Educ Stud Math (2015) 89:267–281
DOI 10.1007/s10649-015-9597-z

Multiple representation instruction first versus traditional algorithmic instruction first: Impact in middle school mathematics classrooms

Raymond Flores · Esther Koontz · Fethi A. Inan · Mara Alagic

Published online: 22 March 2015

Abstract This study examined the impact of the order of two teaching approaches on students' abilities and on-task behaviors while learning how to solve percentage problems. Two treatment groups were compared. MR first received multiple representation instruction followed by traditional algorithmic instruction and TA first received these teaching approaches in reverse order. Participants included 43 seventh grade students from an urban middle school in Midwestern USA. Results indicated gains in knowledge from both treatment groups; however, the differences between groups were nonsignificant. Comparisons of effect size however, indicated larger growths in abilities to solve among students who received multiple representation instruction first. In addition, statistical differences between on-task behaviors were found in favor of the traditional algorithmic approach.

Keywords Multiple representations · Algorithms · Rational numbers · Middle school

1 Introduction

Research suggests that the ability to solve complex problems and transfer skills to new situations is related to how well students' procedural and conceptual knowledge have developed (Hiebert & Carpenter, 1992). Procedural knowledge in mathematics refers to the ability to execute algorithms and encompasses knowledge of procedures, symbols, and domain

R. Flores (✉) · F. A. Inan
College of Education, Texas Tech University, 3008 18th St., Lubbock, TX 79409, USA
e-mail: raymond.flores@ttu.edu

F. A. Inan
e-mail: inanfethi@gmail.com

E. Koontz
Horace Mann Dual Language Magnet School, 1243 N. Market, Wichita, KS 67214, USA
e-mail: ekoontz@usd259.net

M. Alagic
College of Education, Wichita State University, 1845 Fairmount, Wichita, KS 67260, USA
e-mail: mara.alagic@wichita.edu

conventions (Hiebert & Lefevre, 1986). Conceptual knowledge, on the other hand, is more networked, connected, and rich in relationships between the concepts of a domain. For mathematics and other domains, both kinds of knowledge have been hypothesized to contribute to *procedural flexibility* or the ability to solve a range of problems flexibly and efficiently (National Research Council [NRC], 2001). Howe (1999) proclaimed that there is no serious conflict between procedural knowledge and conceptual knowledge. In fact, many leaders in mathematics education today support the idea that students must have a balance of both conceptual understanding and procedural fluency in all areas of mathematics (National Council of Teachers of Mathematics [NCTM], 2000). In addition, the writers of Common Core State Standards for Mathematics (CCSSO & NGA, 2010) share the belief that both conceptual understanding and procedural fluency are essential for student mathematics learning, and Hiebert and Lefevre (1986) add that there are many benefits when conceptual and procedural knowledge are linked.

The teaching of many mathematical topics in the USA however, often relies heavily on algorithmic approaches that emphasize procedural skills (Ma, 2010; NRC, 2001; Van de Walle & Lovin, 2006). For example, instruction on rational numbers (e.g., fractions, decimals, and percents) and their manipulations is traditionally algorithmic and rule-based and relies on sets of procedures aimed at making students quick and accurate when solving problems (National Research Council, 2001). Traditional algorithmic instruction, a form of direct instruction, often begins with teachers stating an algorithm (e.g., "to divide by a fraction, invert and multiply"), teacher-led demonstrations of how the algorithm works by presenting several examples, and then student practice, independently or in groups, on similar exercises. While algorithmic approaches have been found to be efficient methods for teaching students how to solve problems (Newton & Sands, 2012), major issues arise when, as a result of these approaches, students begin to view mathematics as sets of rules and give up their own mathematical sense making while carrying out the steps of an algorithm (Fosnot & Dolk, 2002). The National Research Council (NRC) finds that the "rules for manipulating symbols are being memorized but students are not connecting those rules to their conceptual understanding nor are they reasoning about the rules" (National Research Council [NRC], 2001, p. 234). The unintended consequences are that many students are not engaged in their learning of mathematics, forget important mathematical concepts from year to year, and are not fully prepared for higher-level mathematics (Rasmussen et al., 2011). For teachers, this causes concerns as without the connections that reasoning and sense making provide, a seemingly endless cycle of re-teaching may result (NCTM, 2009).

In efforts to promote deeper understanding of mathematical topics, various methods have been used and recommended. One successful teaching approach for helping students make better sense of mathematics and develop deeper conceptual understanding is the use of multiple representations (Fosnot & Dolk, 2002; Ng & Lee, 2009; Van den Heuvel-Panhuizen, 2003). Research suggests that engaging students in mathematics through multiple representations (MRs)—such as diagrams, graphical displays, and symbolic expressions—helps them to better visualize, simplify, and make sense of abstract mathematical topics, and using representations flexibly is a key characteristic of skilled problem solvers (Lamon, 2001; NCTM, 2000; NRC, 2001). Using representations however, not only to the product or student created model, but to the process and act of capturing mathematical concepts or relationships using student created models (NCTM, 2000). Van den Heuvel-Panhuizen (2003) proclaims, "it is not the models in themselves that make the growth in mathematical understanding possible, but the students' modeling activities" (p. 29). Therefore, when using MRs, it is important for teachers to place emphasis beyond the model and more on students' focus on sense making while using the models, their justifications, and the use of multiple methods to find solutions.

Lamon (2001) found that by using different representations of rational numbers students gained a deeper understanding of them and were better able to transfer their knowledge from one model to another. Hence, representations are recommended as an essential component of mathematical activities and means for capturing mathematical concepts (e.g., Goldin & Shteingold, 2001).

