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The secondary-tertiary transition viewed as a change in mathematical cultures: an exploration concerning symbolism and its use

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Abstract Secondary-tertiary transition issues are explored from the perspective of ways of doing mathematics that are constituted in the implicit aspects of teachers' action. Theories of culture (Hall, 1959) and ethnomethodology (Garfinkel, 1967) provide us with a basis for describing and explicating the ways of doing mathematics specific to each teaching level, according to the "accounts" provided by the teachers involved in this research project. To borrow from Hall (1959), the "informal" mode of mathematical culture specific to each teaching level plays a key role in attempts to better grasp transition issues.

Keywords Secondary-tertiary transition · Mathematical cultures · Ethnomethodology · Ways of doing mathematics · Mathematical symbolism

1 Introduction

Several studies have shed light on students' difficulties during the secondary-tertiary transition and what advanced mathematics requires in relation to elementary mathematics (e.g., Gyöngyösi, Solovej, & Winsløw, 2011; Robert, 1998), or on a comparison of a content common to both levels (Corriveau, 2007; Corriveau & Tanguay, 2007; Sawadogo, 2014; Stadler, 2011; Vandebrouck, 2011b; Winsløw, 2007). While this work has added considerably to our knowledge about this transition, as amply demonstrated by Gueudet's (2008) survey, various international studies[1] have, nevertheless, highlighted the complexity of this subject,

[1]Espace Mathématique Francophone (EMF) 2003, 2006, 2009, 2015; PME 2011; PME-NA 2012; CERME 2010

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which clearly stands to benefit from a variety of angles so as to better fathom the various issues confronting students. A previous study suggests that one such aspect warranting further consideration is the ways mathematics is done by teachers on a daily basis (Corriveau, 2007). These ways of doing mathematics, an unconscious component of daily practices, play a pivotal role in the change in mathematical cultures mentioned by Artigue (2004) concerning this transition. They constitute the focus of our research project, which we conducted with teachers at both levels.

Before situating the goals of this study, we review transition research so as to bring out the importance of focusing on the informal mode of mathematics culture at both the secondary and tertiary levels (Section 1). We then examine the conceptualization of this informal mode as this takes form in the ways of doing mathematics shared by teachers (Section 2). The importance of taking teachers' perspectives into account weighs critically on this study's methodological orientations, particularly in terms of having opted to conduct a collaborative research study (Section 3). The analysis of results charts the "territory" (of ways of doing mathematics) that emerges at each level concerning mathematical symbolism and its uses (Section 4). The shared territories associated with each level evince different mathematical cultures that provide us with a basis for discussing secondary-tertiary transition issues (Section 5).

2 Transition issues: approaching the informal mode of mathematical culture via teachers' ways of doing mathematics

Following an analysis of French secondary school curricula and borrowing from Hall's (1959) concept of culture, Artigue (2004) hypothesized that the secondary mathematical culture is characterized by "the encounter of numerous fields and problems; but an encounter that can remain superficial, as students do not have enough time to truly operationalize, stabilize and structure their knowledge" (p. 4 [our translation]). According to Artigue, the fragility of final-year secondary school students' knowledge is due in part to the highly contextualized nature of their knowledge. In a previous study, we studied the transition from secondary to tertiary level via curricula and textbooks, focusing on proofs and languages used (Corriveau, 2007, 2009; Corriveau & Tanguay, 2007). Our analysis brought out how this transition entails having to meet heightened requirements regarding proof and formalization, thus compelling the appropriate use of the syntax specific to mathematical language combined simultaneously with control of the semantic contents (Weber & Alcock, 2004). The secondary-tertiary transition also coincides with a fracturing[2] of the meaning of symbols, thus making it difficult for many students to understand and use formal mathematical language (Corriveau & Tanguay, 2007). This work bears out the findings of other studies regarding proof and formalism, a major source of failure among first-year university students (Robert, 1998; Sawadogo, 2014). The insights afforded by these studies make it possible to grasp the numerous breaks confronting students and help foster receptiveness to attempts at developing bridges between teaching levels (see De Vleeschouwer & Gueudet, 2011; Praslon, 2000).

[2] For example, if at the secondary level, letters represent essentially real numbers; at the tertiary level, particularly in the linear algebra course, letters represent various mathematical objects: real numbers, complex numbers, matrices, geometrical vectors, algebraic vectors, polynomial expressions, etc. This makes the formal mathematical language difficult to understand and use for many students (Corriveau & Tanguay, 2007)

That said, until now, little attention has been devoted to transition issues from the perspective of teachers.[3] The same is true in respect to the research performed by us in the past, which has viewed symbolization as a key issue (Corriveau, 2007, 2009), but primarily from the angle of the proposed tasks and their related requirements. It has not grappled with this issue in terms of what occurs in the teachers' action involving the symbolism at hand. And yet, several studies have shown that as students transition from one teaching level to the next, they are confronted with different ways of approaching mathematics (Bednarz, 2009; Durand-Guerrier, 2003; Stadler, 2011) that stem not only from institutional requirements but also from teachers' ways of doing things. Thus, in an exploratory study, we brought out how the difficulties experienced by tertiary teachers involved ways of doing mathematics (e.g., using symbolism, involving students in proofs) as much as they did the new mathematical content being dealt with (Corriveau, 2007; Corriveau & Tanguay, 2007). These observations are consistent with other research studies (De Vleeschouwer & Gueudet, 2011; Stadler, 2011), which have shown that key elements of the transition involve these different ways of approaching mathematics. Stadler (2011) has noted the difference between the task solution routines used by students (which are rooted in their previous experience at the secondary level) and what their university teacher proposes instead. One can discern differences in ways of explaining mathematical content (even if Stadler does not use such terminology herself)[4]: "These different approaches can be interpreted as an expression of the transition, where the teacher focuses [on] the general character of the mathematical content, while the students are primarily interested in a specific routine for finding a solution of the task" (p. 4).

