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We all as a family are graduating tonight: a case for mathematical knowledge for parental involvement

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Abstract The researchers in this study investigated the impact of mathematics-focused parental involvement on Kindergarten to Grade 8 children and parents as well as factors prompting that impact. Qualitative analysis consisting of parent, child, and teacher interviews and 3-year quantitative testing showed significant improvements in students' mathematics understanding and achievement. Moreover parents and teachers improved on measures of content knowledge. We hypothesize that improved parent content and pedagogical knowledge and improved parent-child interactions around mathematics stimulate children to learn at school. Furthermore, we found aspects of parent knowledge and dispositions gained to be analogous to teacher MKT; we termed these aspects *Mathematical Knowledge for Parental Involvement*. Such aspects include content knowledge, valuing students' own strategies, and listening to students' explanations and may boost student achievement in mathematics.

Keywords Mathematical knowledge for teaching · Mathematical knowledge for parenting · Parental involvement · Parent involvement · Informal learning · Student achievement · Adult education

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1 Introduction

Parental involvement (PI) has been found to impact student achievement; however, parents are often not accessed as resources for helping children learn mathematics in school (Jackson & Remillard, 2005). Given that teachers' mathematical knowledge for teaching (MKT) appears to impact student achievement, we wondered whether parental knowledge might also play a role (Hill, Rowan, & Ball, 2005). In this report, we describe a study of a PI program designed to enrich schools mathematically. By parents, we imply guardians or mentors internal to children's lives. We asked,

1. *Does parental involvement in mathematics improve children's understanding and achievement in mathematics?*
2. *How might this improvement occur? In particular, do parents develop mathematical knowledge for teaching? If so, in what ways?*

Although this study researched parents, teachers, and children, in this paper we focus on *parents*.

With initial funding from the National Science Foundation (NSF)[1] during the period from 1999 to 2003, the Math and Parent Partners (MAPPS) program was developed to engage K-8 parents in collaboratively exploring concepts and pedagogies behind the mathematics that their children are learning in school. The target population of MAPPS from the outset has been parents, viewed as learners *and* teachers, initially with children in economically disadvantaged schools (Civil & Bernier, 2004; Remillard & Jackson, 2006). Qualitative evidence has suggested improved student performance in mathematics and participant interactions (Henderson & Mapp, 2002). Further outcomes have been improved teacher and parent confidence with mathematics, parent enjoyment of mathematics, and parents encouraging high-level performance (Civil 2000, 2001, 2002; Civil, Andrade, & Anhalt, 2000; Civil, Guevara, & Allexsaht-Snider, 2002). This study built on the 1999–2003 project by focusing quantitatively on student achievement and inquiring about how that improvement might come about.

2 Literature review

In order to frame literature about PI in mathematics, we examine how PI has been defined. Epstein put forth six categories of PI including parenting, communicating, volunteering, learning at home (helping with homework), community collaboration, and decision making that has been widely adopted by many researchers (Epstein, 1994; Sanders, 2015; Willems & Gonzalez-DeHass, 2012). However, it may be problematic to apply the framework to all families. For example, studies show that Asian parents are less likely to participate in school-based activities but are highly involved in home-based activities (Siu & Feldman, 1996). For another instance, Winter, Salway, Yee and Hughes (2004) and de Abreu (1995) found that there is a fundamental difference between the mathematics students experience in school and the mathematics they experience outside of school. Winter et al. concluded that games and

[1] NSF is an independent federal agency created by the United States (US) Congress in 1950 to promote research and education in non-medical science and engineering areas (NSF, n.d.).

authentic family activities such as mathematics involving shopping, and cooking, formed the two main kinds of mathematical activity in the children's out-of-school lives. Other parents hold an unfriendly view towards schools that may impede their school-based PI activities. For example, immigrant parents such as those who left countries of the former Soviet Union for Germany have been found to be dissatisfied with the receiving countries' mathematics education (Hawighorst, 2005; Li, 2006).

Never-the-less, studies have shown that parent involvement is strongly linked with children's academic performance (Epstein, 1994; Kellaghan, Sloane, Alvarez, & Bloom, 1993). Henderson and Mapp (2002) said, "The evidence is consistent, positive and convincing: families have a major influence on their children's achievement" (p. 7). Contrary to this evidence, some schools give increased PI a token effort. Token efforts fall short in that, "there is serious need to conceptualize, as well as enact, practices that value what parents bring to reforms in terms of their own mathematical understandings and histories" (Remillard & Jackson, 2006, p. 232).

