Educ Stud Math (2015) 89:357–373
DOI 10.1007/s10649-015-9605-3

Research mathematicians' practices in selecting mathematical problems

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Published online: 21 May 2015

Abstract Developing abilities to create, inquire into, qualify, and choose among mathematical problems is an important educational goal. In this paper, we elucidate how mathematicians work with mathematical problems in order to understand this mathematical process. More specifically, we investigate how mathematicians select and pose problems and discuss to what extent our results can be used to inform, criticize, and develop educational practice at various levels. Selecting and posing problems is far from simple. In fact, it is considered hard, complex, and of crucial importance. A number of criteria concerning personal interest, continuity with previous work, the danger of getting stuck, and how fellow mathematicians will respond to the findings are considered when mathematicians think about whether to approach a specific problem. These results add to previous investigations of mathematicians' practice and suggest that mathematics education research could further investigate how students *select and develop* problems, work with *multiple problems over a longer period of time*, and use the solutions to problems *to support the development of new problems*. Furthermore, *the negative emotional aspects of being stuck* in problem solving and students' *conceptions of solvability* and *relevance of or interest in* a mathematical problem are areas of research suggested by our study.

Keywords Mathematical practice · Mathematicians · Problem choice · Problem solving

1 Introduction

In this article, we explore how mathematicians select mathematical problems to work with. We show that they consider the selection of problems to be difficult and, moreover, that the

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selection of problems is strongly related to considerations about own ability and interest, as well as about what the mathematical society or a certain community of peers finds interesting. The mathematics education community has been interested in processes relating to generating, selecting, and posing mathematical problems for many years (Brown & Walter, 2005; English, 1998; Kilpatrick, 1987; Pólya, 1945; Silver, 1994). Inquiry trends in science and mathematics education continue and augment this research agenda (Artigue & Baptist, 2012; European Commission, 2007; National Research Council (U.S.), 1996).

Despite discussions in educational research and policy about having students actively participate in finding, formulating, and selecting mathematical problems to work on, we do not yet have an adequate understanding of how mathematicians (and other relevant societal groups) handle this part of their working process. This article contributes to developing such knowledge by empirically investigating how researchers in the field of pure mathematics select the problems they want to work on.

Our argument builds on the assumption that an improved understanding of the mathematical profession and the competences needed to practice mathematics at a professional level are relevant for mathematics education. Of course, schooling and research are different domains; schooling involves obligations other than the creation of future mathematicians, but we do suggest that an understanding of mathematicians' work processes concerning the posing and selection of problems is needed. In the article, we describe the literature on posing and selecting mathematical problems in classroom settings, as well as among mathematicians. After that, we describe the methods and results of an interview study, suggesting that mathematicians balance concerns relating to interest, ability, and likelihood of obtaining results, as well as the values and interests of peers, when they make decisions about research paths. Finally, we discuss the possible influence these insights about cognitive, metacognitive, and social aspects of doing mathematics might have on everyday practice in mathematics classrooms.

2 The state of the art

In this section, we describe the state of the art with respect to knowledge about the problem formulation and problem selection processes in the teaching and learning of mathematics and how this relates to knowledge about practices among mathematicians.

Students' participation in finding, posing, and selecting problems to work on has been investigated as a question of "problem posing" (Brown & Walter, 2005; Cai, Moyer, & Wang, 2013; English, 1997; Kilpatrick, 1987; Silver, 1994). Problem posing allows students to engage mathematically with different situations. Silver (1994) reviews the reasons for and possibilities in addressing problem posing in the teaching of mathematics. He observes that problem posing can be used as a window into students' mathematical thinking and that problem posing is a rich instructional arena for bridging cognitive and affective aspects of mathematics.

Brown and Walter's *The Art of Problem Posing* (2005), first published in 1983, stands as both a coherent contribution to problem posing research and an extensive description of educational practice building on problem posing and selection. The authors take as a point of departure the observation that mathematical problems in educational settings are typically given by an authority to the students to solve on their own. Brown and Walter's ambition is to explore alternatives to this model. They argue that students should engage individually and collaboratively both in the creation and in the solution of mathematical problems.

The main strategy that they adopt consists in supporting students' divergent thinking when they are faced with a mathematical situation. This is described as a "what if not" strategy, where conventions, unspoken and spoken, and the mathematical facts of situations are challenged and systematically ignored. This approach results in a multitude of potential mathematical situations that, as argued by Brown and Walter, significantly enriche the classroom.