These studies show that in order to understand the issues, it is necessary to account for the ways of doing things that are specific to each teaching level. Indeed, these practices are inextricably bound up with the very mathematical culture that we are striving to grasp. Over and above learning content, mathematics refers to shared convictions, know-how developed in action, and so on, which do not necessarily come within the explicit dimension of mathematics teaching:

> […] A considerable portion of our knowledge and know-how is located at the informal level of mathematical culture. It is acquired through action, experience and imitation; it is not verbalized and is often not verbalizable. (Artigue, 2004, p. 6 [our translation])

According to Hall (1959), the greatest cultural differences fall within this informal mode of a culture—namely, the often unspecified ways of doing things and the activities that have been unconsciously incorporated into daily practices. From this perspective, secondary and tertiary teachers' ways of doing mathematics constitute the fundamental elements of this change in

[3] Tertiary mathematics teaching has been the subject of several research studies (see the "How we Teach" project reported in Jaworski & Matthews, 2010). Some studies bearing more directly on transition issues have focused more specifically on the perspective of teachers regarding transition (Hong et al., 2009) or that of students regarding their teachers (De Guzman, Hodgson, Robert, & Villani, 1998), but they shed light partially on specific issues linked to transitional issues.

[4] Students had to solve the following system of equations for all values of the constants a and b, with the help of the teacher (in linear algebra)

$$
\begin{aligned}
x+y+z&=5\\
x-y+az&=3\\
2x+y+z&=b
\end{aligned}
$$

mathematical cultures referred to by Artigue. And, it is this informal level, to borrow from Hall, that has been of particular interest to us.

2.1 Research goals

The main goal of this research project is to explore the secondary-tertiary transition (tertiary referring to the "cégep level"[5] in this instance) with teachers from both levels, from the perspective of their ways of doing mathematics as teachers.[6] Specifically, it is not just a question of what differentiates the ways of doing mathematics characterizing secondary and postsecondary teachers; it is, importantly, a question of how, when teachers explore transition issues together, these ways of doing mathematics come to be seen in terms of the particular meanings with which they are endowed in each practice setting.

3 Theoretical bases of our research

Following Artigue's (2004) line of thinking, our study first draws on Hall's (1959) trimodal theory of culture. We have also relied on ethnomethodology to refine the angle of approach used to analyze the ways of doing mathematics typifying this culture in terms of its informal aspects. The value of ethnomethodology was apparent because, on the one hand, ways of doing mathematics (hereafter referred to as WDMs) fall within the realm of action and, on the other hand, we are interested particularly in shared WDMs, in which teachers recognize themselves. Precisely so, ethnomethodology[7] focuses on ways of doing things as they are deployed in everyday activities and are shared by the "members" of a given group.

3.1 The notion of culture according to Hall's anthropological perspective

Hall (1959) conceives of culture in terms of three major modes (namely, the formal, informal, and technical modes) that make up the basic organization of all human activities and characterize every culture.

Thus, from Hall's (1959) perspective, the *formal mode* of culture is bi-polar, consisting largely in precepts and admonitions ("one may do this; one may not do that").

[5] *Cégep* is a French acronym for *Collège d'enseignement général et professionnel* (referred to in English as *General and Vocational College*). In Quebec, the "cégep level" lasts 2 years (grades 12 and 13, students of 17 to 19 years old) in the case of pre-university programs and 3 years in the case of technical/vocational programs. This level is, like the university level, part of the province's system of higher education. "Cégep" institutions are independent of both secondary institutions (grades 7 to 11) and universities and lead to a degree specific to that level. Teachers receive formal training in a given discipline (e.g., masters in mathematics) and have access to research grants (in mathematics education), but they are not required to do research. So mathematics teachers at the Cégep are not doing mathematics for the purpose of scholarship and publication in mathematics.

Please note that while the general focus of our research concerns the secondary-tertiary transition, for the purposes of clarity, we shall describe those teachers and institutions participating in our study as "postsecondary"—in reference to the specific level of the Quebec higher education system concerned.

[6] These ways of doing mathematics are situated in the context of teaching and therefore include thinking about and planning how to do mathematics with students and how to represent mathematical concept for the purpose of teaching.

[7] Ethnomethodology refers to "ethno-methods" and "-logy"—i.e., the study of the methods used by a particular sociocultural group in its everyday activities for understanding and producing the social order in which its members live.

It corresponds to values, convictions, and unreflected assumptions that require no justification. In the context of mathematical culture, this mode can be observed in those elements that teachers consider as “going without saying.” As such, it delimits what is acceptable and what is unacceptable, with no justification being required in order to do so. Teachers do mathematics within the framework of their classroom practice on the basis of self-evident assumptions specific to this culture and that they never quite throw open to question—for example, a secondary teacher could use a generic example to illustrate some properties, and convince students of the validity of these properties, and this kind of examples would be inappropriate for a postsecondary teacher who would use a general example.

The *informal mode* of culture corresponds to what is a regular part of everyday action and yet also remains implicit—that is, a “doing” that is framed by implicit rules of action. In the context of mathematics education, this mode brings into play the ways of doing things that are implemented by teachers but that, in contrast with the formal mode, are not necessarily explicit—for example, ways of using a definition in mathematics, of using representations, of justifying, of introducing or using mathematical symbolism, and so on.