Extensive reviews of evaluations in the United States comparing the effectiveness of programs with and without parent components (White, Taylor, & Moss, 1992), and programs geared toward promoting stronger parent involvement (Mattingly, Prislin, McKenzie, Rodriguez, & Kayzar, 2002), have indicated that efforts to increase parents' involvement have had mixed results. On one hand, studies indicate that parents' aspirations of children's education and communication between parents and children are found to be effective predictors of children's educational outcomes (Aldous, 2006; Robinson & Harris, 2014). On the other hand, other types of PI such as helping with homework, are found to be negatively linked to children's educational outcomes (Hampden-Thompson, Guzman, & Lippman, 2013; Robinson & Harris, 2014). This lack of consistent evidence for the benefits of PI in mathematics gives rise to the rationale for this study. We agree with Peressini (1996) who advocated making efforts to understand PI in mathematics education as opposed to making blanket calls for further PI.

3 Theoretical framework: mathematical knowledge for teaching

The framework of MKT relates to the knowledge and habits of mind needed to teach mathematics well (Ball, Thames, & Phelps, 2008). MKT is loosely analogous to the Knowledge Quartet that "provides a repertoire of ideal types that provide a heuristic to guide attention to, and analysis of, mathematical knowledge-in-use within teaching" (Ruthven, 2011, p. 85). More specifically, Ball et al. (2008) categorize MKT with six constructs; we focused on four of the more commonly referenced constructs in this study. Common content knowledge (CCK) involves the ground-level knowledge of the mathematics that people in other disciplines would know—how to take 43×26 for example. CCK would also involve knowing if an explanation is accurate and sufficient (Hill, Schilling, & Ball, 2004). Specialized content knowledge (SCK) on the other hand is unique for the work of teaching mathematics, such as for recognizing patterns of student errors or constructing accurate, understandable representations (Ball et al., 2008). Knowing several ways to take 43×26 would be an example. Knowledge of content and students (KCS) indicates a teacher's knowledge about how students think in mathematical contexts; this is knowledge related to research on student thinking in mathematics education. For example, the teacher would know the most common error students make when solving 43×26. Knowledge of content and teaching (KCT) indicates a teacher's knowledge of the most operative examples and sequences for arranging content. For example, the teacher would know what representations would be best to teach 43 × 26. Although the literature on MKT is about classroom teachers, we hypothesized that MKT in some form may be developed with parents.

The second research focus for this study was to quantitatively ascertain *how* a math-focused PI program might prompt student understanding and achievement. Jensen (2009) emphasized the parental role as teachers in that primary caregiving such as bathing or serving snacks often turns into informal instructional time for parents with their children. This added instructional time may then contribute to student achievement. Further, Pan, Gauvain, Liu, and Cheng (2006) stated, “Parental involvement helps children learn mathematics concepts and how to carry out mathematics operations. However, it is the nature of this involvement. . . that matters” (p. 32). Both Jensen (2009) and Pan et al. (2006) point to the model that parents often in some way teach their children. Furthermore, the first author conducted an action research project in which she taught her 4th-grade child using tasks from *Investigations in Number, Data, and Space (Investigations)* (Akers, Tierney, Evans, & Murray, 1998). She, as a parent, developed aspects of MKT as she played mathematical games such as *Close to 100* with her child. Several of these tasks subsequently were incorporated as MAPPS childcare tasks. Finally, MKT has been linked to student achievement (Hill et al., 2005), leading us to further inquire, *In what ways do parents develop MKT*? We framed our quantitative data analysis using this construct.

From its founding, MAPPS has been grounded in the socio cultural theory of Funds of Knowledge/Communities of learners (Allexsaht-Snider & Bernier, 2003; Moll, Amanti, Neff, & González, 1992; Rogoff, 1994). Although differing from theories of knowledge construction, sociocultural theory also informed and motivated our study and data analysis. Sociocultural theory involves the development of knowledge that takes place as members of a community interact with one another (Rogoff, 1994). Cobb, Wood, and Yackel (1990) asserted that cultural development appears on both a social plane and a psychological plane. Confrey likewise wrote, “We experience ourselves both as biologically developing beings and as productive members of a collective enterprise…It seems obvious that we must consider both when we study our view of humanity and its development” (1995, p. 46). Thus, we see the two approaches as complementary. Our choice to analyze data about MAPPS through an unrelated lens allowed theory triangulation of the initial studies about the program (Denzin, 1978).

