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Engaging pre-service middle-school teacher-education students in mathematical problem posing: development of an active learning framework

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Abstract Although official curriculum documents make cursory mention of the need for problem posing in school mathematics, problem posing rarely becomes part of the implemented or assessed curriculum. This paper provides examples of how problem posing can be made an integral part of mathematics teacher education programs. It is argued that such programs are a good place to start the process of redesigning mathematics curricula so that school mathematics will pay more than lip service to problem posing. Data are presented and analyzed showing that teacher education students can recognize the need for problem posing both in their own programs and in school mathematics curricula. Examples of problems posed by teacher education students are presented and discussed. An active learning framework for interpreting the role of problem posing in mathematics classrooms is presented.

Keywords Problem posing · Problem creation · Pre-service mathematics teacher education · Problem solving · Active Learning Framework

1 Introduction

To state that problem posing is as fundamental to mathematics as problem solving should be to state the obvious—after all, one cannot solve a problem unless first, a problem has been posed. Yet in school mathematics, developing skills in problem *solving* has been placed at the heart of the curriculum and classroom practice, and problem *posing* receives scant attention.

School students learn to focus on the outcomes of their problem-solving efforts and often have little or no opportunity to be involved in any problem-formulation processes. Thus begins an enculturation process of accepting problems that others create as those which need to be solved. Some maturing students learn to love mathematics and decide to become mathematics teachers; pre-service teacher education students enjoy the challenge of solving more complex mathematical problems, learn classroom strategies to help the next generation

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of students to become skilled problem solvers, and find that there are plenty of ready-made problems available on the Internet and in books. The enculturation process is complete once newly graduated teachers start to give the students in their very own classes mathematics problem-solving (but not problem-posing) experiences for the first time. That is precisely what all of their experiences have led them to believe that mathematics is all about.

It is not a trivial matter to intervene in any enculturation process, particularly in this case when most within the community believe in the paramount importance of being able to solve mathematics problems. Resistance to change in education is well documented (e.g., Taylor, 2002), and reforms are typically imposed from outside the school (as, for example, through the introduction of Common Core State Standards), or from inside the school, where the change agent may be school management, or the Principal. Young teachers who are potentially change agents are particularly vulnerable to being molded to fit the status quo (e.g., Lortie, 1975).

This study examined the impact on pre-service middle-school students of integrating problem-posing activities into the curriculum of a basic *Algebra for the Middle-School Teacher* course taught in a large mid-western university in the USA. The research investigated only the impact of the problem-posing activities on students at the time they were taking their courses—research which followed these students into their classrooms once they became teachers was not possible. The research reported here therefore represents a first step in understanding pre-service students' responses to being confronted with problem-posing activities directly linked to the curriculum.

2 Literature review

Students tend to believe that someone else always provides the mathematics problems that they work on. Even mathematics teachers tend to rely on outside sources for their mathematics problems, although some will also create their own problems by adapting those in other sources. Pre-service teachers tend to assume that they will always be able to rely on outside sources (such as textbooks or the Internet) to provide them with the mathematics problems that they will give to students. It is important, therefore, that mathematics teacher education students be given the opportunity to pose problems for formal school mathematics settings. Unless this happens, young learners will continue to believe that creating mathematics problems must always be "someone else's business."

In real life, we are all confronted with having to frame questions every day, and many of these may have mathematical components. Often, relatively simple arithmetical calculations associated with transport or with purchases are implicit in the problems that are created. Sometimes these questions may be much more complex, and will need to be carefully formulated. It is therefore widely recognized and espoused that the posing of problems should be part of the school mathematics curriculum (see, e.g., National Council of Teachers of Mathematics (NCTM), 2000).

One of the earliest references to involving students in problem posing was made by Belfield (1887) when he listed 13 suggestions for teachers immediately after the Preface in his book *Revised Model Elementary Arithmetic*. In his final suggestion, Belfield wrote: "Children become interested in making their own problems. Let some abstract examples be assigned for the children to change to concrete problems" (p. 4).

Over 70 years ago, Einstein and Infeld (1938) noted that "the formulation of a problem is often more essential than its solution, which may be merely a matter of mathematical or experimental skills. To raise new questions, new possibilities, to regard old questions from a new angle, requires creative imagination and marks real advance in science" (p. 92). But it took until the 1980s and 1990s for problem posing to emerge as important for mathematics education when research about mathematical problem posing began to appear (e.g., Brown & Walter, 1983; Ellerton, 1986; Hashimoto, 1987; Silver & Cai, 1996; Silver & Mamona, 1989; Stoyanova & Ellerton, 1996), and reports such as those by Kilpatrick (1987), and Silver, Kilpatrick, and Schlesinger (1990) were published. Repeated calls were made to give problem posing a more prominent role in the teaching and learning of mathematics. However, as Silver, Kilpatrick, and Schlesinger lamented: “Problem posing is almost always overlooked in discussion of the importance of problem solving in the curriculum” (p. 15). Silver et al. went on to say: “Mathematics teachers have a particularly difficult time with problem posing because it is open ended. Nevertheless it needs to be given the same emphasis in instruction that problem solving is beginning to receive” (p. 16).