Finally, the *technical mode* of culture corresponds to an explicit, organized system of justifications. In the case of mathematics, it refers to the “institutionalized” elements of curricula that bring into play explicit mathematical content, or to textbook contents that are explicated, supported, and justified. For example, solving an equation in algebra (of the form $ax + b = cx + d$) is introduced explicitly and justified using equivalent transformations on algebraic expressions preserving equality.

In fact, all three modes occur simultaneously in the situations experienced by social actors; however, as Hall (1959) noted, in some cases, one of these three modes will predominate.

3.2 Ethnomethodological foundations

Ethnomethodology is a sociological theory focusing on “ethno-methods”—namely, the methods used by actors in the pursuit of their everyday activities. In the words of Garfinkel (1967):

> Ethnomethodological studies analyze everyday activities as members’ methods for making those same activities visibly-rational-and-reportable-for-all-practical-purposes, i.e., “accountable,” as organizations of commonplace everyday activities. (p. vii)

“Ethno-method,” coupled with other related concepts, provides an angle of approach to the WDMs that are constituted in teachers’ professional activity.

In Garfinkel’s (1967) view, these ways of doing things are describable, intelligible, and observable in the action of actors, in a variety of ways, for example action that is effectively performed, the manner of discussing this action, interaction with other actors, and so on This property known as “accountability”—a cornerstone of ethnomethodology—is manifested in various ways. For example, factoring equations by inspection in the classroom, explaining to a student how to do so, and discussing with another teacher how to teach factoring equations by inspection are all “accounts” of what doing a simple inspection is for a teacher. “Accountability” (which here has nothing to do with the idea of “being held to account”) refers to the fact that action is account-able—that is, “storyable,” “picturable,” and “representable” for the actors involved. In addition, it is a vector of reflexivity[8]—in other words, a phenomenon of sense making that can be observed in the action of actors.

Furthermore, ethnomethodology recognizes the inextricably *rational* character of this "account-able" action. Actors use "interpretive procedures" (Cicourel, 1974) to inquire about the world and recognize the particular circumstances that give rise to action. These interpretive procedures are "reflexive instructions that [actors] provide themselves with in order to understand one another and reach decisions concerning their actions" (Coulon, 1993, p. 22 [our translation]).

The concepts of "accountability," "reflexivity," and "interpretive procedures" are framed in relation to the notion of interactions between "members": in ethnomethodology, the focus of interest is not the individual actor but rather actors engaged in interaction with one another. This "membership" brings into play a notion of familiarity with ways of doing things in which actors recognize themselves, particularly in terms of mastering a common language (Garfinkel & Sacks, 1970), not to mention common WDMs. In interactions with other actors, this familiarity can be observed in the capacity to remedy incomplete (i.e., vague or ambiguous or truncated) expressions (Cicourel, cited by Coulon, 1993).

Therefore, in order to understand the verbal interactions occurring between actors, they must be "indexed" in relation to particular situations and circumstances. Accordingly, "indexicality" is a key concept, for it refers to the incompleteness of words and actions, which only acquire their full meaning in the context of their production (Coulon, 1987).

These various theoretical concepts (i.e., accounts, rationality, interpretive procedures, and indexicality) will also serve to shed light on the WDMs associated with the informal level of culture, *to be approached in terms of shared elements* (i.e., those ethno-methods in which members recognize themselves).

4 Methodology

As is apparent from the preceding considerations, it is vital to approach several teachers belonging to the same teaching level in order to gain access to shared WDMs. Furthermore, efforts to facilitate dialogue between teachers from both levels hold considerable promise for gaining access to the culture of others and making the implicit explicit. Indeed, a most useful contrast emerges from the encounter of teachers at both levels who do not all identify with the others' WDMs.

Six postsecondary and secondary teachers (three from each level) agreed to participate in a collaborative research project (Bednarz, 2013, Desgagné, Bednarz, Couture, Poirier, & Lebuis, 2001), whose goal was to deepen their understanding of their classroom practices and, consequently, the implications of these practices with regard to students' transition issues.

A reflexive activity—a pivot of collaborative research—thus developed around certain common mathematical content or processes (selected by the teachers), on the basis of which verbal interaction took shape (e.g., function, proof).

[8] Ethnomethodology is rooted in this reflexive and interpretative capacity of each social actor, inseparable from action: « Le mode de connaissance pratique c'est cette faculté d'interprétation que tout individu, savant ou ordinaire, possède et met en œuvre dans la routine de ses activités pratiques quotidiennes (…) procédure régie par le sens commun, l'interprétation est posée comme indissociable de l'action et comme également partagée par l'ensemble des acteurs sociaux… » (Coulon 1993, p. 15)

4.1 Reflexive activity

Reflexive activity can be thought of as a meeting ground or clearinghouse for the teachers and a researcher who were involved in this study; this so-called "interpretive zone" (Davidson, Wasser, & Bresler, 1996) serves to elicit an interpretation from such actors (Desgagné et al., 2001). Reflexive activity is rooted in ethnomethodology and, specifically, the closely related concepts of reflexivity and "accountability." More precisely, based on ethnomethodology, the reflexive activity is seen as "an activity" where members, in questioning their practice, make explicit its "code of practice" and "rules of functioning." Underlying this reflexive activity, there is an idea that members recognize themselves in ways of doing that allow them to understand each other and interact. In terms of a methodological point of view, these foundations stress the importance of placing teachers in a situation of action so as to allow them to "provide an account of" their ways of doing things in the very process of constituting these practices.

