing the exercises. However, it seems that there are big gaps in Paul's conceptual knowledge. This is evidenced by his frequent inability to recognise formal statements of theorems, his inadequate justification of solutions to exercises, and his struggle with providing coherent paraphrasing or explanations of any of the proofs of properties in the textbook.

Paul's tendency to only read sections of the text as (implicitly) directed by the exercises indicates a non-engagement with the structures and genre of the textbook. Similarly, Paul's inattention to the warnings and the pedagogical directives in the text reveal a disregard for the voice of the text. Overall, Paul's primary focus on information relevant to the exercise that he is doing makes him an archetypical scanner.

Skimmer Tom is a below-average performing students in the Honours mathematics courses. Like Paul, Tom has an undergraduate degree in Education, specialising in Mathematics Education. He is a full-time pre-service student who has not yet taught in schools. He tells the researcher that he has studied logarithms both at school and at university.

Like Paul, Tom starts by reading the exercises and only looking at the text on a "need to know" basis. That is, when stuck with an exercise, he looks back in the text for a relevant worked example or property of logarithm (theorem). Unlike Paul, he is not always able to access an appropriate worked example or theorem in the text. For example, he gets stuck on simplifying $5^{\log_5 42}$. Although he looks for theorems and worked examples which he thinks are relevant to this exercise, he does not seem to see the appropriate theorem ($a^{\log_a M} = M$). Sometimes Tom does look at the appropriate theorem or worked example but is unable to use what he sees productively. For example, when doing an exercise (Q56) which requires him to "express powers as factors" he looks at appropriate worked examples where powers are expressed as factors. But this does not help. As he says: "I seem to be not understanding what they want me to do" (line 49). Tom indicates that he is having difficulty using the text productively: "What maybe might make it difficult for me to understand was that the way I wrote it... It's quite different to what was done in the book. And it makes it difficult for me to understand what the textbook has explained to me" (line 31).

Tom makes many errors when he is doing the exercises and coherent justifications and conceptual connections are largely missing from his explanations of his solutions. Tom mostly refers to worked examples (as opposed to properties, proofs or explanatory text) when he gets stuck on an exercise. Occasionally, he skims over explanatory text. He rarely looks at the statements of any theorems or their proofs. When Tom tries to explain what he is doing, his explanations are often incoherent. His attempted paraphrasing of what he is reading is also often muddled.

As with the scanner, Tom mostly organises his reading of the text according to the perceived demands of a particular exercise. However, unlike the scanner, Tom is frequently unable to locate suitable worked examples, properties, or explanatory text to help him solve the exercises. Even when he does find the appropriate text, Tom seems unable to usefully interpret and apply it.

Tom has obvious difficulties with exploiting the genre of the book (a textbook written for self-study). He also does not engage productively with either the structure or voice of the text. Tom's reading style, with its inattention to detail and substance, exemplifies a skimmer.

The avoider Like Tom and Paul, Judy has an undergraduate degree in Education, specialising in Mathematics Education. She has no experience in teaching in schools. At various stages of the reading session, Judy tells the interviewer that she has studied logarithms previously, both at school and in her mathematics education degree.

Of all the students, Judy is the most inconsistent in her style: She reads some sections carefully but refuses to read others. She does some exercises and worked examples correctly; in others, she makes many errors of which she is not even aware. Although Judy starts off by reading the chapter sequentially (unlike Paul or Tom), she will not read any proof or worked examples when the property or worked example looks familiar to her. She explains why she will not read the proof of the property, $\log_a a^r = r$: "I think I understand. I understand because I know this rule" (line 162). Judy says that proving theorems, when she already "knows" the rules, is "a waste of time" (line 183). It seems that, to Judy, "understanding" a proof or concept is the same as knowing a rule. When a property or worked example is new to her, Judy tries to prove it or to solve it, without looking at the textbook. However, she often gets very confused and her explanations become incoherent. For example, she glibly (and incorrectly) replaces $x \log a$ in a particular proof, with $\log_a x$ without any justification. She also does not check her proofs or worked examples adequately against those in the textbook. Indeed she makes many errors in both of these.