Recent studies have ranged from a focus on the use of problem posing in mathematics instruction for young school students (Cankoy & Darbaz, 2010), to studies of teaching with variation in Chinese mathematics classrooms (Cai & Nie, 2007), to studying prospective teachers’ ways of making sense of mathematical problem posing (Chapman, 2011), and to designing pedagogical strategies for university-level mathematics instruction (Staebler-Wiseman, 2011). Studies such as these have helped to draw the attention of mathematics educators to the fundamental nature and far-reaching consequences of problem posing at all levels of education.

But the emphasis on problem solving in mathematics classrooms has continued, in spite of calls for the incorporation of problem posing into school curricula or for curricula designed for teacher education students in both the *Principles and Standards of School Mathematics* (NCTM, 2000), and the *Common Core State Standards* (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2012). Although these documents appear to give equal status to posing and solving problems in statements such as “Mathematically proficient students at various grade levels are able to identify external mathematical resources, such as digital content located on a website, and use them to pose or solve problems” (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2012, p. 7), this is the only place in this document that explicitly refers to the posing of mathematics problems, yet the word “solve” is used frequently in the same document.

Del Campo and Clements (1987) presented a model which contrasted the nature of teachers’ and students’ roles as they are displayed through various classroom activities, ranging from passive to receptive to active on the part of students. Students’ role as problem posers is “active,” in contrast to their “receptive” role when teachers (or other students) model a solution to a problem, or their “passive” role when they read examples in a textbook.

At present, although state systems, schools, and pre-service teacher education institutions may embrace official documents that use the deceptively simple words “pose” or “frame,” no sense of how and where the posing of mathematics problems might fit into their teaching plans has been provided. There is clearly a need for a framework linking the ideas associated with active learners, problem solving and problem posing.

Although some teachers may have incorporated problem posing activities into their classrooms, little has been reported in the literature about how pre-service middle-school teacher education students view problem creation. The literature is also silent on the extent to which students may or may not need supportive scaffolding to begin what for many would be an excursion into a hitherto unexplored world of problem posing. The relevance of the following comment by Bernardo (1998) seems to have gone almost un-noticed:

> A deeper level of understanding of the problem structure [can be] achieved by the problem solver…[as he/she] explores the problem structure while attempting to create

> an analog, …[and] of correctly mapping the problem structural information to create a true analog of the original problem. (Bernardo, 1998, p. 7)

This paper describes an exploratory study, designed to identify and document the effect of incorporating problem posing as part of the mathematics content curriculum for students in an undergraduate middle-school mathematics teacher-education course. To help integrate the findings of this study, a conceptual framework for positioning problem posing within the context of mathematical learning will be proposed.

3 The study

In the research reported in this paper, the focus is on understanding some of the implications of consciously and systematically engaging pre-service teacher-education students in mathematical problem posing. The following research questions guided the research:

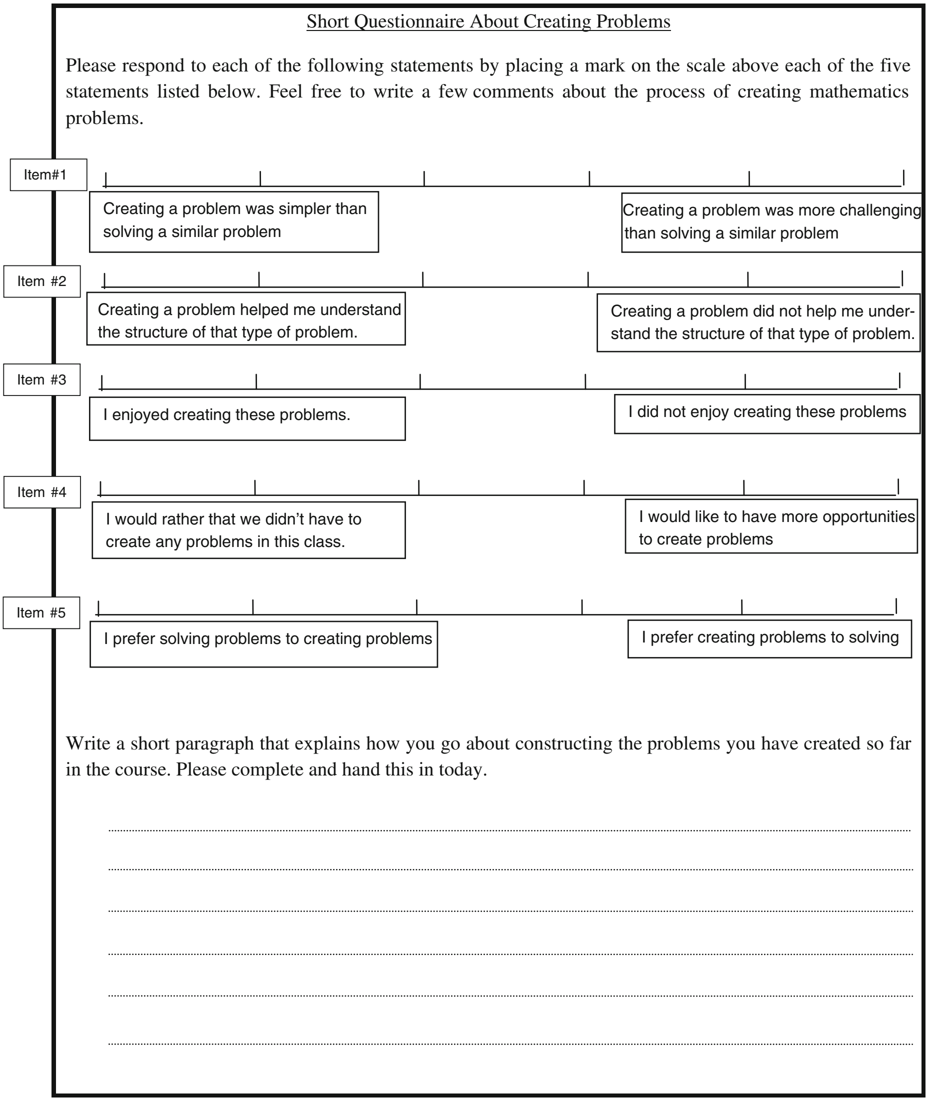
1. To what extent did pre-service middle-school students see mathematics problem posing as more of a challenge than solving similar problems?
2. To what extent did pre-service middle-school students find the problem-posing process helpful in understanding the mathematical structures of problems?
3. To what extent did pre-service middle-school students enjoy creating mathematics problems?
4. In what ways did the mathematics problems created by students as part of their mathematics teacher education course support students' learning of mathematical structures?

A sample of 154 students from six undergraduate pre-service teacher education mathematics content classes—*Algebra for the Middle-School Teacher*—two classes in each of three semesters—participated in the study. Three classes were taught by one instructor, and three by another, with all classes following closely aligned syllabi. Students regularly worked in groups of three or four students, with some activities specifying that they work in pairs. Two types of problem-posing activities—"routine" and "project" problem-posing activities—will be described in this paper. Details of these activities will be outlined in the section which follows. Examples of problems created by students from each instructor's classes during both routine and project problem-posing activities will be presented in Section 4, as will comments and reflections by the students.

At approximately week 12 of each 16-week semester, students in all classes were invited to complete a short questionnaire which is reproduced in Fig. 1. All students had been involved in both routine and project problem-posing activities at that point in the semester. Numerical values (from 0 to 5) were assigned to each response given by students according to where they marked the scale for each of the five items. Students were also asked to comment on how they went about constructing mathematics problems.

3.1 Problem-posing tasks

In all of the classes, students were asked to create mathematics problems at approximately two-weekly intervals, as a normal part of their classroom and homework assignments. These will be referred to as "routine" problem-posing activities. Problem-posing activities given to students as part of the major project that was an integral part of this course will be referred to as "project" problem-posing activities.


Short Questionnaire About Creating Problems

Please respond to each of the following statements by placing a mark on the scale above each of the five statements listed below. Feel free to write a few comments about the process of creating mathematics problems.

Item#1
Creating a problem was simpler than solving a similar problem
Creating a problem was more challenging than solving a similar problem

Item #2
Creating a problem helped me understand the structure of that type of problem.
Creating a problem did not help me understand the structure of that type of problem.

Item #3
I enjoyed creating these problems.
I did not enjoy creating these problems

Item #4
I would rather that we didn't have to create any problems in this class.
I would like to have more opportunities to create problems

Item #5
I prefer solving problems to creating problems
I prefer creating problems to solving

Write a short paragraph that explains how you go about constructing the problems you have created so far in the course. Please complete and hand this in today.

Fig. 1 Questionnaire on problem posing completed by students

3.1.1 Routine problem-posing activity based on the structure of the handshake problem

As part of the course, students had been introduced to the handshake problem ("How many handshakes will there be if there are n people in a room ($n>2$), and each shakes hands with every other person exactly once?"). Students were also asked to solve the following two problems:

> If there are n football teams in a competition ($n>2$), and each team plays each other team exactly once, how many matches altogether are played?
> If there are n distinct dots on a circle ($n>2$), and each dot is joined to each other dot, how many chords altogether are formed?