While traditional algorithmic instruction and multiple representation instruction are both useful for helping students achieve a balance of conceptual understanding and procedural fluency, questions still remain as to how these approaches should be integrated to best meet the learning needs of students. For example, should teachers begin with teaching approaches that help students develop conceptual understanding first (e.g., using multiple representations) or teaching approaches that help students develop procedural fluency first (e.g., using traditional algorithms). Furthermore, questions remain as to the impact the order of these teaching approaches may have on student learning outcomes. The research questions that this study will investigate are as follows:

1. Does the order of teaching approaches (MR first versus TA first) impact students' abilities to solve mathematics problems that involve fractions, percents, and decimals?
2. Does the teaching approach (MR versus TA) impact students' on-task behaviors?

1.1 Theoretical perspectives

For many decades, researchers have attempted to examine how conceptual and procedural knowledge influence and impact each other (Byrnes & Wasik, 1991; Canobi, Reeve, & Pattison, 1998; Rittle-Johnson, Siegler, & Alibali, 2001). From this body of research, various theoretical perspectives have emerged (see Rittle-Johnson et al., 2001; Schneider & Stern, 2010). *Concepts first* theory conjectures that students initially gain conceptual knowledge and then derive and develop procedural knowledge from it through repeated practice with solving problems (Halford, 1993). Empirical evidence supporting the concepts first perspective has been found for the teaching of various mathematics concepts including simple arithmetic and proportional reasoning (Byrnes, 1992; see Rittle-Johnson et al., 2001). The concepts first theory has been used to justify teaching conceptual knowledge before procedural knowledge (Putnam, Heaton, Prewat, & Remillard, 1992). *Procedures first* theory, on the other hand, states the opposite and suggests that students first learn procedures and from practice with those procedures gradually develop conceptual knowledge (Karmiloff-Smith, 1994). Similarly, empirical evidence has also been found in support of the procedures first approach for teaching various mathematical concepts such as counting and fraction multiplication (Briars & Siegler, 1984; Byrnes & Wasik, 1991).

While these two perspectives continue to be debated, the importance of these perspectives is their implications for how teaching approaches should be sequenced to best meet the needs of students. In support of the concepts first theory, the researchers of this study hypothesize larger gains in student learning outcomes for those presented with MR approaches before TA approaches.

1.2 Purpose of the study

The purpose of this study was to examine and compare the impact of the order of two teaching approaches (e.g., multiple representation (MR) and traditional algorithmic (TA)) on students' abilities to solve mathematics problems that involve fractions, decimals, and percents. Additionally, this study sought to examine whether the teaching approaches (e.g., MR versus TA) impacted on-task behaviors while learning. To be successful in algebra, students should be fluent with rational numbers, their operations, and the ability to convert between equivalent forms (i.e., decimals, fractions, and percents) (Bottoms, 2003). Due to the importance of these mathematical skills, the population of interest for this study was middle school-aged pre-algebra students, and the mathematical topic of interest was percentage problems that involved decimals and fractions.

2 Method

2.1 Sample

This study was conducted in an urban middle school in the midwestern region of the USA. Almost half (44.17 %) of students in the middle school are classified as economically disadvantaged; a small percentage, 3.39 %, are English Language Learners (ELLs), and African Americans make up the largest percentage with 43.16 %, followed by Whites with 36.04 %.

Participants for this study included 43 advanced skills seventh graders enrolled in two pre-algebra sections. Students in the given pre-algebra sections were identified by former teachers as “advanced” and more quickly advancing than “on-grade” peers. This classification was determined by the school based on a combination of classroom performance, motivation, and scores on standardized tests. Over half were males (56 %), and all students were 12–13 years old. Students came from very diverse ethnic backgrounds including 19 % African American, 9 % were Asian American, 5 % were Hispanic, 37 % were Whites, 26 % were mixed race, and 5 % indicated other. No participant involved in the study had an Individualized Education Program (IEP) that outlines accommodations that the teacher should make to better meet the learning needs of individual students.

The teacher involved in this study taught both classroom sections. At the time of the study, the teacher had over 6 years of teaching experience at the urban middle school. Further, while her instruction included traditional algorithmic approaches, she was known for emphasizing teaching for understanding, providing students with diverse real-world experiences where mathematics was used, posing problems to students that could be solved using multiple strategies, and encouraging students to use and create multiple representations to solve problems. This teacher had also been involved in professional development on using multiple representations in her teaching and was part of a district committee that developed lesson plans for mathematics teachers in the school district. Throughout the school year, her students had been exposed to both teaching approaches (i.e., use of algorithmic approaches and multiple representations) in her classroom along with other mathematics concepts.

2.2 Content

The content for this study included a seventh grade pre-algebra unit on percentage problems involving fractions and decimals. Problem types included (1) finding the unknown part of a number represented by the percent of a whole number, (2) finding the unknown whole number when the percent and the part were known, (3) finding the unknown percent when the part and the whole numbers were known, and (4) combinations of the three types.

2.2.1 Teaching approaches

For research purposes, two teaching approaches also guided the design of instructional activities of lessons within modules. These two approaches are explained below.

Traditional algorithmic (TA) approach The TA teaching approach relied heavily on the use of commonly used algorithms and mnemonic devices for solving percent problems in US mathematics textbooks (Bennett et al., 2007). For example, students were given the mnemonic device, “is over of; percent over 100” and instruction/practice with setting up proportions for the various problem types (see Table 1).

