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Developing teachers' subject didactic competence through problem posing

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Abstract Problem posing (not only in lesson planning but also directly in teaching whenever needed) is one of the attributes of a teacher's subject didactic competence. In this paper, problem posing in teacher education is understood as an educational and a diagnostic tool. The results of the study were gained in pre-service primary school teacher education. Students were asked to pose problems containing some given data (namely fractions). The subsequent analysis of the problems posed by the students revealed shortcomings in their conceptual understanding of fractions. Classroom-based joint reflection became the means of reeducation.

Keywords Professional development · Primary school teachers · Problem posing

1 Introduction

Our experience from primary school teacher education and research associated with teachers' professional development has shown us how these can be perceived as processes of cultivation of teachers' competences. In this paper, we focus on pre-service mathematics teacher education.

We are convinced that pre-service teacher education students enter university with naive ideas about the nature of mathematics and mathematics education. Many believe that the content of primary mathematics is simple and straightforward. During their teacher education programs, they need to acquire a deep understanding of mathematical concepts. They need to realize that knowledge alone of mathematical procedures is not sufficient, but that they must penetrate in depth the essence of, development of, and relationships between mathematical concepts included in primary school curricula. In the course of their study,

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every student should grow aware of the necessity to change the structure of his/her own knowledge with respect to the demands of his/her future profession; they need to change their perspective from seeing mathematical learning as additive to a view that is characterized by conceptual restructuring (Greer & Verschaffel, 2007).

Our research has shown that such change can be fostered by activities associated with problem posing, particularly when these activities are followed by joint reflection on the problems posed. The range of the posed problems made it possible to open group discussion on the various components of subject didactic competence: mathematical content, didactic elaboration (language, topicality, realness of the problem, links to other subjects, and potential for modeling).

The aim of this paper is to investigate the potential of problem posing in the education of future primary school teachers. By analyzing examples of problems posed by pre-service teachers we illustrate different shortcomings in prospective teachers' conceptual understandings. We conclude by discussing the educational and diagnostic outcomes of including joint reflection as a supplement to problem-posing activities.

2 Theoretical background

2.1 Teachers' competences

We interpret professional teacher's competences to mean the complex qualifications, skills, and dispositions that are prerequisites for successful performance of the profession. We use the word "competence" to highlight the dynamic and conative perspectives, that is the ability to act adequately, to respond to situations and stimuli that come up during lessons, and to capitalize on these in order to improve the quality of their students' education.

Helus (2001) formulated four basic teacher's competences, which form the basis of a teacher's self-confidence:

> Pedagogical competence consisting of (1) creating conditions for development of students' prerequisites by effective organization of educational influences, by motivating students' own educational activities and by exploiting their potential, (2) removing mental blocks and barriers, (3) mastering diagnostic operations, (4) getting an insight and empathy, and (5) designing procedures for effective pedagogical intervention.
> Subject-didactic competence consisting of a skilled orientation towards the educational meaning of teaching a specific subject, and putting this into action in relation to specific students. This competence encompasses mastering the scientific basis of the subject and its teaching, as well as didactic creativity.
> Pedagogical-organizational competence consisting of a skilled orientation of controlling the relations and activities in the classroom aimed at creating an effective educational environment, together with a supportive and stimulating climate.
> Competence in qualified pedagogical (self-) reflection with an emphasis on the analysis of the teacher's own thinking and dealing with students in a way suitable to their ability to plan their own lifelong education. (p. 26)

In the context of pre-service primary school mathematics teacher education, we are particularly concerned with the notions of *subject–didactical competence* and of the *competence of reflection*.

Subject–didactical competence consists of skilled orientation towards the educational meaning of the teaching of a specific subject, mastering the scientific basis of teaching of the subject, as well as didactical creativity. We believe that the concept of subject didactic competence connects several aspects of a teacher's work and pinpoints its complexity. With respect to the tradition of European didactics, we prefer this concept to the widely used concept of a *knowledge base for teaching* (Shulman, 1986).

The competence of reflection mirrors one's need to develop an awareness of unceasing work on oneself (to sustain and refine one's own professionalism). The benefits of qualified joint reflection and aspects associated with its development have been summarized by Ticha and Hošpesová (2006).

