Educational Studies in Mathematics (2019) 102:417–434
https://doi.org/10.1007/s10649-018-9834-3

Collaborative design of a reform-oriented mathematics curriculum: contradictions and boundaries across teaching, research, and policy

Despina Potari[1] · Giorgos Psycharis[1] · Charalampos Sakonidis[2] · Theodossios Zachariades[1]

Published online: 23 August 2018

Abstract
The reported study is situated within the process of developing a reform-oriented national mathematics curriculum for compulsory education in Greece by a design team that involved teachers, academic researchers, and policy-makers. From an activity theory perspective, we identify the activity systems of mathematics teaching, research in mathematics education, and educational policy interacting in the design process. We focus on the contradictions between the three activity systems and how these were dealt with. We based our analysis on email exchanges during the curriculum design, field notes from whole-team sessions, and interviews with key persons. Our results highlight that the emerging contradictions primarily concerned the teaching and research activity systems. Members of the team who acted as brokers between the different activity systems and facilitated their interaction played an important role in overcoming the contradictions.

Keywords Mathematics curriculum design · Activity system · Boundary crossing · Boundary objects · Educational policy · Mathematics education research · Mathematics teaching

1 Introduction

Educational innovations in centralised educational systems are usually introduced in a top-down way. However, the success of the innovation in large-scale contexts depends on how well it is integrated into the existing teaching reality. In mathematics education, a few studies have attempted to shed light on this complex process. For example, Krainer and Zehetmeier (2013) reported on their theoretical perspective of a large scale and long-term innovation

✉ Giorgos Psycharis
gpsych@math.uoa.gr

1 National and Kapodistrian University of Athens, Athens, Greece

2 Democritus University of Thrace, Xanthi, Greece

programme in mathematics and science teaching (IMST). They discussed establishing a culture of innovation and assumed that teachers and schools become owners of the innovation and innovations are “continuous processes that lead to a natural further development of practice, as opposed to singular events that replace an ineffective practice” (p. 877). Within this perspective, taking the triangle practice-research-policy into account is important in any innovation scaling.

Curriculum resources constitute fundamental bearers/agents of any innovation and the vehicle of its implementation. Several studies have highlighted the difficulties inherent in the process of enacting curriculum resources in actual teaching. Teachers tend to interpret the philosophy underlying these resources differently according to their epistemologies and teaching priorities (Choppin, 2011; Remillard, 2005). Although the relationship between curriculum resources and teaching practice has been an object of study for the last two decades, the process of designing innovations and curriculum resources that takes up teaching reality has received little attention (Adler, Ball, Krainer, Lin, & Novotna, 2005). Yet another gap in the existing literature relates to how the educational policy at the ministry level frames curriculum design, the outcome, and its enactment. Our study attempts to address the aforementioned research challenges by studying the practice-research-policy triangle in the process of transforming a national educational innovation initiated by the Greek Ministry of Education into mathematics curriculum documents and materials aiming to improve mathematics teaching and learning.

The reform-oriented national mathematics curriculum concerned compulsory education in Greece (5–15 years old), and we were members of its design team (the first author as coordinator). From an activity theory (AT) perspective (Engeström, 2001), we identified the following activity systems interacting in the mathematics curriculum design process: mathematics education research, mathematics teaching, and educational policy. The interaction across and within these activity systems was evident in negotiations among the design team members. The team consisted of academic researchers (predominately in mathematics education), classroom teachers (from preschool up to secondary school), and ministry and school advisors. All the participant teachers and school advisors had long teaching experience and had also been involved in mathematics education research (at PhD or Master’s level and/or collaborations with academic researchers in research projects). Negotiations concerned primarily formulating the mathematics curriculum principles, objectives, content and structure, and brought to the fore important issues where research, practice, and policy intertwined (Hoyles & Ferrini-Mundy, 2013). Our study aims to investigate this intertwining during the curriculum design process.

We structure our paper in the following way. It begins with an outline that frames theoretical ideas, concentrating first on successful scaling innovations and then on AT as a framework for studying mathematics education reforms, while paying attention to the concepts of boundary crossing and boundary objects. After a methodological interlude, we report the results of an empirical study and then proceed to the discussion and the conclusions.

2 Theoretical background

2.1 Educational innovations at scale

Innovations in mathematics education have been manifold. In many countries over the past two decades, ministries of education or agencies employed by governments have prescribed national standards for mathematics that result in new demands on mathematics teachers. In this context, balancing innovations to meet the needs of the system and the needs of individual teachers constitutes one of the major challenges (Krainer, 2014). Considering these innovations from within a teacher's perspective, the starting point has to be the daily teaching practice (Cochran-Smith & Lytle, 1999; Roesken, Hoechsmann, & Toerner, 2008). When viewed from a systemic perspective, there is a need to scale innovations while maintaining the original vision of building on teacher needs (Roesken-Winter, Hoyles, & Blömeke, 2015).

Coburn (2003) claimed that "the issue of 'scale' is a key challenge for school reform, yet it remains under-theorised in the literature" (p. 3). In an attempt to conceptualise scaling, she identified four interrelated dimensions: depth, sustainability, spread, and shift in innovation/reform ownership. Effective scaling requires teachers to obtain ownership of the reform and ultimately transfer the new approaches into an "internal reform" in their own school.

The challenge of scaling innovations has come increasingly into the focus of mathematics education over the last years (e.g. Adler et al., 2005). There are at least three key players recognised as having an influence on shaping successful scaling interventions: educational administrators and policy-makers, teachers, and mathematics or mathematics education academics (mathematics educators, mathematics researchers and/or mathematics education researchers). Key players are quite often members of more than one of these groups. In this paper, we study the design of a reform-oriented curriculum seen as a scaling innovation and how the involved participants' different memberships shaped the process.