During the TA approach students were provided with direct instruction on how to solve problems using algorithms. The steps usually followed during the algorithmic approach were as follows: (1) identify the problem type, (2) set up a proportion to represent the problem, (3) solve the problem, and (4) check and verify solution. During TA lessons, students were provided with guided, independent, and group practice on how to solve these problems step-by-step. In the TA approach, emphasis was placed on using the proportion set-up algorithm (see Eq. 1). Using the mnemonic device, students set up proportions for problems where *is*, *of*, and *percent* were cue words to guide them in their placement of the known and unknown values.

$$\frac{\text{is}}{\text{of}} = \frac{\%}{100} \tag{1}$$

An example problem that students could solve with this algorithm is, “What is 20 % of 4000?” Students would use “what is” to place an unknown variable (x) in the first numerator. They would use “of” to place the 4000 in the denominator. Finally, they would place 20 (%) over 100 (see Eq. 2). To solve the unknown value in the proportion, students used the already familiar “cross multiply and divide” algorithm from a previous unit. Basically, students would multiply 20 times 4000 and 100 times x. The result would be the equation $100x=8000$. Students would then divide 8000 by 100 to find their solution.

$$\frac{x}{4000} = \frac{20}{100} \tag{2}$$

Multiple representations (MR) approach The MR approach included opportunities for exploring multiple representations and multiple solution methods and mathematical communication. In the MR approach, students learned and were expected to solve percent problems by using various representations and models, explaining their methods in writing, discussing strategies with their peers in groups, and rationalizing methods verbally with their teacher and peers. Examples of representations and models used by students included chunking, number lines, double number lines, percent bars, ratio tables, and writing equations. While students were introduced to these MR approaches, students were encouraged to explore, create their own models, and given the choice of which representation they wanted to use to solve problems. A major aspect of the MR approach was that students justified the representation they used. An additional aspect of the MR approach was that students were encouraged to estimate and self-

Table 1 Proportion algorithm setup by the major problem type

Problem type	Example problem	Algorithm for setting up the proportion
Finding the percent of a number	15 % of 240=n	$\frac{15}{100} = \frac{n}{240}$
Finding the percent one number is of another	p% of 240=18	$\frac{p}{100} = \frac{18}{240}$
Finding a number when the percent is known	15 % of n=18	$\frac{15}{100} = \frac{18}{n}$

evaluate whether their answers made mathematical sense. For example, one MR method, "chunking," was used to calculate or estimate percentages by using the benchmarks, 10 and 1 %. To calculate 32 % of 200, students could add three times 10 % of 200 (or 20) plus two times 1 % of 200 (or 2) to get the correct answer 64.

2.3 Design

This study involved a quasi-experimental design using two intact groups, namely two pre-algebra classes which students had already been assigned to based on their individual scheduling needs. Students from both groups received each teaching approach (i.e., MR and TA); however; they were offered in different order dependent on which treatment group the students were placed into. The MR first group ($N=22$), experienced MR lessons first in module 1 and then TA lessons in module 2. The TA first group ($N=21$), received TA lessons in module 1 and then MR lessons in module 2 (see Table 2). This counterbalanced design is ideal for students in an academic setting where the teacher wants to ensure that all students have had an optimum opportunity to interact with the subject area (Best & Kahn, 2006).

MR versus TA lessons Lessons within the two approaches differed in the type of instruction provided and the types of activities that students were asked to engage in. For example, one lesson topic was finding percentages of numbers (e.g., what is 20 % of 20?). In the TA treatment, the teacher provided students with step-by-step instructions including multiple examples of how to set up proportions and equations to find the solution. For example, in the TA treatment, to find 20 % of 20, students were taught how the set up the proportion $20/100=n/20$ and then shown how to solve the proportion. In the MR treatment, on the other hand, sense making through the use of multiple representations was emphasized. In the MR lesson, the teacher provided instruction, examples, and guidance to students on how to use visual percent bars to find 20 % of 20 (see Fig. 1).

Using percent bars allowed students to make sense of the meaning of 20 % of 20 and also helped them connect percentages to fractions. In other words, finding 20 % of a number is the same as finding 1/5 of a number.

Regardless of whether the lesson integrated TA or MR approaches, all lessons had an introduction where the teacher introduced the topic and provided teacher-led instruction, work time where the students received guided practice and then independent practice, and a closing where the teacher and students reviewed the lesson's objectives and discussed things that they learned during the lesson. In summary, while the TA approach followed the structure of the course textbook, the MR approach was a compilation of techniques aimed at creating more opportunities for students to make sense of mathematics as they solved problems. Rather

Table 2 Study design

Group	Pre-data collection	Module 1	Module 2	Post-data collection
	Day 1	Days 2–6	Days 7–10	Day 11
MR first	Pre-knowledge test	MR lessons Observation (day 3)	TA lessons Observation (day 9)	Post-knowledge test
TA first		TA lessons Observation (day 3)	MR lessons Observation (day 9)	

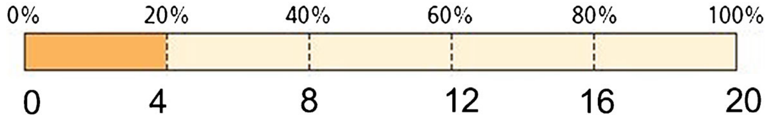

Fig. 1 Using percent bars to find 20 % of 20

than relying on algorithms, MR students created representations or visual models of what fractions, percents, and decimals should look like. Lessons plans for both treatment groups were prepared and reviewed by a team consisting of the teacher and two mathematics educators prior to implementation.