Teacher education students initially find these problems difficult, and very few recognize the structural isomorphism involved. Many students prefer to avoid the algebraic challenge by giving *n* a definite value. But, after group discussion, the notions of structurally similar and structurally dissimilar problems begin to emerge. The "inverse" problem of finding the number of people, given a certain number of handshakes, is then discussed, and students are invited to suggest "inverse" problems for the other two contexts.

Students were then asked to create other problems, and their "inverses," that have a similar structure to the handshake problem, but within different contexts. Students were encouraged to share their ideas (and created problems) with others in their groups, and to make adjustments to their problems based on any discussion that had ensued.

3.1.2 Project problem-posing activity based on the structure of assigned problems

A major project was also assigned to pairs of mathematics teacher education students. This assignment consisted of mathematics problems with each pair being asked to solve one of them. Students were then asked to create two different mathematics problems which embodied the same mathematical concepts, but which used a different context from the problem they had been assigned. Each pair of students made a class presentation in which they became the class teacher, assigning at least one of their newly created problems to the class and inviting students to share their approaches to the problems and their solutions. The project required the students to document what they had done, present their solutions to all three problems, and provide written reflections on the whole experience. One of the instructors asked students to create two short-answer questions for the project, but the other instructor wanted one of the two posed problems to have a multiple-choice format. No direct instructions were given by either instructor about how students might go about creating their own problems.

4 Results

4.1 Questionnaire data

The means and standard deviations for student ratings for each of the five items shown in Fig. 1, for each of the four classes, have been summarized in Table 1, in which the overall means and standard deviations for all 154 students have also been given.

Table 1 Means and standard deviations for questionnaire items

	No. of students	Item 1 (mean, SD)	Item 2 (mean, SD)	Item 3 (mean, SD)	Item 4 (mean, SD)	Item 5 (mean, SD)
Class 1 (Instructor A)	28	3.13, 1.11	1.35, 1.07	2.19, 0.96	2.57, 0.99	1.38, 1.16
Class 2 (Instructor A)	20	2.60, 1.14	1.23, 0.85	1.50, 0.86	2.86, 0.95	1.56, 1.08
Class 3 (Instructor A)	28	2.70, 0.84	1.00, 0.62	1.00, 0.67	3.09, 0.85	1.58, 1.08
Class 4 (Instructor B)	27	3.28, 0.96	1.21, 0.77	2.17, 0.84	3.04, 0.73	1.32, 1.09
Class 5 (Instructor B)	19	2.71, 1.24	1.13, 0.90	2.16, 0.91	2.59, 1.05	1.56, 0.98
Class 6 (Instructor B)	32	2.94, 0.71	1.41, 1.08	1.68, 0.82	3.19, 0.90	1.76, 1.08
Aggregate of all classes	154	2.92, 0.90	1.23, 0.89	1.87, 0.93	2.92, 1.08	1.50, 1.08

The relative consistency of the means and standard deviations for the five items among the students from six different classes is of interest. Students' responses and comments on each of the items will be discussed in turn.

4.1.1 Item 1

Item 1 asked students to indicate if they felt that creating a problem was simpler or more challenging than solving a similar problem. Analysis of the responses generated data in relation to the first research question. With means between 2.60 and 3.28, there was a trend for students to feel that creating a problem was more challenging than solving a similar problem. About 30 of the 154 students marked 4 or above on the scale, but this was balanced by 34 students who marked 2 or below. The spread of responses is reflected in the high standard deviation.

One student noted next to her circled tick mark (which corresponded to 3.0): "Creating a problem can be difficult because it's hard to find numbers that will work out simple in the end." In contrast, a student who indicated her response at a point corresponding to 1.4 on the line, wrote next to her response: "Once I already know how to solve, I can create a problem using the same components and steps." These responses were echoed by other students, and suggest that the level of a student's understanding of the problem is likely to influence their response to Item 1.

4.1.2 Item 2

Item 2 was designed to provide data in relation to the second research question. Students were asked to indicate the extent to which they felt that creating a problem helped them to understand the structure of that type of problem. With means between 1.0 and 1.41, students in all classes indicated clearly that they felt their understanding of the structure of problems was helped by creating problems. As one student stated spontaneously next to her mark (at 0.4) on the line: "Making a problem makes me think backwards which helps me understand solving." Other students made similar comments: one student, who indicated 0.5 on the line (i.e., half way towards the first tick mark), wrote: "Totally! Once you create a problem you have all the information and knowledge to solve."