We consider as scaling both the products and the processes of scaling over time (Hung, Lim, & Huang, 2010). The products are the tangible outcomes of the reform (e.g., better learning outcomes), while the processes are the main actions taken for the products to be developed. Within this framework, innovation scaling is conceptualised as a set of processes that need to be "re-created/re-instantiated/re-enacted" in the milieu of the reform products (Clark-Wilson, Hoyles, Noss, Vahey, & Roschelle, 2015). For this recontextualisation to happen, different groups collaborate with a view to establishing shared language and understandings through objects at the boundaries between communities. This collaboration leads to either "legitimate" or "lethal" mutations of the intended reform, that is, mutations which are either consistent or inconsistent, respectively, with the learning principles and the teaching practices informing the reform (Hung et al., 2010). In our study, the curriculum documents and materials are considered as products in relation to the scaling dimension of the innovation and function as objects at the boundaries in the design team's collaborative actions. Our emphasis in this paper is on the actions taken for developing these products, that is, on the process dimension of scaling.

2.2 Activity systems, contradictions, and boundaries

Innovations develop in socio-political contexts shaped by the activity of the people concerned and thus, by the communities involved. The term *community* has been used to mean groups of people who engage together socially, professionally, corporately, or officially. Community has been recognised (e.g. by Engeström, 1999) as central to seeing work and practice as *activity*, deriving from the work of Vygotsky and Leont'ev.

Community is also a key element in Engeström's (1999) third generation framework of AT where he presented his "Expanded Mediational Triangle" in the form of two interacting activity systems. Here, the *subject* achieves the *object* of the activity system through the mediation of an instrument or artefact. As well as the mediation of *tools* (textbooks, digital tools, or mathematical symbols), Engeström suggested that *rules*, *community*, and *division of labour* are also important mediators in an activity system. Objects in the two interacting activity systems move from an un-reflected and situationally given goal, to a collectively meaningful object, and to a potentially shared or jointly constructed object (Engeström, 2001). This process is characterised by contradictions being seen as historically accumulating structural tensions within and between activity systems. A new element adopted by an activity system often leads to a contradiction, where some old element collides with the new one. This generates not only disturbances and conflicts but also drives attempts to change the activity.

When activity systems' participants are collaborating, they bring their respective communities' practices and discourses together and enable continuity across the corresponding activity systems at their boundaries defined as "socio-cultural differences leading to discontinuity in action or interaction" (Akkerman & Bakker, 2011, p. 133). Boundaries are dynamic constructions denoting co-location of practices and co-existence of competing discourses. Efforts by individuals or groups at boundaries to restore continuity in action or interaction across practices trigger dialogical engagement and collective reflection, compelling people to reconsider their assumptions and look beyond what is known and familiar.

Through collaboration/negotiation at boundaries between activity systems, new and hybridised ideas and practices emerge where mutual understanding of shared tasks and problems develops (Edwards & Fowler, 2007). Described as *boundary crossing* (Engeström, Engeström, & Kärkkäinen, 1995), this process involves moving into unfamiliar territories and requires cognitive retooling. People who cross boundaries are called *brokers* and are simultaneously members (full, peripheral, partial) of multiple communities (Wenger, 1998). This enables brokers to act as conduits for introducing elements of one practice into another. These elements include artefacts, discourses, and processes and are referred to as *boundary objects* (Engeström et al., 1995).

It is evident from the above that boundaries are sites for new understandings, identity development, change of practices, and institutional development. Akkerman and Bakker (2011) discerned four types of learning through which these take place at boundaries: (a) identification of a boundary leading to renewed insight into what the diverse practices concern; (b) coordination of activity flow acknowledging diverse sites and making transitions between practices smoother; (c) reflection on the specificities of two sites and the existence of the boundary between them while actors become aware of their own perspectives by redefining them in relation to the perspectives of others (perspective making) as well as by taking a new look to their own perspectives through the eyes of others (perspective taking); and (d) transformation where actors from different sites engage in some constructive activity leading to significant changes in the existing practices, potentially even the creation of a new practice.

The present study combines AT and boundary crossing to study the design process of a reform-oriented mathematics curriculum by concentrating on contradictions emerging within and across three communities of practice: mathematics teachers, mathematics education researchers, and policy-makers. We see teaching, research, and policy as situated within three distinct activity systems, as they meet the five principles of an *activity system* defined by Engeström (2001). They are collective, tool-mediated, and have a motive and an object; involve multiple points of view, traditions, or interests; are transformed over lengthy periods of time; contradictions exist as sources of change and development; and expansive transformations occur when individuals begin to question and deviate from the established norms. By looking at contradictions at boundaries of the three activity systems in particular, we also use boundary crossing to scrutinise the participants' efforts in the direction of maintaining or overcoming the encountered boundaries. Our research questions are as follows:

- What were the emerging contradictions during the interaction of the team members and how were these related to the elements of the three activity systems?
- How did the participants deal with these contradictions between the three activity systems?

3 Methodology

3.1 The systemic context

This section provides information concerning how the three activity systems function in the Greek context in relation to curriculum design.

3.1.1 The educational policy context

The Greek educational system is highly centralised and innovations are mainly introduced through a top-down approach coordinated by the Ministry of Education after advising by an educational board (Institute of Educational Policy—IEP). The Ministry of Education initiated a curriculum reform through the *New School* act at the beginning of 2010. It focussed on active engagement of students, openness of the education to society, multiplicity of resources (digital and non-digital), and new roles for teachers as active agents of the curriculum. After the Ministry's initial invitation to the coordinator, a committee was comprised of the subject matter coordinators and ministry representatives and IEP. There were collaborative meetings over 6 months among the members of this committee aiming to prepare a document highlighting the overall philosophy and objectives that would frame the curriculum design for each school subject. During the mathematics curriculum design process, the coordinator acted as a broker between the educational policy activity and the designing activity.

