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Learning to solve addition and subtraction word problems in English as an imported language

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Abstract This paper reports an intervention phase of a design study aimed to assist second-grade Filipino children in solving addition word problems in English, a language they primarily encounter only in school. With Filipino as the medium of instruction, an out-of-school pedagogical intervention providing linguistic and representational scaffolds was implemented with 17 children. Pre-intervention, children experienced linguistic difficulties and were limited to conceptualising and solving simple additive structures. Post-intervention interviews revealed improved performance and understanding of more complex structures, but only when linguistic difficulties were minimised. The study identified socially and culturally driven barriers to learning: superficial strategies, children's engagement, and learning in an urban poor context.

Keywords Word problems · Imported language · Word problem interview · Pedagogical intervention · Filipino students

1 Background

In the Philippines, English is the language of instruction for mathematics beginning in Grade 3. However, because mathematics textbooks and assessments are in English even in Grades 1 and 2, the benefits of mathematics instruction in the first language are limited. This situation is problematic because many Filipino teachers are not fluent in English. Further, Filipino children, especially those from disadvantaged socioeconomic backgrounds, have limited access to engaging in English outside school (Gonzalez, 2006). As Bernardo (2008) writes,

> The cognitive disadvantages brought about by using English in instruction among students with near-zero English language proficiency and who live in non-English speaking environments converge with the oppressive and marginalizing effect of English on the lives of the poor. The overwhelming majority of Filipino children find

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their limited English a major stumbling block in their efforts to learn in the various domains of knowledge. (p. 36)

This problem deserves particular attention because the language of instruction may be more a barrier rather than a tool for communication and may contribute to low levels of mathematics achievement (Brock-Utne, 2007). There is a need to examine the extent to which young children's mathematics learning difficulties may be attributed to the language of instruction, and how their learning may be supported.

We can investigate these difficulties closely in the context of children's solutions to arithmetic word problems where linguistic factors and semantic understanding are critical. Word problems form an integral part of the Philippine mathematics curriculum (Department of Education Bureau of Elementary Education [DepEd], 2003). The curriculum documents are quite explicit about how children should solve word problems. Before solving problems, children should be able to (1) state what is asked, (2) state what is given, (3) identify word clues, and (4) specify the correct operation to be used. Textbooks and school assessments reflect this stringent procedure for solving word problems from as early as Grade 1.

The problem arises that many Filipino children who have completed 2 or 3 years of schooling are unable to solve even simple addition and subtraction word problems in English (Bernardo, 1999), a result replicated by our own studies (Bautista, Mitchelmore, & Mulligan, 2009; Bautista, Mulligan, & Mitchelmore, 2009). While language problems are often blamed for such poor performance (Philippine Executive Report on the TIMSS, cited by Carteciano, 2005), it remains unclear as to what extent other factors impede word problem solving. We provide insight into these issues through a pedagogical intervention aimed to assist Filipino children in solving additive word problems in English. This study addresses the following research questions:

1. What are the obstacles that impede Filipino children's strategies for solving additive word problems in English?, and
2. What aspects of a pedagogical intervention are effective for developing Filipino children's word problem-solving performance in English as an imported language?

These questions are investigated within the sociocultural context of a developing country where mathematics education is also deeply entangled with broader constraints (Bansilal, 2011; Nebres, 2009). Language barriers, large class sizes, inadequate resources, and low teacher qualifications are often implicated as the "what-else-is-new" causes for failure (Carteciano, 2005). Thus, Nebres argues that addressing micro-problems (i.e., concerns about curriculum, professional development, textbooks) should begin in the context of understanding and working on macro-problems (i.e., the social, political, and economic situation of schools). Children's learning obstacles should also be conceptualised in view of their home environment and the opportunities made available to them (Skovsmose, 2005).

2 Mathematics learning in a second language

Early research on mathematics learning in bilingual populations was motivated by the work of Jim Cummins (1979). While the belief until the early 1970s was that bilingualism handicapped learning, Cummins suggested that bilingualism was not an obstacle and may even offer cognitive advantages when proficiency in two languages has been attained. According to his theory, problem-solving difficulties in a second language (L2) are not due to bilingualism per se but to the child's limited proficiency in the language of the problem (Clarkson, 1992; Ní Ríordáin &

O'Donoghue, 2009; Secada, 1991; Whang, 1996). Thus, children who could solve word problems in their first language (L1) are also expected to solve the same word problems in whatever other language they are proficient in (Bernardo, 2000).

A large amount of the research on bilingualism and word problem solving involves language minority children who nevertheless are able to use their L2 in social situations. For example, the students documented and interviewed in the studies of Clarkson and colleagues (Clarkson, 2007; Clarkson & Dawe, 1994) were Australian immigrants who could converse in their L2 (English), judging from the fact that it was possible to interview them in English.

A different situation, however, may hold for children for whom the L2 is an imported language. Despite the extensive mathematics education research on multilingual classrooms (Barwell, 2009), few studies have focused on children who learn mathematics in an imported language that is not widely spoken in the immediate community. There are comparatively few mathematics education researchers working in this situation, such as those from Papua New Guinea (Lean, Clements, & Del Campo, 1990), South Africa (Setati & Barwell, 2006), and Australia (Jorgensen, 2011). Children in this context need substantial scaffolding from their teachers in understanding words that native speakers ordinarily use in social conversations (Lim & Presmeg, 2011).

Proficiency in the language of the problem may thus explain the contrasting results drawn from the research on additive word problems. On one hand, some studies indicate that children find word problems in their L1 easier than equivalent problems in their L2, as was found among Filipino and Nigerian children (Adetula, 1990; Bernardo, 1999). On the other hand, some studies suggest that children are not necessarily disadvantaged when solving problems in L2, as was found among children from language minority communities in the USA (Secada, 1991; Whang, 1996).

While it is not possible to directly compare language proficiency across the samples in the abovementioned studies, it is quite evident that the samples in these studies were drawn from different sociolinguistic settings. While the language minority American children used their L2 (English) in their social interactions with the wider community, the same could not be said for the Filipino or Nigerian children who typically encountered English only in school. However, if more advanced problems with more complex linguistic structures were considered, even second language learners who have access to the L2 in the community may experience difficulties (Fernandes, Anhalt, & Civil, 2009; Fillmore, 2007) as they have to contend with the technical linguistic features of the mathematics register (Kazima, 2007).

It is thus important to distinguish between children who have access to the L2 in the wider community and those for whom the L2 is an *imported language*. While children for whom the L2 is an imported language have restricted opportunities to develop conversational fluency in L2, language minority children are likely to communicate easily in their L2 and their primary difficulty is with the academic aspects of L2 proficiency (J. Cummins, 2000).