Thus, in this study, the selected reflexive activity embraced a series of elements designed to enable the teachers to offer an account of their "shared code of meaning":

1. Interaction between teachers at both levels: practices that are familiar to teachers at one level are more likely to come to the surface during interaction with teachers who do not share such familiarity.
2. Situations (used as a basis for discussion) drawn from the everyday actions performed by teachers—for example ascribing meaning to a problem or question raised in a textbook; specifying the way they would make use of the problem or question in the classroom, commenting on students' solutions to this problem; narrating a lesson; and so on. The idea is to use these various accounts to identify how teachers at both levels do mathematics. One of the researchers previously observed final-year secondary and postsecondary classes for the purpose of identifying familiar situations of this kind.
3. "Breaching" situations: we also sought to subtly disrupt familiarity through the introduction of foreign elements having the ability to catch teachers off balance and compel explication. In ethnomethodology, this procedure is referred to as "breaching" (Garfinkel, 1963). Teachers' reactions to these situations are thus useful for bringing out not only what is meaningful for them but also their usual ways of doing mathematics in the classroom.

In short, the reflexive activity brought together teachers from both levels who were prompted to produce accounts of the WDMs through situations drawn from their professional actions. Six meetings, each lasting a day, took place between January and November 2011.

4.2 Analysis of the body of data: some guideposts

The main body of research data is made up of verbatim transcriptions of the meetings (36 h). In keeping with an emergent analytical framework as well as research rooted in the principles of grounded theory (Glaser & Strauss, 1967), all transcriptions were grouped together according to clusters of episodes centering on three main emergent themes: the use of symbolic representation, the use of contexts, and work surrounding functions. In this article, we will explore the theme of symbolism in particular.

Once the transcriptions concerning the use of symbolism had been grouped together, all the episodes associated with this theme were then coded. Categorization proceeded along two levels.

The purpose of the first level of categorization was to report the voice of actors concerning these WDMs, "as revealed by [the actors themselves]." The point here was to offer an account of what teachers could show us about various WDMs and about their underlying circumstances and rationale. The researcher's role is to play spokesperson to teachers, allowing them to be heard in their own words while also, at this stage, "refraining from interpreting" this voice in terms of some theoretical approach (Desgagné, 2007).

At a second level, we grouped together and characterized these shared WDMs, as formulated in the accounts of the teachers involved in this research project. This interpretation brought to light a "territory"[9] of mathematical ethno-methods at each level. Here, we adopted an analytical posture as such, in which the researcher proposes an interpretation of the first level of analysis so as to be able to delve more deeply into what is constituted by teachers at both levels. We shall now explore this second level of analysis.

5 WDMs pertaining to the use of symbolism at each teaching level

5.1 Shared WDMs among the secondary teachers

The territory of secondary teachers' WDMs can be described in terms of three characteristics: a progressive symbolism, a transparent symbolism, and a chosen symbolism.

5.1.1 A progressive symbolism

This heading refers to a symbolization that is gradually implemented, occasionally through intermediate notations.

Working from a basic/familiar symbolism and transforming it The secondary teachers said they worked from a simple, familiar symbolism and transformed it. For example, when they worked on functions, they started out by writing a basic function (e.g., exponential function, quadratic function), using the basic notation (e.g., $f(x)=x^2$) to study its properties and then transform it in order to introduce parameters. Teachers were familiar with this parameter-based notation—($f(x)=a(bx-h)^2+k$)—which they gradually introduced into the classroom.

This symbolism-related WDM appears in other cases as well. Thus, whenever teachers introduced trigonometric relationships into a right-angled triangle, they used a word-based notation (i.e., the relationship between the length of the opposite site the length of the hypotenuse), a notation in fractional form (opposite length/hypotenuse length), or even an intermediate notation based on symbols (*m*AB/*m*AC, a sine table, etc.). They only introduced the "sin" symbol when they were required to operate on it. A second WDM will further support this idea of progressive symbolization.

Introducing and maintaining intermediate notations The secondary teachers stated that they avoided losses in meaning by maintaining intermediate notations and by refraining from introducing the standard symbolism too quickly. Thus, as shown in the following excerpt,

[9] "Territory" is an evocative metaphor serving to convey the idea of a "land" that people organize so as to be able to "live" in it (Raffestin, 1981). This space is continually undergoing organization.

secondary teachers were in agreement that they postponed introducing the term “sine” and the symbol “sin” as much as possible:

> In sub-groups the teachers[10] discussed courses dedicated to introducing new concepts in mathematics. Sam presented his way of introducing trigonometric relationships, and in particular the introduction to “sine”. As he explained to the postsecondary teachers, he waited as long as possible before introducing the term “sine” and the symbol “sin”. During discussions among the whole group, he reiterated:
> Sam:… And even try to introduce “sin” as late as possible. What I have noticed is that as soon as you bring up the sine, it’s as though everything you’ve done before – that it’s a relationship between two lengths [in the minds of students] – has disappeared. “Sin” is more important than the idea that it’s the relationship between two sides and that this relationship remains constant when there is an angle…
> Serge: I remember having tried to stay for a long time – we’re talking about several weeks – on a table [of relationships] in which the words “sine”, “cosine” and “tangent” didn’t appear. It was length of opposite, length of hypotenuse, etc. So, like, students truly recognized [the relationship] and it worked pretty well. But there’s, like, a small break. Like it or not, there comes a time when you have to deal with the name of that relationship. It’s a form of notation that you operate on.

