Educ Stud Math (2018) 97:21–37
DOI 10.1007/s10649-017-9777-0

Conflicting frames: a case of misalignment between professional development efforts and a teacher's practice in a high school mathematics classroom

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Published online: 11 July 2017

Abstract We examine the case of a lesson planning session within the context of professional development for dialogic instruction, and the lesson enacted following this session, which was intended to provide opportunities to 11th and 12th grade algebra students to explore polynomial functions in terms of their roots and linear factors. Our goal was, through the close analysis of the planning and enactment of the lesson, to gain deeper understanding of how the two participants were framing mathematical learning and how such different frames may explain the disparity between the planned lesson and its outcome. The analysis and discussion point to the complexities of supporting teachers in transitioning from a "doing" frame to an "exploring" frame.

Keywords Professional development · High school algebra · Frames · Discourse · Lesson planning · Secondary school teaching · Dialogic instruction

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1 Introduction

Much has been written about divides between educational research, which is conducted in/by the world of Academia, and educational practice, which is enacted in the world of teachers and schools (e.g., Coburn & Stein, 2010), and about the need to bridge between these worlds. During the last 20–30 years in mathematics education, some areas of the field have seen considerable bridge-building, as researchers, mathematics educators, and teachers have worked to build upon research findings about how students learn to enhance teaching practices and students' outcomes (e.g., Cobb & Jackson, 2011; Tate, King, & Anderson, 2011). In other areas, however, differences remain. This article is about such a difference—a misalignment between Munter (CM; second author), a secondary mathematics teacher turned mathematics education researcher, and Ms. Q,[1] a high school algebra teacher who participated in a professional development (PD) effort, including one-on-one planning sessions with Munter. Specifically, we shall focus on one planning session that, despite its apparent positive and collaborative atmosphere, and despite its outcome, which was a seemingly well-planned lesson, resulted in the enactment of a lesson that was very far from the visions that either of the parties had for it. Our goal is, through the close analysis of the planning and enactment of the lesson, to gain deeper understanding of how the two participants (Ms. Q and CM) were framing mathematical learning and how such different frames may explain the disparity between the planned lesson and its outcome.

2 Preview: the illusive nature of CM and Ms. Q's planning session

Before explaining the theoretical foundations of this work, let us first illustrate the problem at hand through a short excerpt from CM and Ms. Q's planning session. The context, very briefly, is as follows: the session began with Ms. Q describing the lesson they had co-planned the week before, expressing satisfaction with the increase in classroom discussion: "we were able to explore like, 'How did you get that, and you got this? Like, walk us through it.' They were going off of each other. It was the best lesson ever." In this episode, CM and Ms. Q (more on her, her school setting, and the broader project in the methods section) were planning a curriculum-based lesson that would center on identifying the zeroes (or, x-intercepts) of polynomial functions, with an emphasis on factoring. Students had explored this topic for first- (linear) and second- (quadratic) degree polynomial functions but not for third-degree (cubic) functions or beyond. Although the textbook put it off for a couple more lessons, Ms. Q expressed her desire to connect the idea of zeroes to graphical representations in this lesson.

CM and Ms. Q decided to pose a task involving a cubic function and invite students to generate their own ideas and strategies for identifying its zeroes without a demonstration of any particular approach. For the day's primary task, they chose "What are the zeroes of $f(x) = 6x^3 - 19x^2 - 9x + 36$?"—which, in factored form, is $f(x) = (x - 3)(2x - 3)(3x + 4)$—and identified at least two strategies that students would likely employ to begin solving the problem: (a) trial and error or (b) graphing it on a calculator to find x-intercepts. CM imagined that, employing either of those methods to find an initial root, students might then write a factor "around" that root and use long division (a procedure that Ms. Q reported demonstrating in the previous day's lesson) to rewrite $f(x)$, for example, $f(x) = (x - 3)(6x^2 - x - 12)$. In order to

[1] All names, except Munter's, are pseudonyms.

set up that group work and the eventual discussion of students' strategies, CM suggested that the lesson might be launched with a warm-up task asking students to "find the zeroes of the quadratic function by factoring: $g(x) = x^2 - 7x + 12$," which would promote factoring polynomials for a purpose (finding the roots) and increase the likelihood that students would write a binomial factor (e.g., $x - 3$) for the first root they identify in the day's primary task (likely $x = 3$). Here is an excerpt from the conversation between CM and Ms. Q about this plan, in which they are discussing the whole-class conversation towards the end of the lesson:

Excerpt 1 (see transcript conventions in Appendix 1).
[1.01] CM:And then, in the whole class conversation… *((if students spent their time using a graphing calculator to estimate the roots))*… the whole, the point you can bring from that is like, "you kept trying out numbers, and your <u>goal</u> was to make it equal to <u>ze</u>ro."
[1.02] Ms. Q: Mm-hm.
[1.03] CM: "Yeah, that's- that's the point." You know, "That's great. Did anyone come up with a way that you can, that's <u>not</u> having to keep putting in other numbers and trying them out, that's a little more efficient?"
[1.04] Ms. Q: Yeah.
[1.05] CM:And then, bring on the next group who um, did long division and, uh, figured out that when they factor this out it leaves a quadratic, that- you know, structure the conversation so that you're linking strategies that are-
[1.06] Ms. Q: so I think that when I do the, uh, when I do the warm up, I should definitely bring <u>out</u> the fact, ok, so the x – 3, ok, the answer is, you know, getting to the, **they solve it** the 3, "so what would <u>hap</u>pen **if I put 3 back** in this equation?" I think I need to make sure I bring that <u>out</u> so I get **somebody trying to do exactly that**. "Well, **let me put in** 1, **let me put in** this," cause without specifically bringing that [up,
[1.07] CM: [yup, yup
[1.08] Ms. Q: that that makes it zero, they might not try that.
[1.09] CM: I think that's a good point, I agree.