3 Theoretical framework

This study integrates Vergnaud's (1982, 2009) theory of conceptual fields with the text comprehension theories of van Dijk and Kintsch (1983), later extended by Nathan, Kintsch, and Young (1992). Vergnaud provided a framework for analysing the knowledge required for solving additive word problems. He emphasised that solving the various classes of problems in the field of additive structures requires "very important mathematical concepts" (Vergnaud, 1979, p. 263). These various concepts of number are implicit in the child's activity and are what Vergnaud (1982) calls *concepts-in-action*.

Vergnaud (1982) also drew attention to the different strategies that may be used for solving word problems. The range of strategies reflects the different propositions that children may implicitly hold to be true, or what Vergnaud calls *theorems-in-action*. For instance, the recognition of subtraction as an appropriate strategy for solving a problem involving $a+\square=b$ requires a relational calculation or an understanding of commutativity or inversion (Nunes & Bryant, 2009; Verschaffel & De Corte, 1997).

From the perspective of van Dijk and Kintsch's (1983) text comprehension theories, problem-solving consists of three inter-related mental representations—the *textbase*, *situation model*, and *problem model* (English & Halford, 1995; Nathan et al., 1992). The textbase is a content- and structure-preserving paraphrase of the text, and is distinct from the surface form that represents the exact wording of the text (Fletcher & Chrysler, 1990). The situation model is an elaboration of the textbase on the basis of the reader's prior knowledge, resulting in a representation of the *situation* described by the text (Kintsch, 1994). It reflects a deeper level of understanding as compared to that obtained from solely a textbase representation of the text. The problem model is the representation of the problem's mathematical structure from which a formal arithmetic operation may be applied to generate a solution (Nathan et al., 1992).

Van Dijk and Kintsch's (1983) model may be related to Vergnaud's (1982) conceptual field theory in two ways. First, children may fail to construct accurate situation models not only if their linguistic skills are weak, but also when they do not have the concepts-in-action necessary for conceptualising the situation described by the text. Second, children may fail to map their situation model to a problem model if they do not possess the theorems-in-action to perform the relational calculations required for the task.

Although there is considerable empirical support for the mental representations proposed in Vergnaud's (1982) and van Dijk and Kintsch's (1983) models (Carpenter & Moser, 1984; De Corte & Verschaffel, 1987), it is also recognised that cognitive activity takes place within a particular sociocultural setting and is a product of an individual's participation in social interactions (Perret-Clermont, Brun, Saada, & Schubauer-Leoni, 1984). Vinner (1997), for example, argued that a cognitive analysis of children's strategies is not adequate for understanding their behaviour. He proposed the notion of *pseudo-analytical* thinking which is "based on the belief that a certain act will lead to an answer which will be accepted by society…(mathematics teachers in our case)" (p. 115). To achieve this goal, children may often (although subconsciously) be guided by the *minimal effort principle*, where they use the easiest procedure that seems to lead to the correct answer. From this perspective, errors may be due not only to misconceptions but also to children's ways of coping with a given task (Leron & Hazzan, 1997).

4 Research on additive word problems

The classification of addition and subtraction word problems according to their semantic structure has formed the basis of a long tradition of research on addition and subtraction word problems (Verschaffel, Greer, & De Corte, 2007). Problem structures involve increases, decreases, combinations, or comparisons. Table 1 presents examples of these problems.

Empirical findings show two remarkably consistent results. These relate to (a) the influence of semantic structure on problem difficulty and (b) the range of children's strategies for solving word problems. First, the semantic complexity of the problem contributes to problem difficulty. In Table 1, for instance, the Missing Addend, Part Unknown, and Compare problems are more difficult than Join, Separate, and Combine problems (Lean et al., 1990).

Table 1 Types of addition or subtraction word problems

Problem type	Problem
Join	Alvin had 3 coins. Then Jun gave him 8 more coins. How many coins does Alvin have now?
Separate	Dora had 11 mangoes. Then Dora gave 6 mangoes to Kevin. How many mangoes does Dora have now?
Combine	Tess has 5 hats. Rodel has 8 hats. How many hats do they have altogether?
Missing Addend	Jolina had 7 pencils. Then Alma gave her some more pencils. Now Jolina has 12 pencils. How many pencils did Alma give her?
Part Unknown	Jimmy and Mia have 11 marbles altogether. Jimmy has 4 marbles. How many marbles does Mia have?
Compare	Rica has 12 books. Luis has 7 books. How many more books does Rica have than Luis?

Adapted from Carpenter and Moser (1984)

While there is agreement that problem difficulty is influenced by a problem's semantic structure, the explanations provided by researchers vary according to the relative emphasis they place on the linguistic and mathematical demands of the task. Some researchers contend that word problem-solving difficulties mostly originate from linguistic or text comprehension errors, thereby preventing children from mapping the textbase onto existing mathematical knowledge. For example, D. D. Cummins (1991) showed that children often interpret the statement, "Mary and John have 5 pencils altogether" as "Mary and John have 5 pencils each." Children may also associate certain words (such as "more" and "altogether") with addition, and certain words (such as "less" and "taking away") with subtraction because they lack the linguistic skills to handle semantically complex statements (Lean et al., 1990).

A second group of researchers, however, emphasise the significance of non-linguistic barriers to problem comprehension and solution. Strong part-whole knowledge and a flexible understanding of number meanings are seen as essential for recognising the structure of additive word problems (Nunes & Bryant, 1996). Children may find it difficult, for example, to comprehend and solve the Missing Addend problem in Table 1 if their concept of number is necessarily tied to the activity of counting. This prevents them from conceptualising relationships involving quantities with an unknown measure. In Vergnaud's (2009) terms, these children lack essential *concepts-in-action*.

The second consistent finding drawn from the research on additive word problems is that young children often solve word problems using situation-based strategies (Brissiaud & Sander, 2010). That is, they solve problems by directly modelling the action or relationships described by the text (Carpenter & Moser, 1984; De Corte & Verschaffel, 1987). These strategies could be described as "a complex, interrelated whole in which the addition and subtraction meaning is taken directly from the problem situation and modelled with entities" (Fuson, 1992, p. 251).

The problem with many young children, however, is that they can only solve problems that can be directly modelled (Brissiaud, 1994; Carpenter & Moser, 1984). While a situation-based strategy may often work, its application is limited if it is not linked with a problem model. For example, while a problem involving $2+\square=53$ is efficiently solved by carrying out a subtraction, some children may resort to trying different values until a correct solution is found. Still, others may erroneously add the two given numbers. The type and range of strategies reflect children's theorems-in-action. The recognition of subtraction as an appropriate strategy requires a relational calculation or an understanding of commutativity or inversion (Nunes & Bryant, 2009; Verschaffel & De Corte, 1997). By contrast, the incorrect addition strategy previously described may indicate that the theorem-in-action guiding the child is likewise incorrect (e.g., all word problems that describe an increase in quantity signify addition).