4.1.3 Item 3

Item 3 asked students to indicate the extent to which they enjoyed creating mathematics problems. Analysis of the responses to this item generated data in relation to the third research question. The aggregate mean for all classes was close to 2, suggesting that students were fairly ambivalent about the task and did not have a strong sense of enjoyment or dislike for the task. Many made comments like: "I was indifferent when creating problems"; "Creating problems was good practice and something that all teachers *need* to know how to do"; "I think I enjoyed creating problems because it's what we will have to do as a teacher and it gave us some experience"; "Even though it was difficult creating problems, I feel it helped me to understand the material more thoroughly. I believe it will help me greatly for future tasks as a teacher"; and "Overall I did not mind creating two math problems. It allowed me to evaluate my knowledge because if I can create a problem I should be able to solve it."

Thus the students seemed to accept the challenge of posing problems—even though they did not particularly like doing it—because they felt a sense of purpose for the task in the light of their future roles as teachers.

4.1.4 Item 4

Item 4 was included to seek feedback from students about whether they would prefer more or fewer opportunities for creating mathematics problems during this course. Most students indicated that they would like to have more opportunities to create problems. Many included comments like: "I will always have to create problems as a teacher, so this is good practice," "I like the number of problems we create in class now. We do it enough to help each other understand a concept," and "It's challenging, but it's my future career." Students seemed to acknowledge that problem posing would become part of their professional lives, as teachers.

This item was included on the Questionnaire to gain additional data in relation to the third research question. If students enjoyed creating mathematics problems, then one might expect them to prefer more opportunities to create problems.

4.1.5 Item 5

Students across all six classes were particularly consistent in their responses to Item 5 which sought their preferences for creating versus solving mathematics problems. With a mean of 1.50 and standard deviation of 1.08 for this item, students made it clear that they prefer solving problems. Some students seemed to feel that creating problems was not actually "doing" mathematics. One student noted: "I enjoy actually doing math more than creating math," while another wrote: "This [problem solving] is a more traditional way of thinking that I really enjoy." Other students commented that problem posing was something they had not done before—so it is hardly surprising that these pre-service students were more comfortable with problem solving than they were with creating mathematics problems.

The Questionnaire data pointed to several contrasting, perhaps conflicting, observations: most students could see the merit of posing problems (problem posing helped them to understand the structure of mathematics problems), and indeed enjoyed creating problems and would be happy to spend more time creating them. Yet, on the whole, students preferred problem solving to problem posing. Thus any framework which sets out to describe students' learning experiences in mathematics classrooms should be capable of helping the reader interpret such contrasting observations.

4.1.6 Student comments about constructing problems

In addition to the five items on the Questionnaire, students were asked (see Fig. 1) to explain how they went about constructing mathematics problems. Their responses provided data in relation to the fourth research question. Several student comments have been quoted below from students who indicated that they preferred solving problems to creating problems, as well as comments from students who indicated that they preferred creating problems to solving problems. It is worth noting that students often stated that they worked backwards when creating their problems—regardless of whether they stated that they preferred to create or to solve mathematics problems.

Comments made by students who preferred to solve rather than create mathematics problems

- "I decided that the best way to start would be to start backwards";
- "We first looked at other problems that we had already solved. This was in order to make sure we knew exactly what to do. Making a problem that does not make sense would have been pointless. After we figured out what our question was, we each solved it to

make sure it worked. We tried to be as creative as possible when coming up with our problem";

- "I like to pick a solution that I would want the answer to be of my problem. Then I work backwards to see if my solution has a somewhat similar starting point. If it looks like it would be a messy and challenging problem, then I alter numbers and revise it, so other students are able to solve it."

Comments made by students who prefer to create rather than solve mathematics problems

- "When creating a problem, I usually begin at the end. That is to say, I start with an answer and work backwards to create a problem";
- "In order to create this problem, we actually started with the answer. This helped us to know what criteria we would have to make using the answer first";
- "I enjoyed creating problems … we made up a problem on inequalities and that's what we were learning about in class. Creating the problem really helped me to fully understand inequalities."

4.1.7 Correlations between responses to the five questionnaire items

Since the response data for the five items could be given a meaningful numerical score between 0 and 5, the data can be considered to be on an interval scale. Pearson product–moment calculations therefore provided a useful tool to check for possible correlations between each pair of items. Correlations were calculated for the data that were taken literally from the scored marks placed on the response lines in the Questionnaire. In other words, no adjustments for having positive or negative responses on the left of the score line were made. The results (rounded to two decimal places) are summarized in Table 2.

Perhaps the most interesting features of Table 2 are the low correlations between items. In other words, the items were addressing quite different aspects of students' thinking and affect about posing mathematics problems. The strongest correlation (−0.40) was found between Items 3 and 4. Thus, students who enjoyed creating mathematics problems tended to want to have more opportunities to create problems than students who did not enjoy creating problems. Although this particular result is hardly surprising, it serves to help validate the instrument.

Of particular note was the zero correlation between Items 1 and 4. Thus there was no correlation at all between how students felt about whether creating a mathematics problem was simpler than solving a similar problem, and whether or not these same students preferred to have less or more opportunities to create mathematics problems in the class.