4 Methods and methodology

4.1 Design-based research

Conceptualization of our study relied on underlying assumptions that 1) families can bring a wealth of knowledge to a learning environment and individuals’ experiences and strategies should be valued, 2) participants can learn intrinsically when given alternative and multiple approaches to understand concepts, and 3) participants’ learning evolves through social interaction, in particular through the MAPPS community of learners. Specifically, our study followed the design-based research model characterized as situating in a real context, focusing on a design and testing of a significant intervention, using mixed methods, enacting multiple iterations, partnering between researchers and practitioners, and having practical impact on practice (Barab & Squire, 2004; McKinney & Reeves, 2013).

For the first research question, "Does parental involvement in MAPPS improve student understanding and achievement," we chose to employ a quasi-experimental design because parents and teachers in this study self-selected to the program (Shadish, Cook, & Campbell, 2002; Vogt, 2007). For the second research question about how such improvement might occur, we employed a multi-tiered teacher development experiment (TDE). The teacher development experiment was founded on the principles of design research (Gravemeijer, Cobb, Bowers, & Whitenack, 2000; Lesh & Kelly, 2000; Presmeg & Barrett, 2003). The TDE takes a global approach to studying teachers' development as well as observing and analyzing the learning of the teachers' students. We extended the methodology to study parents' development and the learning of their children. The researchers coordinated the analysis of the levels of the experiment by engaging in an iterative model of reflection and interaction following each year.

4.2 Context for the study

The study was conducted on a high-poverty, suburban MAPPS program in the Southeastern US during 2008–2011 (Knapp, Jefferson, & Landers, 2013). The core MAPPS activity on which data for this study were collected and analyzed was the 8-week, 2 h per week MAPPS *Math for Parents Mini-courses*. The five-Mini-course curriculum was developed in the initial NSF MAPPS grant and focused on number, geometry, algebra, and data. Mini-courses engaged parents and teachers in doing mathematics and considering pedagogy by using hands-on materials, working in small groups to solve problems, and presenting their solutions.

4.3 Participants

A majority of attendees were single parents, and most attended the Mini-courses with one to three children. Most of the parents had graduated from high school with some technical training, and they typically held low-income jobs. Attendees were approximately 40% African-American, 40% Caucasian, and 20% Hispanic. Schools were selected for participation based on their Title I[2] status, willingness of the principals, and superintendent advisement. All families, teachers, and paraprofessionals from the schools were invited to participate, regardless of their past participation in MAPPS. While their parents were in class, children aged preschool-Grade 3 were invited to play educational games from the MAPPS curriculum or from the *Investigations* curriculum. Children in 4th -8th grade attended the Mini-course sessions alongside their parents. Eight separate 8-week Mini-courses were offered during the 3-year study. Mini-courses were managed through a local university, and instructors were practicing teachers that were taking graduate courses in mathematics education; some of these teachers were also parents themselves. Data analyses focused on a group of treatment parents, teachers, and children who attended regularly, meaning at least half of an 8-week Mini-course: 115 children, 59 parents, and 33 teachers (5 served as facilitators) from primarily four Title I schools. Notably, nearly double that number attended at least once. A matched-comparison group of children ($n = 89$) were chosen by availability from the same four Title I schools' after-

[2] Title I is a provision of the Elementary and Secondary Education Act that passed in 1965. The US Department of Education provides financial assistance to local educational agencies and schools with high numbers or high percentages of children from low-income families (US Department of Education, 2015).

school programs or during the school day. Students from the same communities attended the local Title I schools, leading to a comparative sample.

4.4 Data collection and analysis

Data collected for the study included pre/post surveys and pre/post tests of MKT (Hill et al., 2004) given to parents and teachers before and after each Mini-course. Ninety-five interviews of willing parents, teachers, and children were collected as well; most interviews lasted approximately 15 minutes and were audiotaped and transcribed. Approximately 45 of the interviews were of parents, 34 were of teachers, and 16 were of children. Finally, children's mathematics test scores from the state Criterion Reference Competency Test (CRCT) were collected for years 2008–2011. Following the design-based research model, our analysis occurred on three levels: the micro, or individual Mini-course level, the yearly level, and the cumulative, or macro, level. For quantitative analysis of CRCT scores, individual students were the unit of analysis. CRCT scores in mathematics were compared using paired samples t-tests.