3.1.2 The mathematics teaching context

In Greece, mathematics is compulsory for pupils aged 5 to 15. There is a national mathematics curriculum and one mathematics textbook for each year. Teachers as participants in the mathematics teaching activity system use the textbook as the main tool for their lessons and follow its structure, suggested tasks, and exercises strictly. However, teachers often use other materials from commercial exercise books to provide pupils further practice.

Pre-primary, primary, and secondary school teachers constitute distinct communities in terms of their rules and division of labour. They have different educational backgrounds and professional status. Mathematics teachers at pre-primary and primary schools have degrees in education and also teach other subjects. At secondary school, mathematics teachers have degrees in mathematics, but do not gain teaching qualifications during or after their undergraduate studies. Since official channels for professional development (PD) are limited, few teachers look for opportunities to develop professionally by participating in Master's programmes, seminars, and PD initiatives. At the time of the curriculum design, the financial crisis negatively affected teachers' professional status (e.g., reduction of salaries, no new appointments and cuts in school resources). Under these circumstances, teachers were rather reluctant to participate in innovative top-down initiatives.

The centralised character of the Greek educational system frames the rules of mathematics teaching. It is traditionally characterised by an emphasis on covering specific contents with a rather expository approach, especially in upper years. Testing is also central in all years, so students' mathematical achievement in test-type tasks is a main teaching goal. Therefore, there is little space for open tasks emphasising inquiry approaches in mathematics. Kynigos, Philippou, Potari, & Sakonidis (2009) discussed the historicity of mathematics teaching in the Greek context and indicated contextual constraints and resistance to innovative teaching and learning practices.

3.1.3 The mathematics education research context

Mathematics education research activity in Greece has been developed during the last 30 years. Researchers in this activity system are academic researchers, PhD graduates, and teachers occasionally involved in research, usually in collaboration with academic researchers. The main object of the research activity system constitutes research development, its dissemination in the national and international research communities, and its impact on improving mathematics teaching and teacher education. Academic researchers participate in multiple communities, including teacher educators in teacher education and PD programmes. They often receive invitations from educational authorities to act as project evaluators and as members of national examination boards and/or curriculum resources and materials advisory committees. IEP mainly regulates curriculum developments in collaboration with representatives of scientific societies like the Greek Mathematical Society. In the curriculum development considered in this study, it was the first time that the socio-political conditions prioritised a significant role for the research community. One research asset of this community has been the established collaboration of many academic researchers with teacher networks and schools through PD programmes, seminars, workshops, and research projects.

3.2 The mathematics curriculum design team

The mathematics curriculum design team was formed based on the coordinator's recommendations to the Ministry. It consisted of 15 academic researchers (two in mathematics and 13 in mathematics education), 11 classroom teachers (one in kindergarten, four in primary school, and six in secondary school), two ministry, and six school advisors (four for primary and two for secondary schools). Most members knew each other and some had previously collaborated in the context of other research activities. In terms of Wenger's (1998) classification of membership in communities, the academic researchers can be seen as full members of the research activity system, classroom teachers of the teaching activity system, and ministry and school advisors of the educational policy system. However, eight teachers and all school and ministry advisors had partial membership in research activity as they held a PhD or a Master's degree in mathematics education. Five academic researchers had a peripheral membership in educational policy activity due to past collaboration in the development of curriculum materials. The coordinator could also be considered as a peripheral member of the educational policy activity system due to her invited participation in the meetings with the educational policy-makers.

The team worked collaboratively for about 9 months (December 2010–August 2011) and prepared the first version of a mathematics curriculum document, including the main mathematics education objectives, its structure, and the content unfolded in the form of learning trajectories transcending different educational levels. Exemplary materials and teachers' guides were also developed, including tasks addressing specific learning goals and cross-curricular projects accompanied by digital resources and teaching ideas. The team finalised a revised version of the curriculum document and resources 2 years later after a pilot phase evaluation.

3.3 Data collection and analysis

The collaboration took place through six whole-team sessions, 20 face-to-face meetings of thematic working subgroups (on philosophy, structure, number-algebra, geometry, statistics, technology, and assessment), and numerous online exchanges through email and/or Skype. After submitting the final version of the curriculum, the authors also interviewed 11 key design team members (two ministry advisors, two academic researchers including the coordinator, two school advisors, and five classroom teachers). We structured the interviews along four main axes: the meaning of innovation and its relation to research, teaching reality, and policy. The data analysed in this paper come from the design of the first draft curriculum document and were the email exchanges during the design phase, the field notes from the whole-team sessions kept by the first author, face-to-face meetings, and the transcribed interviews.

The analysis was carried out in two steps. Firstly, we adopted grounded theory techniques (Charmaz, 2006) to identify discussion themes. Through open coding of the email messages, we identified discussion themes and searched for these in the field notes and interviews. We grouped themes in four categories: the formulation of the central principles and objectives of the curriculum; the specification of mathematics content and the corresponding learning outcomes; the development of implementation resources; and the design of assessment tools and strategies. Here, we analyse data related to the first two categories and two specific discussion themes: (a) defining the objectives of the mathematics curriculum and (b) organising the algebra content and the learning outcomes across educational levels. Within each theme, we arranged the corresponding parts of all data sources chronologically.

In the second step of the analysis, we used the AT framework to identify and trace actions and goals participants undertook and negotiated in the dominant discussion themes underlying the activity systems at work (research, policy, and teaching). Under this perspective, we searched for emerging contradictions fuelled by the enactment of the three activity systems in the design context. We characterised these contradictions in terms of their content, the involved activity systems, and the elements of Engeström's interconnected triangles (see Fig. 1).