2.3.1 Instruments and measures

Data were obtained through performance tests, scales, and observations during the spring 2013 semester. These instruments are described below:

Pre-/post-knowledge test In order to measure students' abilities to solve problems involving fractions, decimals, and percentages; show their work; and explain their thinking process, locally developed parallel assessments for pre- and post-knowledge assessments were used (see Appendix). Each test consisted of ten open response items. These included two decontextual-based and eight contextual-based problems.

Examples of decontextual-based problems included: *what is 25 % of 32?* and *list the following numbers from greatest to least*: *0.43, 297 %,* $\frac{4}{5}$*, 17 %*. Examples of contextual-based problems included: *Jena is eating at Red Robin. Her total bill comes to $28. If she decides to leave a tip that is 15 % of the total bill, how much should she leave for the tip?* and *40 seventh grade girls are trying out for the basketball team, but only 15 can make the team. What percentage of the girls will make the team?*

To differentiate students' abilities to solve and show their mathematical processes and sense making skills, students were told to solve the problem and to show their problem solving process; however, the approach that students took was not prescribed (i.e., students could use TA approaches or MR approaches). For each item on pre- and post-knowledge tests, students' responses were scored on a 0 to 2 points scale based on the appropriateness of their strategies for solving and the correctness of solutions obtained by those strategies. On individual items: 0 points were given for incorrect strategy (or no work shown) and incorrect solution; 1 point partial credit was given for correct strategy but incorrect solution due to minor arithmetic error, or correct solution but no work shown; and 2 points full credit were given for both correct strategy and correct solution. The maximum score on each of pre- and post-knowledge tests was 20 points. The open-response format of the items and the associated scoring allowed researchers to gather deeper information on students' reasoning and sense making in contrast to commonly used dichotomously scored multiple choice items which often conceal those details. Pre- and post-knowledge tests were administered before and after the treatments. The purpose of these instruments was to examine the impact of treatment conditions on students' abilities to solve and to investigate whether treatment conditions impacted students differently.

On-task behaviors observation form An adapted version of the *Basic 5 Observation Form* (Sprick, Knight, Reinke, & McKale, 2006) was used as a measure of students' on-task behaviors. On-task behaviors observed included students writing or taking notes, tracking the teacher with their eyes, talking with their partners about relevant topics, asking questions, drawing MR models, and/or following directions. Examples of observable off-task behaviors included off-topic conversations, students being out of their seats, students not tracking the teacher or staring into space, and/or not following directions. During the observation, for 5 min, the trained observer focused on students' observable behaviors during the lesson. After 5 s of observation, the observer would record a tally for that student's behavior and then move on to the next student in the row. If the student was on-task the observer would tally a "+" on the form. Alternatively, a "−" was tallied if an off-task behavior was observed. A total of 60 tallies were made during each observation session. The percentage of on-task behaviors was calculated by dividing the total number of on-task tallies by the set total number of tallies (60) and multiplying by 100. For each module, the MR first and TA first received a total percentage of on-task behaviors. This form was used to measure and assess students' on-task behaviors within their treatment groups while learning with each distinctive teaching approach (e.g., MR or TA).

Teacher reflection notes Each day after class, the teacher wrote down her reflections on how the lesson went and students' reactions to the lesson. To organize these notes, the teacher used a reflection notes template that consisted of a blank table with three columns: module day, MR first notes, and TA first notes; and rows for each module day. The purpose of the reflection notes was to gather additional data to support and further explain students' on-task behaviors within their treatment groups while learning with each distinctive teaching approach (e.g., MR or TA).

2.4 Procedures

For both treatment groups, instruction over percents, fractions, and decimals was broken into two modules. Before module 1, students completed a pre-knowledge test. During module 1, students experienced MR or TA lessons depending on the group that they were in. On day 3 of module 1, an outside observer (i.e., trained instructional coach from the school district) completed the On-Task Behaviors Observation Form for each group. On day 7, students began module 2. During module 2, groups were presented with the other teaching approach (i.e., MR first experienced TA lessons and TA first experienced MR lessons). On day 9, a trained observer recompleted the On-Task Behaviors Observation Form for each group. Following the completion of module 2, students filled out a post-knowledge test on day 11 (see Table 2). For each module day, students were in class for approximately 50 min.

2.5 Data analysis

The scores from all instruments were entered and analyzed using SPSS v21. Inferential statistics were then used to examine the impact of the teaching approach order on student learning outcomes (i.e., abilities to solve), and the impact of teaching approach on student engagement. Data analyses included ANCOVAs, chi-square tests, and qualitative data analyses. These are described in Table 3 below.

2.6 Limitations

Methodological limitations for this study include the small sample size, the short duration of treatment conditions, and the fidelity of treatment approach implementation. Consequently,

Table 3 Summary of research questions, variables, and data analyses

Research Questions/Variables	Instruments	Data Analyses
Does the order of teaching approaches (MR first versus TA first) impact student learning outcomes (SLO1) when working with math problems that involve fractions, percents, and decimals?		
SLO1. *Abilities* to solve	Pre-/post-knowledge tests	ANCOVA with pre-knowledge as a covariate
Does the teaching approach (MR versus TA) impact students' on-task behaviors (SLO2)?		
SLO2. *On-task behaviors* while learning	On-task behaviors Observation form Teacher reflection notes	McNemar chi-square Qualitative data analysis

while results may provide valuable insights, they are suggestive and may not generalize to all middle school student populations. Specifically, the use of a sample of convenience may limit the study to middle school pre-algebra students. Further, because the duration of treatment groups was short (i.e., 11 days), students may not have been exposed to the treatments long enough for them to have an impact on their student learning outcomes. Finally, while efforts were made to ensure fidelity in the implementation of both treatment approaches with careful planning, documentation, and reviews, researchers did not observe the teacher and their in-class behaviors and actions during these lessons.