At first glance, CM and Ms. Q might seem to be well-aligned. They agree with and affirm each other's ideas ("Yeah," [1.04] "yup, yup" [1.07] "I think that's a good point, I agree" [1.15]), with no obvious misunderstanding or disagreement between them. Indeed, CM reported having had a good feeling about the planning session and expecting it to produce a productive lesson (author's notes). We acknowledge that for fresh readers of the transcript, problematic aspects may be immediately evident (more on these in later sections). Yet, at least for CM, who was a participant in the interaction, it was surprising how different the teacher's enactment of the lesson was from what he thought had been planned. Most of the students chose to "solve" the quadratic equation by using the quadratic formula, a procedure that drew them away from seeing roots as related to factors. The lesson progressed very slowly, with Ms. Q. walking from group to group, pushing the students to identify the roots of the polynomial function by providing more and more hints. Finally, one student was able to divide $f(x)$ by $(x - 3)$, after which Ms. Q suggested that others do that as well. Students struggled to perform this procedure, and the lesson ended with no discussion of strategies, or even a statement of the solutions to the day's primary task. Throughout the lesson, students complained about Ms. Q not telling them if they were right or wrong and not demonstrating strategies that they could follow.

Though an initial explanation for this unfortunate turn of events might be that students were simply unprepared for the task, a deeper analysis of the classroom event will show that even if that was at least partially the case, the way in which Ms. Q posed the task and guided the students did not align with the vision that CM had for this lesson and the assumptions that he was making about what learning mathematics requires. We propose an explanation of this miscommunication on the basis of CM and Ms. Q's different *frames* of mathematical learning.

In what comes next, we first put forward the theoretical framework that we will be using for analyzing the lesson planning session and the lesson itself. We will then examine some more instances of alignment and misalignment between CM and Ms. Q's frames of mathematical learning, with regard to both the mathematical content and expectations for students' actions. We will complement this analysis with some excerpts from the enacted lesson that exemplify the ways in which it diverged from CM's understanding of the plan. Finally, we will discuss implications for teacher learning and PD efforts that attempt to mobilize teachers towards implementing dialogic types of instruction.

3 Theoretical background

Inherent to the enterprise of teacher professional development—whether in formal training programs or in more informal interactions such as those comprising the focus of this article—is the underlying assumption that there is something that could be improved. Efforts that focus on pedagogy likely aim for some change, often approaching teachers as being "in transition" (Nelson, 1997)—from old forms of practice to new.

In mathematics education, for at least the last 20 years, this transition has been vaguely characterized as moving from "traditional" instruction to more "reform"-oriented practices (Munter, Stein, & Smith, 2015). Based on a series of conversations with nationally recognized experts who held opposing points of view, Munter et al. specified two distinct instructional models: *direct* instruction, in which students are provided demonstrations and scaffolded phases of guided and independent practice, accompanied by close monitoring and corrective feedback, and *dialogic* instruction, in which students engage collaboratively in solving novel tasks and, through discussions, draw connections among their collective invented strategies and representations. The models were offered not as a dichotomization of all possible pedagogies, merely two contrasting approaches. Still, the explication of differences between them helps to clarify that, most often, the aim of US mathematics education PD is, likely, actually a move from instruction that is *conventional* (i.e., poor enactments of direct instruction that too often over-emphasize procedures without conceptual understanding) to instruction that is *dialogic*.

Professional development in this vein has pursued a great number of goals. One example that is especially relevant to our analysis—and to Ms. Q's efforts to use a new, more conceptually oriented curriculum—is that of supporting teachers in implementing more cognitively demanding mathematical tasks. Smith and Stein (1998) distinguished between tasks of "low" and "high" level demand.[2] The former require only memorization or using

[2] In this case, conceptual distinctions such as Skemp's (1976) "instrumental" vs. "relational" understanding or Lithner's (2008) "creative" vs. "imitative" reasoning are also relevant. However, we chose to employ Smith and Stein's (1998) mathematical tasks lens because it more directly relates to an expectation for Ms. Q's practice at the time—implementing a more conceptually oriented curriculum.

procedures, mostly based on manipulation of symbols. The latter require students to construct a solution based on conceptual relations. By that, it enhances the *objectification* of mathematical talk in the classroom, enabling the progressive buildup of mathematical complexity (Sfard, 2008). An important distinction between conventional and more dialogic instruction is that in the latter, students are positioned to take initiative in making sense of a novel task and in generating and justifying a sequence of actions to accomplish it. Such a commitment implicates far more aspects of teachers' practice than mere task selection (NCTM, 2014); it requires teachers and students to play different roles from those that are typical in the mathematics classroom and therefore points to the significance of the "transition" that teachers are asked to make.

A number of researchers have taken up the question of what underlies such transition (or non-transition), with some locating it in changes in epistemological perspectives (e.g., Simon, Tzur, Heinz, Kinzel, & Smith, 2000); degrees of appropriation of pedagogical tools (e.g., Grossman, Smagorinsky, & Valencia, 1999); or teachers' identity (e.g., Gresalfi & Cobb, 2011), to name a few. In this article, we examine the question from the standpoint of teachers' (and researchers') *framing* (van de Sande & Greeno, 2012; Hammer, Elby, Scherr, & Redish, 2005). To further explicate the meaning of framing and to locate frames in the discourse of teachers and in classroom discourse, we draw on Sfard's (2008) notion of mathematical routines and meta-rules.