Judy indicates that she will not read the "warnings" on p. 300 because "it's just the rules" (line 370). Ironically, she makes the exact mistake that a "warning" warns her against and writes $\frac{\log a}{\log b} = \log \frac{a}{b}$ several times during her activities.

Judy is able to do several exercises correctly but occasionally she gets stuck. When she gets stuck she sometimes tries, successfully, to find help in the text (including re-reading statements of theorems, i.e., the properties). Other times, she does not try using the text to help her.

Sometimes, she battles through an exercise for a long time before returning back to the text. With some exercises, Judy does not realise that she has made errors in her solutions and so she just carries on. When doing the exercises, Judy talks out loud describing what she is doing. She occasionally makes explicit connections between her workings and the text (properties or other worked examples).

Judy's reading of the text, in its avoidance of reading large sections, is explicitly at loggerheads with the structure of the text; similarly her explicit refusal to read the warnings in the text is in opposition to the pedagogical voice of the text. Accordingly, her reading is not in sympathy with the genre of the textbook. Given Judy's insistence on avoiding structural and voice elements of the text, her reading style is classified as that of an avoider.

7 Conclusion

In this article, we see how various students read and study the "same" sub-chapter of a textbook. Some of these ways of reading (e.g., those of Abby and Solly) are similar; other styles of reading are very different. Similarly, the way that these five students do the exercises attached to the sub-chapter range from "all correct with many or few connections" to "very few correct with very few connections". Although we cannot posit a causal explanation between style of reading, making connections, and quality of solutions to exercises, there certainly seems to be a correlation between a close reading of the text with plenty of explicit connections and well-reasoned solutions to the exercises (as in the case of Abby). It seems likely that this latter student and Solly were able to do the exercises so well because of their close reading and/or good quality prior knowledge (itself possibly due to good prior reading skills) and that Tom, Paul, and Judy did the exercises quite weakly with few coherent justifications because of poor reading skills and/or poor prior knowledge (again a possible result of poor prior reading).

As discussed in Section 6, aspects of the categories of skimming and scanning relate to Österholm's (2008) discussion around students searching for keywords and students focusing on mathematical symbols while ignoring written words. In relation to the doing of exercises, skimming and scanning involve "reasoning based on identification of similarities" (Lithner, 2003, p. 35). Close reading of the text involves plausible reasoning as discussed by Lithner (2003).

Although aspects of the different categorisations in the different styles resonate with description of various strategies discussed in the mathematics educational literature, this research is original in that it constructs "styles" of reading. Specifically, five different styles of reading mathematics textbooks for the purpose of self-study have been analytically delineated in this research. The categorisation of styles draws on the informal literature around academic reading skills but is inductively elaborated in the context of a mathematics self-study course. In the real world, and as is sometimes apparent in the actual transcripts of the five readers, different reading styles may be apparent in one reading. That is the skimmer may sometimes scan, the scanner may sometimes avoid, the close reader may sometimes scan, and so on. Despite the real-world blurring of categories, the important contribution of this research is that a description of different reading styles (even if it does not describe every possible reading style) gives the educator or researcher an easily accessible framework within which to view the way different learners read. If necessary, such a framework may be expanded to include other reading styles. Understanding that different readers do have different reading styles and what some of these different reading styles are, is a first step in promoting a culture of productive reading of mathematical text and of helping learners exploit the learning opportunities embedded in various mathematics textbooks.

At this juncture, it is relevant to revisit some of the limitations of this study. Since the empirical observations consisted of five (purposefully selected) students, its results cannot be generalised without further empirical research. Also the results are limited by the fact that only one textbook was used. Further research with different students and different mathematics texts will increase the generalisability of the characterisations and may also result in a further refinement of the categorisation of styles and the generation of additional reading styles. In this sense, we can regard the five described styles of this article as a starting point for a typology of mathematical reading styles.