This also demonstrates the limit of the problem posing approach, however, because the what if not strategy might very well allow mathematicians to create a lot of mathematical problems, but it will not help them to judge whether they are worth solving or too difficult. As Brown and Walter (2005, p. 167) put it:

> *[I]t is frequently difficult to judge the value of a question. Sometimes we do not know how simple, revealing, delightful or foolish a question is until after considerable analysis has taken place.*

Although Brown and Walter notice this difficulty, they do not seriously address the problem. However, in a different contribution, Brown (1984) suggests that the conception of mathematics as a collection of technical means to solve already existing problems is misguided. Rather, mathematics should be considered as a way to make meaning in various situations related to mathematics. Similar critiques of the viewpoint that mathematics consists of chains of posing, choosing, and solving problems can be found in the context of curricular documents for mathematics (Appelbaum, 1999), in the relation between local culture and mathematics teaching (d'Ambrosio, 1985), and in the context of the nature of mathematical research (Davis & Hersh, 1981; Thurston, 1994). Furthermore, Kirshner (2002) shows how different ways of conceptualizing and making sense of the goal of teaching mathematics give rise to different conceptions of what a "problem" is and also of the relation between the mathematical practices of researchers and of school children.

In their book *Thinking Mathematically*, Mason, Burton, and Stacey (2010) take on the task of teaching mathematical thinking processes. They describe mathematical work as a dance wavering between an entry phase, an attack phase, and a review phase. Entering a mathematical problem means internalizing the question by really reading the question in order to understand it (Mason et al., 2010, p. 26). This description suggests that the work related to attacking and reviewing mathematical questions or concerns is a major source for entering new mathematical problems and that choosing which problem to "enter" is relevant for students of mathematics. Mason et al. (2010) describe the problem of being stuck, and they suggest that unpleasant as it may seem, being stuck is mainly an opportunity to learn (p. 46), even though it might result in abandoning a problem and consequently addressing another question.

With this article, we aim to contribute to the discussion concerning problem posing and selection as educational activities by providing insights into how professionals in the area of pure mathematics conduct problem posing and selection.[1] We focus on the process of problem *selection*. Hence, we give more attention to considerations and criteria for choosing directions for research than to the skill of stating a problem in a mathematical fashion.

[1] A sampling strategy considering that pure but not applied mathematics can of course be contested, because applied mathematics—with its attention to models and real-life situations—could be more suitable as inspiration to and mirror of the teaching and learning of mathematics. However, there are a multitude of practices in society that can serve as such inspiration (engineering, science, and economy to mention a few). Furthermore, in this investigation, we have chosen a narrow focus and aim only at relating the selection of research problem in the area of pure mathematics, to problem posing and selection when teaching and learning mathematics.

The largest investigation into professional mathematical practice and its relation to mathematics education is Burton's (2004) interview study of 70 research mathematicians. Her findings suggest that mathematics is increasingly a collaborative effort and that the discipline is highly competitive. Interestingly, Burton also finds that emotions play a vital part in the mathematical practice (Burton, 2004, p. 88). These emotions include the joy of working together and sharing discoveries with other mathematicians, but importantly, they also include the excitement of exploiting new mathematical territories and satisfaction—or even euphoria—when a problem is solved or a new discovery made. Burton acknowledges that mathematicians also get stuck (Burton, 2004, p. 59, 190), but she maintains that it is a positive state and quotes the conclusion from Mason et al. (2010, p. 46)[2] that being stuck is "an honourable and positive state, from which much can be learned" (Burton, 2004, p. 59).

According to Burton, positive emotions acts as motivating factor for doing mathematics and emotions and feelings form the basis of mathematicians' intuitions, including their intuitive understanding concerning whether they can get results from a given problem. Thus, although Burton does not explicitly make this conclusion, it seems clear from her work that the emotive side of mathematics forms part of the process of problem choice. This aspect of emotions are important to this paper, and we will add to the discussion about how emotions and feelings—both positive and negative—shape mathematicians' intuitions concerning whether they can get results from a given problem.

The research questions we explore in this article are as follows:

- How and under what circumstances do mathematicians choose the problems they work on?
- How is their choice conditioned and what criteria do they use to guide their choice?

3 Investigating mathematicians' problem choice

We collected data using a semi-structured approach in our interviews (Kvale, 1996). After conducting the first four—rather explorative—interviews (R2, R5, R7, and R9),[3] we transcribed the interviews, completed an initial analysis, and revised the interview guide (Strauss & Corbin, 1990). We have used grounded theory to analyze the transcripts obtained from interviewed mathematicians. Grounded theory (Charmaz, 2006; Strauss & Corbin, 1990) allowed us to develop the categories and themes used to classify the mathematicians' utterances from the transcript through an iterative process of coding and collecting codes to categories and themes.