3.1 Features of mathematical and school discourse

Following Sfard (2008), we assume that one view of the gist of mathematical activity, both in the classroom and over history, is the exploration of "mathematical objects." These discursive objects, like numbers, functions, or matrices, are not existing objects in the physical world, yet the "metaphor of object" is used by any skilled mathematician to talk about them as though they were indeed such physical objects. They are often "explored," "transposed," or "pulled apart" and they have "attributes" and "characteristics" just as physical objects do. Mathematical objects can have many realizations or physical symbolic representations. A function, for instance, can be represented as a graph, a set of paired numbers, a table, an algebraic expression, or a verbal statement. In the process of learning, according to Sfard, students come to see all these realizations as signifying one object. Once they do that, they start talking about it as an object existing of itself, a process termed "objectification."

Sfard (2008) defined discourse as a "special type of communication made distinct by its repertoire of admissible actions and the way these actions are paired with re-actions; every discourse defines its own **community of discourse**; discourses in language are distinguishable by their vocabularies, visual mediators, **routines** and **endorsed narratives**" (p. 297; emphasis in the original). Routines are rule governed. Sfard divides these rules into two types: object-level and meta-level. Object-level rules are "narratives about regularities in the behavior of objects" (p. 201), in our case mathematical objects. Meta-rules regard the actions of the people performing the routines or in Sfard's words "define patterns in the activity of the discursants trying to produce and substantiate object-level narratives" (p. 201). Meta-rules may be explicitly endorsed (as in stating "this is how one should multiply fractions") but are more often tacitly enacted.

Such tacit meta-rules have been described by different authors as "norms," "sociomathematical norms" (Voigt, 1995; Yackel & Cobb, 1996), and "frames" (Goffman, 1974; Tannen, 1993). In the context of mathematical learning, meta-rules are often tied to object-level rules. For example, given a quadratic function $f(x) = ax^2 + bx + c$, the object-level rule is that the roots of the function $f(x)$ fulfill the identity: $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$. A related mathematical meta-rule could be "whenever encountering a quadratic equation solve it by plugging in *a,b,c* into the quadratic formula" (this meta-rule of course constrains other possibilities, such as solving the equation by factoring). We shall refer to these meta-rules as mathematical meta-rules, to differentiate them from *social meta-rules* which do not deal with specific mathematical objects but rather with more general people-related interactions. For instance, when a teacher asks "and how do we solve a quadratic formula?", the students are supposed to bring up a previously rehearsed procedure (indicated by the words "and" and "we" and by tone of voice, which signals this is a rehearsal of previously learned material). Thus, in social meta-rules, the expected action and re-action are unrelated to the mathematical object at hand (quadratic function) but rather pertain to the roles and responsibilities of the participants (e.g., teacher vs. students).

3.2 Frames of mathematical learning

Frames lie somewhere in between a specific meta-rule or norm and the much more general set of meta-rules that describe a discourse. They are based on regularities that can be observed in repetitive human interactions, yet most authors describe them in their internalized form, as cognitive structures. For example, Hammer et al. (2005) define them as follows:

> By a "frame" we mean, phenomenologically, a set of expectations an individual has about the situation in which she finds herself that affect what she notices and how she thinks to act. An individual's or group's framing of a situation that can have many aspects, including social ("Whom do I expect to interact with here and how?"), affective ("How do I expect to feel about it?"), epistemological ("What do I expect to use to answer questions and build new knowledge?"), and others. (p. 9)

Building on this definition, and operationalizing it in discursive terms, we define a "frame" in mathematical instruction as a set of meta-rules, both mathematical and social, which includes appropriate questions, answers, justifications, and other discursive actions in a situation of solving a mathematical problem or performing a mathematical task. Phenomenologically, frames mean a set of an individual's expectations originating in repetitive interactions.

3.3 Ritual vs. explorative participation

Within the view of learning as becoming a participant in a certain type of discourse, Sfard and Lavie (2005) theorize the process of learning as moving from a ritual, peripheral phase, where activity is first and foremost aimed at pleasing the experts of the discourse (e.g., the teacher), to an explorative phase where new mathematical narratives are produced by oneself for the sake of the activity itself. In contrast to explorative participation, the focus of ritual participation is *activity* itself, not the mathematical narrative produced by it (Lavie & Sfard, 2016). A student participating ritually will be satisfied with the accurate execution of a specified set of prescribed calculational steps. Dialogic instruction attempts to offer opportunities for explorative participation, reducing the focus on "doing" and instead shifting attention to the production of valid mathematical statements (by reasoning) and the exploration of mathematical objects.

Whereas frames of explorations would be sets of meta-rules that cohere around the goal of producing mathematical narratives based on logical justifications, frames of "doing" would be more aligned with ritual goals such as performing a procedure accurately according to a prescribed set of steps.

3.4 Questions of the research

In the next section, we describe our methods for addressing two guiding questions: (1) What were the indicators of differences between Ms. Q and CM's frames of mathematical learning, including their talk about mathematical objects and their talk about students' actions? (2) How can the differences in frames explain the gap between the lesson as envisioned by CM and the lesson as enacted by Ms. Q?

4 Methods

4.1 Study context

The episodes that we analyzed for this report occurred within a broader, semester-long (January–April 2013) professional development effort that included four Algebra 2[3] teachers in an urban public high school in the northeastern region of the USA. The school served approximately 500 grade 9–12 students, roughly 80% of whom were African-American and 20% were white. The school was designated as 100% "low-income."