In the excerpt, the secondary teachers (Sam, Serge, and Scott, who nod in approval of what Sam has explained) recognized themselves in a shared WDM. They stressed that they had intermediate ways of representing the object referred to as “sine” (e.g., words, table of relationships). Serge also brought to light a limitation of this WDM: from a mathematical point of view, there was a need to move on to the standard symbol in order to be able to operate with it; so doing, he highlighted the mini-break that he perceived in the transition from a familiar symbolism (that maintains the meaning of the relationship) to a standard symbolism.

The rationale underlying this WDM is unrelated to any official prescription. It is instead of a didactic nature, the point being to avoid losses in meaning and to maintain a meaning (in terms of relationship) among students as long as possible.

The same intention also drives the secondary teachers’ decision not to use symbols when introducing the definitions. For example, when they spoke of increase and decrease in the context of functions, they did so on the basis of a graph (“it goes up or down”), or they relied on everyday language in reference to a context (“the longer time goes by, the more bacteria there are”). Teachers proceeded through intermediate forms (in words that are “indexed” to a context or to the graph).

5.1.2 A transparent symbolism

Secondary teachers opted to approach symbolization with a view to making the meaning of symbols readily accessible to students. This aspect of the “territory” was noticeable not only in the WDMs brought out above but also in the following WDMs.

Determining the symbolism of choice Intentionally, and over a long period of time, the secondary teachers made consistent use of certain symbols. Thus, in the context of functions, they

[10] The secondary teachers were Sam, Serge and Scott and the postsecondary (cégep) teachers were Colin, Colette and Corinne.

always used the same letter (*a*, *b*, *h*, and *k*) to represent each of the parameters, across all the functions studied during the last year of secondary school. Moreover, each parameter was associated with a transformation in the graph. The multiplicative parameters *a* and *b* were, respectively, horizontal and vertical dilations or contractions, whereas the additive parameters *h* and *k* were, respectively, horizontal and vertical translations. At the same time, this quest for meaning and consistency appearing in the systematic use of the same letters to represent parameters was deemed "dangerous," according to these teachers. Indeed, they were aware of the limitations of this way of doing mathematics: students tended to associate the letters used for parameters with specific transformations in the graph and became confused when these same letters were used in other contexts that did not permit such association. Their comments also serve to highlight one of the reasons underlying the use of a specific symbolism associated with one of these four parameters; the symbolism referred to is that which is used in any of a number of textbooks: "Well… it's what you find in the textbooks; I think you'll find only that" (Serge). When a dozen or so [textbooks] say that "the highest point of the quadratic function is (*h*, *k*) [shrug of shoulders as though to state the obvious]" (Sam). In what amounts to another underlying rationale, however, these teachers were also aware that this WDM spared students' difficulty and indeed provided a foundation: "Students are quite comfortable" (Serge) with this way of doing mathematics.

Linking the symbolism to the graphic register Another WDM comes into focus in the comments of the secondary teachers: "progression" in writing the function occurs by means of linking to the graph ("Well, so we transform it now? At that point, you enter the parameters, you draw it [i.e., you plot the graph], and once it has been drawn [you say]: Well now, what's of interest in this?"). Sam's comments put in light the idea of a writing process which, with the addition of parameters, is linked to graph-related work. In short, we are dealing here with a WDM that brings into play a linkage between the algebraic register and the graphical register. This WDM (tacitly) implies that relationships are established between the way a function is written and the way it is graphed, with the graph serving to support the symbolism.

Choosing a symbolism that links to contexts The secondary teachers had a way of choosing symbolism depending on the context at hand. Thus, whenever it was a question of functions in relation to a given context, the teachers chose those letters that referred to the context, as can be seen in the following excerpt:

> Sam: If I have a situation, for example, like, I will make sure I come up with the letters that are associated with this situation. If we're talking about height, then it's going to be *h(x)*. If I'm talking about some function or another, then it's going to be *f(x)* and *g(x)*. If I'm talking about a variable that is the distance, like, then I'm going to have a little 'd' in order to emphasize the situation.

Selecting a symbolism that links to various contexts is a way of highlighting the situation for students and attaching meaning to symbols.

5.1.3 A chosen symbolism

Our analysis evinces the idea that symbolism is chosen by the teachers. For example, they exercised the option of not introducing any formal symbolic definition of increase and decrease in the context of functions. They instead chose to proceed through intermediate notations. In the context of functions, it is worth noting that even though all textbooks employ a symbolism using the four parameters, mathematicians or practitioners in other disciplines do not use this symbolization. We are dealing here with a way of symbolizing that is specific to the secondary level.

5.2 Shared WDMs among the postsecondary teachers

An analysis of transcriptions offers a basis for mapping a "territory" of WDM for postsecondary teachers around three ideas: an explicated symbolism; a determined, exterior symbolism; and a compact symbolism.

5.2.1 An explicated symbolism

Three WDMs came under this heading: translating into symbolism and translating symbolism, explicating/specifying what is represented by symbols, and "giving voice to symbolism."

Translating into symbolism and translating symbolism Verbal interaction concerning increase and decrease in the context of functions brought out the postsecondary teachers' concern with mathematically symbolizing definitions—a process they explicated in terms of translation:

> The researcher asked the teachers how they "talked" about increase and decrease. Sam replied "when it goes up, when it goes down," with Serge adding that they also do so in context. Following this, Corinne responded.
> Corinne: For example, increase goes up [she looks at Sam], but how do you write it in symbols? I try to bring them [the students] around to that. When it goes up, well, you have to go from left to right [along a curve in a graph]. You know, I'm a stickler about all the bits and pieces of the definition… I think they're not used to this kind of translation – how to write things… In differential calculus, that's the course in which you should introduce it slowly and surely, because in integral calculus, you use theorems.