We interviewed 13 male mathematicians in total. We met all of the interviewees in their workspace (typically in their office, and on one occasion in an adjacent meeting room). The interviews took between 30 and 60 min, except on one occasion, where the interview lasted almost 100 min. All interviews were recorded and transcribed. Pictures or scans of visual representations produced or referred to by the mathematicians in the interview were collected. As sampling criteria, we used the interviewees' academic position and portfolio, age, field, mother tongue, and institutional distribution. We deliberately selected mathematicians who were mature enough in their career to have experience with different aspects of mathematical

[2] Burton quotes an earlier version (from 1982—in this edition, the quotation is on page 49) of Mason et al. (2010), cited in this article. Leone Burton is a co-author of the book.

[3] To secure the anonymity of the interviewees, we will refer to them using a sequence of random numbers (obtained from www.random.org) throughout the paper. Thus, the order of the numbers might not reflect the order in which the interviews were made.

work. Consequently, we only interviewed researchers in tenured positions. Regarding age, we aimed at interviewing mathematicians between 30 and 50 years of age. The lower bound was dictated by the criterion mentioned above, whereas the upper bound was dictated by an intention to interview mathematicians who were still active researchers. We ended up including some interviewees who were in their 50s. We chose to limit our investigation to researchers of pure mathematics, but we included interviewees from various fields such as algebra, topology, and analysis. We aimed to conduct interviews in the mother tongue of both the interviewee and the interviewer, or at least with a minimum language barrier.

The interviews all followed the same structure. After a short briefing about the research project, we explained that the purpose of the interview was to learn about the practices of mathematics researchers. Furthermore, we explained our expectation that the knowledge gained in the project could inform the philosophy of mathematics, as well as mathematics education research, but that the specific purpose of the interview was to obtain the best possible understanding of the interviewee's work as a mathematician.

The interview guide posed questions about the mathematician's work and problem-solving processes, specific working papers relating to research activities, and a functional "process model" of the written activities relating to mathematical research. This model was taken from Misfeldt (2005).

The questions relating to mathematicians' work and problem-solving process differed in the first and second rounds of interviews, since a question relating to selecting problems (*How do you find a suitable mathematical problem to work on?*) was included as a consequence of our initial analysis of the first four interviews. Later in the interview, we asked the mathematicians to: *Try to describe how you start to work on a problem. Which stages are there in your work, or how would you explain the process? What do you imagine and what do you write down/ draw when you start working on a problem? Can you show me examples from recent work?* Furthermore, we asked about the institutional concerns relating to their choice of problem. More specifically, we examined whether *department management*, *collaborators*, and *funding options* were taken into account when choosing a mathematical problem. The interviews continued with a number of questions relating specifically to the use of external representations, individual drafts, or papers.

All interviews were transcribed and coded in NVivo 10. The analysis presented here is based on the subset of codes that relate in a broad sense to the selection of mathematical problems. The coding that is not reflected on in the analysis mainly revolves around the use of computers and external representations in various aspects of the mathematicians' work.

The codes relating to problem selection were condensed to a more manageable number of categories, as described in Table 1. The extracts from the interviews that are included in the following analysis were translated and cleaned up in order to remove redundancies resulting from transcribed spoken language. The coding and analysis were based on a word-by-word transcription of the interviews. In the coding scheme included below, we show how the codes relevant to understanding the process of problem selection were condensed into the categories that were used to structure the analysis presented in the next section.

The initial interview guide reflected the following three main areas of concern:

- How does a mathematician come to work on a new problem?
- Which artifacts support this work and how?
- What is the relation between the investigated aspects of mathematical activity and the mathematical writing process?

Table 1 Coding, categories, and themes

Codes	Categories	Main themes
Ideas as social	Living with mathematical problems	Choosing problem is important
Long time frame in relation to problem		
How to address as problem		
Work with many problems in parallel		
Several kinds of problems		
Sensing problems		
Mathematician and his problems		
Problem solving		
Problem choice as the most difficult	Problem choice as important	
Problem choice as high stakes		
Problem choice as important		
Problem choice as new/surprise		
Problem choice as management	External conditions	Conditions and criteria for choosing mathematical problems
Problem choice and education		
Problem choice and computers		
Problem choice and fame	The values of the community	
Social criteria for problem choice		
Problem choice as a social process		
Hunting the big problems		
Problem choice is not about funding and university politics	Unimportant factors	
Problem choice is not about usefulness		
Personal interest	Interest	
Problem choice and metacognition	My mathematical horizon	
Possible solution criteria		
Appropriate difficulty		
Continuity with previous work		
Continuity in methods		
Continuity and problem choice		

As described above, we realized from the first four interviews that understanding how mathematicians approach new problems is intimately connected to understanding how problems are selected. Hence, the theme of this article was developed. Apart from the analysis presented in this paper, these data have allowed us to describe mathematicians' use of technology and representations in their work (Johansen & Misfeldt, 2014), as well as the values that they enact when conducting and publishing mathematical research (Johansen & Misfeldt, n.d.).