2.2 Problem posing in teacher training

Problem posing in mathematics teacher education is perceived in research through several different perspectives. It is usually understood as the objective as well as the means of mathematics education as it helps to develop mental models, to model phenomena, and to apply various representations (Brown & Walter, 1990; Tichá, 2008). Other research has stressed the diagnostic power of problem-posing activities (Silver & Cai, 1996; English, 1997; Harel, Koichu & Manaster, 2006). Several authors have suggested making problem posing a central element in mathematics classes (Singer, Ellerton, Cai & Silver, 2009). Interesting ideas have been presented by researchers who paid attention to the classification of problem-posing tasks (an overview is presented by Pittalis, Christou, Mousolides & Pitta-Pantazi, 2004).

However, most literature on problem posing points out that more research on the nature of students' problem posing is needed. For example, Christou, Mousolides, Pittalis, Pitta-Pantazi, & Shiraman noted that "little is known about the nature of the underlying thinking processes that constitute problem posing and schemes through which students' mathematical problem posing can be analysed and assessed" (Christou, Mousolides, Pittalis, Pitta-Pantazi & Shiraman, 2005, p. 150). And Crespo (2003) added "… while a lot of attention has been focused on teacher candidates' own ability to solve mathematical problems, little attention has been paid to their ability to construct and pose mathematical problems to their pupils" (p. 267).

We began to explore the development of students' ability to pose mathematics problems as an extension of earlier studies that focused on concept development (Koman & Tichá, 1998). Our objective was to help pre-service and in-service teachers to see mathematics in the world that surrounds them, in different "mathematical" and "non-mathematical" situations. We presupposed that the teacher usually (a) looks for situations and contexts which facilitate the development of particular mathematical methods and concepts (one mathematical topic may be anchored in a number of different situations) or (b) reflects on which mathematical topics may be developed in a particular situation that is interesting, stimulating or challenging for the pupils (a number of mathematical topics may stem from one situation). In the follow-up research, we changed conditions of the experiments and analyzed the posed problems containing specific data (Hošpesová & Tichá, 2010), the problems whose solution led to a given result (Tichá, 2009), and those problems whose solution can be found by carrying out a given (simple) calculation (Tichá, 2003; Tichá & Hošpesová, 2010).

2.3 Interpretation of fractions

We have focused our research on problem posing in the content area of fractions, an area that is often perceived by pre-service primary teacher education students as problematic. Research has shown that although pre-service teachers understand (are able to use) appropriate algorithms for calculations with fractions, they lack conceptual understanding of the concepts involved (Mewborn, 2001; Ma, 1999).

One possible reason for many misunderstandings may stem from the fact that fractions can be interpreted in different ways (for example, Freudenthal, 1983; Behr, Lesh, Post & Silver, 1983; Pitkethly & Hunting, 1996; Charalambous & Pitta-Pantazzi, 2007; Toluk-Uçar, 2008). The characteristics of individual interpretations of fractions (subconstructs) are based on the work of Kieren (1976). He suggested that the concept of fractions includes the following subconstructs: measure, ratio, operator, and quotient. He expressed the view that to understand the concept of fractions more fully means that all of these subconstructs and their mutual relations are understood. He ascribed an important role to the part–whole relationship, which is understood not as a separate subconstruct, and showed that this relationship permeates all four subconstructs thus establishing the basis for their understanding.

The idea of Kieren (1976) was further developed by Behr et al. (1983). They connected different interpretations of fractions with basic fraction operations and problem solving and noted that the diagram suggests that:

> (a) partitioning and the part-whole subconstruct of rational numbers are basic to learning other subcontructs of rational numbers, (b) the ratio subconstruct is most natural to promote the concept equivalence, and (c) the operator and measure subconstruct are very useful in developing and understanding of multiplication and addition. (p. 98)

Other researchers have also investigated students' conceptual understanding and procedural knowledge of fractions (see, for example, Charalambous & Pitta-Pantazzi, 2007; Baker, Czarnocha, Dias, Doyle & Prabhu, 2009).

3 Empirical study

3.1 Participants

Data for this study were gained from 56 pre-service primary teacher education students who participated in seminars on the Didactics of Mathematics.[1] This seminar series is compulsory in the second half of students' undergraduate studies (usually in their third year). It follows courses in arithmetic and geometry. The goal of the series is to lay the foundations of subject didactical competence—in other words, the seminar series is designed to raise students' awareness of the need for them to enhance their initial ideas about mathematics and mathematics education. In the seminar, problem posing is frequently used as a tool. At the same time, problem posing is also understood to be a goal of pre-service teacher education (since students, in their everyday teaching practice, will have to be able to pose problems).