Table 2 Pearson product-moment correlations between pairs of questionnaire items

Item No.	Item number				
	1	2	3	4	5
1		−0.19	0.11	0.00	−0.14
2			0.10	−0.12	−0.15
3				−0.40	−0.28
4					0.28
5					

Such a result is entirely consistent with students' written comments. Many students indicated that, even if they found creating problems more difficult and challenging than solving similar problems, they welcomed the opportunity to pose problems because they would need to have these skills when they graduated as a teacher. The long-term teaching aspirations of students would appear to have overridden any negative feelings they had about posing mathematics problems.

4.2 Problems constructed by students

In response to the two types of problem-posing activities (routine and project activities) used in the course *Algebra for the Middle-School Teacher*, students created quite different mathematics problems. Both activities are consistent with the concept of semi-structured problem-posing (Stoyanova & Ellerton, 1996), although project activities encouraged students to create more extended tasks which, by their very nature, demanded particularly careful consideration of the structure of the original problem. Data from these two types of problem-posing activities related to the fourth research question.

4.2.1 Problems posed during routine problem-posing activities

As outlined in 3.1.1, one of the activities in which students were involved when they considered the handshake problem was to create problems that had a similar structure to the handshake problem, but with a different context. In class, the handshake problem had been considered in two parts, where one part began with the number of people shaking hands (and asked "How many handshakes?"), while the "inverse" began with the number of handshakes (and asked "How many people?"). Groups of students were asked to choose what they considered to be their "best" problems—one with a structure similar to the handshake problem, and one with the "inverse" structure.

Five pairs of problems created by different groups of students in one of the six classes included in the study are shown in Fig. 2. Although students were also asked to solve the problems they had created, the solutions have not been reproduced here (all groups presented correct solutions to their own problems).

4.2.2 Problems posed during project problem-posing activities

Problems posed by students as part of their assigned projects were inevitably longer, more complex problems than those created by students during a class session. Students had longer to reflect on the project problems, and since their projects involved class presentations, they spent longer working on both the problem they were asked to solve as well as the posed problems they were to present to the class. Problems would still be described as semi-structured since they were to be modeled on a given problem. Designing a different context for a problem that has a similar structure to the given problem was challenging for most students.

Examples of problems posed as part of students' projects are shown in Fig. 3, and examples of multiple-choice problems posed by students are presented in Fig. 4.

4.3 Joint reflections of the instructors

One of the challenging questions facing the instructors as they discussed incorporating problem posing into the curriculum for this course was: "Why have we engaged students actively in problem posing rather than place a stronger emphasis on problem solving?" The

Problems Created by Group 1

Suppose there are 12 boys playing video games. Each boy must play each other once. How many games will be played total?

A group of boys meet at their friend's house. Each boy plays each other boy in a video game once. Altogether, there were 105 video games played. How many boys were there at the house?

Problems Created by Group 2

Gordon's class is having a Valentine's Day party. She has 20 students in her class and asked each of them to exchange 1 valentine with each person in the class. How many exchanges occurred?

A family gets together for Christmas. It is a tradition that each family member exchanges one present with each person. A total of 190 exchanges occur. How many family members are there?

Problems Created by Group 3

Suppose there are 8 different students at a Pre-K screening. Each student must partner with another student to play a game. How many games were played altogether?

A group of preschoolers are at a preschool screening. Each preschooler plays a game with every other preschooler. Altogether there were 1326 games played. How many preschoolers were there?

Problems Created by Group 4

There are 20 kids in a daycare. Each child has to play with each other child once. How many different play group interactions will there be?

In one day, there were 990 playgroup interactions. How many children showed up to daycare that day?

Problems Created by Group 5

In a dance class, there are 12 dancers all together. Each dancer has to dance with every other dancer once. How many dances will take place?

A group of adults attend a speed dating event. Each adult has to introduce themselves to each adult once before the dating activities take place. In all, 190 introductions were made. How many adults were there total?

Fig. 2 Routine problem pairs created by groups of students

instructors were both convinced that before someone can pose a non-trivial but solvable mathematics problem, it is essential for that person to begin the exercise with a sound understanding of the mathematical concepts involved. The instructors recognized that, in posing a mathematics problem, students needed to be aware of boundary conditions, as well as the structure of the task to be defined within those boundary conditions.

In the past, much emphasis has been placed on the posed problems themselves, rather than on the *process* of engaging learners in active problem posing. When emphasis was refocused on problem solving (e.g., Schoenfeld, 1989), teachers and researchers alike realized that the *process* of problem solving needed much greater emphasis than simply finding a correct solution to a problem. In mathematics teaching and learning, the notion of focusing on a *process* is not, in itself, new. The framework shown in Fig. 5 situates active problem posing in mathematics classroom in the context of mathematics teaching and learning. This framework builds on the notion of a continuum ranging from passive to active (from the student's point of view), and places emphasis on actions (or processes) rather than on passive outcomes (or

Original Clockface Problem

Suppose, on a circular clockface the time shows exactly 12 noon. Assuming the clock is working well, how many minutes (to 1 decimal place) will it take before the minute hand and the hour hand are pointing in exactly the same direction again?