The postsecondary teachers spoke of "translation," "how to write things using symbols," or "translating symbolism when it is given." In the latter case, they were referring to those definitions that had been previously symbolized (in books, especially), noting that that was the form of writing to which they introduced their students. Individual WDMs emerged in relation to the way such translation should be conducted: for example, writing with mathematical symbolism and "giving voice to it"—that is, by clarifying the meaning of all the "bits and pieces of the definition" (Corinne); or writing in everyday language and then translating this with mathematical symbols (Colin). Regardless of the particular WDM opted for, we are witnessing here to the idea of translation—namely, of a shift to a (mathematical) symbolic language to which students must be provided an introduction. The teachers emphasized translation while also noting that students were not familiar with the writing involved and experienced difficulty with this symbolism.

The secondary teachers handled this difficulty differently, as was seen above. Taking a progressive approach to symbolization, they emphasized the use of intermediate notations. Among the postsecondary teachers, the idea was to explicate and specify what is represented by the symbolism, which is pre-given. From their perspective, the rationale underlying this WDM was strongly linked to a notion of preparing students for advanced studies as is illustrated in the following comments:

> In reaction to what Corinne asserted, the secondary teachers noted that at the secondary level there is not enough formalism. Regarding this reaction, the researcher asked Corinne:
> Researcher: But is formalism overdone at the cégep level? You seem to be saying that [increase and decrease] are well understood informally. Is it necessary to be so formal at the cégep level?
> Corinne: I've wondered about that before. I've wondered: those students enrolled in science, where are they headed? I've wondered, there are surely a number of them who are going to enrol in pure science… in chemistry… My role is to prepare them for university. They're going on university afterwards… At that point, I say to myself, at university, things are pretty formal. […] I think they're not used to that translation [process], how to write things. So, differential calculus is the course in which it really should be gradually introduced, because in integral calculus, you use theorems, and… And I imagine that in linear algebra, you use [it] even more [looks at Colette].

Explicating/specifying the meaning of symbols The postsecondary teachers mentioned that they introduced new mathematical objects through their associated symbols or spelled out the meaning of a symbol when first introducing it (e.g., let u and v be two functions). In a linear algebra course, when the topic was matrices, they stated: "After a few examples, I define the matrix, it's a rectangular table of real numbers. I then introduce a notation – a capital letter for the matrices, a lower-case letter for the elements" (Colette, seconded by Colin). This WDM was thus explicated under two types of circumstances: when teachers introduced the objects manipulated in definitions, theorems, and demonstrations and when teachers introduced new objects for study. They conceived of this way of doing things in such terms because, in their view, that is what the mathematical game or tradition consisted of: mathematicians attempt to symbolize everything as much as possible.

> Corinne: Do you know what I say to students? Mathematicians are lazy. They are always trying to symbolize everything. They got tired of writing… But it's true, because they're mixed up, too: what are constants? Are they variables? That's where I say no, u and v were defined as two functions. When I set out the derivation rules, I let u and v be two functions. It's like that just about everywhere: A and B, they have to be defined as points on a Cartesian plane…

"**Giving voice**" **to symbolism** This WDM is closely bound up with the preceding one. For example, in the context of introducing limits, the postsecondary teachers took an intuitive approach to introducing the notion of limit and then presented the associated notation that is "lent or given a voice," so to speak:

> Corinne: Well, when you want to talk about it [i.e., the limit], when you approach a value without ever reaching it, you get as close as you want to… Like, I'm in the middle of giving the course [laughs]…
> Colin: Right…

Corinne: At that point, I introduce the notation.
Colin: Right.
Corinne: The limit, when x tends to 2 [lim(x→2)]. At that point, I repeat it, [the limit] not at 2, when you approach 2. That's how I go about introducing the notation.

Corinne and Colin were familiar with students' difficulties in connection with the concept of limit (e.g., students calculated the value of the function in terms of $x = 2$ in order to evaluate the limit), and consequently, they "give voice" to the symbolism in order to minimize this obstacle.

5.2.2 A determined, exterior symbolism

The postsecondary teachers used a conventional symbolism—that is, one that is recognized by the scientific community. All in all, our analysis suggests that while the symbolism used by these teachers may be arbitrary, it is not one that they have chosen.

Using a pre-given symbolism In their interactions with the secondary teachers, the post-secondary teachers spoke of their ways of symbolizing in terms of a symbolism that is exterior to themselves, conventional, and pre-given. For example, when the topic was matrices, Colin noted the difficulties students experienced when using dual indices, but also suggested that the notation used was pre-given:

Colin: It's true that, initially, they see twelve [12, rather than the indices 1 and 2]. It's really dumb, too, but that's the way it is; we weren't the ones who invented it; they should have invented something else. That said, it's true that it [this mode of writing] causes them all kinds of little problems.

This same way of handling symbolism is encountered in other circumstances—that is, naming in advance what will be manipulated in proofs or properties, such as that occurs when teachers say: "In the following, such a symbol will be such an object." The same holds for whenever the symbols used (meaning, whenever it is a question of applying them) differ from what teachers use as a mode of writing in definitions ("But you start by showing that x can be called x + delta, $5 + h$"). Similarly, when the postsecondary teachers presented derivation rules using Leibniz's notation, they used the lowercase letters u and v to represent functions, as will be seen below (u and v are given symbols)—a move that seemed self-evident to them.