4 Results

In the following section, we present our findings concerning the development and choice of mathematical problems. The section is organized according to the way in which we have condensed our coding into themes, as described above.

5 Choosing problems is important

"Try to describe how you start to work on a mathematical problem" was the first question in the pilot interviews. When asked this question, all four interviewees changed the subject and began to talk about how to choose problems. One answered as follows:

> R2: *Well, in fact I believe it all starts a bit earlier, because you also need to find the problem you wish to attack. There are so many problems, so you have to make some kind of selection.*
>
> Another interviewee made the following comment:
>
> R5*: Well, of course the most important part is … The most essential part, which comes earlier, is that you have to ask some questions, right, and then you can ask where those questions come from.*

As a result of our analysis of the pilot interviews, we formed the hypothesis that choosing which problem to work on is both difficult and important in mathematical research; thus, we decided to investigate this process further. The remaining interviews confirmed this hypothesis: None of the interviewees considered the choice of problems to be a trivial part of the working process. On the contrary, all of them were very much aware of the importance of finding and choosing good problems to work on. Several of the interviewees spontaneously noted that the process of finding and/or selecting a suitable problem is both hard and important. When asked how he chose problems, R13's response began with the following statement: "As a matter of fact, I believe… in fact, it is the hardest part." Another interviewee answered as follows:

> R12: *It is not easy. The great art in mathematics is to find a problem that you have ideas about how to solve.*
> Interviewer: *Yes?*
> R12: *And in fact have a chance of solving. […] The greatest art is not to solve problems, the greatest art is to find the problems we can solve.*

An obvious reason for seriously considering which problems one chooses to approach is the fact that these problems might remain for years. Several of our interviewees expressed this situation in terms of "having" a problem for many years, and they gave two different reasons for why this situation occurs.

First, the right idea for solving a problem cannot be forced. Sometimes, it is necessary to let the problem rest while waiting for the right idea to appear. Not having the right idea can challenge one's progress as a mathematician; hence, our interviewees saw a strategic advantage to working with several problems at the same time. Second, sometimes problems are not simply given to be solved but are rather developed as part of the problem-solving activity. One interviewee made the following statement:

R1: *Often the problem is not a given. It will develop along the way as you develop new methods, develop new angles of attack, and get new questions that might shed new light on old problems.*

Later in the interview, he commented as follows:

R1: *Often the solution to the problem is not the most important thing. The most important thing is the methods that are developed in order to solve the problem.*

For this mathematician, a long-term relationship with a mathematical problem does not simply occur due to the lack of the right idea. His task as a mathematician involves more than just solving problems; it also involves working with problems, developing mathematical methods to address them, and representing or refinding the problems in different contexts. In that sense, the mathematicians' working process not only involves problem solving but also *working with* problems. Hence, the notion of mathematical problems clearly resonated with the mathematicians' experience in working with mathematics. R10 described this with reference to his wife's interpretation of social situations among mathematicians as follows:

R10: *My wife makes very clear that she doesn't know anyone who looks for problems the way mathematicians do. She hears us in social situations asking each other, 'What problem are you working on?' We ask not because we wish to help with the problems, but because we wish to know what problems other people are working with.*

Thus, "a mathematical problem" is not just a thing to be solved; it is also an area of interest, a source for developing new methods and for finding new problems.

6 Criteria for choosing mathematical problems

Most of the mathematicians in our study used a set of implicitly or explicitly formulated criteria or concerns to guide their choice of problems. In this section, we present these criteria and address how the criteria balance the individual mathematician's personal interests with the conditions in which he is working. In our organization of the data, we see clusters of criteria evolving around the following three main themes: (1) personal motivation and interest, (2) suitable problems of the right difficulty following previous work, and (3) values of, and position in, various mathematical communities.

6.1 Personal motivation and interest

References to pure interest as the main reason for following a research path are sparse in the data. When personal interest is present, it is typically mentioned in combination with other criteria, as it is clearly expressed by one mathematician, who combined interest with a consideration about what it is possible for him to obtain, as follows:

> R11: *[On choice of problem]: It is of course a combination of what you feel ... what you are interested in, and what you feel you have a chance to solve.*

Other interviewees also took the interests of other mathematicians working in the field into explicit consideration. When asked what made him choose a specific problem, one interviewee (R3) said, "It has mainly to do with what I personally find interesting. When that is said, it has also to do with what other people find interesting." Hence, personal interest and engagement with a problem is seen in balance with community-orientated criteria.