For the research focus on parental development of MKT, we utilized domain-specific multiple-choice Content Knowledge for Teaching Mathematics (CKT-M) measures as pre- and posttests corresponding to the content for each Mini-course (Hill et al., 2004). Reliability on the CKT-M measures was previously established for in-service elementary and middle school teachers (Hill, 2007). We conducted paired samples t-tests on the scaled scores from the CKT-M tests generated through Item Response Theory (IRT) to assess for improved domain-specific knowledge.

The 95 interviews and pre/post surveys were qualitatively and quantitatively analyzed for development of MKT, evidence of student understanding and achievement, and other factors seemingly related to student understanding and achievement. Interview questions were such as these: 1) Have you learned anything about mathematics that you did not know before? Explain. 2) Have you learned anything in MAPPS that helped you help your child or students with math? Explain. The MKT framework was used to analyze data quantitatively for instances of *developing* MKT using CCK, SCK, KCT, and KCS as codes. Other factors arising from the PI literature that might prompt student achievement were analyzed through open coding. Open coding was also employed to ascertain how the MKT domains were developing. Several of the other 59 codes arising from the data were these: *valuing math, school math vs applied math, nature of math, confidence, family interactions, value of program, school math versus parent math, desire, authority, sibling math, knowledge of self, life constraints, parent-child interaction, learning community,* and *enjoyment* (See Table 1 for primary and secondary codes).

The first and second authors, with a Ph.D. and Master's in mathematics education respectively, as well as a graduate student in mathematics education coded the qualitative data so that each interview was coded (MKT and open codes) by at least two people (see examples of this first level of analysis in Results). Coders then engaged in consensus-building by comparing coding results and resolving discrepancies in coding for each interview (Creswell, 2012). Each year (second level of analysis), the list of open codes was revised that were then clustered and compressed. Primary and secondary codes were separately identified for parents, teachers, and children. At the end of each year and at the conclusion of the study (third level of analysis) we conducted cross-case analysis (Coffey & Atkinson, 1996).

Table 1 Aggregate results from 95 interviews

Code	Freq
Primary:	
Improved parent-child interaction	103
Knowledge of content and teaching	87
Content knowledge CCK(32) SCK(29) other (26)	87
Enjoyment of/Valuing MAPPS	75
Learning community	43
Student learning/achievement	42
Secondary:	
Confidence/Motivation	31
Continuing education	23
Broader impact of MAPPS	18

5 Results

5.1 Research question #1: did student understanding and achievement of mathematics improve?

Although students are not the primary focus of this paper, we backdrop our parent results by providing evidence of improved student understanding and achievement presented in the form of CRCT scores (Knapp et al., 2013). The primary quantitative result was that MAPPS students taking at least one Mini-course over the 3 years improved significantly on the mathematics portion of the CRCT. A paired samples *t*-test was used ($n = 39$, $p < .001$, $d = 0.766$). Comparison students *did not* improve significantly ($n = 36$, $p = .331$) (See Table 2.)

Further, coded data reported by all three groups of participants–parents, teachers, and children–revealed student understanding and achievement as a top code (See Table 1). Lastly, qualitative data revealed that MAPPS improved classroom learning, and ultimately student achievement over time. Our next research question concerned how these improvements might have occurred.

Table 2 Three year mean CRCT changes 2008–2011

	2008	2011	Difference in CRCT scores
Comparison ($n = 36$)	817.4 sd 31.7	823.1 sd 29.9	+5.7 sd 33.2
Treatment ($n = 39$)	807.4 sd 22.5	825.5 sd 25.34	+18.1 sd 19.2

The 95% confidence interval for mean improvement in CRCT scores for the treatment group was [−24.35, −11.9]. Both 2008 and 2011 data sets were checked for normality using the Anderson-Darling test ($p = .855$, $p = .128$ respectively)

5.2 Research question #2: how might this improvement occur? In particular, do parents develop mathematical knowledge for teaching? If so, in what ways?

5.2.1 Baseline

Consistent with the findings of Jackson and Remillard (2005), many parents attending MAPPS desired to assist their children with learning mathematics on some level. Parents reported considerable consternation with the homework process. For example, one mother reported incorrectly helping in the following interview.

> Int: OK. So, do you help him with his math homework sometimes?
> Parent A: Yes. But lately he doesn't want me to help him. Remember a couple of weeks ago I was telling you about the tenths and ten?
> Int: Yes
> Parent A: And I did it for him, but I was doing the tenths instead of ten. And we got all of them wrong.