In this excerpt, the discussion concerns the equality: $d/dx\ (u + v) = du/dx + dv/dx$
Researcher: [In equality, are letters] numbers? What is the d, what is the x, what is the u and the v?
Corinne: It's because the u and the v can be functions. When that notation is used, you say u and v are functions, $u = f(x)$ and $v = g(x)$…
Colin: That's right.
[…]
Researcher: Why don't you write "of x," $u(x)$ or $v(x)$?
[Colette and Corinne try to explain to the researcher that u and v are functions. They do not understand what the researcher means by u "of x"]
Researcher: But why don't you write the "of x" if it's a function?
Colette: OK.

Colin: Because it's defined … You'll say, let *u* and *v* be two functions of *x*… and then you begin.

In the preceding dialogue, the teachers showed that the symbolism they manipulate in derivation rules is given, named, and specified prior to performing any manipulations. Thus, in the equality $d/dx(u + v) = du/dx + dv/dx$, the symbols *u* and *v* are "indexed" to functions. Furthermore, Colette and Corinne's reaction to the researcher's question brings out how, for them, writing goes without saying in this context. Indeed, when the researcher asks them why not $u(x)$ and $v(x)$, to see if they can explicate their underlying rationale, it is so clear to Corinne and Colette that *u* and *v* are functions that they do not understand the question.[11]

This way of doing things among the postsecondary teachers can also be discerned in the reactions they shared vis-à-vis the circumstances described by secondary teachers. When confronted with the secondary teachers' vision of the symbolization of functions (involving a progression from the basic symbolism to one involving four parameters), the postsecondary teachers' reactions showed that they shared a different WDM concerning the symbolism—that is, that of a pre-given system of notation. "Well, I thought it was written that way from the start" (Colin); "Me, too" (Corinne); "We're studying that function today!" (Colin).

Employing a symbolism that varies according to the circumstances surrounding its use The symbols associated with mathematical concepts may vary in accordance with the use to which they are put. For example, the postsecondary teachers used $f(x)$, $g(x)$, or $h(x)$ to represent a function along with its rule (e.g., $f(x) = x^3 - 3x^2 + 4$). When the topic was functions in applications, the letters used were those given by the situation and were used in science (accordingly, the same letters were not always used; Colin gave the example of the use of $v(t)$ in a problem focusing on speed as a function of time). The proposed symbolism linked, in this instance, to the symbolism used in scientific fields (e.g., physics).

Thus, when teachers discussed limits and introduced the properties of limits, they used *f* and *g* to represent functions. To state the properties of derivatives or perform calculations using Leibniz's notation, they instead used *u* and *v* to represent functions, whereas they used *f* and *g* with the prime notation for derivatives. These different ways of symbolizing were all compact; in addition, they were presented in textbooks and in other science and mathematics courses.

Substituting the symbolism In the context of functions, the postsecondary teachers used a general symbolism to define, prove, and present theorems. In applications, they "played with this symbolism," with the result that what was represented by a single symbol could be substituted by a more complex expression. This game was important to the extent that they expected a certain degree of flexibility on the part of their students.

Corinne: I say that "subject to confirmation," but that's what interests us. You set *b* to *x* [b^x], it's a quick reminder, something like that. You work with that for the entire duration, when you give the derivative and you prove all of that. Then you can have *a* to something, not necessarily just *x*. You can play with it, you can set *a* to the sine of *x* [$a^{\sin x}$].

The symbolism in question is by no means fixed or rigid; on the contrary, flexibility in the use of symbolism is what is sought after, as is suggested by the comment: "That's what

[11] However, when the topic at hand is limit, *f* and *g* are instead the letters used to symbolize the associated functions.

interests us." Moreover, Corinne brought to light the circumstances in which she used this way of symbolizing. First b^x served to recall previous learnings for the purpose of presenting the derivative of an exponential function and performing the associated proofs. Then, in the applications, however, she varied the use she made of symbolism—for example, $a^{\sin x}$. Thus, there is a kind of backdrop against which teachers can be perceived as playing deliberately with writing, refraining from associating it with a specific attribution (as was the case at secondary level), but instead opting to introduce variation in it (Colin says: "x can be called $x + \text{delta}$, $5 + h$ or it can become sin x").

Thus, the postsecondary teachers also provide evidence of a shared "territory" in the process of being constituted, as can be seen from various WDMs enacted under particular circumstances and in keeping with a rationale, and that is implicitly present in teachers' work with students on various mathematics contents (e.g., continuity, derivatives, matrices).

6 Discussion

These findings open up some important perspectives. They shed light on aspects of mathematics different from those that have, until now, been explored in research on transitions from secondary to postsecondary instructional settings. Indeed, such research has directed little attention to the theme of symbolization. And when it has done so, this theme emerges only indirectly, through an analysis of the tasks proposed at each teaching level in a way serving to highlight the additional demands confronting postsecondary students (e.g., formalization, breaking up of the meaning of symbols). By grappling with the informal mode of culture encountered at each level, the present research offers another perspective. It brings to light the complexity of the transition from one mathematical culture to another, in particular by showing up the "different rules of the mathematical game" (Drouhard, 2006). These cultures correspond not only to different WDMs, as witnessed in teachers' accounts, but also to the different logics underlying these WDMs, which were explicated in relation to their underlying circumstances and rationale. Now, by setting out the previous results side by side, we will attempt to provide a basis for reviewing a certain number of key issues concerning this transition in relation to symbolism and its uses.

6.1 Giving form to a symbolism versus acting on an existing symbolism

We note that the secondary teachers formulated theirs in a way that suggested gradually giving form to symbolism whereas the postsecondary teachers formulated theirs in terms of actions performed on a pre-given symbolism (see Table 1).