6.2 Problems of the right difficulty

To progress as a researcher, it is important to choose problems that are neither too difficult nor too easy. One interviewee expressed this as follows:

> R10: *Typically, you will look for an interesting problem. It should be something others find interesting. It should not be too easy.*
> Interviewer: *No.*
> R10: *Because then it is nothing. There is nothing to work with. Sometimes you start doing something, and then it turns out to be trivial.*
> Interviewer: *Yes.*
> R10: *On the other hand, it should be something you can solve within a reasonable timespan.*

Another interviewee made the following comment:

> R3: *There are lots and lots of mathematical problems, but most of them are so difficult that we do not have the slightest chance of solving them. And then there is a group of problems that we can solve, and it might be that a large part of them are uninteresting. Then, we have a tiny slice of things that are interesting and at the same time within our reach. And it's all about searching for the border [of that slice].*

Searching for "the relevant slice" of mathematical problems—not too easy and not too difficult—is important. Moreover, being able to distinguish between problems that are too easy and too difficult is an expertise acknowledged by the respondents; one made the following statement:

> R5: *So you can say that something changes with the amount of experience you have, right. Today I am much faster at determining whether I can do something about a problem or not. Is there a chance? [...] I have a much better feeling for this today than I had maybe 20 years ago.*

Hence, for R5, the expertise related to choosing a problem develops with experience and over time. It is also noteworthy that the experience needed to choose wisely has an emotional component for R5, as he describes that his "feeling for it" has improved with time.

Finding problems of the right difficulty not only relates to whether a problem is solvable for the mathematician. It also relates to knowing whether other mathematicians are better suited to handle certain aspects of it. When explaining why his group had chosen a specific problem, one interviewee (R13) explained that the topic "is very hot right now. People are interested in this. But we have an angle that gives us a head start. For that reason, I believe our group can, you know, solve [the problem].” In connection to this, R13 also specified that he was not the right person to address a specific part of the work, making the following statement:

> R13: *It is not that I am hopeless at it, right? But some people simply have an extra gear. So, typically I actually try to stay away from it because I am not really good at it. Typically, what I do … well, this is rather specialized, so I know who is good at it.*
> Interviewer: *Okay.*
> R13: *So I’ll talk to them. Or I’ll boss a PhD-student around to do it.*

Another aspect of finding suitable problems of the right difficulty is the attention to previous work and how that determines the types of new problems a mathematician can take on. Almost all the interviewees made it clear that the new problems they took on were often continuations of previous work in some way. One interviewee (R12) explained how new problems “kind of crop up from the things you have previously worked on.” Others explained how the knowledge gained by solving one problem naturally led to other problems. For example, R5 stated that the research questions “typically come from the research you are already involved in,” and in a similar vein, R8 explained, “My latest result opens the possibility that now I can … If I can do this, then I can also do something more, right? That’s usually how it is.”

The continuation can be less direct and more a matter of seeing similarities to other fields or issues. One respondent made the following statement:

> R9: *Another possibility is that you see this problem and think that it is very similar to something else. It might be a different context, but it reminds me of something I know from previous work, and I believe I have the pieces, I just need to put them together in the right way.*

Several interviewees described similarity in method as an important factor in their process of choosing problems. For instance, one interviewee commented as follows:

> R13: *I can see a connecting thread in everything I have done since I was a PhD student. A new tool emerged, and I was in the right place for that. I learnt it and wrote a paper about it that was successful. This has been my main tool ever since.*

Hence, the continuation of prior research seems to be a typical way of finding suitable problems. The continuation can consist of applying a familiar method to a new area, addressing a question that comes as a natural consequence of an obtained result, or simply seeing similarities between prior research and other areas, and using these similarities as a source of inspiration for generating new questions.

6.3 The values of the community

Many different concerns play a role in directing a specific mathematician to pursue a specific problem. In our interviews, we explicitly asked about several different external factors, such as management, applicability of the envisioned results, collaboration with colleagues, and funding possibilities. In general, none of these factors were considered important by the interviewees. A few of them considered some of these factors important, while others did not consider any of them important at all. Only one external factor was persistently and prominently mentioned in all of the interviews, namely the interests and values of the mathematical community. For example, one respondent made the following statement:

R12: *[When I choose problems] I consider what we, in a very narrow community of peers, believe to be important and interesting. But outside ... There are people who consider what things should be used for outside, but as a matter of principle I have no opinion on that.*

Another commented as follows:

R4: *[My supervisor] told me several times that it's very important to think about what other people are interested in. Because that's the only way anyone is ever going to know about the results you get. Because even if you do something and you get some results, it's hard to understand them, nobody is going to invest time in them. They are not interested. And so you might as well not have done it.*

Without any hesitation, the same interviewee (R4) clearly stated that he had exactly three criteria for choosing problems, as follows: (1) the problem should be interesting to him, (2) it should be interesting to other people in the mathematics community, and (3) he should have some expectation of obtaining results. Contemplating what he considered a good mathematical problem to be, another interviewee (R13) answered, "First of all, there has to be someone who finds it interesting, right […] There are some people whose attention I want. Some of the people who are leading figures [in mathematics]."