The tension between a desire to help but lacking the ability to help was further evidenced by the following interview with a father who said, "You know it's going to be a day when she comes home, and I really ain't going to know what to say or do." Parents such as this father reported having low content knowledge. Parents also repeatedly reported feelings of strong dislike and avoidance of mathematics, yet they attended MAPPS in desiring more for their children. Another motivator for parents to attend MAPPS was to strengthen their own content knowledge. Aside from coding about MKT, we found from open coding interviews that parents valued mathematics, were aware of its usefulness, and had a self-awareness of their own knowledge, strategies, and limitations. One mother stated, "You want to help your children and help yourself…why not?" She continued, "In 5 years, you might be in a university for real".

5.2.2 Qualitative results following the intervention

As we sought to measure impact from the MAPPS intervention, we looked at quantitative coding tallies (See Table 1). Top codes related to improved parent-child interaction as well as MKT domains of content knowledge and knowledge of content and teaching (KCT). We then looked at the related data with a qualitative lens to analyze how these constructs may have prompted student achievement.

5.2.3 Parent-child interaction

The top theme that presented from interview data involved parent-child interactions around mathematics, and it confirms and extends prior qualitative research that MAPPS strengthens family relationships (Civil et al., 2002). To arrive at this theme, the following open codes were compressed into one "Parent-child interaction" code: *Parent-child interaction, improved homework interaction, family interaction, child helping parent, and games*. We provide several examples from the data exemplifying this theme.

Whereas some parents previously had expected their children to work on mathematics homework in isolation, they began assisting their children and engaging them in mathematical thought at home, as seen in a child interview:

> Int: Are they [your parents] better at explaining now that they've come to MAPPS?
> Child: Yes
> Int: How?
> Child: They tell about the shapes and the stuff that I do at school. They compare it to here.

Instead of shying away from helping their children, parents began enjoying the challenge and felt confident enough in their mathematical skills to figure out mathematics problems and tasks. Parent D said, "I feel much more confident working with Sarah (pseudonym), because even though it's new and it's a different way of presenting the material, this class is helping me to learn how to help her." Parents' focus began to shift from their children *completing* mathematics homework to *understanding* mathematics homework. For some, this time of homework interaction evolved into "family time." Moreover, parents and children shared MAPPS games, activities, and technology resources at home to reinforce the concepts and skills learned in the Mini-courses. Parents were provided cut-out manipulatives such as base-ten blocks, pattern blocks, and tangrams for this purpose. Parent D additionally said,

> …those tangrams? I LOVE those. Those were our [her and her daughter] favorites. Trying to put the pictures. Figure out how they go. We had the best time with those. We played with those all the time, even though it wasn't homework, you know?"

Another parent explained that MAPPS helped her *listen* to her child and thereby improve homework time. She said, "It showed me to listen at her as to how she's trying to tell me, and then I can see whether or not she's getting to the right answer or not, or going about it the right way." For this parent, instead of trying to explain a concept using the parent's own strategy, which may have been different from how the child was thinking about it and independent of how it was taught at school, the parent listened to the child's strategy.

A final aspect of improved parent-child interaction occurred on-site during the MAPPS Mini-courses. Numerous parents expressed that the MAPPS environment provided enjoyable "family time." One teacher observed about children, "…they really enjoyed getting it [the problem] before their family member did and impress them with their knowledge and all the other parents. They enjoyed getting up and showing how they found the answer." At times, parents were surprised to see their "shy" children boldly sharing knowledge with the group. Families engaged in playful competition in seeing who could get the problems correct. One mother exemplified the family aspect of MAPPS by stating, "We all as a family are graduating tonight".

5.2.4 Learning community

Parents, teachers, and children repeatedly reported assisted one another, leading to "learning community" as one of our primary codes (See Table 1). At times, *children helped parents* figure out problems, which became a source of pride and motivation for the children. Bonding between parents and teachers formed because they learned to know and value one another's roles in children's lives instead of in a negatively-connoted position of power, with teachers telling parents what to do or not to do in regard to their children. Teachers and parents enjoyed learning on the same page in which both parties were invested in helping children succeed mathematically. Moreover, parents' and teachers' interaction with the 4th-8th grade children during Mini-courses provided them an on-site clinical experience for facilitating children's knowledge construction. The enhanced relationships with regard to mathematics resulting from MAPPS sessions appeared to create spaces at home for furthering the learning community involving parents and children.