To address our second research question, we used the construct of boundary crossing to study the ways in which the management of the contradictions by the team members contributed to the design process. In particular, the identified contradictions indicated the boundary that was traced in the data in relation to the involved boundary objects and/or brokers and their role in boundary crossing. Next, we coded the process of dealing with this boundary by using the four types of learning at the boundaries.

We use the acronyms R (researcher), T (teacher), P (policy-maker), or their combinations (PR, TR) after participants' names to denote membership in specific instances.

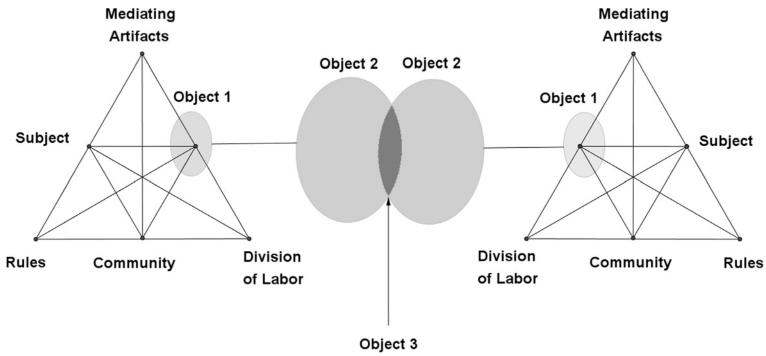

Fig. 1 Interacting activity systems (Engeström, 2001, p. 136)

4 Results

We structure the presentation of results based on the main contradictions that emerged while defining the curriculum objectives and organising the content and the learning outcomes of algebra across the educational levels. We identified four main contradictions and coded them: (1) educational innovation vs. teaching reality, (2) theoretical vs. practice-oriented ideas, (3) research-informed teaching resources vs. traditional teaching resources, and (4) arithmetic vs. algebra in primary school teaching. The first three emerged in defining the curriculum objectives, whereas the fourth one appeared while the team was working on organising the algebra content and the learning outcomes across educational levels. In each of the four following subsections, we present through illustrative examples the identified contradictions, the involved activity systems and their elements, and how these contractions were managed in terms of boundary crossing.

4.1 Educational innovation versus teaching reality

The first contradiction concerned the relation between educational policy and teaching activity systems. It emerged in the design team's first meeting when they negotiated the mathematics curriculum philosophy and objectives. At this phase, the role of educational policy was critical in formulating the design team's goals, as the Ministry had established specific educational aims and provided tools to achieve them. For example, the vision of educational reform was a central aim at policy level and supported through specific tools, such as legislation, media advertisements, digital resources, and publications. In particular, the new curriculum aims and principles in the *New School* context attributed to the teacher an active role in the process of enacting the curriculum resources. The teachers could decide which of the curriculum resources to use and also develop their own everyday teaching materials. This goal contrasted with the existing situation in Greece where teachers mainly plan their lessons based almost exclusively on the prescribed textbook. Moreover, the curriculum aims and principles formulated during preceding meetings between subject coordinators and ministry advisors expressed reform-oriented views, such as students' engagement in inquiry processes, connections between school and society, differentiation of teaching, formative assessment, and informal settings as areas of learning.

Below, we provide two examples to illustrate the role of the activity systems involved in the contradiction and the process of dealing with this in designing the mathematics curriculum objectives. Example 1 is based on data from the first team meeting, and example 2 is from an interview with a secondary school teacher.

4.1.1 Example 1

In this meeting, the coordinator presented the above-mentioned aims and principles. The participants expressed concerns as to what extent these could be concretised in the current educational and social contexts. Academic researchers questioned the meaning of expressions appearing in policy documents, such as "*the student first*," "*trusting the teachers*," "*digitalisation of the classrooms*," and "*multiple resources*" in relation to the state's capacity to support them. Another issue academic researchers and teachers addressed was the distance between the intended reform's vision and the reality of the current educational situation. For example, "Can these targets be reached under the current financial problems?" and "Can we talk about digital resources, since many classrooms do not have Internet access?" Other team member concerns were, "What are the teacher's motives to be engaged in the innovation?" and "How does the educational policy plan to secure the innovation and its implementation in schools?"

4.1.2 Example 2

While reflecting on the active role of the teacher in selecting and/or designing curriculum materials for teaching targeted by the educational policy, Victor (T) commented in his interview that this was not possible in the current teaching reality. He considered that the educational policy was far beyond the actual school reality, as it was expected that teachers would take up the role of designers on an everyday basis:

> It is a good idea to provide the teacher with responsibility of selecting and working with different resources. However, it is unrealistic for the teachers to develop their own materials for each lesson. It is good to provide suggestions about how this can be done. It requires small steps. The teachers do not trust what comes from the top and they are right to some extent. There are not appropriate educational plans … This way the teacher develops stereotypes that anything coming from the top is bad. (Victor, interview)

In the design team's first attempts to explore how the object of the educational policy activity mediated the activity of defining the mathematics curriculum objectives, the community of teachers and the existing conditions in this community appeared to be central in participants' actions and decisions. In example 1, it seems that the emerging contradiction is between the educational policy goals and the availability of tools (e.g., funding, PD programmes) to support a reform-oriented teaching activity. In example 2, the emerging contradiction is between the object of the educational policy activity (i.e., teachers developing their own materials on an everyday basis) and the rules of the school and the teacher communities (e.g., acting along specific directions provided centrally), as well as the available tools for the teacher (e.g., time and resources). This contradiction seems sustained, since it is rooted in stereotypes built over the years and concern teachers being suspicious of educational policy motives and object. The policy-makers' goals seem to be in contrast to those of the broad community of teachers and its rules. Our analysis indicates the distance between educational policy goals and teachers' goals, as well as the gap between the resources educational policy provided and those teachers traditionally used.