Problems with a Similar Structure Posed by Students

1. If the time is 3:10, how long would it take before the two hands will be pointing in exactly the same direction?

2. If the time is 9:50, how long would it take before the two hands will be pointing in exactly the same direction?

3. How many times in any 12-hour period will the two hands point in exactly the same direction?

4. Suppose after noon the hour-hand moved backwards (i.e., counter-clockwise) at its normal rate, but the minute-hand moved forward at its normal rate. How long would it take before the two hands were pointing in exactly the same direction?

5. On a long straight road, *Car A,* which is traveling at 60 mph, is 5 miles behind *Car B,* which is travelling at 40 mph. Assuming that they continue to travel at those speeds, how long will it take for *Car A* to catch *Car B*?

6. On a long straight road, *Car A,* which is traveling at a mph, is d miles behind *Car B,* which is travelling at b mph. Assuming that $a > b$, and that they continue to travel at those speeds, how long will it take for *Car A* to catch *Car B*?

7. Suppose *Train A* leaves City *A* and travels at a mph toward City *B,* which is d miles from City *A*. At the same time, *Train B,* began traveling at b mph toward City *B*. If the two trains continue to travel at those speeds, how long will it be before *Train A* and *Train B* meet?

Fig. 3 Project problems created by groups of students

products). It is consistent with a model which contrasted the nature of teachers' and students' roles that are displayed through various classroom activities, ranging from passive to receptive and active on the part of students (Del Campo & Clements, 1987).

1. A cyclist, traveling at 15 mph, is 15 miles ahead of a car, traveling at 60 mph, on a long straight road. Given those speeds remain, how many minutes should it take for the car to catch the cyclist?

(A) 15 minutes
(B) 17.5 minutes
(C) 20minutes
(D) 25 minutes
(E) None of (A) or (B) or (C) or (D)

2. Two trains are at Town *A* and Town *B*, respectively, on the same railway line, but 400 miles apart. At exactly the same time, the train at A started and traveled towards *B* at an average speed of 45 mph, and the train at *B* started and traveled toward *A* at an average speed of 55 mph. When they pass each other, how far will they be from Town *A*?

(A) 180 miles
(B) 200 miles
(C) 220 miles
(D) 240 miles
(E) None of (A) or (B) or (C) or (D)

3. Suppose that there are n teams in a football competition ($n \geq 2$), and each team is to play each other team exactly once. How many matches will there be altogether?

(A) $n(n + 1)/2$
(B) $n(n + 1)$
(C) $n(n - 1)/2$
(D) $n(n - 1)$
(E) None of (A) or (B) or (C) or (D)

4. Suppose the symbol S_n is used to represent the sum of the first n natural numbers [so that $S_n = 1 + 2 + 3 + ... + (n - 1) + n$]. Which of the following would represent the sum of all natural numbers that are greater than 50 but less than 100?

(A) $S_{100} - S_{50}$
(B) $S_{99} - S_{50}$
(C) $S_{100} - S_{49}$
(D) $S_{99} - S_{49}$
(E) None of (A) or (B) or (C) or (D)

Fig. 4 Four multiple-choice problems created by groups of students as part of their projects