Hence, the MAPPS environment forged a Parent-Teacher-Child triangle of knowledge and respect (See Fig. 1). The arrows in the figure represent interactions within the MAPPS learning community. The MAPPS instructor is in the foreground, impacting and facilitating the learning community. Parents, Teachers, and Children in the inner triangle interacted with children, teachers, and other parents. Knowledge was impacted, constructed, and shaped by interactions among participants during MAPPS sessions and during follow-up at home (family homework and games). In the MAPPS learning community, the values, experiences, and strategies of parents and children played a part as their knowledge developed through what Rogoff (1994) refers to as a *transformation of participation*.

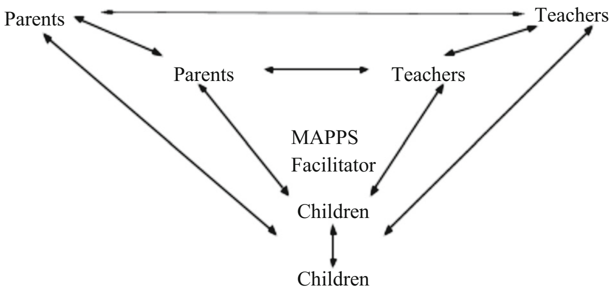
5.3 Research question #2: how might this improvement occur? In particular, do parents develop mathematical knowledge for teaching? If so, in what ways?

In this section, we discuss the development of "parental" aspects of MKT based on interview and test data. We focus on three aspects of MKT based on their emergence in the data, CCK, SCK, KCT, following the first, or coding, level of Mini-course analysis.

5.3.1 Common content knowledge

Parents' development occurred more in the area of CCK than SCK. During the interviews, parents gave numerous examples of new content (CCK) they had learned such as turning percents into fractions, calculating the volume of a cylinder, and knowing that a nonzero number to the zero power is one. One activity required participants to form collections of square inch color tiles based on percentages (Knapp et al., 2013). Families were given the task of creating a model that was 10% blue, 15% green, 50% red, and 25% yellow (Griffin, 2007). When a child and his parents tried create the model based on one percent, they ran out of tiles. They modified their conjecture to each tile representing two percent. The child then asserted that he could multiply by two to get five blue tiles and 25 red tiles. When the parent questioned how to figure out 15%, the child suggested using seven green tiles and cutting another tile in half. Another group in the learning community wanted to let each tile be worth 5%. Through social interaction, the problem was solved, and a child presented the solution of two blue tiles, three green tiles, ten red tiles, and five yellow tiles. A similar homework task followed.

Fig. 1 MAPPS learning community



In another example, Parent A, from the baseline section, said the following:

Parent A: For example, one night we had this conversation: A half…what is the half of a quarter?
Int: oh.
Parent A: and would you believe that for years I didn't know that half of a quarter…
Int: half of a quarter
Parent A: It is one-eighth.
Int: yes.
Parent A: and that you keep cutting it [the fraction strip]…ummm…one-half of one-eighth…
Int: so…you know. Ok
Parent A: and even on this test [technical college entrance exam] that I got, they asked me that question, one half of a quarter, and I could answer

Parent A learned that one- half of one-quarter is one-eighth while engaging in a fraction strip activity, and she subsequently was able to answer a related question on her college entrance exam (Griffin, 2007). Parents shared that their increased content knowledge prepared them to assist their children with specific homework tasks and also strengthened the parents' confidence to assist. Throughout the 3 years of the study, some participants shared a desire and new confidence to continue their education (See Continuing Education code in Table 1). Toward the end of the program, we saw numerous participants act on that desire and begin college. We mention this result here because student achievement has been linked to higher education levels of parents (Choy 2001), thus addressing the question, *How might student improvement occur?*

The qualitative result that parents improved their content knowledge was substantiated by the CKT-M test. The same Number and Operations test (or alternate form) was given before and after each *Number and Operations* Mini-course (Years 1 and 2) and *Fractions, Decimals, & Percents* Mini-course (all years). Most individual 8-week Mini-courses produced increased means. Significant changes of the parent and teacher group was noted when the scores from the first time a participant took the test to last time were compared (See Table 3). The content knowledge tests were designed such that a well-prepared elementary teacher would get 50% of the questions correct, an Item Response Theory (IRT) scaled score or standard deviation of 0.

Average scores for parents and teachers increased, showing greater CCK and SCK (See Table 3).