3.2 Educational experiment

In this study, the students created problems that included specific data. We usually asked students to pose three to five problems (Tichá, 2009; Tichá & Hošpesová, 2010; Tichá, 2008). The reason was that (a) we wanted to find out whether the respondents were aware of

[1] Primary school teachers in the Czech Republic must complete 4–5 years of undergraduate courses designed especially for primary school teachers. Their undergraduate teacher education program includes courses in all subjects taught at the primary school level.

the various interpretations of fractions—diagnostics; (b) we wanted the respondents to become aware of the need to take the various interpretations of fractions into account—(re)education.

The students were asked to do the following: "*Pose three word problems which include fractions 1/2 and 3/4. Solve the posed problems*." The task was carried out in several steps. Students were asked to:

- pose problems
- solve some of the posed problems
- produce individual written reflections (both about problem(s) created by that individual, and about problems created by other students)
- take part in joint reflection with classmates and the teacher educator

At the end of the seminar series, students were asked to state how they perceived the inclusion of problem posing into the teacher education program.

3.3 Data and analysis

In this research study, we obtained different types of data: problems posed by students, individual written reflections on given problems, field notes from joint reflection, and written reflections on the overall problem-posing activity.

When analyzing the problems, we used semantic analysis of their text. We tried to classify the posed problems according to how they might represent students' understanding of fractions, and assigned codes that corresponded to the subconstructs mentioned above. During data analysis, we took into account the responses made by the authors of each problem, as well as their responses to classmates' comments, which we had recorded in our observation of the joint reflection. Then, we classified the posed problems on the basis of the characteristics identified. For some problems, we identified several characteristics. In the first phase of our analysis, each of us worked independently with her students; in the second phase, we discussed and combined our views.

For individual student's written reflections, we chose posed problems in which we had identified weaknesses in students' understanding of fractions. Students were asked to comment on whether the problems corresponded to the task assigned, and if not, why not.

Finally, we asked students to reflect on the overall problem-posing activity. In these reflections, we looked for common features through coding.

4 Analyses and results

4.1 Examples of conceptual flaws in posed problems

Through our analysis of the characteristics of the problems posed by the students, we have formulated categories to describe the types of problems created. Here, we will discuss two categories that we believe to be the most fundamental to interpreting and addressing students' misconceptions of fractions.

4.1.1 Monothematic nature of problems

What could be observed in the work of most students was a markedly monotonous nature of the situational context (cakes, marbles, …), of the properties of the environment (either discrete or continuous, rarely both, …), and of the interpretation (fraction as operator, quantity, etc.). A tendency to use stereotyped contexts was also observed in our earlier research (Hošpesová & Tichá, 2010).

Students often began to pose problems in one particular environment or began with one problem type and adhered to this environment or type when posing the rest of the problems (problems 1–3).

Problem 1. Jirka has 1/2 cake, Zdeněk has 3/4 cake. Who has more?
Problem 2. Eva has 1/2 cake, Jirka has 3/4 cake. How much do they have together?
Problem 3. Jan ate 1/2 cake, Pepík ate 3/4 cake. Was 1 cake enough for them?

Some triads of problems showed that their author gradually gained deeper insight into the task and posed problems in a purposeful manner—for example, creating cascades of problems of growing difficulty. This was the case with problems four to six or problems in which the whole (unit) changed in the process of solution (problem 7).

Problem 4. Mum bought 1/2 kg tomatoes and 3/4 kg red peppers. How much vegetable did she buy?
Problem 5. Mum bought 1/2 kg tomatoes and 3/4 kg red peppers. Of what vegetable did she buy more and by how much?
Problem 6. 1/2 kg tomatoes costs 20 CZK. How much is 3/4 kg tomatoes?
Problem 7. There were 16 cakes on a plate. Jirka ate 1/2 and Pepa 3/4 of the remaining cakes. Who ate more? How many were there left?

What we found alarming was the fact that most students failed to interpret fractions in different ways, and failed to use different representations and translations between them (Behr et al., 1983)—in spite of the fact that in the assigned task, they were encouraged to pose different problems. We expected some variety both in the context and interpretation. However, this was not the case—many of the students posed problems of similar structure that differed only in minor details (see the above mentioned triad of problems 1–3 or 4–6). Problem 6 has a different nature from problem 4–5. The author probably assumed that the solution would involve using the rule of three, but the fraction here is again used to indicate quantity. In problem 7, both the interpretation and the whole were changed. The author worked with operators—as in problems 1–3.