The fine-grained analysis of children's strategies for additive problems provided a strong theoretical basis for the design of a professional development program, Cognitively Guided Instruction (CGI; Carpenter, Fennema, & Franke, 1996). CGI provides teachers with models of the development of children's mathematical thinking, which are then used to frame their pedagogical practice. In CGI classrooms, children solve problems in a way that makes sense to them. They are encouraged to share their strategies as they also learn from listening to strategies shared by other children.

While a cognitive framework is essential to analyse children's strategies in approaches such as CGI, a large amount of research has also pointed to the sociocultural influences that shape children's strategies (Ellis, 1997). Children may solve word problems even without first understanding the situation and the relevant relations in the problem. Instead, they carry out superficial strategies such as looking for key words or using the operation most recently taught in class (Sowder, 1988). Verschaffel and De Corte (1997) suggest that children may be "playing the game of school word problems" (p. 80). This kind of reasoning reflects Vinner's (1997) notion of pseudo-analytical thinking in that solutions are driven by what is perceived to be accepted in the classroom. In terms of van Dijk and Kintsch's (1983) model, these strategies are based on an incoherent textbase without any situation model. It therefore becomes problematic to assess children's concepts-in-action because superficial strategies may often appear meaningful and correct (Vinner, 1997).

5 Method

We adopted a design research methodology (Gravemeijer & van Eerde, 2009) because we aimed not only to investigate the impact of our intervention but also to identify aspects or "conditions for success" within the intervention that promoted conceptual change (Dede, 2004). Additionally, the formative nature of design research resonates with Nebres' (2009) call to engage "in an iterative process of acting, reflecting, and learning, and acting again, to find a path to success" (p. 240).

This study comprises three aspects (written tests, interviews, and pedagogical interventions), each of which was carried out in two or three iterations. This paper reports on the third iteration of the pedagogical intervention which aimed to support children's solution strategies in solving additive word problems in English.

5.1 The setting and participants

The study took place in a parish-based mathematics intervention program serving public school children from urban poor communities in metropolitan Manila. The parish recruited volunteer tutors who had at least completed high school to teach the children. They were welcomed into the program less on the basis of formal application letters but more on a demonstrated spirit of volunteerism and commitment to serve.

In implementing the intervention, the Researcher[1] coordinated with the head of the parish tutorial project and volunteered to tutor a group of Grade 2 children in mathematics. The Researcher trained two volunteers on the pedagogical approach. Both volunteers worked as teaching assistants and their primary role was to help manage the teaching sessions. One

[1] The Researcher refers to the first author.

volunteer was always available in each session. Neither teaching assistant had any teaching qualifications. The Researcher provided the teaching assistants with a teaching plan and discussed this with them before and after each session. The teaching set-up was quite informal. In the middle of a task for all children, for example, the Researcher and teaching assistant may have decided to split the group with respect to each child's ability to solve the task, so that each child could be given more attention.

The teaching assistants as well as the Researcher had been volunteering in different capacities in disadvantaged areas for several years. The Researcher's familiarity to the participants facilitated the recruitment process, as a trust relationship was especially important in a developing country such as the Philippines (Davis, Seah, & Bishop, 2009).

While some children dropped out of the program during the first three weeks for various reasons, 17 children (6 boys, 11 girls; mean age, 7 years 10 months) had consistent attendance and were present during the time of the pre- and post-intervention interviews. These children's main exposure to English came from school. They attended the teaching sessions in four pre-determined groups on the basis of their availability.

The Researcher conducted 1-h teaching sessions focused on additive word problems carried out twice a week over 7 to 10 weeks (June–September; the variability is accounted for by some unavoidable circumstances). Also, four teaching sessions were converted to review sessions where the Researcher prepared children for their school examinations. While the word problem intervention concluded in September, ethical reasons motivated the Researcher to stay with the parish program for one entire school year.

5.2 Pedagogical approach

Consistent with features of a design study, a set of instructional tasks based on an envisioned learning process was implemented (Gravemeijer & van Eerde, 2009). Our main conjecture was that word problem solving could be improved by addressing the three components of word problem solving *simultaneously,* rather than sequentially. In practical terms, this meant not having to wait for children to acquire skills necessary to construct a coherent textbase before providing them with opportunities to develop concepts-in-action relevant to additive structures.

In connection, our approach extended the method recommended by the Philippine curriculum (DepEd, 2003) as described earlier, which emphasises the development of a coherent textbase (i.e., stating information from the text) and problem model (i.e., choosing an arithmetic operation), but not the construction of a cohesive situation model. Thus, we provided opportunities for children to develop this component of problem solving, as well as appropriate mappings between components (Fig. 1). To achieve these, it was necessary to use Filipino as the primary medium of instruction because our prior interviews revealed that many children were not familiar with several high-frequency English words (Bautista & Mulligan, 2010). Additionally, we conjectured that using Filipino to convey mathematical concepts would not prevent them from accessing the same concepts in English once they have acquired proficiency in the language (Bernardo, 2000; J. Cummins, 2000).

We also conjectured that constructing an appropriate situation model required more than developing language comprehension skills. Thus, we strengthened children's concepts-in-action by utilising a range of representations (see Table 2), with particular emphasis on Wright, Martland, and Stafford's (2000) screening tasks, that allowed us to model the structural relationship in a given problem. Each problem structure was presented one at a time in the sequence provided in Table 1 and was the focus of three to four teaching sessions.

A typical lesson began with a verbal task involving a particular additive structure. This introductory task was presented in the form of a narrative close to children's everyday

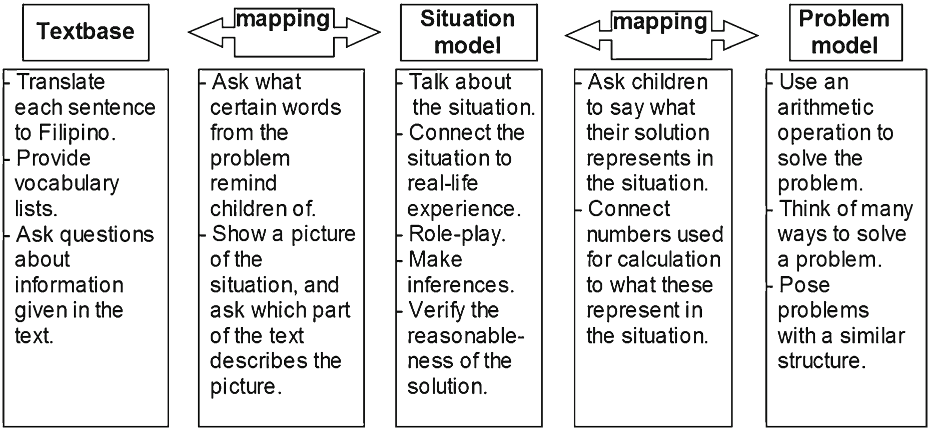

Fig. 1 Pedagogical approaches

experiences. Solutions and errors were then discussed as a group. Children then worked individually on tasks involving the same additive structure through representations that became increasingly more demanding with respect to the linguistic or mathematical knowledge required for solution (that is, from pictorial to symbolic to Filipino and English textual representations in Table 2).