Several of our interviewees had experienced choosing research agendas that did not catch the interest of other mathematicians. One of them (R13) had spent a lot of time on something only to discover that nobody found it interesting, "and that was not much fun." Another (R3) had experienced difficulties having a result published, and consequently, he had put a line of research on hold, even though he and his collaborator found it very interesting. To spend time on a project, it has to "resonate" with a larger group of people. When contemplating this episode, R3 stated that "in a way, mathematics is a social activity. It is important to communicate with each other, both in research publications, but also at conferences, and on the internet." The need to gain the attention of other mathematicians can be interpreted as a strategic consideration (if one works with problems that other finds interesting, one can pass reviews more easily and publish papers), but it can also be viewed as an emotional statement, in the sense that mathematicians need to belong to and resonate with a social group in order to do mathematics. This last interpretation is supported by the use of emotive expressions (such as "that was not much fun"), but in general, our data do not allow us to single out one of these interpretations as the most adequate.

Asked whether they considered other external factors, such as funding, management expectations, and possible application of the results, most of the interviewees answered "not really" or something similar. There were, however, a few notable exceptions. When asked whether he considered funding possibilities when choosing problems, one interviewee replied as follows:

R13: *Well, it's not that I just think: 'Let's do something that can get us some funding.' Or well ... As a matter of fact, I might just have done that. I've just applied for funding, and in that process I designed a project that had a slightly different angle to the one I would have chosen. So if I get this money—which is not likely—then I'll just have to do it, right?*

Apart from R13, none of our interviewees reported that they had used funding possibilities as a specific criterion for problem choice. However, this does not mean that they were indifferent about funding. Another interviewee (R11) explained that he considered the funding issue covered by following the interests of the mathematical community: If one does research that lives up to the values and interests of the mathematical community (or a relevant subgroup thereof), one will also get funding.

Concerning possible applications, we received one clearly positive response, as follows:

> R4: *Now that I'm here at this technical university, then I do think about it [applications], because I would like to find problems that students here want to work on [...]. But if I were working at the [a classical university], then I wouldn't think about whether it was useful. Personally, I think mathematics is part of our culture, and I think that all mathematics as a body is useful. And when you're working on problems which mathematicians are interested in, then you are contributing to that.*

Although most of the interviewees did not use applicability as a specific criterion for problem choice, they were not all indifferent about the issue. Some of them were in direct contact with people who applied their results in broader contexts; others expressed the hope or even conviction that their work was useful to a broader audience (such as R4 in the quotation above); and some were indifferent (R12).

7 Discussion: implications for teaching and learning mathematics

The notion of problem resonates with the mathematicians' way of describing their work, and in a sense, working as a mathematician means working with mathematical problems. Typically, mathematicians work on the same problems for years, and their problems to some extent shape their identity as a mathematician. In other words, the problems a mathematician chooses play a dominant role in his or her professional life, and consequently, the choice of problems is considered an important strategic choice (as also confirmed by Burton, 2004).

Most of the interviewees worked on several problems simultaneously. This created a situation where they could always progress on at least one of the problems, even if they did not have the right idea to progress on others. In this sense, working with more problems simultaneously minimizes the risk of getting completely stuck in a way that might set back one's career.

Previous work done by a mathematician is often a good source of new problems. The connection between new problems and previous work can have different qualities. It can be a matter of applying a known method to a different area or of seeing similarities and using these as a source of inspiration.

In their deliberate choice of problems, our interviewees balanced a number of concerns and criteria. As a prominent criterion, they had to find a problem interesting if they were to work on it. This criterion, however, was balanced with several other criteria, primarily in the form of considerations such as the following: Is this a fruitful problem? Can I progress on this? Is there a good match between the required work and my skills and competences as a mathematician? Do I have a head start compared to others who might work on the same problem?

The deliberations on these criteria, however, were set in the context of an overarching external condition, namely the interest of other mathematicians. Our interviewees would only pursue a problem if it was considered interesting by fellow mathematicians. They expressed the need to have an audience—the right audience—for their work.