However, in some problems we may be observing the author's growing confusion. For example, in problem 8, the author posed an artificially complicated problem.

Problem 8 Katka was playing with Play-Doh. She formed one cake from it. She divided it into four pieces. She made a butterfly from 1/2 and an airplane from the other half. Then she took another piece of Play-Doh and made a cake from it again (which she again divided into four pieces). She only used 3/4 of this cake to make a dragon. How many pieces of the second cake remained on the table?

This could be interpreted as the result of the students' effort: *I want to teach something so I teach pupils to solve a greater number of problems of the same type* (*structure*). However, it is more likely that this was the result of how pre-service teachers remembered mathematics textbooks in which one problem type is often followed by a number of similar problems for practice.

In most problems posed by the students, fractions function as quantity (measure) or operator (as a rule either one or another). Moreover, students often put in the assignment quantitative data as for example in problem 7. They usually ask "How much?" (only rarely "What part?") since they tend to believe that mathematics is about numbers, quantities, and counting.

Students failed to realize that a seemingly minor change in the wording may play a very important role (for example, problems 9 and 10). Some students even assumed that it was just a "fruitless play with words." Many students firmly believed that problems 9 and 10 were identical.

Problem 9. Petr got 3/4 of a cake. He ate 1/2 of the cake. How much of the cake is there left?

Problem 10. There were 3/4 kg biscuits in the box. Eva ate 1/2 of them. How many biscuits are there left?

Furthermore, joint reflection revealed that students failed to discern the differences between the problems. For example, one of the solvers of problems 9 and 10 wrote: "*Easy problems. Very similar.*"

Comparison of problems 9 and 10 provides a useful example of the aspects discussed in joint reflection. Through the joint reflection, students grew aware of the possibility of different interpretations (measure, operator) and learned how to tell the whole purely on the basis of explanation and clarification.

4.1.2 What is the whole?

It is vital that students be aware of the importance of being able to determine the whole in each individual situation.

Problem 11 illustrates that some students posed problems in which they were at a loss to know how the whole may be divided into parts. They did not understand the process of division of the whole into equal parts (or of formation of the whole from parts). The authors' conceptions of a fraction were seemingly erroneous (the extent of these shortcomings is more fully revealed in the discussion in joint reflection).

Problem 11. Mum bought a pizza. She cut it into 8 pieces. Of the pizza, 1/2 was eaten by Dad and 3/4 by Lucy. How many pieces of pizza were there left for Mum?

The wording of problem 11 results led to two different interpretations by other students. In joint reflection some students pointed out a major mistake (1/2+3/4 is more than 1 whole). However, some students tried to overcome this obstacle and pointed out defects in the way the problem had been written. For example one student wrote: "*The problem is assigned badly. Its author probably meant that Dad ate one half and Lucy ate 3/4 of what remained. That is what I was solving.*" And he illustrated the problem with the picture shown in Fig. 1.

The author of problem 12 also failed to realize the two possible interpretations of the problem he created.

Problem 12. Petr and Mirek are eating cakes that Granny has baked. Petr ate 3/4 of a cake, Mirek 1/2 more. How much did Mirek eat?

The above quoted solver of problem 11 took on the role of the "healer", i.e. the person who is able to point out mistakes and misunderstandings. It was, in fact, very

Fig. 1 Sketch to help interpret another student's problem

common for some mistakes and misconceptions to appear in problem solutions but come to the surface only during joint reflection. For example, the solver of problem 13 came to the conclusion that Jirka spent 5/4 of the total time of the match in play. His calculation is illustrated by the picture shown in Fig. 2. He realized his mistake during the joint reflection.

Problem 13. Jirka plays football. In the first halftime he played 1/2 of the time, in the second halftime 3/4. What proportion of the total time did Jirka spend in play?

The above mentioned examples illustrate how joint reflection can develop students' awareness of the need for change in their own conceptions.

4.2 Importance of joint reflection on the posed problems

When carrying out joint reflection with the posed problems, it was clear that sharing different points of view was beneficial, albeit a different focus from the individual reflections (context, difficulty, possible representations). These differences depended on a range of factors, including previous education (type of secondary school), etc.