3 Results

3.1 Order of teaching approach: impact on students' abilities to solve (SLO1)

Using pre-knowledge score as a covariate, an ANCOVA was used with group (e.g., MR first versus TA first) as a between-subjects factor and ability to solve as the dependent measure. Although MR first had a slightly higher post-knowledge tests (M=15.41, SD=4.47) than TA first on post-tests (M=14.71, SD=3.69), no significant differences on ability to solve were found between the two groups ($F(1,40)=.931$, $p=0.34$). The covariate, pre-knowledge, was significant ($p=0.05$), indicating that treatment groups differed in prior knowledge (see Table 4).

To examine the differences between the pre-knowledge (PreK) and post-knowledge (PostK) test scores for each of the two treatments groups, two paired sample t test were conducted. Both paired sample t tests were statistically significant for both groups, indicating that MR first (PreK-M=9.68, PreK-SD=2.95; PostK-M=15.41, PostK-SD=4.47), $t(21)=$

Table 4 ANCOVA Results for Teaching Approach Order on Ability to Solve

Source	SS	*df*	MS	*F*	*p*
Pre-knowledge	63.881	1	63.881	4.071	0.050*
Group	14.615	1	14.615	.931	0.340
Error	627.723	40	15.693		
Total	696.791	42			

*Significant covariate at p=0.05

−6.048, $p<0.001$) and TA first (Pre-M=11.05, PreK-SD=3.89; PostK-M=14.71, Post-SD=3.69), $t(20)=3.708$, $p=0.002$) showed improvement on abilities to solve scores from pre-knowledge to post-knowledge tests. Although both groups improved significantly in their abilities to solve, MR first students exhibited larger growth. Comparison of effect sizes, calculated by using the within-subject calculator for means and standard deviations method presented by Morris and DeShon (2002), indicated that the effect size for MR first ($d=1.51$) was much higher than for TA first ($d=0.965$).

Further item level examination of the test scores revealed that all problems on the post-test were answered correctly by more than 50 % of the students except for items 8 and 9 for the TA group. On these items, although the MR group students' success rate was comparably low on the pre-test, they had a much higher success than students in the TA group on the post-test. Analysis of the problem characteristics shows that these items differ from other test items in that they require a two-step process involving an additive operation after the multiplicative one to find the correct answer. For example, to solve item 9 using the traditional algorithm, one must first find 20 % of 250, and then subtract that amount from $250 to find the reduced price. Similarly, to solve item 8 using the traditional algorithm, one must first find what 16 % of 80 is and then add that to 80 (see Table 5 and 6 in Appendix).

3.2 Teaching approach: impact on students' on-task behaviors (SLO2)

In order to explore whether there was a relationship between students' engagement and each distinctive teaching approach used (i.e., MR approach versus TA approach), a McNemar chi-square test was performed. Results indicated a significant relationship was found, indicating that students exhibited higher levels of engagement when TA approaches were used (MR first 78 %; TA first 80 %) compared to when MR approaches were used (MR first 55 %; TA first 57 %) (χ^2 (1, N=240)=11.358, $p=0.001$).

From the teacher's perspective, there was also a noted difference in observable behaviors exhibited by students during the two treatment approaches. In her reflection notes, the teacher perceived and documented more on-task behaviors when TA approaches were used. Specifically, the teacher states that with TA approaches students were more engaged with the task at hand, wrote or took notes, tracked the teacher with their eyes, engaged in discussions with their partners related to the lesson topics, asked more questions related to the tasks, and followed directions. Conversely, the teacher perceived and documented more off-task behaviors when MR approaches were used. The teacher states that students had more instances of off-topic conversations not related to the lesson, falling out of their chairs and laughing, not tracking the teacher during instruction, students staring into space, and students not following instructions.

Students' perceptions towards TA instruction were also more favorable compared to MR approaches. For example, teacher written reflections showed that the MR first group struggled with the multiple representations initially but when introduced to the algorithms in the second module, they were very grateful and made comments such as: "You should have shown us this sooner!" Students felt like the algorithms helped fit together the concepts from what they had learned or struggled with during the MR treatment. Furthermore, the teacher noted that it was hard to motivate the TA first group to engage using multiple representations after being taught the algorithms. During the second treatment (MR treatment) students in the TA first group made statements like: "Can't we just do it with *is over of* [proportion algorithm]?" Teacher reflection notes also suggested that the teacher perceived that students felt like the algorithms were easier than the use of multiple representations. Further, the teacher indicated that she also experienced difficulty with motivating TA first students (TA first) to engage in the modeling (MR) activities having already received instruction on algorithms. The teacher states that students in TA first "already knew a method for solving the problem, and during module 2 they asked whether they could use proportion algorithm instead." This preference may be due to the novelty of the MR approach as the teacher noted that students "complained about the newness of the modeling approaches [MR]" and vocally expressed that they "experienced difficulty with trying something new."

4 Discussion and implications for future research

The goal of this study was to test the hypotheses that students taught with the multiple representation (MR) approach first before traditional algorithmic (TA) approaches would improve their learning and that when students were taught with MR approaches, they would be more engaged in their learning. To test these hypotheses, we used two treatment groups: MR first received MR approaches first and TA first received TA approaches first. The primary goal of this study was not to test which teaching approach is better but rather to investigate how teachers can best organize these two approaches to better meet the learning needs of students and get the most effective student learning outcomes.