The intent of the PD—of which co-planning lessons with CM was one part—was to support the implementation of Accountable Talk® in the high school mathematics classroom,[4] which the sponsoring research team was hoping to observe and study. The research team asked CM (himself previously a high school mathematics teacher of 8 years, including algebra 2) to facilitate in-person PD on Accountable Talk®, to which the teachers were first introduced through a series of online modules facilitated by an Accountable Talk® expert. Representatives from the research team approached the school's principal, who expressed her desire to see more discussion in the school's mathematics classrooms. Following the principal's suggestions, it was decided that the PD would include the algebra 2 teachers and would begin with Ms. Q, who was in her ninth year teaching at the high school, and in her first year as an official teacher-leader,[5] with the intention of then collaborating with her in designing and leading future sessions with the other algebra 2 teachers.

Given that the teachers were facing a number of significant policy changes around that time, including implementing a new district-adopted textbook that focused more on conceptual understanding than did the more conventional text they had been using, as well as a new, comprehensive, district-wide system for evaluating their annual performance, CM was interested in situating the research team's "implementation" effort within the current school and district context in which the teachers were working. To aid in this, all four teachers and the

[3] An "Algebra 2" course in the USA is a second, year-long algebra course that typically follows a first algebra course and a geometry course and focuses largely on expanding work with functions to polynomial, rational, radical, and possibly trigonometric functions.

[4] For more information on Accountable Talk, see http://ifl.pitt.edu/index.php/educator_resources/accountable_talk.

[5] In her "teacher-leader" role, Ms. Q's teaching load was reduced to allow time for her to provide direct instructional support to other teachers (e.g., co-teaching, observing, and other instructional "coaching" activities).

principal were interviewed several weeks before the beginning of the PD by a member of the research team who was not involved in the PD. The purpose of the interviews was to understand the goals, challenges, accountability expectations, and forms of support that contributed to the institutional setting in which they worked, as well as their views about high-quality instruction and the students they served.

In the interview with Ms. Q, her description of a high-quality mathematics lesson generally aligned with recent consensus efforts in the USA (e.g., NCTM, 2014), including emphases on "[students] doing the work, not the teacher," "getting the kids to draw out connections between the math concepts by getting kids to activate their prior knowledge," using "tasks that have various entry points, … connect different representations …, [and] require students to explain their thinking," and students "challenging each other's thinking and listening to each other." She also reported that she considered the newly adopted textbook and curriculum to be appropriate for all of her students, adding that although particular sections were very challenging, "a lot of the book is about 'do what you can.' You don't necessarily have to do all of it. All of the problems go to your level, which makes it good for differentiating."

Following Ms. Q's "instructional vision" and perspective of the textbook's affordances, CM attempted to situate the focus on Accountable Talk® within a broader set of considerations for promoting explorative participation and rich mathematical discourse through planning and enacting lessons that (a) focus on a cognitively demanding task; (b) offer and encourage multiple solution paths; (c) give time for students to work on the task themselves, while the teacher is walking around the groups and advancing students' thinking through "assessing and advancing" questions; and (d) invite students to participate in discussions in which they compare and connect their strategies and representations in order to build shared understandings (Stein, Engle, Smith, & Hughes, 2008).

4.2 Data collection

The PD session that is the focus of our analysis occurred early in the second half of the school year and included only CM and Ms. Q. The classroom lesson occurred the following day and was observed and recorded by other members of the larger research team. Every interview, PD session, and observed classroom lesson were audio recorded and transcribed. We also collected relevant district artifacts, including curriculum materials and teacher observation and evaluation rubrics.

4.3 Analyzing frames and meta-rules of discourse

In order to analyze how CM and Ms. Q were each framing the expected lesson, we attended to the ways they described students' activity. In particular, we paid attention to the underlying social meta-rules, such as the way the students' activity would be organized, how students would talk with each other, who would ask questions, and what the role of the teacher would be in the activity. For the mathematical meta-rules, we attended to how CM and Ms. Q described mathematical objects, which words and narratives they used and what routines they referred to (such as "solving" or "finding the roots"). We then conducted a similar analysis on the lesson that was enacted by Ms. Q, comparing the mathematical meta-rules envisioned by CM and Ms. Q in the planning session with those that were enacted in the lesson. We also attended in the lesson itself to the social meta-rules, in particular to how Ms. Q positioned the students and what positions the students took up. Within our search for meta-rules of mathematical activity and of students' actions, we attempted to group the different meta-rules into frames, that is, sets of meta-rules that cohere and reinforce each other.

5 Findings

5.1 The planning session

CM and Ms. Q's mathematical discourse Early in the planning of the lesson on factoring and zeros of polynomials, CM and Ms. Q considered her textbook's proof of the polynomial remainder theorem, which, as stated in the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), is "For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$" (p. 64).