In line with the CGI approach (Carpenter et al., 1996), children were encouraged to use their own strategies and to share their strategies with others. If children could not begin a solution strategy, a range of linguistic and mathematical scaffolds were provided (see Fig. 2). While word problems primarily involved small number combinations (in the range 1–20), problems involving multidigit numbers were presented to children who were successful with smaller number combinations.

Table 2 Various representations for the Missing Addend task 8+□=14

Mode of representation	Typical tasks or activities
Concrete	Screening task (Wright et al., 2000): Briefly display 8 blocks. "I will join some blocks to the 8, but I will not tell you how many." Join 6 blo cks to the original 8, without showing the child the number of additional blocks. "Now, there are 14 blocks altogether. How many blocks are in the bag?" [presented in Filipino]
Pictorial	
Verbal-pictorial	"*Wish ko lang* [I wish I had]" task (Kolson, Mole, & Silva, 2006): Show 8 dots. "I have 8 dots. I wish I had 14. How many dots do I need?" [presented in Filipino]
Textual	Gina had 8 bags. Ramon gave her some more bags. Now, Gina has 14 bags. How many bags did Ramon give Gina? [presented in English or Filipino]
Symbolic	8 + □ = 14

These representations were presented in the order prescribed above, which correspond to increasing levels of abstraction

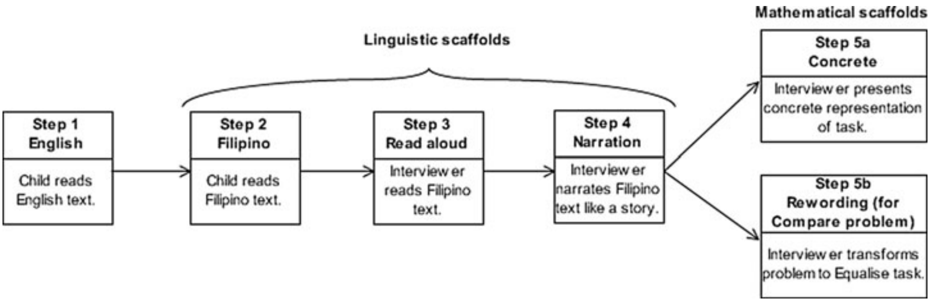

Fig. 2 Steps in the interview process

On the basis of the pedagogical approach outlined above, we developed a total of 10 lessons for the six problems in Table 1. A pilot intervention was trialled with four classes of 20–30 children during the summer school break (April to May 2009).

5.3 Data collection

5.3.1 Individual interviews

The main data source for this report was the video recordings of the pre- and post-intervention interviews of the 17 children who consistently took part in the program. The Researcher conducted all interviews. The interviews consisted of bare number tasks followed by six word problems. The number tasks included 9 + 6, 4 + 8, 17 − 9, 17 − 15, and 13 − 7 (Wright et al., 2000). Pre-intervention word problem tasks were those presented in Table 1. The surface characteristics and number triples for these tasks were modified and used during the intervention and the post-interview. All number triples were based on those used in Carpenter and Moser's (1984) study.

Unlike interviews that sought to determine the stage in the problem-solving process where children commit errors (Newman, 1983), our interviews provided structured scaffolds (see Fig. 2) aimed to reveal the mathematical knowledge that may be obscured by initial, usually linguistic, obstacles (Bautista & Mulligan, 2010; Fernandes et al., 2009).

The English problem text was first presented to the child, who was asked to read the problem aloud (step 1). If the child failed to solve the problem, a series of linguistic scaffolds (steps 2–4) was provided. These included presenting the task in Filipino (step 2), reading the problem aloud for the child (step 3), and building a narrative (in Filipino) while utilising questions about the situational and structural aspects of the problem (step 4; Depaepe, De Corte, & Verschaffel, 2010; Setati & Barwell, 2006).

If linguistic scaffolds still did not facilitate success, a mathematical scaffold in the form of a concrete representation of the problem's mathematical structure was provided (step 5a; see Table 2). For the Compare problem, however, the mathematical scaffold involved transforming it to an Equalise situation (Step 5b; Bernardo, 1999): *Rica has 12 books. Luis has 7 books. How many more books does Luis need to have the same number of books as Rica?*

To further investigate children's difficulties, the Researcher asked the child to identify any word that was difficult to understand or to retell the situation being described by the text. The Researcher also asked the child to explain their strategy for obtaining their numerical answer. Post-intervention, further probes were given to verify whether solutions were based on appropriate situation models. These included asking the child what their answer represented, or asking questions about the text even after a correct solution had been produced. These additional probes were necessary because it became evident during the course of the intervention that children's solutions often did not reflect correct understanding of the text.

There were times during the interview when it became necessary to bypass some steps in Fig. 2. For example, after presenting two problems where the child could not provide even segmented recollections of the English text, only Filipino versions were presented in subsequent problems. Although not ideal, this was necessary to prevent the child from becoming too stressed or frustrated to continue with the tasks.

5.3.2 Teaching sessions

The Researcher conducted semi-structured interviews with the teaching assistants and recorded observations and reflections in fieldnotes after each teaching session. A total of 88 fieldnotes (which included the teaching assistants' interviews) across the four tutorial groups was collected. Additionally, we collected 59 audio recordings of the teaching sessions.

5.4 Data analysis

5.4.1 Data preparation

The pre- and post-intervention interviews involved 17 children solving six problems each, resulting in 204 episodes. We partitioned each episode with respect to the steps in Fig. 2 (i.e., English, Filipino, Read aloud, Narration, Concrete/Rewording). We then encoded a detailed description of each episode segment in an Excel spreadsheet where it became possible to view various combinations of episodes at the same time and to search for commonalities and patterns (Meyer & Avery, 2009). We also collated audio recordings and fieldnotes associated with each problem type across all sessions, resulting in a folder containing all data related to a specific problem type.