When set out side by side, these results make it possible to discern the difficulties that are likely to confront students, as the same symbolism does not at all have the same status at each teaching level. Thus, the beginning of postsecondary education is a challenge for students not merely because of heightened requirements relating to formalization (Chellougui, 2004; Corriveau, 2007; Najar, 2011; Sawadogo, 2014)—that is, stemming from the introduction of new symbols and new rules. The way of conceiving of the use of symbolism is constituted in two distinct cultures. In a way, these results mesh with the findings of Stadler (2011), whose research focused on the help that a postsecondary teacher provided to students in relation to problem solving. If, in Stadler's study, one can discern differences in the ways used to explain

Table 1 Various formulations used by teachers at both levels in their accounts of WDMs

WDMs explicated by secondary teachers
• Starting out with a basic or familiar symbolism and transforming it
• Introducing intermediate notations
• Determining the symbolism of choice
• Linking the symbolism to the graphic register
• Choosing a symbolism linked to the context
WDMs explicated by postsecondary teachers
• Presenting the pre-given symbolism
• Translating (into) the pre-given symbolism
• Explicating/specifying the meaning of the symbolism
• Giving voice to the symbolism
• Substituting the symbolism
• Using a symbolism that varies according to the circumstances

mathematical contents, in our study, one can discern differences in the ways symbolism is used at each teaching level. Symbols and their use may initially appear to be similar, but the WDMs that bring such symbols into play are very different.

6.2 A symbolism that "speaks" to students versus a symbolism that is "given voice"

The "transparency" of symbolism is not expressed in the same way at both teaching levels. The secondary and postsecondary teachers agreed that students have difficulty grappling with the opacity of symbolism. However, the WDMs constituted are different. On the one hand, the secondary teachers chose a transparent symbolism that "spoke" to students and that referred to the context (they maintained intermediate notations, settled on the same symbolism over a long term, etc.). In contrast, the postsecondary teachers "gave voice" to the standard symbolism (they translated it, presented it as pre-given, etc.).

Once again, the difficulties for students can be readily perceived. For instance, the use of a constant symbolism to which they were accustomed all of a sudden turned into a field of possible variations. This finding is consistent with other studies that have highlighted that any form of manipulation of symbolism at the start of postsecondary education results in errors (Corriveau, 2007). Whereas most studies have emphasized that these errors stem essentially from the fact that students do not reason with the symbols they manipulate (Corriveau & Tanguay, 2007; Weber, 2001), the results of the present research instead bring out how this issue is more complex. If students do reason with symbols the way they were accustomed to doing so at the secondary level, this will also become a source of difficulty for them.

6.3 A certain generalization versus clear-cut generality in the use of symbolism

Through their actions, teachers "create" a certain generality. For example, $f(x) = b^x$ is neither more general nor more particular than $f(x) = ac^{b(x-h)} + k$: it is all a question of what one does with it.

The WDM observed among the secondary teachers and referred to as "Working from a basic or familiar symbolism and transforming it" endowed this transformation with a certain notion of expansion: the issue here is to make visible all the possible cases within a given family of functions. A process of generalization is implicitly involved. The teachers did not seek to emphasize the process of generalization itself; they sought instead to extend the family (of exponential functions). The implicit effect of this manner of presenting the function was to make the function presented in the form of $f(x) = c^x$, the basic function, and to allow the form $f(x) = ac^{b(x-h)} + k$ to represent all the possible elements of the family of functions. In terms of the way symbolization was performed at the secondary level, the latter approach to presenting the function was thus more "general" than the former.

At the postsecondary level, in the context of functions, x, or any other letter inherently contained all possibilities. Even from the start, there was a notion of generality whenever teachers stated that x can be $x +$ delta, $5 + h$ (obviously, they had in mind the formal expression of the derivative as limit). They said that they then went on to play with the expressions and that only at that point did they mention that x could become sin x in the work to follow. Thus, in a way, this manner of writing the function $f(x) = c^x$ included the entire family of exponential functions and even extended beyond it. In addition to representing the entire family of functions of the type $f(x) = ac^{b(x+h)} + k$ (in which letters represent numbers), it could also be combined and could open more broadly onto functions other than of the exponential variety, such as $f(x) = c^{sin\, x}$.

When set side by side, these results bring to a light a series of "micro-breaks" (to borrow from Praslon, 2000): what was, in the one case, only a specific function turned out, in the other case, to be general, representing an entire class of functions. While Vandebrouck (2011a) has shown that the shift between "working domains" (Robert, 2003) is not the same during the transition to the postsecondary level (i.e., a more algebraized domain occurs at the postsecondary level), there are also, in addition, differences that are more implicit in nature and that are related more to the meaning of symbolism and the generality that arises from the circumstances and the ways it is used.

This work with teachers led us to areas of the secondary-tertiary transition that have been little explored until now. The contribution of our research with respect to previous studies is thus twofold: first, it sheds light on a previously unexplored key issue of transition. Secondly, it provides a basis for grappling more specifically with WDMs concerning symbolism and its use, the organization and underlying logic of WDMs, as well as the informal level of mathematical culture surrounding these practices. At this point, there is still a need to examine the linkages between these two cultures so as to smooth the transition for students from the secondary to the tertiary level. An initial exploration of this kind was conducted with the teachers involved in the present research project. It has shown that worthwhile perspectives are opened up by a problematization of the use of symbolism at a given teaching level, in which teachers at one level are able to gain access to the "territory" of teachers at the other level and appropriate some of the others' investigative approaches (Corriveau & Bednarz, 2016). Further research will be required in order to more fully examine such perspectives.

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