In some cases, students posed a very interesting and substantial problem without being aware of what needed to be considered in the solution process or which methods or concepts were prompted by the problem. For these problems, joint reflection proved to be of crucial importance. In the class discussion on problem 14, students suggested adding the following question: *Is there any red fish with a large tail fin in the aquarium*? *If so*, *how many specimens are there*? This proposition led to a discussion on the suitability of problems involving multiple attributes at the primary school level.

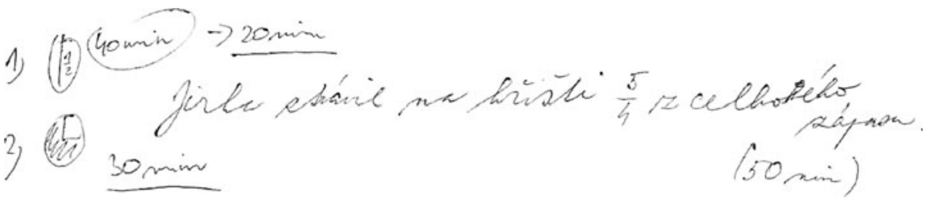

Fig. 2 A student's solution to problem 13. The translation of the solution reads: "Jirka spent on the playground 5/4 from the total time of the match"

Problem 14. There are 40 fish in the aquarium. Of it, 1/2 is red and 3/4 has a large tail fin. How many specimens of fish are red and how many have a large tail fin?

It is also true that individual reflection on the posed problems often involved shortcomings and misconceptions which came to surface through joint reflection and were then amended.

4.3 Joint reflection and reflections on the overall problem posing activity

The students often stated that joint reflection on the posed problems helped them realize that their current state of knowledge was insufficient, and in some cases helped to motivate them to a more careful analysis of the mathematical content. The motivation for change needs to stem from each student's personal acceptance that change is needed.

> I also realized that my imagination and ideas at the primary school level were quite likely much more developed and richer for problem posing than they are now.

Work with posed problems can be enriching for all the parties involved and offers a unique chance for natural differentiation of pupils, e.g., pupils with higher levels of knowledge pose mathematically rich problems, and they notice other problem characteristics. During joint reflection, our students were encouraged to think about the knowledge of mathematics that they needed for teaching because they realized their own shortcomings in understanding. We observed that problem posing played a strong motivation role for the students.

Many pre-service and in-service teachers tend to regard problem posing as a very unusual activity. If asked to formulate questions, or to pose problems based on a given situation, they find themselves in a strange, unexpected position. Some even believe that this position does not fall within their capabilities, and feel that we have asked too much of them. Often they feel completely helpless. We can illustrate this by the following students' statements from their reflections on the overall problem-posing activity:

> I've never had the chance to pose problems. I didn't anticipate how difficult it would be.
> I realized I take problems for granted but their posing is far from easy. Having tried it I grew aware of a number of difficulties.
> Problem posing—difficult but very enriching.
> I've never been asked to pose a problem. … I think I learned a lot through it and started to see things.
> Problem posing showed me that many of the things I've carried out more or less automatically are in fact unfamiliar to me and I don't understand them. I know how to do it but not why.

5 Conclusions

The research presented in this paper leads us to conclude that problem posing is first of all an appropriate way to introduce pre-service primary school teachers to the teaching of mathematics. Students who had had little teaching practice so far, and who were generally used to problem solving, were able to transcend gradually to the position of teachers who pose mathematics problems, modify the problems, offer help in their solution, and evaluate solution procedures. The usual students' standpoint is the position of the solver. However, for teachers the solution of a problem represents only a small part of their work. They must be aware of the multidimensional nature of the problem, and of the many aspects associated with the process of problem posing.

The data from our studies provide strong evidence that problem posing can be a significant motivational force resulting in deeper exploration of the mathematical content. If pre-service teachers are unaware of possible deficiencies in their content knowledge they soon realize that their knowledge does not meet the required level when they attempt to pose problems. During problem posing, students must learn to identify important mathematical ideas, and plan how to react to valuable pupils' suggestions.

A significant role in this process is played by joint reflection about the posed problems. In this reflection pre-service teachers gain a deeper understanding of concepts, and an overview of different possible approaches; they realize the need for various representations, thereby extending their repertoire of interpretations. In our opinion, problem posing, supplemented by joint reflection, should be one of the central themes in mathematics teacher education.

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