The strong attention to what is valued in the community, along with the obvious possible career impact of working with "popular" issues, could paint a picture of the mathematicians as non-emotional persons, simply maximizing their career output while minimizing effort. This analysis holds some truth, since such explicit "economical" considerations where prevalent among the interviewees. However, the interviewees also suggested that emotions play a role in the decision-making process. The realization that a potential research path was not acknowledged by the community was described in negative emotional terms, and expertise in selecting problems was referred to in terms of feelings with the phrases "gut feeling" and "feeling for," regarding whether it would be possible to make progress on a given problem. To us, this suggests that the wish to attract the attention of other mathematicians is not only a strategic choice dictated by the need to publish papers but also has an underlying emotional component: One cannot do mathematics in isolation but instead needs to belong to and resonate with a social group.

We conclude this article by discussing how the increased understanding of mathematicians' problem selection activities could and should affect mathematics education research and practice.

Despite the obvious differences between students' and researchers' practices, the results presented in this article suggest a number of implications for education. The results described above indicate that the choice of a problem is considered very important to research mathematicians and that they view their own interests, the interests of others, and the perceived chance of obtaining results when they make their choice. All of these aspects of problem choice have implications both for mathematics education research and for teaching practice.

In a first approximation, this study suggests that students should spend significant energy on choosing the right problems to address and should discuss these choices extensively with their peers and teachers. Such discussion must support the process of negotiating what is interesting and important, taking each student's perception and judgment into account; at the same time, it should address how the individual enquirer should subordinate his or her mathematical interest to an existing value system that is, for instance, suggested by the curriculum, by historical sources, or negotiated in the class. Furthermore, students might work with several problems simultaneously in order to avoid the negative effects of getting stuck in their attempts to address a particular mathematical problem. The mathematicians in our investigation invite us to rethink the very notion of problems. Students can work with problems for other reasons than solving them. Some problems can have the purpose of guiding a longer investigation into a mathematical area, and such a problem can be very valuable even if it is never solved in full. However, it is important to judge and discuss the anticipated progress on problems with which one works. It could be important to discuss questions like the following with the students:

1. Can this problem be solved? (By me? By the class? By anyone?)
2. What kind of subproblems can we solve?
3. What can we learn from this problem?
4. Can we develop some kind of method when working with this problem?

Finally, the mathematicians suggest that the results obtained when working with a mathematical problem represent one main source for posing new mathematical problems. This insight suggests an educational practice that consistently asks for new questions, investigative directions, and possible applications whenever obtaining a mathematical result.

The educational suggestions derived from understanding mathematicians' practice with selecting problems echo both existing educational practice and mathematics education literature. However, it also complements, contrasts, and augments this. The mere fact that the mathematicians invest significant consideration in the process of problem selection—and find these considerations to be very important—contrasts with mainstream educational practice, where attention is rarely given to processes of posing and selecting problems. However, noticing this discrepancy is by no means unique or new. Discussing the four questions described above with students is close to a practice endorsed by Brown and Walter (2005) advocating for a problem posing perspective in mathematics. And, the discrepancy between the open investigations of mathematicians and the closed tasks performed by their students has already been described by Burton (2004) in her investigation of mathematicians' practice. Mathematics education research has developed ways of allowing pupils to ask questions and discuss their value (Brown & Walter, 2005; Mason et al. 2010). Furthermore, the educational value of such activities has been supported by empirical evidence (English, 1998; Kilpatrick, 1987; Silver, 1994, 2013). Our study confirms and augments the existing knowledge on the value of discussing, selecting, and posing problems in the classroom. Mathematicians consider similar activities to be important and difficult in their research, but they consider the selection of problems to be an autonomous part of this work. Furthermore, our results imply that the selection must be the hardest and most important part when developing a problem to work on and that selecting problems is serious since a mathematical problem can be something that you work on for a long time. This points to new directions for research in mathematics education, focusing on students' selection processes, as well as their reasons for choosing to work with specific mathematical problems in addition to their ability to pose mathematical problems.

Mathematicians balance concerns of personal interest, the interest of others/the community, and considerations about the feasibility or probability of reaching a result when they choose to address a certain mathematical problem. If we turn to an educational context, this could suggest that we should allow students to express why a specific problem is mathematically interesting for them and discuss such questions in the classroom. Such a practice is in line with Burton's (2004) educational conclusion supporting enquiry and collaboration in the classroom. Our results suggest that such practice could be augmented with discussions about or considerations concerning why a certain mathematical problem is interesting to others, both in an abstract and in a very concrete sense. In this respect, our results can be relevant for discussions related to classroom socio-mathematical norms (Cobb, Stephan, McClain, & Gravemeijer, 2001) and the didactical contract between students and teachers (Brousseau, 1997). Even professional mathematicians align their personal interest in a mathematical problem with interests, norms, and values of specific other mathematicians and the larger community, and such social concerns might very well be even more important in a teaching situation where the students are likely to be less independent in doing mathematics than research mathematicians.