5.4.2 Coding

A series of data analysis procedures were performed on the data corpus. First, we coded each child's response during the interview with respect to (1) the scaffolds, if any, that were necessary for solution success (Fig. 2) and (2) the mathematical strategies employed by the child (Carpenter & Moser, 1984). We recorded minor computation errors that were corrected after the children had been asked to repeat their solution as correct (7 instances before, and 5 after the intervention). These codes formed the basis for the graphs shown in Figs. 3, 4 and 5.

While the interviews provided snapshots of children's knowledge pre- and post-intervention, we also aimed to investigate the processes by which our intervention promoted conceptual growth. Thus, the analysis was extended to include the entire data corpus. With the research questions in mind, the interview episodes, fieldnotes, and audio recordings were reviewed multiple times to gain an overall sense of the data and identify recurring themes. The data generated an initial list of 40 recurring themes, 12 of which related to linguistic and mathematical difficulties. The remaining themes were searched for similarities, resulting in three broad themes; namely, superficial strategies, children's engagement, and learning in an urban poor setting.

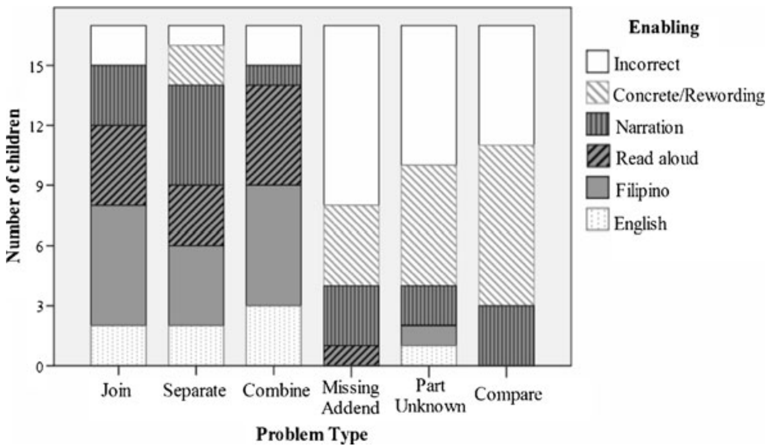

Fig. 3 Interview scaffolds leading to success, pre-intervention

6 Results

The first two parts present the findings in relation to the two research questions, largely obtained from an analysis of the children's solutions pre- and post-intervention, with a particular focus on how the scaffolding techniques in Fig. 2 influenced children's strategies. The third part presents the three broad themes resulting from the qualitative analysis of the entire data corpus.

6.1 Obstacles to solving word problems

The scaffolding techniques in our interviews were used to investigate to what extent linguistic or mathematical factors impeded word problem solving. If difficulties in solving word problems were primarily linguistic, then linguistic scaffolds, as shown in Fig. 2, would facilitate correct solutions. However, if children had poor understanding of number concepts and part-whole relations, then even substantial linguistic support in the form of narration would fail to help them construct appropriate situation models.

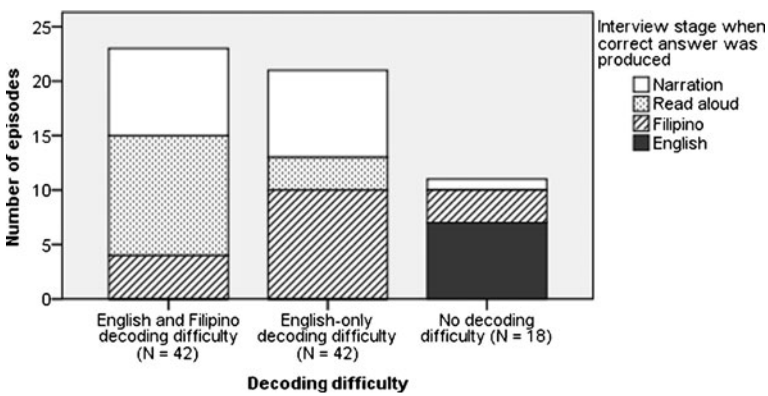

Fig. 4 Necessary linguistic scaffolds (steps 1–4), by decoding ability

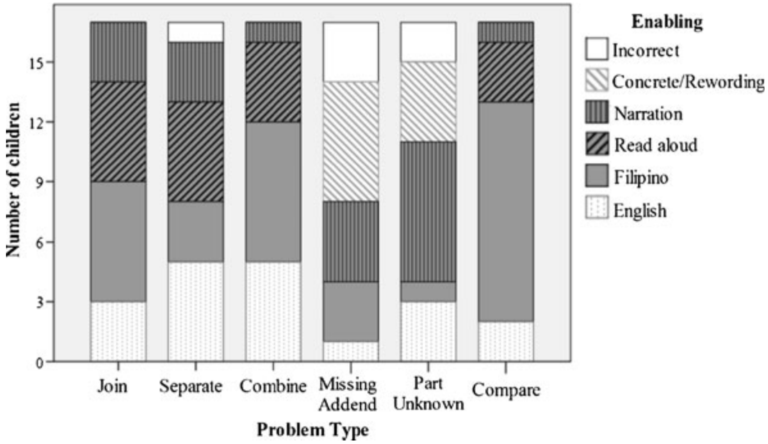

Fig. 5 Interview scaffolds leading to success, post-intervention

Figure 3 shows the type of scaffold that facilitated success, pre-intervention. The darker areas in the graph represent instances when linguistic scaffolds were necessary and sufficient for success. Pre-intervention, the extent of the dark regions provides evidence for linguistic difficulties—very few children could solve problems in English, and most required linguistic scaffolds. However, the linguistic scaffolds were primarily helpful for the Join, Separate and Combine problems. As one teaching assistant mentioned, "*Pagkapaliwanag ko* [Once I explain], *alam na nila yung gagawin* [they would know what to do], *pag yung mga* [but only for] simple addition *lang* [only]." In contrast, the linguistic scaffolds facilitated correct solutions for only three or four children for the Missing Addend, Part Unknown, and Compare problems, an indication of underlying mathematical difficulties. The following section elaborates on children's linguistic and mathematical difficulties.

6.1.1 Linguistic difficulties (pre-intervention)

Children's linguistic difficulties were reflected by their struggle to form a coherent textbase. One reason for this was with the way they decoded text. One child could not read at all, and 6 others decoded both English and Filipino text one syllable at a time. Of the 10 children who fluently decoded Filipino text, 7 decoded English text the way they would read in Filipino; that is, in a manner that strictly corresponded with word spellings (reading "now" as "no," and "gave" as "gav"). Their decoding abilities were associated with the type of linguistic scaffold that was necessary to facilitate success (Fig. 4). Children with decoding difficulties failed to solve any problem in English. The read aloud scaffold primarily benefitted those who had difficulties in decoding both English and Filipino text, while presenting problems in Filipino was mostly helpful for those who could fluently decode Filipino but not English text.