Dealing with the contradiction "educational innovation versus teaching reality" challenged the team, since it was often present in the design process. The boundary between the two activity systems explicit in both examples signals an identification process demarcating the two communities' goals, tools, and responsibilities in the educational system. The questioning of educational policy's effectiveness to support the innovation in example 1 keeps discontinuity at stake without implying an overcoming. By referring to the distance between "teacher-as-designer" targeted by the curriculum and "every teacher" in example 2, Victor identified a boundary that is not easy to cross. It requires long-term actions from educational policy to support teachers in adopting this new role through appropriate tools and structures. However, the variety of resources the design team provided (e.g., exemplary tasks, digital tools, teacher guides) can be seen as boundary objects that support the teacher as designer and facilitate overcoming the boundary under the existing teaching reality.

4.2 Theoretical versus practice-oriented ideas

Integrating research and theory in curriculum design and taking into account teachers' needs and goals appeared to be central forces driving the design team's work in all design phases. Here, we focus on how the research and teaching activity framed the curriculum principles and objectives. This was evident from the beginning of discussions on the mathematics curriculum philosophy. The subgroup working on preparing a draft on the philosophy and subsequent whole team discussions revealed different points of view that sometimes conflicted. Epistemological and learning and teaching aspects of the mathematics curriculum were brought to the fore, indicating an emerging contradiction concerning the gap between theory and teaching. We illustrate in our analysis of example 3 the interaction of the activity systems involved in this contradiction and the process of dealing with it while designing the mathematics curriculum philosophy.

4.2.1 Example 3

The issue discussed in the second team meeting was if and how theoretical and analytical ideas and the general principles of *New School* could become operational tools for classroom teachers. Participants questioned how theoretical ideas, research-informed curriculum aims, or teaching practices could provide a basis for developing useful tools for teachers, such as tasks promoting inquiry/creativity, descriptions of mathematical ideas, and learning goals. Members of the philosophy subgroup supported the view that theoretical constructs could be made clear through examples:

> I will prepare a draft of the philosophy document describing in detail the basic competences that transcend all educational levels through examples clarifying terms such as critical or creative thinking. These notions are too general to be operationalised in practice. (Elena (R), second whole-team session)

In contrast, other participants suggested using less theoretically overwhelming language while prioritising accessibility of the ideas to classroom teachers:

> It is necessary to decide on the degree of details we provide and the language we use in relation to the teachers. In my view, we should describe the mathematical ideas precisely with a language that is comprehensible, concise and terse. (Margaret (R), second whole-team session)

In subsequent emails, the above dilemma was further discussed mainly among the three members of the philosophy subgroup and the coordinator. Philip (PR), a primary school advisor, pointed out that, “The philosophy document needs to take into account teachers’ beliefs, expectations, and feelings if we want them to accept it.” Phillip also not only mentioned the need for PD programmes to support teachers but also pointed out that the philosophy text should be meaningful for teachers, “I tried to see the text through the lens of a person who is not involved in research and I made some comments on it.”

In Example 3, we see that the emerging contradiction concerns the object of the research activity system (theoretical ideas) and the tools of the teaching activity that mediate classroom teaching. Research in mathematics education as an activity system has a historicity in developing theory based on the actual practice. However, as the example illustrates, researchers do not share a common view on the balance between theory and practice. Bringing closer theoretical ideas to classroom teachers constitutes a challenge, especially for the researchers who also feel responsible for bridging this gap.

In dealing with this contradiction, the participants propose exemplifying the theoretical ideas with references to specific learning and teaching phenomena, simplifying the language and shortening the document to make it more meaningful and operational for teachers. In the decision-making process, the philosophy document seems to play the role of a boundary object in the intersection of research and teaching. The two academic researchers propose ways to establish continuity between the “big” theoretical ideas underlying the curriculum philosophy and the language that can be meaningful to teachers. The school advisor, also a researcher, acts as a broker bringing ideas from both practices. This way, the participants’ multiple membership in the two activity systems promoted boundary crossing through coordination.

4.3 Research-informed teaching resources versus traditional teaching resources

The challenge of integrating research into the curriculum was also present in all subsequent phases of the design process. This was often evident in the development of exemplary teaching resources aimed to illustrate the expected learning outcomes and to support teachers in classroom enactment. Most of the academic researchers in the team had long experience of collaborative work with teachers (e.g., teacher education courses) and/or policy institutions (e.g., Ministry committees). Thus, they felt more or less responsible for finding ways to design teaching resources taking into account school reality and systemic deficiencies. In general, as the participating teachers had also been involved in the research activity, they valued the contribution of research outcomes to the curriculum design. However, they often raised issues of the resources’ relevance to the classroom teacher. The emerging contradiction here concerns whether the research-informed teaching resources would be relevant to the teacher, or if they were too far from existing resources that the teacher is familiar with using. This involved mainly the research and teaching activity systems, although the educational policy activity was present concerning actions needed for supporting curriculum enactment.

Below, we provide two examples to illustrate the role of the activity systems involved in the contradiction and the process of dealing with it in designing the teaching resources. Both are from the participant interviews: the first with an academic researcher and the second with a classroom teacher.