The different styles of reading (close reading with or without connections, skimming, scanning, and avoiding) are not intended as a simple and uncritical way of judging the quality of any mathematical reading. For any evaluative statements to be made about the reading style, there needs to be consideration of the goal of reading and the student's prior knowledge. For example, if the goal of the reading is to complete the exercises and the student has good prior knowledge of the content, scanning and even avoiding may well be a very productive strategy. In this article and as previously stated, the goal of reading was for the student to study the mathematical topic on their own (self-study) and to provide solutions to the exercises based on conceptual understanding of the mathematical topic.

As mentioned previously, the students' reading of the textbook may be regarded as a transaction between the text and the reader (Rosenblatt, 1982; Weinberg & Wiesner, 2011). Although this characterisation is implicit in other literature around reading of mathematics textbooks (e.g. Österholm, 2006, 2008; Rezat, 2013), it is not usually made explicit. In accord with the underlying notion that reading is an interaction between reader and text, the categories of reading are linked specifically to the structure, genre, look, and voice (Remillard, 2012) of the book. This is one of the important contributions of this article to the research on reading mathematics text. It suggests a valuable analytical method for furthering our understanding of relationships between reader and mathematical text.

In particular, the two close readers with connections engage satisfactorily with all the structures of the book and with its pedagogical voice (in terms of paying attention to warnings and other directives in the text). The reading style of close readers with connections is thus appropriate for the genre of this specific textbook which is written purposefully for the student to read and study. In contrast, the scanner, skimmer, and avoider do not engage adequately with the structure or the voice of the book. Specifically, the scanner and skimmer's tendency to only read sections of the text as (implicitly) determined by the exercises indicates a poor engagement with the structure of the textbook: in the case of the skimmer, this non-systematic engagement with the structure is exacerbated by a seeming difficulty with the productive usage of the structure; the scanner, despite frequently being able to execute the exercises, does not seem to make any conceptual connections between the theory and the exercises. The avoider explicitly refuses to read many sections of the text. On a different level, the scanner and avoider's explicit inattention to the warnings and the pedagogical directives in the text reveals a disregard for the voice of the text; the skimmer seems oblivious to the directives and warnings in the textbook. Given these various shades of difficulty with the productive use of the structure or voice of the text, the reading styles of the skimmer, scanner, or avoider are not in harmony with the genre of the textbook.

On a more practical level, it is hoped that other researchers expand this nascent typology into a full-blown typology of reading styles. But even without this extension, the varied reading styles as presented here are useful tools with which to interrogate students' ways of reading mathematics text. Such an interrogation (like a medical diagnosis) is necessary before educators can help students develop more productive reading styles and appropriate ways of moving between different reading styles, depending on the particular context. Being able to read mathematics text for understanding and learning is surely a very valuable and empowering skill.

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References

Adler, J., & Davis, Z. (2011). Modelling teaching in mathematics teacher education and the constitution of mathematics for teaching. In K. Ruthven & T. Rowland (Eds.), *Mathematical knowledge in teaching* (pp. 139–160). Dordrecht: Springer.

BBC. (2011). *Skillswise: English and maths for adults*. Retrieved from http://www.bbc.co.uk/skillswise/factsheet/en05skim-e3-f-skimming-and-scanning. Accessed 6 Jan 2017.

BBC & British Council. (2008). Intensive reading. Retrieved from https://www.teachingenglish.org.uk/article/intensive-reading. Accessed 6 Jan 2017.

Berger, M. (2016). Reading and learning from mathematics textbooks: An analytic framework. In C. Csikos, A. Rausch, & J. Szitanyi (Eds.), *Proceedings of the 40th conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 83–90). Szeged: PME.

Berger, M. (2017). Reading mathematics text: A study of two empirical readings. *International Journal of Science and Mathematics Education*. https://doi.org/10.1007/s10763-017-9867-6

Bruner, J. S. (1985). The role of interaction formats in language acquisition. In J. P. Forgas (Ed.), *Language and social situations* (pp. 31–46). New York: Springer-Verlag.