Many children failed to retrieve information explicitly stated in the text. Pre-intervention, 12 children failed to understand the English statement, "Alvin had 3 coins." It was also common for them to make sense of only isolated words from the text. For example, after reading the English Join problem, Helen[2] said, "*Alam ko lang yung* [All I know is] give *eh.*"

[2] All children's names are pseudonyms.

These difficulties in retrieving textual information also occurred for Filipino problems. For example, four children could not retell the situation described by the Filipino Join problem.

6.1.2 Mathematical difficulties (pre-intervention)

Children's strategies for solving the Missing Addend and Part Unknown problems confirm the finding that they had not developed the concepts-in-action required for conceptualising part-whole relationships. Some indications of this difficulty are the need to know a set's cardinal measure before reasoning about it, the inability to conceptualise a set as being part of another set, and the conception of number solely as cardinal measures of sets. In the pre-intervention interview, Sharon constructed two disjoint sets, instead of one set having a subset for the Part Unknown problem, even when a corresponding concrete task involving smaller numbers ($2+\square=6$) had been provided. Similarly, during a teaching session, Jenny and Rose persistently joined the two given numbers in the Missing Addend problem. They managed to obtain a correct solution only when they thought of trying out different numbers.

Results also indicate that some children had limited theorems-in-action. Children whose strategies were limited to counting the cardinal measures of sets (suggesting weak theorems-in-action) solved fewer problems than those who demonstrated a wide range of strategies. A particular example relates to the association between children's ability to solve the number task "17–15" by counting up and the ability to successfully solve Missing Addend and Part Unknown problems. Of the five children who efficiently solved the 17–15 task, four could solve both the Missing Addend and Part Unknown problems. Also, 9 of the 10 children who used inefficient strategies for solving 17–15 failed to solve either or both word problems.

The third word problem where linguistic scaffolds did not seem to help was the Compare problem. Of the 10 children who correctly solved the Compare problem pre-intervention, 8 managed to solve only the corresponding Equalise task.

Children's strategies were predominantly based on counting (56 of 75 correct solutions), and there were only four instances where a child cited an arithmetic operation as the strategy used. Instead, most strategies reflected the action in the problem. For example, the strategy of removing a subset from an initial set of blocks and counting what remained was more common for the Separate problem (11 of 16 correct solutions) than for the Missing Addend problem (0 of 8 correct solutions).

6.2 Pedagogical outcomes

6.2.1 Linguistic difficulties (post-intervention)

Figure 5 presents the type of scaffold that facilitated success, post-intervention, at which a correct solution was achieved. Similar to pre-intervention results, the extent of the dark regions reveals the indispensability of linguistic scaffolds. However, there were more instances post-intervention when linguistic scaffolds were sufficient for correct solutions.

Still, there were very few instances when a problem in English was solved. Children's unfamiliarity with the language continued to impede problem solution. For example, when Dante was asked if there was any word he did not understand, he looked at the text and said, "*Lahat 'yan* [All of them]." Similarly, Ana encircled the whole word problem when asked to encircle the words that were not clear to her. Seven children explicitly mentioned that they did not know what *gave* meant, even when this word was repeatedly translated to Filipino during the intervention. It appeared that the words directly taught during the intervention were largely just memorised. When Sarah was asked if there was any word she could not understand, she said, "*Nakalimutan ko na yung altogether eh* [I forgot what altogether means]." Fred thought that "have now" meant "how many" and that "how many" meant "number." Some children associated words with similarly-sounding words, such as Jenny who interpreted the name "Tess" as children having a test.

Knowledge of only isolated words from the text resulted in strategies based on fragmented situation models. For example, when the Separate problem was read aloud to Fred in Filipino, he replied, "Minus," and his reason was, "*Binigyan eh* [It's given]." Similarly, some children constructed a situation model based on their recollection of some of the situations encountered during the intervention. For example, after reading the English version of the Missing Addend problem, Helen asked, "*Ilan yung lagpas* [How many more]*?*" because she confused it with the Compare task that she remembered from the intervention.

In contrast, there was one child (Monica) who successfully answered four and six tasks in English pre- and post-intervention, respectively. She may not have known all words (e.g., during one lesson, she asked what 'altogether' meant), yet unlike the other children, she knew enough words to solve some problems even before the intervention.

6.2.2 Mathematical difficulties (post-intervention)

While performance generally improved post-intervention, children continued to rely on the Researcher's concrete representation more for the Missing Addend and Part Unknown problems than for the other problems. The Compare problem was where the most notable progress was observed. All 17 children correctly solved the non-reworded version of this problem, post-intervention. Seven children solved the Compare problem by forming two sets and counting how much one set exceeded the other, a strategy learned during the intervention.

Although children developed better strategies for Missing Addend, Part Unknown, and Compare problems, they found it difficult to connect these problems to a subtraction equation. One teaching assistant regularly showed a corresponding number sentence after a correct solution was reached, "hoping *na sa susunod* [afterwards]*, gagawa na sila ng* [they will construct a] number sentence. *Pero hindi eh* [But they did not]." Even higher-achieving children such as Monica and Carol struggled to make these links. Fieldnotes revealed that they consistently solved these three problems by counting up from the smaller number, even when a direct subtraction was more efficient. When encouraged to verify whether subtraction would have yielded the same solution for a Compare problem involving 37 and 63, both wrote and solved an incorrect subtraction expression ($37–63=34$).

The pedagogical strategy of presenting various representations while encouraging children to model the action in the problems was instrumental in conveying the mathematical structures that children initially could not conceptualise. For example, in one teaching session, Jenny, Sharon, and Linda were solving a word problem involving the structure $4+\square=9$ in the context of increasing the quantity of a type of bread. Even when the Researcher narrated the problem to them in Filipino, they produced 9 as the answer because as Linda explained, "*Bumili siya ng nine* [She bought nine]." They clung to their answer even when the Researcher prompted them to draw 4 pieces of bread and a bag and to think how many pieces are needed in the bag to make 9. It was only when the Researcher drew a piece of bread inside the bag and asked them if there were already 9 pieces that they began to conceptualise a set having an unknown quantity.

The non-textual representations (see Table 2) also provided opportunities to communicate mathematical concepts when linguistic difficulties interfered with problem solution. In one teaching session, Helen, Sharon, and Ana gave incorrect responses for the Filipino textual representation of the Missing Addend problem, but successfully solved the corresponding verbal-pictorial and symbolic representations.