No significant differences were found between the two treatment groups (MR first versus TA first) in terms of abilities to solve on post-tests; however, results from follow-up paired sample t tests suggested a larger gain in abilities to solve for students introduced to MR approaches before TA approaches. In this study, students in MR first began with lower abilities to solve (e.g., pre-knowledge) than TA first and yielded higher abilities to solve on post-tests when compared to TA first. Furthermore, the result suggests that MR first could have an observable effect on students' reasoning skills when they are asked to solve problems that require multiple steps and deeper reasoning as the solution is not simply found by setting up and solving the algorithm. This result is consistent with previous research that suggests that MR approaches help students make a better sense of mathematics (Fosnot & Dolk, 2002; Ng & Lee, 2009; Van den Heuvel-Panhuizen, 2003). This finding also suggests that students may improve their learning if they are first introduced to MR approaches that were designed to emphasize mathematical sense making and justification. After being presented with various MR representations, students may then be more ready to supplement those initial problem solving skills with more efficient algorithms (Donovan & Bransford, 2005; Van de Walle & Lovin, 2006).

The sequence of MR first is also aligned with a number of empirical studies that support Bruner's theory (Bruner, 1966) which recommends that instruction be sequenced from grounded representations to more abstract ones with numbers and symbols (Goldstone & Son, 2005; Nathan & Koedinger, 2000). While the results of this study suggest support for the MR first approach, it is important to highlight that much empirical support has also been found for the procedures-first approach (Briars & Siegler, 1984; Byrnes & Wasik, 1991). In addition, Ma (2010) describes the development of a "profound understanding of fundamental mathematics" as a well-organized mental package of highly connected concepts and procedures. This suggests that perhaps an integrative teaching approach in which both procedural and conceptual knowledge are taught together may be preferred. Future research, therefore, should continue to investigate how both TA and MR teaching approaches can be integrated to best support student learning outcomes and with other mathematical content.

Results further suggest that, in terms of teaching, using MR approaches before TA approaches may also be advantageous for teachers. In this study, the teacher noted that although most students struggled and were challenged by MR approaches, it was especially difficult to engage TA first students who had already been presented with traditional algorithmic approaches on how to solve given problems. Furthermore, the teacher noted that TA first students expressed that they did not find much value for exploring alternative methods for solving problems having already been introduced to efficient algorithms that could be used. This also caused the teacher a few challenges.

Although the two approaches were presented in different order, observed students' on-task behaviors were also similar for MR first students. During module 2 where MR first students were presented with TA approaches, students were observed to be more on task. Teacher reflections also noted that students even expressed preference for step-by-step algorithmic approaches. For example, teacher written reflections showed that the MR first group struggled with the multiple representations initially but when introduced to the algorithms in the second module, they were very grateful. While there may be many reasons for these results, this may be attributed to the ease of using algorithms. Furthermore, students' preferences may have been due to students' familiarity with this type of teaching approach that is often emphasized in US mathematics classrooms and textbooks (Ma, 2010; National Research Council [NRC], 2001; Van de Walle & Lovin, 2006). While the teacher in this study was known for teaching with multiple representations and students in her class were familiar with both approaches, years of experience in previous courses may have had an impact on their learning and preferences related to the specific topic of fractions, percents, and decimals. Further, a major assumption of using MR approaches is that they will lead to increased student on-task behaviors because students are required to make sense of their various representations and models, explain their methods in writing, discuss strategies with their peers in groups, and rationalize their methods verbally with their teacher and peers. Unfortunately, as this study suggests, this may not always be the case. Future research should continue to investigate students' on-task behaviors but also students' preferences for various problem solving strategies and tools. Furthermore, it is important to highlight the importance of teacher knowledge and preferences as well. Previous research has found that students often use the same tools and models as their teachers use (Cai, 2004; Cai & Lester, 2005). If teachers stress TA approaches over MR approaches, this may explain students' preferences and comfort with TA approaches. This also has links to how teachers are trained. Unfortunately, if teachers have been taught, prepared, and trained solely with TA approaches (Wu, 2011), they often only use this method to teach mathematics.

Just as students find MR approaches challenging, research has found that teachers may also lack the deep mathematical understanding and proficiency to use MR approaches (Ma, 2010). Future research should continue to investigate not only how students can be supported, but also how their teachers can be better prepared to support and help their students. In addition, research should involve gathering observations of teacher behaviors during implementation of TA and MR approaches. As the likelihood of student success with either approach rests on the teachers, they should be able to successfully implement and facilitate students' learning with either approach. For those teachers who are not familiar with MR methods and are more accustomed to traditional teaching, guidelines on how to scaffold student learning with MR approaches should be provided to guide their implementation. Future research might also investigate teachers' beliefs and values about teaching and learning as these could affect the implementation of teaching strategies (Clark & Peterson, 1986). Thompson (1992) found that teachers who held traditional beliefs about what it means to "know mathematics" tended to teach traditionally by introducing new algorithms, providing students with step-by-step instructions, and then assigning them to practice these algorithms and procedures. Alternatively, teachers who held more inquiry-oriented beliefs engaged students in activities to construct mathematical concepts, use reasoning, and communicate ideas (Ball, 1993; Stipek, Givvin, Salmon, & MacGyvers, 2001).