Gerofsky, S. (1999). Genre analysis as a way of understanding pedagogy in mathematics education. *For the Learning of Mathematics, 19*(3), 36–46.

Glaser, B. G., & Strauss, A. L. (1967). *The discovery of grounded theory: Strategies for qualitative research*. New Brunswick: Aldine Transaction.

Kilpatrick, J. (2014). *From clay tablets to computer tablet: The evolution of school mathematics textbooks*. International Conference on Mathematics Textbook Research and Development 2014 (ICMT-2014) 29–31 July 2014, University of Southampton, UK.

Lithner, J. (2003). Students' mathematical reasoning in university textbook exercises. *Educational Studies in Mathematics, 52*(1), 29–55.

Massey University. (2012). *The online writing and learning link*. Reading styles. Retrieved from owll.massey.ac.nz/study-skills/reading-styles.php. Accessed 10 Jan 2017.

Merriam-Webster Dictionary. (2016). *Merriam-Webster Dictionary*. USA: Merriam-Webster dictionary. Retrieved from https://www.merriam-webster.com. Accessed 28 May 2017.

Österholm, M. (2006). Characterizing reading comprehension of mathematical texts. *Educational Studies in Mathematics, 63*, 325–346.

Österholm, M. (2008). Do students need to learn how to use their mathematics textbooks? The case of reading comprehension. *NOMAD -Nordisk Matematikkdidakticc, 13*(3), 53–73.

Österholm, M., & Bergqvist, E. (2013). What is so special about mathematical texts? Analyses of common claims in research literature and of properties of textbooks. *ZDM Mathematics Education, 45*, 751–763.

Remillard, J. (2012). Modes of engagement: Understanding teachers' transactions with mathematics curriculum resources. In G. Gueudet, B. Pepin, & L. Trouche (Eds.), *From text to "lived" resources: Mathematics curriculum materials and teacher development* (pp. 105–122). Dordrecht: Springer.
Rezat, S. (2006). The structures of German mathematics textbooks. *ZDM, 38*(6), 482–487.
Rezat, S. (2008). Learning mathematics with textbooks. In O. Figueras, L. Cortina, S. Alatorre, T. Rojano, & A. Sepulveda (Eds.), *Proceedings of the 32nd conference of the International Group for the Psychology of Mathematics Education and PME-NA XXX* (Vol. 4, pp. 177–184). Morelia: PME.
Rezat, S. (2013). The textbook-in-use: Students' utilization schemes of mathematics textbook related to self-regulated practices. *ZDM, 45*, 659–670.
Rezat, S., & Straesser, R. (2014). Mathematics textbooks and how they are used. In P. Andrews & T. Rowland (Eds.), *MasterClass in mathematics education: International perspectives on teaching and learning* (pp. 51–62). London & New York: Bloomsbury.
Rosenblatt, L. M. (1982). The literary transaction: Evocation and response. *Theory Into Practice, 21*(4), 268–277.
Schleppegrell, M. J. (2004). *The language of schooling: A functional linguistics perspective*. Mahwah, NJ: Erlbaum.
Shepherd, M. D., Selden, A., & Selden, J. (2012). University students' reading of their first-year mathematics textbooks. *Mathematical Thinking and Learning, 14*(3), 226–256.
Sierpinska, A. (1997). Formats of interaction and model readers. *For the Learning of Mathematics, 17*(2), 3–12.
Sullivan, M. (2012). *Precalculus* (9th ed.). Boston: Pearson Education.
Valverde, G. A., Bianchi, L. J., Wolfe, R. G., Schmidt, W. H., & Houang, R. T. (2002). *According to the book: Using TIMSS to investigate the translation of policy into practice through the world of textbooks*. Dordrecht: Kluwer.
Vygotsky, L. (1978). In M. Cole, V. John-Steiner, S. Scribner, & E. Souberman (Eds.), *Mind in society*. Cambridge: Harvard University Press.
Weinberg, A., & Wiesner, E. (2011). Understanding mathematics textbooks through reader-oriented theory. *Educational Studies in Mathematics, 76*, 49–63.