4.3.1 Example 4

Stephen (RP), a mathematician who had also been a member of educational policy committees, questioned if it is possible to bring about changes in mathematics teaching, while emphasising the need to consider a variety of systemic factors that impede any innovation. In his interview, he prioritised the need for more realistic targets, since ambitious plans are often not applicable. He criticised the position of another researcher who considered that the team's goal was to design research-informed materials without being constrained by the teaching reality. He added:

> It is difficult to transform what is reported in international conferences in the design of the curriculum, especially when you know that the policy will not take care, the teachers will resist etc. … We need to be moderate. We introduced geometrical transformations in geometry …. I doubt that students could understand the mathematical ideas behind this. I favour small steps if we want a learning outcome. (Stephen, interview)

4.3.2 Example 5

Victor (T) felt that sometimes, academic researchers could not understand the actual problems, such as time pressure:

> I often felt in the team that the academic researchers are on the other side. I felt closer to the teachers, as the researchers are far from the actual practice of classroom teaching. For classroom teachers, time is a serious constraint and only teachers can make sense of this. (Victor, interview)

Interpreting Stephen's position, we recognise that the contradiction is between the object and rules of the research activity system (i.e., research-informed tasks, materials, and teaching strategies) and the tools that are common in the teaching activity system (i.e., procedural tasks, traditional teaching approaches). He makes explicit different obstacles of bridging research and teaching. Based on his prior experience as an education policy-maker, he refers to the community of policy-makers' rules and goals and to the teachers' community motives and goals that are not necessarily aligned with those of a reform-oriented curriculum. Victor, though, considers that researchers as participants in the research activity system are far from the specificities and constraints that teachers face every day. The contradiction raised in this example can be seen between the communities of teachers and researchers and in particular, between the object of the research activity and the rules of teaching.

Concerning the process of overcoming this contradiction for both Stephen and Victor, continuity between research findings and school practice cannot be achieved, and this eliminates the opportunities for boundary crossing. The designed resources acting as boundary objects allowed team participants to identify the boundary in many instances in their interaction. Although in these two examples the boundary remains, the team had to deal with this in the design process. The designed resources included research-informed tasks based on the use of manipulatives, digital tools, and realistic contexts that were too far from the resources that the teachers were familiar with. However, the design team attempted to overcome the boundary by making explicit in the teaching guides the rationale underlying the developed resources and by providing ideas for classroom implementation. The latter indicates boundary crossing in terms of perspective making and taking.

4.4 Arithmetic versus algebra in primary school

The contradiction concerned whether algebraic ideas will be included in primary school curriculum, or only in secondary school curriculum. Deciding which specific content areas and learning outcomes would be included in the curriculum document and their place across educational levels triggered a lot of discussion in the team. Main issues addressed in the interactions related to the content, its structure, conceptual demands, and requirements at the didactic level. The discussion brought to the fore divergent views about the balance between arithmetic and algebra in primary school.

The analysis of example 6 provided below illustrates certain interactions between the teaching and research activity systems involved in this contradiction and the ways that the design team managed it.

4.4.1 Example 6

In the email discussions between the members of the “numbers—algebra” group, the focus was on learning outcomes around specific content areas (e.g., integers, equations, functions). Victor (T) wondered if it was a good choice to introduce algebra in primary school:

> Integers could be introduced in the 6th year (the last year of primary school), but in this case the primary mathematics curriculum would be overloaded with algebraic concepts. So, the learning of arithmetic would be neglected … I think that we need to reduce the content both in primary and lower secondary school. (Victor, email)

Ken (TR), responding to Victor’s concerns, claimed that it is possible to include algebraic concepts in an intuitive way in the primary mathematics curriculum, “I think that introducing negative integers and their order intuitively through the number line in primary school could be without any problems” (Ken, email). In contrast, Olive (TR), a primary school teacher, opposed Ken’s claim by arguing that this overloading of content would not allow pupils to actively construct mathematical knowledge, “It is good not to bring new content in primary school unless we’re sure there will be no negative impact at all. Let’s not forget the pupils first (stressing it)” (Olive, email). Ben (R) argued that we need to offer warrants about suggestions towards any direction and any change certainly has some cost, “I think there is a dilemma here, do we want something to be realistic, or do we offer a continuous challenge of the knowledge borders?” (Ben, email). Olive claimed that there is a danger in teaching this new content traditionally like it is taught in other grades, so this would be an extra burden for the pupils. The discussion continued with new arguments in both directions. Philip (PR) agreed that transferring new content to primary school has to be well justified. He suggested that, “In primary school, we must not use algebraic symbols for some concepts (e.g. variable, equation), but make use of appropriate materials in order for the “algebraic character of arithmetic” (e.g. the different meanings of the equal sign) to emerge.” (Philip, email).

Victor questioned primary teachers' capacity to teach algebraic concepts, 'Can they handle the generalisation of a pattern? Can they teach a number system of base two?' (Victor, email).

Ben placed the discussion in the broader perspective of the nature of pupils' mathematical activity:

> The distribution of content across the school years does not matter so much. For me it is more important to consider which aspects of mathematical thinking we want pupils to develop. If, for example, we include demanding word problems we need to specify the targeted mental processes for the pupil, the future citizen that we envision. (Ben, email)

In example 6, we see that teachers and researchers bring tools, objects, and rules from the activity systems in which they participate. The mathematics education research activity comes into play from academic researchers, or teachers who had been engaged in research and support the introduction of algebra in primary school. Rules underlying research activity such as "designing decisions take into account research findings" and "claims need to be justified through theoretical or empirical evidence" introduced by Ben and supported further by Philip also indicate the role of the research activity in the design process. Victor and Olive oppose the idea of introducing algebra in the early years and the emerging contradiction here refers to time constraints, the importance of arithmetic, and the primary teachers' limited expertise in teaching algebra. In terms of the elements of the AT triangle, the rules of the teaching activity (e.g., teaching time) and the tools available to the teachers (e.g., prior experience and teaching materials) seem to contradict the goals and rules of the research activity.

In dealing with the contradiction, Ken and Philip operate as brokers between research and teaching offering ideas and arguments for overcoming the constraints (e.g., intuitive models, informal language). In the end of the extract, Ben seems to make an effort to coordinate both activities offering a common educational goal that targets the quality of students' mathematical thinking and their future citizenship. Overall, the design team attempted to overcome the boundary by embedding the algebraic activity in arithmetic; providing a variety of models and materials (e.g., algebra tiles, digital applets, integer model) that could facilitate developing algebraic thinking in primary school and provide teachers with texts describing the role of algebra in primary school. The latter indicates boundary crossing in terms of perspective making and taking—at least for the group of participants who believed that overcoming this boundary is an important step in making the reform possible. For instance, the mathematics coordinator in her interview made explicit her responsibility to find ways for overcoming the research versus teaching boundary, "I was concerned about how to use research to inform the design of resources and their potential impact on the teaching practice" (Mathematics coordinator interview).