Excerpt 2.
[2.01] Ms. Q:*((Likely examining the final lines of the textbook's proof))* So that turns to- (3s) Oh yeah, and then zero times yeah. (17s) You know, why do we care? What am I missing?
[2.02] CM:Why do we care what?
[2.03] Ms. Q:Why do we care that we can predict the remainder? (9s)
[2.04] CM:That's a good question. (20s) To me again, it uh, what matters more is the- is the behavior of the polynomial and if you factored a binomial that if you put in the number that makes that factor equal zero, then [you understand something about, um, the mechanics of the polynomial,
[2.05] Ms. Q:[Right, the root. Yeah.
[2.06] CM:so if you've, if you've, uh, written a factor- or if you've written a binomial that isn't a factor of the polynomial, meaning that doesn't- [is not, you know, created around a root,
[2.07] Ms. Q:[Oh.
[2.08] CM:then you're going to get a remainder. [You're not going to get zero.
[2.09] Ms. Q:[So you could check it. Is it a factor? "No, I have a remainder." Yeah.
[2.10] CM:Yeah. Yeah yeah. That's right. The reason that, so yeah, if you evaluate f of a and it doesn't equal zero, then [you know a's not a root,
[2.11] Ms. Q:[Right. Mm-hm.
[2.12] CM:which means that when you do x minus a factored out, that there's going to be a remainder and it's going to equal the number you just got.
[2.13] Ms. Q:Mm-hm. Yeah. I mean, yeah.
[2.14] CM:So, it's not- I don't have an answer for- maybe there is one, but I don't have an answer for why we care all that much about predicting the remainder, um, [except that it-
[2.15] Ms. Q:[Except for what you said, if it's not zero, it's not a factor.
[2.16] CM:Right, and so it's a more complete understanding of=
[2.17] Ms. Q:Yeah
[2.18] CM:=tearing apart the polynomial and understanding its roots and what is and isn't and how it behaves.

At first glance, CM and Ms. Q may seem to have been on the same page in this exchange. Though CM was the one who was attempting to answer Ms. Q's question ("Why do we care that we can predict the remainder?"), she supplemented his sentences and echoed them, at least partially [2.09], [2.15], in addition to expressing her agreement through "yeah" and the like [2.05], [2.13]. Yet, a closer look reveals that the two were emphasizing the remainder theorem's implications differently. CM's focus was on a relation between roots and factors, whereas Ms. Q focused more on using the theorem's implications as a "check" for whether a given binomial might be a factor.

Ms. Q's inciting question, "why do we care?" [2.01], implies that the relation established in the theorem is not sufficiently interesting in and of itself; its value is determined by its utility—or what it helps us *do*. CM, on the other hand, implicitly downplayed any practical application when he suggested that "what matters more is the behavior of the polynomial and… understand[ing] something about the mechanics of the polynomial" [2.04]. In suggesting that polynomials *behave* in a certain way [2.04], [2.18] and have certain *mechanics* [2.04] (which evokes images of machinery), he was talking about them in an objectified way. His descriptions of "tearing apart the polynomial and understanding its roots" [2.18] and binomial factors being "created around a root" [implying $(x - a)$ is built around a] point to a focus on exploring those objects, particularly the relation between roots and factors. Ms. Q's focus on utility ("why do we care?") vs. CM's focus on the polynomial object and its "behavior" is thus our first indication that the two interlocutors were framing the mathematical activity at hand differently.

The differences in framing can also be seen in the meaning Ms. Q gave to the remainder theorem, after hearing CM's explanation in [2.06; 2.08]. Though initially, her insertion of the idea of a "root" [2.05] suggests she does not have a problem seeing the relation between "roots" and "factors," her explanation "So you could check it. Is it a factor?" [2.09] suggests that she was searching for the utility of the remainder theorem—that its implications can be employed practically to *do* something. The meta-rule underlying this "doing" was, in this instance, answering the question "Is it a factor?" by performing the long division routine and checking whether it results in a remainder. Though CM affirmed this idea ("That's right"), he again returned to talk about the polynomial parts and the relations between them ("if you evaluate f of a and it doesn't equal zero, then you know *a*'s not a root … which means that when you do x minus *a* factored out, that there's going to be a remainder and it's going to equal the number you just got.") [2.10, 2.12]. The meta-rule, implied by his words "which means" and the reference to "the number you just got," was that the remainder theorem assists in seeing the relation between parts of the polynomial (roots and factors), not necessarily calculating anything. Following that, CM again downplayed the immediate utility of the theorem when he stated, "I don't have an answer for why we care all that much about predicting the remainder" [2.14]. Ms. Q, however, restated CM's words as a potential meta-rule for "testing" if the binomial divisor is a factor: "If [the remainder's] not zero, [the binomial divisor's] not a factor." [2.15].

These differences in the meta-rules derived by CM and Ms. Q from the remainder theorem point to a difference in framing of the learning activity around this task: CM was framing engagement with the theorem as an opportunity for *exploring* polynomial functions, roots, and factors, whereas Ms. Q was employing a *doing* frame, discussing the remainder theorem as a tool for helping her students produce solutions to certain mathematical problems.

Despite the differences in their framing of the mathematical learning activity, neither Ms. Q nor CM expressed any misunderstanding. On the contrary, their talk was filled with approving remarks ("yeah"; "Mm-hm") and even complementing each other's statements. For example, in [2.14] when CM finishes with "except that-," Ms. Q. fills his sentence with "Except for what you said, …." [2.15].

One possibility for the lack of any expression of misunderstanding is that some aspects of the envisioned lesson were, in fact, very similar in both CM and Ms. Q's talk, and this similarity obscured the differences in the framing of mathematical activity. To further explore this possibility, we now move to analyzing the meta-rules of students' engagement, gleaned from CM and Ms. Q's talk, where they envisioned both the mathematical and social aspects of students' activity.