5 Conclusions

Many leaders in mathematics education today support the idea that students must have a balance of both conceptual understanding and procedural fluency in all areas of mathematics (NCTM, 2000, 2009). While conceptual knowledge should not be elevated above procedural knowledge (Howe, 1999), teaching approaches that help students with conceptual understanding are critical especially with newer mathematical standards that require as much attention to be given towards conceptual understanding as to procedural fluency (NRC, 2001). With these new standards, teachers and students may be encouraged to partake in teaching approaches that are unfamiliar, such as the use of multiple representations and mathematical models. With that being the case, as was evident in this study, more research is needed on how to support students with these new approaches and how to organize their instruction in light of the traditional instructional practices to which students have been accustomed.

Appendix

Table 5 Pre-test items and success rates (correct strategy and correct solution) by treatment

Items	MR first (N=22)	TA first (N=21)
1.) What is 25 % of 32?	68.2 %	85.7 %
2.) List the following numbers from greatest to least: 0.43, 297 %, 4/5, 17 %.	50.0 %	52.4 %
3.) Three candidates participated in a school election. Bianca received ¼ of the votes, Chelsea received 0.30 of the votes, and Francisco received the rest of the votes. What percent of the votes did Francisco receive?	72.7 %	71.4 %
4.) Jena is eating at Red Robin. Her total bill comes to $28. If she decides to leave a tip that is 15 % of the total bill, how much should she leave for the tip?	45.5 %	47.6 %
5.) 40 seventh grade girls are trying out for the basketball team, but only 15 can make the team. What percentage of the girls will make the team?	45.5 %	14.3 %
6).An employee earned $40,000 in a year and had $8000 of her earnings withheld for federal income tax. What percent was withheld?	50.0 %	23.8 %
7.) Mike and his 6-year-old son both participate in a 5-K race. Mike quickly ran the race in 28 min. His young son, who ran some and walked some, finished the race in 175 % of the time it took Mike. How many minutes did it take Mike's son to finish the race?	36.4 %	42.9 %
8.) A shampoo company is putting 16 % more shampoo into each of their new bottles. If the original bottles held 80 fluid ounces, how many ounces do the newer bottles hold?	4.5 %	42.9 %
9.) Lu went shopping for a keyboard. At the store, a keyboard originally priced for $250, had a price reduction of 20 %. What was the new, reduced price?	22.7 %	42.9 %
10.) The thresher shark can grow to a length of 18 ft. This is 30 % of the maximum length of a blue whale. Find the maximum length of the blue whale.	22.7 %	28.6 %

Table 6 Post-test items and success rates (correct strategy and correct solution) by treatment

Items	MR first (N=22)	TA first (N=21)
1.) What is 25 % of 36?	95.5 %	85.7 %
2.) List the following numbers from greatest to least: $\frac{3}{5}$, 0.16, 119 %, 65 %.	68.2 %	66.7 %
3.) Three drivers haul wagon loads of people at the fair. Blaine hauls ¼ of the loads before his tractor breaks down. Marcos only hauls 0.15 of the loads because he arrives late. What percent does Jennifer haul if she hauls the rest of the loads?	77.3 %	85.7 %
4.) Kurt takes his girlfriend out to eat for Valentine's Day. The bill comes to $42 dollars and he wants to leave a 15 % tip. How much tip should he leave?	63.6 %	71.4 %
5.) Courtney's team won 15 games this year. If her team played 25 games total this season, what percent of their games did Courtney's team win?	77.3 %	81.0 %
6.) Creighton watches TV for 20,000 min of television a year. Of this time, 17,000 min are devoted to sports. What percent of Creighton's TV time is spent watching sports?	77.3 %	85.7 %
7.) Simon is 50 in. tall. His older brother John is 130 % as tall as Simon. How tall is John?	68.2 %	66.7 %
8.) A package of spaghetti advertises 11 % more for the same price. If the original size was 12 oz, how many ounces are in the new, bigger package of spaghetti?	54.5 %	33.3 %
9.) Cassidy went shopping for an MP3 player. At the store, an MP3 player originally priced at $180,had a price reduction of 30 %. What was the new, reduced price?	68.2 %	33.3 %
10.) 210 students came to the last Stucky home basketball game. Ms. Ellis was impressed because this represented 30 % of Stucky's entire student body. If this is true, how many students make up the student body at Stucky?	63.6 %	52.4 %

References

Ball, D. (1993). With an eye toward the mathematical horizon: Dilemmas of teaching elementary school mathematics. *Elementary School Journal, 93*, 373–397.

Bennett, J. M., Burger, E. B., Chard, D. J., Jackson, A. L., Kennedy, P. A., Renfro, F. L., et al. (2007). *Holt mathematics: Course 3*. Orlando: Holt, Rinehart, and Winston.

Best, J. W., & Kahn, J. V. (2006). *Research in education*. Boston: Pearson.

Bottoms, G. (2003). *Getting students ready for algebra I: What middle grades students need to know and be able to do*. Atlanta: Southern Regional Education Board.

Briars, D., & Siegler, R.S. (1984). A featural analysis of preschoolers' counting knowledge. *Developmental Psychology, 20,* 607-618.

Bruner, J. (1966). *Toward a theory of instruction*. Cambridge: Harvard University Press.

Byrnes, J. (1992). The conceptual basis of procedural learning. *Cognitive Development, 7*, 235–257.

Byrnes, J. P., & Wasik, B. A. (1991). Role of conceptual knowledge in mathematical procedural learning. *Developmental Psychology, 27*, 777–786.