5 Discussion and concluding remarks

Overall, as in Akkerman, Bronkhorst, and Zitter's (2013) study, the interaction of the three activity systems informed and contextualised the curriculum designers' actions, while they engaged in motives and norms established in educational policy, research and teaching. The teaching and the research activity systems had a stronger impact on the design team's work, while the educational policy activity system was present mainly in the transformation of the overall educational goals to mathematics curriculum objectives at the beginning of the process.

Contradictions emerged in the design team across the activity systems and boundaries were explicated. Boundary crossing occurred in many cases in the form of questioning goals and actions in each activity system and occasionally forming new ones that took different perspectives into account. Table 1 synthesises the main contradictions we identified and analysed in the four subsections of the results, the activity systems, the boundary objects involved, and the boundary crossings that occurred.

The teaching activity system was enacted in all emerging contradictions. The mathematics teaching and school reality seemed to pose the designers with dilemmas about how to communicate the outcome clearly to the teachers and their use of the suggested content and resources in everyday teaching. The object of the educational policy activity system seemed to contradict mainly the teaching activity in terms of the existing rules of the teachers and schools' communities and the tools and resources that teachers need to enact the targeted innovation. For example, the teacher's role as a curriculum designer contradicted the existing rules where the teacher is expected to strictly follow the centrally provided teaching guidelines. However, the transformation of the innovation to mathematics curriculum objectives was a rather smooth process because research and educational policy activity systems shared a rather convergent, reform-oriented vision about mathematics education. The coordinator acted as a broker, bringing the design team educational policy products (*New School* documents, general principles of curriculum design, and institutional specifications). These policy products functioned as boundary objects at the borders of the activity systems of teaching, research, and policy. In cases when contradictions emerged, the explicated boundary between teaching and policy remained. Boundary crossing was facilitated by the fact that some of the policy advisors were academic researchers themselves and they trusted the expertise of design team colleagues.

Research activity dominated the design process via tools, resources, and the rules governing the research community. This was expected to some extent due to the strong research-oriented basis of the design team's synthesis. The object of the research activity was to define research and theory informed principles and objectives; distribute mathematics content, taking into account research findings about pupils' learning; enrich teaching with materials promoting inquiry and mathematical understanding; and facilitate teachers' engagement in transforming curriculum resources into the actual classroom teaching. During the development of the mathematics curriculum philosophy, emerging contradictions concerned the object as well as the tools of the research activity and the tools of the teaching activity. For example, the language and theoretical constructs used by the researchers were not meaningful and relevant tools for the classroom teachers. Boundaries were encountered and boundary crossing in the form of coordination was facilitated mainly by the teachers in the design team who were also researchers. Distribution of the algebra content across the educational levels was another area where contradictions emerged and boundary crossing took place at the borders of the research and the teaching activity. Dealing with curriculum structure for algebra, researchers brought research findings, theoretical constructs, research-informed materials, and models to back their choices at the design level. Moreover, teachers brought to the fore practical issues, such as time constraints, accessibility of teaching materials, and primary teachers' capacity to teach the new content. Here, again teachers acted as brokers, adopting occasionally or eventually both teachers' and researchers' perspectives.

When contradictions emerged, using the elements of Engeström's triangle gave us a lens to understand the sources of these contradictions and their role in the design process. The different objects of the activity systems, the tools used within them, and the rules underlying

Table 1 Summary of contradictions in terms of activity systems, boundary objects, and boundary crossing

Contradiction	Activity systems involved (elements)	Boundary objects	Boundary crossing
Innovation vs. teaching reality	Educational policy (object, tools)—teaching (tools)	Principles of *New School*	- Boundary is identified and remains (Identification)
	Educational policy (object)—teaching (tools, rules, object)	Teaching resources	- Boundary is identified and remains (Identification)
Theoretical vs. practice-oriented ideas	Research (tools, object)—teaching (tools)	Curriculum philosophy document	- Boundary crossing smoothens via exemplifying the theoretical ideas and simplifying the language (Coordination)
Research-informed teaching resources vs. traditional teaching resources	Educational policy (object, rules)—research (object, tools)—teaching (tools)	Teaching resources	- Boundary is identified and remains (Identification)
	Research (object)—teaching (rules)	Teaching resources	- Boundary is identified and remains (Identification) - Facilitating boundary crossing by providing a variety of educational materials and resources (Perspective making/Perspective taking)
Arithmetic vs. algebra in primary school	Research (goals, rules)—teaching (tools, rules)	Curriculum structure for algebra	- Boundary crossing via establishing the improvement of students' learning as a common goal (Coordination) - Facilitating boundary crossing by providing a variety of educational materials and resources (Perspective making/Perspective taking)

the activity of the three communities operated as driving forces behind the emerging contradictions and the explication of boundaries. The boundary crossing construct allowed us to gain insight into the nature of interaction across the boundaries of the three systems. It also helped us to describe how the design team dealt with these contradictions and how this influenced the final product. Our findings indicate mainly identification and coordination as two main ways in which the research and teaching activity systems interacted in their attempts to formulate a collectively meaningful object (Engeström, 2001). Participants' dialogical engagement and negotiation around boundary objects (e.g., the philosophy document, curriculum structure of algebra) indicated efforts to coordinate objects or tools of the three activity systems. Teachers and researchers' multi-membership facilitated this coordination. In particular, researchers with substantive collaborative work with teachers emphasised the need to communicate the research-informed curriculum materials to teachers. They also paid attention to the relevance of the developed resources to classroom teaching, acting together with teachers-researchers as brokers that facilitated boundary crossing occasionally in the form of perspective making and taking. The coordinator shared similar perspectives and often supported such crossings between research and teaching activity systems. She also acted as broker between the educational policy community and the design team by sharing policy documents and practices with team members.