Table 3 Number & operations Content Knowledge Tests (CKT-M)

	n	Pre IRT	Post IRT	Change in st dev	Sig?
1st-Last Mini-course Parents & Teachers	60	−1.21361	−0.96921	0.24440	YES $p = .029$ $d = 0.282$
1st-Last Mini-course Parents only	40	−1.35844	−1.18154	0.17690	NO

The 95% confidence interval for mean improvement in CKT-M IRT scores was [−0.462, −0.027] P&T. Both pre and post data sets were checked for normality using the Anderson-Darling test ($p = .504$, .311 respectively-P&T)

5.3.2 Knowledge of content and teaching (KCT)

The third aspect of MKT that developed for parents during MAPPS was KCT (See Table 1). KCT for parents involved improving their teaching efforts toward their children in both formal homework assignments as well as informal day-to-day mentioning of mathematics (Jackson & Remillard, 2005; Winter et al., 2004). In MAPPS classes, parents' own strategies were valued, prompting parents to value their children's mathematical strategies. Likewise, parent and teacher explanations were shared with the entire group, modeling for these participants the importance of eliciting children's reasoning. One parent gave evidence of improved KCT, identified in first level of analysis (coding), in that she learned to explain addition using base ten blocks.

> Int: Can you give me a specific something that you learned in MAPPS that helped you help her [her daughter]?
> Parent C: One thing I learned about was what a units, with the cubes that they used to… as far as working with tens…
> Int: Base ten blocks?
> Parent: Yeah, that right there helped me a lot with her [daughter] because she understand it and before I never understood it [base ten blocks]. I didn't know what it was about, but by coming [to MAPPS], now I know how to help her and she has caught on a whole lot faster.
> Int: So what specifically did she learn better with you just using the base ten blocks with her?
> Parent: The order…let's say in the tens place where she had something like 10 plus 10. Um, a lot of times, she would struggle because I would try to use pennies or little dots on a paper, and she didn't understand it. She would get confused, and I would get upset. . .She was able to catch on and pass and make it to the 2nd grade.

This parent learned that the pre-grouped manipulative, base-ten blocks, better assisted her daughter with place value concepts in multi-digit addition than ungrouped pennies or drawings. Manipulatives such as tangrams, base ten blocks, pattern blocks, and square tiles were specifically introduced throughout the Mini-course, prompting parents to begin using them for the first time. Improvements in parents' KCT appeared to give rise to improved parent-child interaction around mathematics, relating to how improvements in children's understanding and achievement might have occurred. In light of community of learners, parents and children enjoyed more positive interaction around mathematics as a result of the intervention.

Moreover, we found through MAPPS that certain aspects of MKT seemed germane to parents' mathematical work with their children in the home setting. Of course, homework help and informal mathematics instruction such as in games took place in the context of the home environment. But we contend that the crux of the improved mathematical communication at home was due to relationships fostered by the mathematics-focused PI program. *Children's interactions with parents fueled by the MAPPS learning community may have prompted children's student achievement gains*. Children's construction of mathematical knowledge was facilitated and constrained by social interaction with their parents as key players internal to the children's lives. Thus, although several aspects of MKT for parents had a counterpart to MKT for teachers, the critical, math-focused, relationship between parents and children seemed to demand a separate construct. Consequently, we advocate that elements of MKT relating to parents be described as Mathematical Knowledge for Parental Involvement (MKPI), as opposed to “parental” MKT (See Table 4). These eight “parental” aspects of MKPI rose out of the yearly/cumulative, and then aggregate level of analysis (See Table 1).

In using MKPI, we assert that there is important mathematical interaction that need occur between parents/guardians and children that perhaps cannot be replaced by the work that teachers do. The *familial bond* to and with the child may channel learning of mathematics. Indeed, top codes from data analysis revealed improved parent-child interaction around mathematics (See Table 1). A child who can say, “My mom does math with me,” may experience added benefit above a child who says, “My teacher does math with me”.

6 Discussion

In conclusion, we revisit our research questions in light of the literature. For the first research question, children constructed mathematical knowledge while interacting in the learning community during MAPPS sessions and with their parents at home. Further evidence includes that parents reported children’s grades improving as well as better understanding of children’s mathematics homework. Finally, children displayed increased understanding through CRCT scores.