Cai, J. (2004). Why do U.S. and Chinese students think differently in mathematical problem solving? Impact of early algebra learning and teachers' beliefs. *Journal of Mathematical Behavior, 23*(2), 135–167.

Cai, J., & Lester, F. K. (2005). Solution representations and pedagogical representations in Chinese and U.S. classrooms. *Journal of Mathematical Behavior, 24*(3–4), 221–237.

Canobi, K. H., Reeve, R. A., & Pattison, P. E. (1998). The role of conceptual understanding in children's addition problem solving. *Developmental Psychology, 34*, 882–891.

Clark, C. M., & Peterson, P. L. (1986). Teachers' thought processes. In M. Wittrock (Ed.), *Handbook of research in teaching* (3rd ed., pp. 255–296). New York: Macmillan.

Council of Chief State School Officers & National Governors Association. (2010). *Common Core State Standards-Mathematics*. Retrieved July 1, 2013, from http://www.corestandards.org/Math

Donovan, M. S., & Bransford, J. D. (Eds.). (2005). *How students learn: Mathematics in the classroom*. Washington D. C.: The National Academies Press.

Fosnot, C. T., & Dolk, M. (2002). *Young mathematicians at work: Constructing fractions, decimals, and percents*. Portsmouth: Heinemann.

Goldin, G. A., & Shteingold, N. (2001). Systems of representations and the development of mathematical concepts. In A. A. Cuoco & F. R. Curcio (Eds.), *The roles of representation in school mathematics* (pp. 1–23). Reston: NCTM.

Goldstone, R. L., & Son, J. Y. (2005). The transfer of scientific principles using concrete and idealized simulations. *The Journal of the Learning Sciences, 14*, 69–110.

Halford, G. S. (1993). *Children's understanding: The development of mental models*. Hillsdale: Erlbaum.

Hiebert, J., & Carpenter, T. P. (1992). Learning and teaching with understanding. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 65–97). New York: Mcmillan.

Hiebert, J., & Lefevre, P. (1986). Conceptual and procedural knowledge in mathematics: An introductory analysis. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 1–27). Hillsdale: Lawrence Erlbaum Associates.

Howe, R. (1999). Knowing and teaching elementary mathematics [Review of the book *Knowing and teaching elementary mathematics* , by L. Ma]. *Notices of the AMS, 46*, 881–887.

Karmiloff-Smith, A. (1994). Self-organization and cognitive change. In M. H. Johnson (Ed.), *Brain development and cognition* (pp. 592–618). Cambridge: Basil Blackwell.

Lamon, S. J. (2001). Presenting and representing: From fractions to rational numbers. In A. A. Cuoco & F. R. Curcio (Eds.), *The roles of representation in school mathematics* (pp. 146–165). Reston: National Council of Teachers of Mathematics.

Ma, L. (2010). *Knowing and teaching mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. New York: Routledge.

Morris, S. B., & DeShon, R. P. (2002). Combining effect size estimates in meta-analysis with repeated measures and independent-groups designs. *Psychological Methods, 7*(1), 105–125.

Nathan, M. J., & Koedinger, K. R. (2000). Teachers' and researchers' beliefs of early algebra development. *Journal for Research in Mathematics Education, 31*(2), 168–190.

National Research Council. (2001). Adding it up: Helping children learn mathematics. In J. Kilpatrick, J. Swafford, & B. Findell (Eds.), *Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education*. Washington: National Academy Press.

National Council of Teachers of Mathematics. (2009). *Focus in high school mathematics: Reasoning and sense making*. Reston: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston: National Council of Teachers of Mathematics.

Newton, K. J., & Sands, J. (2012). Why don't we just divide across? *Mathematics Teaching in the Middle School, 17*(6), 340–345.

Ng, S.F., & Lee, K. (2009). The model method: Singapore children's tool for representing and solving algebraic word problems. *Journal for Research in Mathematics Education, 40*(3), 282-313.

Putnam, R. T., Heaton, R. M., Prewat, R. S., & Remillard, I. (1992). Teaching mathematics for understanding. *Elementary School Journal, 93*, 213–228.

Rasmussen, C., Heck, D. J., Tarr, J. E., Knuth, E., White, D. Y., Lambdin, D. V., et al. (2011). Trends and issues in high school mathematics: Research insights and needs. *Journal for Research in Mathematics Education, 42*(3), 204–219.

Rittle-Johnson, B., Siegler, R. S., & Alibali, M. W. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology, 93*, 346–362.

Schneider, M., & Stern, E. (2010). The developmental relations between conceptual and procedural knowledge: A multimethod approach. *Developmental Psychology, 46*(1), 178–192.

Sprick, R., Knight, J., Reinke, W., & McKale, T. (2006). *Coaching classroom management: Strategies and tools for administrators and coaches*. Eugene: Pacific Northwest Publishing.

Stipek, D. J., Givvin, K. B., Salmon, J. M., & MacGyvers, V. L. (2001). Teachers' beliefs and practices related to mathematics instruction. *Teaching and Teacher Education, 17*(2), 213–226.

Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 127–146). New York: Macmillan.

Van den Heuvel-Panhuizen, M. (2003). The didactical use of models in realistic mathematics education: An example from a longitudinal trajectory on percentage. *Educational Studies in Mathematics, 54*, 9–35.

Van de Walle, J. A., & Lovin, L. H. (2006). *The Van de Walle Professional Mathematics Series: Vol. 3. Teaching student-centered mathematics: Grades 5–8*. Boston: Pearson.

Wu, H. (2011). The mis-education of mathematics teachers. *Notices of the American Mathematical Society, 58*, 372–383.