Frames of the envisioned lesson When looking at how CM and Ms. Q described students' envisioned activity, one finds striking similarities. For instance, both CM and Ms. Q envisioned students to be working in groups while the teacher walks around and "pushes them" in their thinking. Both of them also talked about the lesson ending with a rich or interesting "conversation" or "discussion." Thus, as far as the social meta-rules in the planned lesson, CM and Ms. Q seemed to be very much aligned. This was corroborated by Ms. Q's statements in the previously described introductory interview, where she spoke enthusiastically about students working in groups, taking on mathematical authority, and discussing problems with each other. The social meta-rules of the exploration frame were thus endorsed both by Ms. Q and by CM.

Yet, when one examines the mathematical meta-rules of the envisioned lesson, CM's and Ms. Q's seemingly well-aligned visions begin to diverge. While CM talked mostly about students "making sense," exploring mathematical principles (such as the reverse distributive property), and preforming acts on mathematical objects (such as "decomposing a polynomial function"), Ms. Q mostly talked about students "solving" or "getting an answer." If we go back to the excerpt in the "Introduction," a close look at Ms. Q's words reveals this emphasis on students' actions, as can be seen in the words in bold:

> so I think that when I do the, uh, when I do the warm up, I should definitely bring out the fact, ok, so the x – 3, ok, the answer is, you know, getting to the, **they solve it** the 3, "so what would happen **if I put 3 back** in this equation?" I think I need to make sure I bring that out so I get **somebody trying to do exactly that**. "Well, **let me put in** 1, **let me put in** this," cause without specifically bringing that up … that that makes it zero, they might not try that. [Ms. Q. 1.12, 1.14]

Implied under these seemingly modest differences in choice of verbs are rather different frames for students' engagement. Whereas according to CM—the activity in the mathematical classroom was about exploring objects and connecting between their attributes—Ms. Q's verbs of "solving" and "finding" indicate there is a specific set of actions she hopes her students to perform and deems herself responsible for bringing about ("without specifically bringing that up …, they might not try that"). More importantly, this focus on "solving" moves away from the product of the activity (which is a known-in-advance "solution") to the sequence of prescribed actions that should be performed for "solving" a sequence that was, as we will show in the analysis of the lesson, quite clear for Ms. Q. In contrast, for CM, the sequence of actions he expected the students to perform was less clear. He imagined the students to be "exploring," "finding by trial and error," and "making sense"—none of which has a fixed set of sequenced actions.

And yet, as was the case in the discussion of the remainder theorem, neither Ms. Q nor CM expressed any disagreement regarding what the lesson would look like. It is therefore not surprising that CM left the meeting feeling the lesson was reasonably well-planned and that Ms. Q was genuinely intending to implement it as they had collectively planned. As we shall see next, however, the enacted lesson diverged from these assumptions and expectations.

5.2 The lesson

Evidence of the differing way in which Ms. Q was about to enact the planned lesson could be seen right from her opening sentence, where she introduced what it was that the class was going to do that day:

> Ms. Q: Today we're gonna do more polynomial division and we're also gonna talk about how to find the roots of a polynomial, so let's work on the warm up right now if you haven't already done so [Lesson 02/19/13, 58]

Notice both of the subjects, polynomial division and roots of polynomials, were introduced as things *to do*. This emphasis on doing was echoed also in her rephrasing of the task given as a warm up. Instead of "Find the roots of the function by factoring," which was the phrasing used in the planning session, it now simply read "Solve $x^2 - 7x + 12 = 0$."

Ms. Q's emphasis on "doing" continued through her interaction with the students while discussing the warm-up task 3 min after the above introduction:

> Excerpt 3.
> [3.01] Ms. Q:First thing, what kind of equation is this?
> [3.02] Shana:Quad.
> …
> [3.05] Ms. Q:Ok, quadratic. And how do we <u>solve</u> quadratics? That's the direction on the warm up, so how do we <u>solve</u> a quadratic equation? …. I've seen two different ways on your papers.
> [3.06] Von: Um, we be doin' the quadratic thingy?
> [3.07] Ms. Q:Okay, what's a quadratic <u>thingy</u>?
> [3.08] Von:Quadratic formula.

In the above interaction, not only did Ms. Q use the "how do we solve?" question [3.05], she also used the plural first person "we" to signal that there is a well-established and agreed upon mathematical meta-rule that everyone in the room is expected to follow for "solving" quadratics. Moreover, even though she reported having seen "two different ways" on students' papers, she made no attempt to connect between those two solutions.

Two minutes later, continuing with her attempts to move from the warm-up problem to the main problem of the lesson, Ms. Q assembled the class and queried students about their solution to the equation. Taking Von's suggestion to use the quadratic formula as a starting point, she wrote the formula on the board, plugging in the numbers he and other students stated and producing $x_1 = 4$ and $x_2 = 3$. Then she went on trying to establish a connection between the solutions produced by applying the quadratic formula and factors of the function:

Excerpt 4.

[4.01] Ms. Q:So, 4 and 3 are the roots of this. Remember, we called them roots, we called them solutions, what's the other word we used? Anybody remember another word? Roots, solutions? *((Teacher hints "Z…" and students try "Zero Product Property." Finally, the teacher provides the answer: "zero"))*

[4.02] Ms. Q:Alright, so, what's the significance—cause we talked about it briefly, do you remember?—if I would graph, if I would graph this, if I graph this quadratic equation, what's my graph looking like?

[4.03] Kassie:A parabola. *((Ms. Q continues querying Kassie until she provides the answer she was looking for: that 3 and 4 are points on the x axis.))*

[4.07] Ms. Q:…. So, Von, what's the significance of these- of the 3 and of the 4?

[4.08] Von: They're points on the x axis.

[4.09] Ms. Q:Thank you. Okay? Exactly.