6.3 Themes emerging from qualitative analysis of interviews, teaching sessions, and fieldnotes

6.3.1 Superficial strategies

A large number of the observed strategies suggested a tendency to use strategies that were not connected to the problem text. Children often produced answers, sometimes even correct ones, even when they could not answer questions about the text or explain what their answer represented (36 % of post-intervention episodes). For example, Dina correctly solved the Combine problem, post-intervention, yet she could not say that her answer represented the total number of items in the problem. Some also preferred to mimic rote procedures taught in school, such as Helen who insisted on first stating what is being asked in the problem by replacing the words "how many" in the question to "the number of".

When presented with word problems in English or Filipino, children frequently focused on the numbers in the problem or on the correct operation to be used. In a Filipino word problem presented during the intervention, for example, children could not proceed because one of the given numbers was spelled out. The teaching assistant observed, "*Parang may hinahanap siya* [She seems to be looking for something], for the problem to make sense." Additionally, children often suggested a solution strategy that they expected the Researcher to validate. For example, Monica remarked, "*Ipa* [I will] plus," after reading a Missing Addend problem. When she was asked why she should add, she interpreted the question as an implication that she was incorrect, and she immediately said, "*Ima* [I will] minus." Similarly, some children solved problems by repeating their previous strategy or by using a default strategy, sometimes even without reading the problem. For example, Ana said, "*Ilan lahat* [How many altogether]?" for all post-intervention interview tasks.

6.3.2 Children's engagement

To promote children's learning and engagement, it was necessary to refine the intervention in order to provide children with tasks and representations suitable to their level, even when these involved more basic concepts than those encountered in school. For example, a game involving ten-complements helped children automate related number facts in an engaging way. Also, a parent commented on tasks related to two- and three-digit numbers, "*Maganda yung ginagawa niyo* [What you're doing is good], *yung may mga* [presenting] pictures, *kasi wala na raw ganun sa* [because there's none in] school."

Children, however, were found to have negative feelings towards solving word problems. During the intervention, it was common for children to express their boredom ("*Kakaantok* [This is boring]!"), difficulties ("*Ang hirap naman* [This is too hard]."), and frustrations ("*Hindi ko na kaya* [I can't do this anymore]!). Likewise, it was difficult to sustain children's attention and to encourage them to listen to other children. Fieldnotes contained entries such as, "One difficulty (again) was how to get their attention… They get restless so easily. No exaggeration, even one minute is a long time for them," and "I asked them to share their solutions but they still can't. It was also difficult to have them listen to others' solutions." Instead, children preferred to engage in tasks which involved writing ("*Ate, sulat tayo* [Let's write]!"), and this was reiterated by the teaching assistants, "it really seems as if the best way to engage children was to give them something to write."

It was important for children to receive affirmation for their efforts and capabilities. For example, children who solved problems quickly often shouted their answers, discouraging other children to proceed with their thinking. Children also insisted that the Researcher write check marks for every correct solution they produced. On the other hand, they also avoided being perceived as incapable. During interviews, children often said that they understood the text even when probing questions revealed they did not. During the intervention, many copied answers from other children when they could not proceed with a task. One fieldnote entry read, "I knew that some kids copied answers, but I found this problem so frequently—in [Research site #1],[3] here, in [Research site #2], everywhere." As well, they tried to conceal their reliance on counting by erasing their tally marks or hiding their fingers under the table when calculating.

6.3.3 Learning in an urban poor setting

A recurring theme obtained from fieldnotes related to the context where the study was situated. Many attended the teaching sessions without having had breakfast or lunch, and we found it appropriate to start each session with a small meal. They lived in densely populated areas with poor living conditions. Children had very few possessions so they generally regarded intervention materials, such as blocks or erasers, as toys. Even the venue for the intervention was typical of areas in urban poor communities. It had no adequate chairs and tables, lacked space and ventilation, and it was liable to regular distractions.

7 Discussion

7.1 Obstacles to children's problem-solving

The first research question concerned the obstacles to solving additive word problems. Children's difficulties for Join, Separate, or Combine problems were primarily due to failures in constructing an appropriate textbase. Some factors contributing to these difficulties were undeveloped decoding skills, lack of English language knowledge, or limited reading comprehension strategies, even when the problems used in this study were not as linguistically complex as those that older children are expected to solve. Indeed, these challenges were more pronounced than those commonly reported in the literature, which tends to relate to difficulties with academic rather than conversational language (Fillmore, 2007; Kazima, 2007). The fact that children who cannot understand simple statements such as "Alvin had 3 coins" are obligated to learn mathematics in English says much about how their school experience must be too far removed from their daily lives. These descriptive accounts are as revealing as the results of any quantitative study in discussions about the language of instruction. These accounts also point to "a large gap between what teaching in English requires and what is possible in the classroom" (Posel & Casale, 2011, p. 445) and to how the language may disadvantage students from low socioeconomic backgrounds (Bernardo, 2008).

When problems with more complex mathematical structures (Missing Addend, Part Unknown, Compare) were presented, however, mathematical difficulties that were similar to those found among monolingual children became evident (Nunes & Bryant, 1996; Vergnaud, 1982). Even when problems were narrated or represented concretely, many children lacked the

[3] These refer to research sites in pilot implementations of the intervention.

necessary concepts-in-action to conceptualise the problem situation; that is, they failed to construct appropriate situation models. This finding is consistent with general text comprehension theories that stress the importance of knowledge of the problem domain (in this case, mathematics) on the construction of cohesive situation models (Hirsch, 2003).

The interviews also revealed that children often solved Missing Addend, Part Unknown, and Compare problems using direct modelling strategies. They did not possess the theorems-in-action necessary to perform the relational calculation needed to connect the situation to a formal mathematical structure (i.e., problem model) (Nunes, Bryant, Evans, Bell, & Barros, 2012; Vergnaud, 2009).

Children often carried out strategies that did not reflect the problem text. One possible explanation is that these strategies were their means to cope as they simply had no other option available. These strategies may conceivably be used even by adults, as when, for instance, a non-Chinese speaker attempts to solve a Join problem in Chinese, and finds it difficult, if not impossible, to comprehend the relatively simple sentence, *Aerwen you* 3-*ge yingbi* [Alvin has 3 coins].

However, another explanation may be that the children in this study have learned how to play the "word problem game" (Verschaffel & De Corte, 1997) using the principle of minimal effort (Vinner, 1997). This assertion is supported by recurring themes from the data. These include the lack of interest in sharing and discussing the problem situation or solution, the satisfaction derived from receiving affirmation, and the aversion to being perceived as incapable. Further research may provide deeper understanding of how these issues impact on any pedagogical intervention and assessment.