Brown and Walter (2005) claim that it is difficult and complex to judge whether a problem or question is valuable until *after* you have worked with it for a long time. This fact aligns with the experiences of the mathematicians in our investigation; however, this does not mean that such a competence is unobtainable. Although the mathematicians in our investigation explained that they only acquire this competence relatively late in their career and after education has ended, this might not need to be the case. The process of judging the value and difficulty of mathematical problems could be included in the teaching of mathematics or at least be developed as an area in relation to problem posing in mathematics. Discussing the value and difficulty of mathematical problems may open a window onto students' mathematical understandings as shaped by their everyday and school experience. Silver (1994) claims that problem posing does the following (p. 24):

> *[It] provides not only a window through which to view students' understandings of mathematics, but also a mirror which reflects their school mathematics experience.*

Discussions of what makes a problem difficult and what makes it important are obvious ways to learn more about students' conception of mathematics and how these conceptions are shaped by conventions of schooling and classroom norms.

Finally, the analysis presented in this article augments our knowledge about the relation between cognitive and emotional aspects of doing mathematics in at least two ways. We find that the satisfaction and positive emotions that mathematicians have when working with mathematical problems are in part related to how the mathematical problem that they work with is perceived by others, and we find that getting stuck in problem solving or pursuing a research direction not acknowledged by others is connected with negative emotions. Hence, mathematicians do what they can to avoid this.

The social nature of mathematics and the desire to share and contribute to the collective project of mathematics is well described by Burton (2004), and our findings in relation to the positive emotions connected to doing mathematics confirm Burton's results. The educational implications of Burton's findings consist of acknowledging the classroom as a community of mathematics enquirers and promoting sharing and discussing mathematical results in this community.

Negative emotions play a larger role in our analysis than that suggested by Burton (2004). In our investigation, we see negative emotions both in relation to being stuck and in relation to working with a problem that does not resonate with the community. Thus, the positive emotions related to sharing and being valued by the community is in our investigation mirrored by negative emotions of equal importance.

Burton (2004) and Mason et al. (2010) both describe being stuck as an honorable state that supports learning. However, we cannot find strong support for this statement in our data. The mathematicians in our investigation know that they will get stuck once in a while, but they do not find it honorable or attractive in any way. They do what they can to avoid it or at least to cope with it. Rather than honor, our respondents expressed danger and frustration in relation to getting stuck. There can be a number of reasons for this discrepancy, including increased competitiveness and pressure on researchers of mathematics (more than a decade has passed since Burton's investigation), cultural differences between the UK and Denmark, and so on.

We think that it is important to acknowledge negative emotions in relation to doing mathematics. If students are developing their own questions and choosing which research directions to pursue, then there is great danger that they will get stuck in their problem-solving process or that they will investigate research questions that no one except themselves find valuable. Hence, such problem posing, selection, and inquiry-oriented pedagogies should be able to cope with students' negative emotions. Such a result is contrary to the results of Brown and Walter (2005), Burton, (2004), and Silver (1994), who all suggest that problem posing and inquiry-based mathematics education represent a way to cope with students' negative emotional relation to mathematics. Of course, we are not suggesting that these scholars are wrong; if done well, changing teaching of mathematics in an enquiry direction can definitely help to overcome some negative emotions. However, we do suggest that other problems related to negative emotions can occur as a result of such a change. A mathematical classroom based on closed tasks formulated by the teacher or the textbook in an abstract fashion can obviously attract negative emotional feedback from students, and more open approaches can overcome some of these problems. Still, we believe that different but equally important concerns with emotional relation to mathematics can occur in these more open situations.

Finally, the results of the investigation suggest that we need to untangle the meaning of "mathematical problem". The data suggest that the notion of problem resonates with the mathematicians' conception of, and jargon around, their work. But, it is also clear that these mathematical problems are different from word problems in a textbook. Mathematicians' problems last for years, and they guide mathematical investigations, allowing mathematicians to suggest new mathematical constructs and to move into different domains of mathematics, meeting and working with new colleagues. Mathematicians' conception of problem is closer to the problems posed by the students and pupils in Brown and Walter's work (2005). But they are "bigger" and more "directional", in the sense that they determine the route for the mathematician's work for a longer period of time.

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