We found in this excerpt that the main meta-rule governing it was a quest to "solve" the quadratic equation. This may raise certain wonderings in the critical reader, since Ms. Q was explicitly searching for the "significance" of the "solutions" and their relation to the points on the *x*-axis [4.07]. Yet a closer examination reveals that a very specific set of mathematical meta-rules was leading students to look for a product named "solutions" or "roots" by applying the quadratic formula to the algebraic expression and then locating them as points on the *x*-axis of the graphed function. The point of locating the roots on the graph was that students would be reminded about the option to graph a function to locate its roots (a procedure envisioned by Ms. Q to be used for factoring the cubic polynomial).

The confusion that the quadratic formula solution brought could be seen much later in the period. In exchanges between Ms. Q and the students as she walked around the classroom attempting to assist them with solving the problem, we saw students getting stuck with trying to use the quadratic formula to find the roots of a cubic polynomial. In her frustration, Ms. Q tried to navigate the students to use their graphing calculators by reminding them of the "significance" of the roots of the quadratic equation, leading them, mostly in a directive fashion, to realize that the roots are points on the *x*-axis. From her leading questions, we draw that she attempted to lead students in the following reasoning:

1. From the warm-up exercise, students should see that for $f(x) = x^2 - 7x + 12$, $x_1 = 3$ and $x_2 = 4$ are the roots of the function, meaning that those are the values for x at which a graph of the function intersects the x-axis.
2. Students should also realize that $f(x) = 6x^3 - 19x^2 - 9x + 36$ is a function that has a graph and that this function has 3 roots, because 3 is the largest exponent that appears in it.
3. Students should then seek to graph the function on the calculator and, from that, determine at least one of the roots (probably $x_1 = 3$, since it is the only whole root).
4. Students should realize the root ($x_1 = 3$) means the function $f(x)$ has a factor $(x - 3)$.
5. Having realized there is a factor, students should think of dividing the function $f(x)$ by that factor to get a quadratic function, which would then be factored by means of the now-familiar quadratic factorization procedure.

Most of these mathematical meta-rules can be derived from Ms. Q's statement at the end of a long series of Initiation, Response, and Evaluation (IRE) interactions with the students, in which she responded to a student's expressed frustration. Again, we highlight the "doing" in her talk:

Excerpt 5.
[5.01] Kassie:I can't do this; this is very difficult.
[5.02] Ms. Q:It is difficult, but I just- Von just **told us what to do**. 3-0 is a root. Leon told us then that x minus 3 must be a factor. So, if you **take** that original polynomial and **you divide it** by x minus 3, **you should get** what's left and that **will help us find** the rest of the roots. Let's try that step and then **maybe you'll see what to do next**. Let's **take** the original polynomial, **divide** it by [what we kn::ow is one of the factors-
[5.03] Von:[Ms. Q, I think it'll be better when you teach it tomorrow to us again.

There are several complications with the path of students' reasoning that Ms. Q seemed to be envisioning, mostly having to do with assumptions about students' prior knowledge that may have been inaccurate. For instance, already at the warm-up stage, many students did not connect "roots" or "solutions" of a quadratic and the points at which a graph intersects the *x*-axis, and even less connection was drawn between "roots" (signified for the students by x_1, x_2) and "factors." Moreover, students had difficulty even connecting between the algebraic expressions $f(x) = 6x^3 - 19x^2 - 9x + 36$ and a graph. None of them thought to graph the function before it was explicitly encouraged by the teacher. Most curious is that, despite the students' expressed difficulties in the warm-up, Ms. Q persisted with an expectation that they would "find out" the solution.

Through CM's "exploration" frame, the focus of the lesson would have been on connecting between the various parts of the polynomial (roots and factors), mostly through the representation of a cubic as a product of binomial and quadratic function (or more generally, a product of binomials). This representation was supposed to be highlighted through at least one of the possible solutions to the first "warm up" quadratic equation, namely by factorization (not by applying the quadratic formula). Yet, within the "doing" frame, these connections were not drawn, and the ways in which one routine implied the next remained hidden. Most importantly, the link between 3 being a root of the function and $(x - 3)$ being its factor remained unclear, being stated as simply a connection between what Von "told us to do" and the fact that Leon stated "*x* minus 3 must be a factor" [5.02]. Yet, it was precisely this link that was one purpose of presenting students with the remainder theorem.

6 Discussion

Our aim in this analysis has been to unearth the differences in the frames of mathematical learning between Ms. Q, the teacher, and CM, the PD facilitator, such that the ultimate failure of the planned lesson could be explained in more sophisticated ways than those which have often been offered for the failure of efforts to enact explorative instruction. Our claim is that there is a clear relation between (a) the differences between Ms. Q's and CM's framing of the remainder theorem and its pedagogical purpose; (b) the differences between Ms. Q's attention to students' "doing" and "solving" vs. CM's emphasis on students' "exploring" polynomials; and (c) the way in which Ms. Q enacted the lesson. All of these strengthen our conjecture that the collection of meta-rules implied by CM and Ms. Q's conversation in the planning session and those seen in Ms. Q's instructional talk in the lesson were far more than a haphazard collection of meta-rules. Rather, there is a relation of mutual reinforcement between meta-rules of mathematical sense making (e.g., the significance of the remainder theorem) and meta-rules of mathematics instruction (e.g., how students are supposed to approach the cubic factorization problem) that coheres around different—and, in this case, misaligned—frames.