Taken together, the results demonstrate the complexity of the factors that impede the problem-solving process. Findings clearly show that children's difficulties are not confined to difficulties associated with lack of English language proficiency. They may not possess the mathematical knowledge necessary to handle more complex mathematical structures. In addition, the results indicate that children's difficulties may not necessarily be due to mathematical misconceptions but on the belief that a certain strategy will be expected by the Researcher (Vinner, 1997).

7.2 Conditions for success

The second research question relates to aspects within the intervention that were effective for improving children's problem-solving strategies in English.

The intervention showed that it is possible to develop concepts-in-action relevant to understanding additive structures even before children have acquired the skills necessary to construct a coherent textbase. However, this progress was not achieved by simply translating problems to Filipino. Indeed, some children required pictorial, symbolic, or verbal representations (see Table 2) before they could conceptualise the more complex additive structures.

Improved concepts-in-action, however, did not necessarily translate to better performance in solving word problems in English. Minor interventions such as providing definitions for English words commonly found in word problems were not effective—children struggled to remember what these words meant. Given that the majority of the children in this study had serious difficulties with the English language post-intervention, it seems highly unreasonable to expect them to solve word problems in English unless adequate English language pedagogical support is provided. There are no short-cuts—even when children develop the necessary concepts-in-action to conceptualise various additive structures, they still need to develop their English language skills before they can solve problems in English.

This study also questions the recommendation by the current Philippine curriculum that children specify "the operation to be used" (DepEd, 2003, p. 5) as an intermediate step for solving word problems. This study has demonstrated that children utilise a range of strategies for solving word problems, and that many find it difficult to link some additive structures to an arithmetic operation. While there was not enough time for the study to investigate specific strategies for achieving this, the literature provides potential solutions for future intervention work. These include developing children's understanding of the inverse relation between addition and subtraction through numerical tasks that are efficiently solved by invoking the inverse principle, or through temporal real-life situations where an addition may "undo" a subtraction (Nunes et al., 2012).

Another persistent difficulty concerned superficial strategies. While these strategies may be children's ways of coping with an undeveloped textbase, the large number of superficial strategies represents an unresolved issue, especially because children were encouraged to solve problems in a way that made sense to them. From a sociocultural perspective, these superficial strategies may reflect the kinds of problems that children engaged in during the intervention (Dominguez, 2011) or the way they perceived themselves in relation to word problem solving (Tobias, 2005). Thus, a sociocultural perspective may guide future research efforts by providing an understanding of the origins of children's pseudo-analytical thinking.

These persistent difficulties, however, should not suggest that the intervention was a futile attempt to improve Filipino children's solutions to additive word problems in English. On the contrary, the results are promising given that the children in this study could not engage in English social conversation. Additionally, on the basis of Jim Cummins' (2000) well-established theory of bilingualism, the children's seemingly modest progress is not to be dismissed—they do not need to relearn mathematical concepts in English, as they are expected to be capable of solving word problems in English once their fluency in the language catches up with the demands of the task.

7.3 Research context

An important factor contributing to the effectiveness of the intervention was the Researcher's flexibility in terms of task selection, pedagogy, and even in the language of instruction. While this study has shown how task selection and pedagogy contribute to promoting children's engagement, the use of the first language is essential if meaningful communication is to take place (Lim & Presmeg, 2011). However, preliminary results from the Researcher's follow-up study involving 23 Filipino public school teachers revealed that using Filipino and code-switching was frowned upon (see also Borlongan, 2009). As one teacher expressed, "I teach word problems in English. It's easier for the children to understand in Filipino than in English but we must explain it in English."

In the absence of such flexibility, classroom discourse would have been fraught by coping strategies, a phenomenon common in African classrooms (Bansilal, 2011; Brock-Utne, 2007). Conversations would have remained at a textbase level of discourse, and teachers would have to resort to procedures that could easily be mimicked by children, just to enable them to solve problems, albeit superficially (as was the case for Helen being taught the strategy of replacing the words "how many" with "the number of").

The use of Filipino, however, still did not promote the rich discussion consistent with CGI. The manner by which productive mathematical discussion is to be achieved may require more focussed interventions (Weber, Radu, Mueller, Powell, & Maher, 2010), especially in developing countries where authoritarian cultural identities dominate (Schweisfurth, 2011). Researchers have indeed challenged the notion that successful programs and processes in one context may be seamlessly transferred to other settings (Nebres, 2009), particularly when children from low-income families or from developing countries are involved (Lubienski, 2000; Schweisfurth, 2011).

The study has also drawn attention to how children's home and learning environments are not conducive to learning. As Skovsmose (2005) notes,

> What seems to be the most obvious learning obstacle to the children in this school: their colour of skin, their dominant father, or the hole in the roof?… The learning obstacles are right there in front of our eyes, and on top of our heads. To me, this hole in the ceiling, not referred to in much research in mathematics education, calls any deficiency theory of the child into question. (p. 5)

In reporting children's urban poor context, it is not intended to suggest that children from poor families are incapable of performing well. Rather, it is an invitation for future studies to investigate learning in terms of children's actual situation and the opportunities made available for them.

8 Limitations and conclusions

This part of a larger study involved a small sample, and it was implemented in an out-of-school environment in a small-group setting. These factors limit the extent to which the findings can be generalised to actual Philippine classrooms. Additionally, there were no formal measures of children's language proficiency (both English and Filipino). While such measures may contribute to the literature on the importance of bilingual competency on academic achievement (Clarkson, 2007; J. Cummins, 2000), they were not undertaken. As discussed earlier, unlike children involved in previous studies, the children in this study evidently did not have communicative competence in English and were unlikely to be academically proficient in two languages. Further, this study was not aimed to investigate the cognitive advantages of having two languages.

Nonetheless, this study provides a rich account of the processes underlying Filipino children's solutions to additive word problems presented in an imported language. On the basis of prior research and findings from this study, an integrated framework of problem solving in second language contexts has been proposed. This framework delineates the various components of the problem-solving process that should be sufficiently and mutually addressed. In conclusion, this study has shown that while growth in individual components of the problem-solving process is possible, such growth would not enable children to solve word problems in English if they are held back by difficulties in other components that have not been developed adequately.

This study has made a unique contribution towards addressing the scarcity of research in word problem solving in an imported language where most other research has either concerned children who have exposure to the language of instruction outside school (Clarkson, 2007; Dominguez, 2011), or has not involved an intervention component (Adetula, 1990; Kazima, 2007; Setati & Barwell, 2006). Adopting an ethical perspective, this study moved beyond ascertaining studies and towards addressing calls for research to investigate effective teaching environments and confront relevant problems in the local context (Gutierrez, 2008; Verschaffel et al., 2007). The unresolved issues and increased insight into children's difficulties provide stimulus for future research efforts.

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