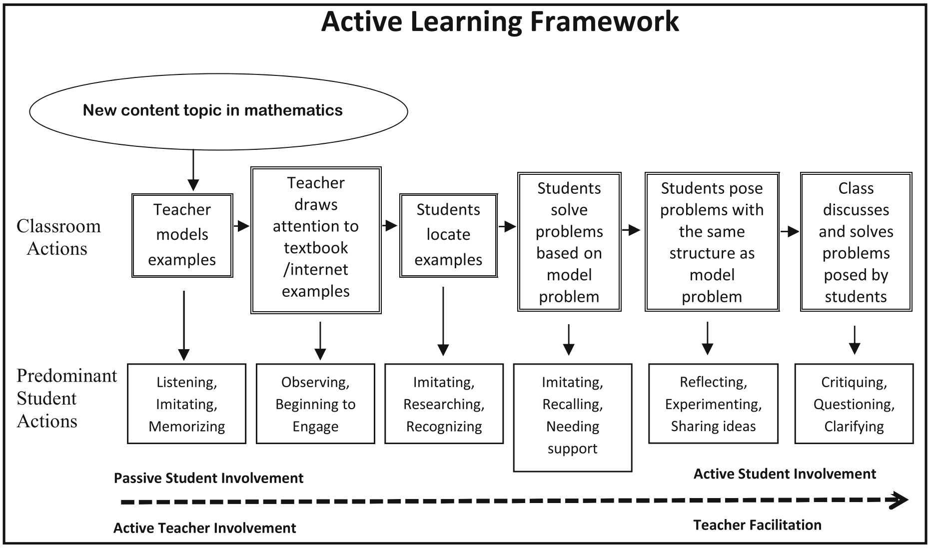


Fig. 5 Framework for locating problem posing in mathematics classrooms

If problem posing is left out of the curriculum, the *Active Learning Framework* illustrates how the students' classroom experiences are, in fact, cut short; their mathematical experiences have been truncated, making problem solving their final mathematics classroom experience. The capstone element of problem posing has been omitted. Most of the pre-service teacher education students in this study had previously experienced only the truncated model in their mathematics classrooms, and had solved rather than posed mathematics problems. Is it any wonder that these students stated that they preferred problem solving to problem creation? It is encouraging, though, that they recognized their own deficiencies with respect to problem posing, and indicated that they wanted to have more opportunities to engage in active problem posing.

5 Discussion

Pre-service teacher education students bring considerable mathematical and pedagogical insight into their involvement with problem posing in mathematics content classes. They are the first to admit that they have had little experience with problem posing, and to acknowledge that they need to have more opportunities to pose problems as they will need these skills as classroom teachers. The majority of students made comments along these lines in their questionnaire responses.

The routine problems students posed during class sessions were rarely polished and often included imperfections in wording or logic. Students are beginners in problem-posing and are experimenting with all of the parameters involved. As a teacher, do we expect perfection from a young student setting out the solution to his or her first mathematics word problem? For example, the first of the two problems created by Group 3 in Fig. 2 is flawed. The answer to the literal problem presented would be four—but the answer intended by those who created the problem was 28. Should this error be pointed out to students? Of course it should—but it is hoped that, by exchanging their posed problems with their peers, students will notice each other's flawed problems and discuss what changes are needed.

One student expressed it this way:

> I would say the most useful parts of this course was working together with other people and creating our own problems. Working with other people was useful because everyone works different ways and everyone makes mistakes. When working in groups of four many ideas will come up on how to solve the problem. Through this class I was able look at the same problem in a different light and some ideas made solving the problem easier. Teamwork also helped verify the right answer. If we all didn't get the same answer we will all look over the answers to check for mistakes. We would also check each other's work and discuss what we did to see if we can spot the mistakes. Going over mistakes reminded me where to pay extra attention to when solving the problem. Creating our own problems also helped because it truly helped me understand the concepts more. I understood all the parts that go into the problem in order to make it. I believe no one can make a problem if they don't understand the concept.

The student's last sentence captured a fundamentally important justification for providing opportunities for problem posing—if one does not understand the concept, then one cannot pose a problem.

In this research the aim was not to compare the quality or creativity of the problems created by different students. Rather, the aim of the study was to investigate the viability of integrating problem-posing activities into the curriculum in parallel with problem-solving activities. Being able to pose mathematics problems lies at the heart of understanding and developing mathematical ideas—yet it is an under-utilized tool in mathematics teaching and learning.

Perhaps the only way that problem posing has a chance of being seriously introduced into school mathematics curricula and classroom practices would be for young teachers to acquire problem-posing skills and confidence in problem posing themselves to the point where they would be capable and willing to help their students to pose problems. The simplest way to move towards achieving this would be to focus attention on this issue in early childhood, primary, and secondary mathematics teacher education programs. The potential is there for problem posing to be integrated relatively seamlessly into the mathematics curriculum of teacher-education programs, with the ultimate potential that the graduates of each class of 30 teacher education students have the potential to influence the introduction of problem posing in many classrooms in many schools over many years. The current enculturation process into a problem-solving mindset among teachers would thus be expanded to include a strong problem-posing component.

The data and analyses presented in this paper, together with the *Active Learning Framework* (Fig. 5), support the incorporation of both routine and project problem-posing activities in mathematics courses for pre-service teacher-education students. The *Framework* can be used to provide a context for interpreting other research studies. For example, Cai and Brook (2006) commented that "Since these problems came from the students themselves, they were more motivated in exploring them further than they were with those posed by teachers" (p. 44). Students' motivation can be linked to their active involvement—their creative acts help to bring together their previous ideas and experiences in ways not possible through more passive participation when working with problems posed by others.

Although further research is needed before this framework can be extended and applied to mathematics classrooms for learners of all ages, the framework is simple and transparent, and was inspired by comments such as the following, made by my own students as they engaged in problem-posing activities: "Creating problems made me feel more like a teacher. It gave me a chance to question my and my students' thinking." For too long, successful problem solving has been lauded as the goal; the time has come for problem posing to be given a prominent but natural place in mathematics curricula and classrooms.

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