Our study offers a dual theoretical perspective involving AT and boundary crossing that can be integrated into the field to highlight contradictions and the process of overcoming them in the interaction of the different communities and the corresponding activity systems during their participation in designing a mathematics curriculum. Our findings indicate issues and areas of tensions that seem to play an influential role in the participants' perspectives, actions and goals, and in the outcome of the curriculum and its development. Although this concerns the design of a specific national curriculum, it opens up new areas of research on the process of developing a curriculum innovation at scale. Furthermore, the analysis highlights curriculum as a product 'under development' and sheds light on boundary crossings between research, teaching, and educational policy that preceded design choices and decisions. What is more, it allows us to gain some insight into how the "shift in reform ownership" (Coburn, 2003, p. 3) needs to be facilitated at the design level of an innovation. In our case, this was realised through allowing the activity systems of teaching, research, and educational policy to interact and starting to understand the role of each in the outcome.

References

Adler, J., Ball, D. L., Krainer, K., Lin, F.-L., & Novotna, J. (2005). Reflections on an emerging field: Researching mathematics teacher education. *Educational Studies in Mathematics, 60*(3), 359–381.

Akkerman, S. F., & Bakker, A. (2011). Boundary crossing and boundary objects. *Review of Educational Research, 81*, 132–169.

Akkerman, S. F., Bronkhorst, L. H., & Zitter, I. (2013). The complexity of educational design research. *Quality & Quantity, 47*(1), 421–439.

Charmaz, K. (2006). *Constructing grounded theory. A practical guide through qualitative analysis*. London, UK: Sage.

Choppin, J. (2011). Learned adaptations: Teachers' understanding and use of curriculum resources. *Journal of Mathematics Teacher Education, 14*(5), 331–353.

Clark-Wilson, A., Hoyles, C., Noss, R., Vahey, P., & Roschelle, J. (2015). Scaling a technology-based innovation: Windows on the evolution of mathematics teachers' practices. *ZDM—the International Journal on Mathematics Education, 47*(1), 79–92.

Coburn, C. E. (2003). Rethinking scale: Moving beyond numbers to deep and lasting change. *Educational Researcher, 32*(6), 3–12.
Cochran-Smith, M., & Lytle, S. (1999). Relationships of knowledge and practice: Teacher learning in communities. In P. Pearson & A. Iran-Nejad (Eds.), *Review of research in education, 24* (pp. 249–307). Washington, D. C.: American Educational Research Association.
Edwards, R., & Fowler, Z. (2007). Unsettling boundaries in making a space for research. *British Educational Research Journal, 33*(1), 107–123.
Engeström, Y. (1999). Innovative learning in work teams: Analysing cycles of knowledge creation in practice. In Y. Engeström, R. Miettinen, & R-L. Punamäki (Eds.), *Perspectives on activity theory* (pp. 377–406). Cambridge: Cambridge University Press.
Engeström, Y. (2001). Expansive learning at work: Toward an activity theoretical reconceptualization. *Journal of Education and Work, 14*, 133–156.
Engeström, Y., Engeström, R., & Kärkkäinen, M. (1995). Polycontextuality and boundary crossing in expert cognition: Learning and problem solving in complex work activities. *Learning and Instruction, 5*, 319–336.
Hoyles, C., & Ferrini-Mundy, J. (2013). Policy implications of developing mathematics education research. In M. A. Clements, A. Bishop, C. Keitel, J. Kilpatrick, & F. Leung (Eds.), *Third international handbook of mathematics education* (pp. 485–514). New York: Springer.
Hung, D., Lim, K., & Huang, D. (2010). Extending and scaling technology-based innovations through research: The case of Singapore. In Organisation for Economic Co-operation and Development (Ed.), *Inspired by technology, driven by pedagogy [electronic resource]: A systemic approach to technology-based school innovations* (pp. 89–102). OECD Publishing.
Krainer, K. (2014). Teachers as stakeholders in mathematics education research. *The Mathematics Enthusiast, 11*(1), 49–60.
Krainer, K., & Zehetmeier, S. (2013). Inquiry-based learning for pupils, teachers, researchers, and representatives of educational administration and policy: Reflections on a nation-wide initiative fostering educational innovations. *ZDM—the International Journal on Mathematics Education, 45*(6), 875–886.
Kynigos, C., Philippou, G., Potari, D., & Sakonidis, H. (2009). Research in mathematics education in Greece and Cyprus. In M. Tzekaki, H. Sakonidis, & M. Kaldrimidou (Eds.), *Proceedings of the 33rd conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 303–322). Thessaloniki, Greece: Aristotle University.
Remillard, J. (2005). Examining key concepts in research on teachers' use of mathematics curricula. *Review of Educational Research, 75*(2), 211–246.
Roesken, B., Hoechsmann, K., & Toerner, G. (2008). *Pedagogies in action: The role of mathematics teachers' professional routines*. Paper presented at the Symposium on the Occasion of the 100th Anniversary of ICMI (Rome, 5–8 March, 2008). Retrieved from http://www.unige.ch/math/EnsMath/Rome2008/WG2/Papers/ROHOTO.pdf.
Roesken-Winter, B., Hoyles, C., & Blömeke, S. (2015). Evidence-based CPD: Scaling up sustainable interventions. *ZDM—the International Journal on Mathematics Education, 47*(1), 1–12.
Wenger, E. (1998). *Communities of practice. Learning, meaning and identity*. New York, NY: Cambridge University Press.