The goal of CM and more generally the "Accountable Talk" PD was to *disrupt* (Ma et al., 2014) the frame of "doing," most often seen in typical forms of instruction, by applying to it meta-rules taken from the exploration frame. This was done by giving students an unfamiliar task, refraining from providing step-by-step directions, and giving them extended time to work on the problem. Yet this disruption was not as effective as hoped for since it was, through the frame of "doing," appropriated only for its social meta-rules and not its mathematical ones. Without the mathematical meta-rules that imply there is a mathematical object to be explored, the whole activity of exploration loses its focus.

The misalignment in frames, surfaced in this study, was not a result of lack of effort on the side of both parties to collaborate. On the contrary, Ms. Q was devoting time from her workday (with CM) and evenings (for online modules) to participate in PD aimed at increasing and improving the mathematical talk in her classrooms—and, based on her recount, perceived that she was experiencing some success as a result of the efforts. CM was trying to work within the current curriculum and accountability expectations that Ms. Q was facing, so as not to burden her or her colleagues unnecessarily. And, we saw both CM and Ms. Q making every effort, at least during the planning session, to communicate well with each other. Rather, our analysis points to the hidden nature of such differences in framings, differences which may sometime require a careful, post hoc analysis to be unearthed.

As we indicated earlier, external observers of the interaction between CM and Ms. Q may wonder how it is that the differences in frames were not surfaced during the planning session itself. How is it, might the critical reader ask, that CM did not see he was in some ways forcing a vision of the lesson onto Ms. Q's lesson plan, a vision that she, apparently, did not share? Yet, it is precisely this unawareness that is the hallmark of differences in framing. Merely criticizing the PD facilitators' actions would not aid in refining future PD efforts; it would simply put the blame on the PD facilitator instead of the more common placement on teachers. Our goal in pointing to the misalignment of frames is to offer an alternative to this deficit-oriented explanation (whether it is pointed at teachers or at PD facilitators) by providing a conceptual tool that looks at what *are* the meta-rules governing the discourse of each of the parties, rather than what they are not.

6.1 Limitations

Before concluding, we wish to acknowledge a few limitations of our study. First, while there are reasons to believe when Ms. Q left the meeting, she was thinking positively about the lesson plan, it is important to note that we did not have access to her thoughts after the planning session. Consequently, CM's perspective (and voice) is likely over-represented in our account. For the reader, this may seem to be exacerbated by the planning session transcripts we included, in which CM did more of the talking. However, this is a consequence of the particular excerpts that we chose to illustrate our arguments, and not representative of the entire session, in which, based on a word count of the transcript, Ms. Q actually spoke approximately 36% more than CM.

Another limitation of this study lies in the focus of the analysis on relatively small excerpts of data. This limitation comes hand in hand with the high resolution of the discourse analytical tools we employed, which constrain the amount of data that can be looked at with such proximity. There very well could be other factors that were involved in the miscommunications between CM and Ms. Q, such as school climate, district policy, the personal histories of the participants, and more. However, these were beyond the focus of the analytical lens we employed in this study.

7 Conclusion

Recent years have seen an increase in mathematics educators searching for understanding the ways teachers (in-service and pre-service) learn. Many researchers have shifted from a cognitive view of teachers' knowledge, which concentrated on teachers' mathematical knowledge, pedagogical content knowledge, and beliefs, to a socio-cultural, situated view of this learning (Borko, 2004; Horn & Kane, 2010). We believe the conceptual tools of framing and meta-rules of discourse offer a step forward in this direction. They provide a set of tools for close-up analysis of teachers' discourse, both outside and inside the classroom, which is not easy to achieve using gross labels such as "reform" or "ambitious" instruction or even teaching for "conceptual understanding."

On a more practical level, we hope that the case studied here would assist teacher educators in seeing that the activity of training teachers to provide opportunities for explorative participation in classrooms is not a simple introduction of forms of discourse and practice where there were previously none. It involves a shift of frames, from frames that have been deeply engrained in school history and teachers' experiences to "explorative" frames that are much newer and less known to most teachers.

Even within the socio-cultural view, it is often easy to fall into the trap of viewing teachers as "ritual" or peripheral to the academic mathematical discourse as embodied by mathematicians and mathematics teacher educators. In the same way, it is easy to dismiss PD efforts as lacking "connection to the field" and being ineffective. Thus, framing is helpful for looking at teachers and mathematics educators as negotiating different discourses. Instead of seeing teachers as peripheral in one, unitary type of discourse (the "mathematical discourse," as dictated by mathematicians or mathematics educators), we see them as appropriating certain frames taken from the "mathematicians' discourse" to their own "school discourse." This appropriation is a complex and delicate task that most likely needs both sides (teachers and mathematics educators) to flexibly apply their old meta-rules to new contexts. Regarding the research-practice divide discussed at the opening of this paper, we believe that the examination of frames and their (mis)alignment can be particularly effective for understanding this divide. As we ourselves experienced through multiple iterations of analyzing our data, the divide between research and practice may often be hidden in the most basic assumptions and forms of talk that we use as researchers, teacher educators, and teachers. Unearthing the misalignment between frames is an important, perhaps even indispensable, step towards bridging this divide.

Appendix

Transcript conventions used in the examples were adapted from those developed by Gail Jefferson (see Atkinson & Heritage, 1984/2006):

[X.Y] Turns at talk numbered for excerpt *X* and turn *Y*

((text)) Descriptions of actions or paraphrases

(text) Transcriber uncertainty

<u>Text</u> Emphatic talk

te::ext Elongated speech

(Zs) Pauses in speech, where *Z* is the duration of the pause in seconds

[text Onsetofoverlappingtalk

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