For the second research question, several factors seemed to indirectly impact student understanding and achievement. Data revealed that parents have a desire to help their children with mathematics, and that they value mathematics learning for their children. However, similar to the findings of Remillard and Jackson (2006), parents may not have the language, tools, content knowledge and/or confidence to support their children’s learning. The collaborative MAPPS environment stimulated parents’ MKPI. KCT strengthened their ability to explain their knowledge to children, especially through improved choice of and appropriate use of manipulatives. Thus, improved parental CCK and KCT couched within situated parent-child interactions strengthened children’s understanding and achievement of mathematics.

On the other hand, many aspects of the work of teaching such as directing the learning process, knowing the standards, choosing curricula, and providing content expertise fall squarely on teachers (National Governors Association Center for Best Practices [NGA], 2010). In fact, one critical aspect of MKT, Knowledge of Content and Students, rarely occurred in our coding of data for this study. We are not advocating that parents become formal educators, nor are we insisting that parents be capable of assisting with all homework tasks in a child’s K-8 schooling. Yet aspects of teaching, such as listening and responding to children’s strategies, appear to fall within the purview of both parents and teachers. Thus, in

Table 4 Aspects of mathematical knowledge for parental involvement (MKPI)

1. Content Knowledge (CCK)
2. Valuing students’ own strategies
3. Listening to students’ explanations
4. Knowing that there is more than one way to solve a problem
5. Knowing to use manipulatives versus solely pencil and paper to solve
6. Knowing *how* to use manipulatives to model problems (SCK)
7. Knowing appropriate games and skill reinforcers
8. Knowing how to support the learning process (i.e. Do not immediately give the answer.)

the absence of a parent/guardian involvement in mathematics, children may not reach their full mathematical potential. Our conclusion is that children's mathematics learning is enhanced when schools embrace and develop parents as intellectual resources (Civil & Bernier, 2004; Jackson & Remillard, 2005). In other words, children's mathematics learning is impacted by both parenting at home as well as teaching at school. Whereas in past years teachers have been expected to carry the lion's share of responsibility for children's mathematical growth, our study implies that PI in mathematics should be elevated as an educational (and by default, funding) priority. Furthermore, MKPI should be encouraged and taught in PI programs, and those programs ought to directly involve children. Finally, we found through this study that parents wanted to help children, and they wanted to help *themselves*. This study verifies that MAPPS did in fact advance parents' knowledge, teaching ability, and confidence to continue their own education.

MAPPS provided a way to open lines of communication that enhanced the mathematics learning culture of schools. Although parents, teachers, and children came to MAPPS with widely varying background knowledge, the community of learners afforded all participants opportunities to learn and develop mathematically in a situated context, and for the parents, to learn and develop pedagogically (Lave & Wenger, 1991). In the MAPPS learning community, the values, experiences, and strategies of parents and children played a part as their knowledge developed through a transformation of participation (Rogoff, 1994). Learning in this study was situated in familial interactions between parents and children in the both the MAPPS environment and in follow-up interactions at home. Parents and children participated interchangeably as the more knowledgeable partner within the parent-child learning community. At times, children brought knowledge of school-based mathematical tools and vocabulary; whereas, parents brought memories (or half-memories) of mathematical concepts to the table. As they communicated about mathematics in a familial learning community, transformation of participation took place.

As implications from this study, we advocate several design principles for programs centered on PI in mathematics (McKenney & Reeves, 2013). First, it is incumbent upon schools to provide substantive opportunities for PI in mathematics. Second, PI programs should promote aspects of MKPI such as content knowledge, valuing students' own strategies, and listening to students' explanations. Third, an enjoyable learning community is critical for improving parent attitudes toward and confidence in mathematics. Fourth, providing manipulatives and technology resources for parents encourages further PI at home. Finally, including children in PI programs proffers a clinical setting for parents to foster positive parent-child interactions and communities of learners at home.

Although this study suggests that student achievement may be linked to MKPI, further study is needed on the constitution and impact of the construct. Additional factors that may have impacted student achievement over the 3-year period include improvements in teacher MKT, although not all teachers of the children attended MAPPS (Knapp & Landers, 2012). Furthermore, it is possible that teachers raised their expectations of what parents and children were capable of, thus impacting student achievement. Teachers utilizing the same MAPPS tasks that parents were using at home may have improved student motivation and understanding as well. It is a limitation to this study that by virtue of self-selection, participants may have already been predisposed to work in productive ways with their children. Thus, we see parental involvement in mathematics, not as a cure-all, but as an avenue for reform to parents, who are most often the strongest advocates for and strongest influence on their children's lives.

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Compliance with ethical standards

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